Mega Publishing Limited
Bulletin of Applied Economics, 2016, 3(2), 43-98 | December 15, 2016

Optimal Budgetary Policies in New-Keynesian Models: Can they help when the Zero Lower Bound is binding?

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Abstract

In case of productivity or taxation rates shocks, monetary policy can perfectly stabilize average variables in all the monetary union when the Zero Lower Bound is not binding. So, the national budgetary policy should only stabilize asymmetric shocks and the differential of these idiosyncratic shocks with their average values in all the monetary union. On the contrary, when the ZLB is binding, monetary policy loses its efficiency to stabilize average shocks in all the monetary union. Budgetary policies should then be expansionary, in order to reduce the recessionary and deflationary tensions due to symmetric positive productivity shocks or to declines in average taxation rates in all member countries. The national budgetary policy should be more active, in order to stabilize not only differentials in the persistence of shocks between the national country and the rest of the monetary union, but also average global shocks. Therefore, budgetary policies could be more useful in a ZLB framework, provided they are not constrained by the fiscal situation and the indebtedness level of the national country.

Keywords: New-Keynesian models, budgetary policy, monetary policy, Zero Lower Bound, monetary union
JEL classification numbers: E62, E63, F45

1 Introduction

With the current economic and financial crisis, the European Central Bank and the Federal Reserve were constrained to reduce their interest rates to levels near zero. So, in these conditions where the Zero Lower Bound (ZLB) was binding, central banks had no more room of manoeuver to sustain economic activity with traditional monetary instruments; therefore, they had to use non-conventional monetary policies. However, the current recessionary and deflationary context also implied a renewed interest for the usefulness of other stabilization instruments, and in particular for budgetary policies. The

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Article Info: *Received*: September 7, 2016. *Revised* : November 18, 2016
Published online : December 15, 2016

question is then the following: can budgetary policies help and complement monetary policies when the ZLB is binding?

In this context, the main limitation of most DSGE models is their weakness in the formalization of budgetary policies. Indeed, they study optimal monetary policy, for example in the framework of a ZLB constraint limiting the decrease of interest rates and the stabilization of negative demand shocks. However, most often, they do no model budgetary instruments and optimal budgetary policies. Therefore, the aim of the current paper is to contribute to fill this weakness in the economic literature, by studying in which ways budgetary policies can assist monetary policy in the stabilization of shocks when the ZLB is binding. Indeed, we will try to define the optimal policy-mix in such circumstances.

Conventional monetary policy is the following. In a recessionary and deflationary framework, the decrease of interest rates can be efficient to decrease real interest rates and to sustain economic growth. However, in a liquidity trap, when the ZLB is binding, when interest rates have already been reduced to levels near zero, they can no longer be reduced, and conventional monetary policy loses its efficiency. Optimal monetary policy is then to commit to maintain a low interest rate for a sufficiently long period, in order to create future inflationary expectations and to reduce the real interest rate, limiting the deflationary tensions. Nevertheless, the problem of such a policy is that it is not time-consistent, and then that it could be difficult to influence private expectations. So, budgetary policies can be helpful in such a context. Temporary increasing public expenditure or raising labour taxation rates can be useful to generate the necessary inflationary tensions.

Indeed, an increase in public expenditure can contribute to decrease labour supply and to raise labour demand, to increase wages and real marginal costs, and then to create inflationary expectations. Therefore, an inflationary spiral contributing to decrease real interest rates, to sustain economic growth and to thwart the deflationary spiral due to a negative demand shock can be launched.

In the economic literature, the optimal policy-mix and the division of responsibilities between the monetary and fiscal authorities is usually supposed to be the following, in the framework of a monetary union: the central bank stabilizes average variables, whereas budgetary authorities stabilize national idiosyncratic shocks. For example, Beetsma and Jensen (2005) consider the optimal policy mix between fiscal and monetary policy in a two-country version of a currency union which is hit by supply shocks. The roles of the policymakers are then clear-cut. Monetary policy should stabilize the aggregate economy while fiscal policy ought to be utilized for stabilizing inflation differences and the terms of trade. This result is confirmed by Gali and Monacelli (2008), who study the policy mix in a currency union made up of a continuum of small open economies.

Ferrero (2009) goes one step further by introducing a government budget constraint in a two-country model of a currency union with staggered price setting and distortionary taxes. Then, he shows that a balanced budget rule (as mentioned by the Stability and Growth Pact) generates welfare losses. Allowing for variations in government debt instead, is a superior policy. In the optimal equilibrium, monetary policy should achieve aggregate price stability following a flexible inflation targeting rule. In parallel, fiscal policies should stabilize idiosyncratic shocks allowing for permanent variations of government debt, without creating inflationary expectations at the union level. Besides, Muscatelli *et al.* (2003) find that the strategic complementarity or substitutability between the fiscal and monetary policy instruments depends crucially on the types of shocks hitting the economy, and on the assumptions made about the underlying structural model. With a closed economy DSGE model, for the US and Germany, between 1970 and 2001, the authors find that monetary and budgetary policies tend to be more complementary following output shocks, whereas they can move in opposite direction in case of inflationary shocks.

However, Colciago *et al.* (2008) use a two-country New-Keynesian DSGE model, with non-Ricardian consumers, who are constrained in their access to financial markets and in their expenditures. They find that fiscal policy can efficiently complement monetary policy in stabilizing output and inflation for the whole union. Besides, model determinacy requires that national fiscal feedbacks on debt accumulation be designed with reference to the debt dynamics of the entire monetary union. Therefore, national fiscal policies should not only react to idiosyncratic shocks.

Nevertheless, as previously mentioned, the usefulness of budgetary policy should be much increased when the ZLB is binding, and when monetary policy can no longer decrease the nominal interest rate. Indeed, Werning (2011) shows that optimal monetary policy is to keep the interest rate at zero in a liquidity-trap (defined as a solution where the natural interest rate is negative), and to increase this interest rate only slowly after exit. Besides, when the ZLB is binding, the author finds that there is a role for a counter-cyclical budgetary policy. Optimal public expenditure is firstly above its natural level, before declining below its natural level when private consumption has recovered its initial amount. Schmidt (2013) also underlines the benefits of an active fiscal policy when the ZLB is binding and when the monetary authority cannot commit, whereas the gains from such a policy would be much more negligible in case of commitment. Indeed, budgetary policy has then a limited weight in the expansionary stimulus, and budgetary expenditure may even begin to decrease when the ZLB is still binding.

Besides, Christiano *et al.* (2011), show that the government-spending multiplier can be much larger than one when the zero lower bound on the nominal interest rate binds, and when the nominal interest rate doesn't respond to an increase in government spending. However, in order to be efficient, the increase in budgetary expenditure must happen with limited delays, when the ZLB is still binding and when the nominal interest rate is still null. Besides, taking into account capital accumulation may increase the size of the budgetary multiplier. Eggertsson and Woodford (2004) also find that tax smoothing is no longer optimal when the ZLB binds. In this case, and when taxes have supply-side effects (example of the U.S sales tax), the optimal policy requires that tax rates be raised during the recession and in a liquidity trap (in order to allow higher public expenditure), while committing to lower tax rates below their long run level later, in times of higher economic growth. Budgetary policies can then help monetary policy by responding in an appropriate way. Therefore, with a flexible and contra-cyclical budgetary policy, monetary policy could avoid to commit and to be fully credible for distant future periods, in order to succeed to stabilize economic variables.

Furthermore, Matveev (2014) shows that the necessity of a balanced budget makes it optimal for the government to rely more on spending instrument. Nevertheless, such a policy implies a deviation of public expenditure from its optimal efficient level. So, the possibility to use variations of the public indebtedness level makes the government to rely more on variations of labor taxation, which is a more efficient fiscal policy. Indeed, a higher debt increases the advantages of a higher future inflation, and then, it creates expectations reducing the current dangers of a deflationary framework. In the same way, Correia *et al.* (2013) show that, in standard New Keynesian models, when the ZLB is binding, tax policy can deliver the appropriate stimulus at no cost and in a time-consistent manner. Consumption taxes should be increase while at the same time, labor taxes are reduced. There is no need to use inefficient policies such as wasteful public spending or future commitments to low interest rates. So, the authors underline the usefulness of making taxation rates much more flexible.

Finally, Eggertsson (2006) shows that in a liquidity trap, like monetary policy, budgetary policy can mostly affect economic activity and inflation through expectations. Therefore, a deflation bias appears if the governments cannot commit (such a policy is difficult to justify as it would be time inconsistent), and if they only use open market operations in short-term government bonds as policy instrument (Quantitative Easing policy, the nominal interest rate is the only instrument). On the contrary, if the governments increase public spending and cut taxation rates, they increase their indebtedness level and they create inflationary incentives and expectations. These expectations about a future higher money supply can then limit the deflationary and recessionary bias, if monetary and fiscal policy are sufficiently coordinated.

However, the size of the increase in public expenditure should be limited even in a recessionary framework and when the ZLB is binding, and it shouldn't be too large.
Indeed, Woodford (2011) shows that in New Keynesian models, sticky prices or wages allow for larger multipliers than in neo-classical models, though the size of the multiplier depends crucially on the monetary policy response. A multiplier well in excess of one is possible when monetary policy is constrained by the zero lower bound, and in this case, global welfare increases if government purchases expand to partially (without fully compensating the recession) fill the output gap that arises from the inability to lower interest rates. Nevertheless, the author insists on the necessity to decrease budgetary expenditure as soon as the ZLB is no longer binding, in order to avoid the danger to crowd-out private consumption and to be harmful to the creation of future inflationary expectations.

In the same way, Erceg and Linde (2014) use a DSGE model to examine the effects of an expansion in government spending in a liquidity trap, in a model environment in which the duration of the liquidity trap is determined endogenously, and depends on the size of the fiscal stimulus. In a liquidity trap, they show that even if the spending multiplier can be quite high for small increases in government spending (which could even be self-financed with fiscal resources increases), it may decrease substantially at higher spending levels (with a growing cost of the public indebtedness). Therefore, the size of public spending should not be excessively large, and it should rationally remain limited, in order not to put sizeable upward pressure on government debt and tax rates and to have limited marginal payoff.

Mertens and Ravn (2014) use a dynamic rational expectations model with nominal rigidities in which monetary policy follows an interest rate rule that prescribes an aggressive response to deviations of inflation from a target. A fundamental shock (for example on households' preferences) may then justify higher budgetary expenditure, in order to create inflationary tensions, to decrease the real interest rate and to crowd-in private consumption. However, such a policy is inefficient in case of a shock on expectations, as higher government spending may then increase deflationary pressure. Indeed, falling prices then result in higher real interest rates which further reduce desired consumption and accentuate the excess saving. According to the authors, the appropriate budgetary policy would then mostly be to cut labor taxation rates. Besides, Burgert and Schmidt (2014) use a model where the economic authorities cooperate to define their economic policies. In this context, they underline the importance of the public debt burden for the efficiency of an active budgetary policy when the ZLB is binding; therefore, the optimal fiscal policy would be time dependent. The optimal level of government spending is also a decreasing function of the public debt level.

Furthermore, Nakata (2015) studies a sticky-price economy where the nominal interest rate is subject to the zero lower bound constraint. He shows that when the government can commit, the optimal government spending policy in a recession is characterized by an initial increase followed by a reduction below, and an eventual return to, the steady state. However, the size of the variation of public expenditure depends on the available tax instrument and on the initial debt level. The variation of government spending and the welfare gain of government spending policy are larger in an economy with a larger initial debt, as the government spending policy has then more to do to reduce long-run distortions. Finally, Bilbiie *et al.* (2014) underline the role of the usefulness of various government spending. Indeed, wasteful government expenditure has no multiplicative effect on private consumption and on output. Even in case of useful government spending, the author mentions that the conditions for welfare improving public expenditure could be quite restrictive regarding the structural parameters of the model and the intensity of the recession related to the ZLB framework. In the same way, Stähler and Thomas (2011) use a DSGE model (FIMOD) which has the particularity not only to model explicitly public expenditure, but also to distinguish between public consumption and investment expenditure. They find that fiscal consolidation is mostly damaging in terms of output and employment losses when performed via public investment cuts. On the contrary, a cut in public expenditure reduces global demand in the short run. However, it can have positive long run consequences via the reduction in distortionary labor income taxation, and positive spillover effects on private economic activity. Reduction in public sector employment or wages would be the most beneficial alternatives, reducing labor costs and increasing international competitiveness.

The current paper is in line with the above mentioned economic literature: it aims at shedding light on the usefulness of budgetary policies to complement monetary policy in a Zero Lower Bound framework. The rest of the paper is organized as follows. Section 2 describes the New-Keynesian model used to analyze the role of active budgetary policies in the policy-mix when the monetary policy can be constrained. Section 3 describes the determination of optimal economic policies. Section 4 and section 5 study the optimal equilibrium respectively when the Zero Lower Bound is not binding and when it is binding. Section 6 concludes the paper.

2 The model

Our study will adopt the standard framework of a small New-Keynesian model [see for example Clarida *et al.* (1999), Galí (2008), or Woodford (2003) for a very extensive presentation], made of a representative household and of a representative firm.

In the context of Dynamic Stochastic General Equilibrium (DSGE) models, individuals make decisions about consumption and labor supply that maximize their economic well-being subject to constraints based on their wealth. Firms set prices that maximize profits and they demand production factors, such as labor and capital, in ways that minimize their costs. Nominal wages and prices rigidities imply that prices are only randomly adjusted according to a markup over current and expected marginal costs. Labor markets are segmented, and labor is supposed to be immobile across countries. Models are also dynamic: economic variables depend on expectations about future outcomes and variables. Nominal rigidities imply a key characteristic of New-Keynesian models: the non-neutrality of monetary policy, and the fundamental role of central banks in stabilizing prices. However, these models often lack a full description of government spending and taxation, contrary to the current paper. Indeed, our goal is also to underline the distortions that inflation causes in a tax system that is not fully indexed, and to study the optimal policy-mix and respective roles of monetary and budgetary policies in stabilizing economic activity.

Besides, our goal is to analyse this optimal policy-mix in the framework of a monetary union. So, our model considers many countries and small open economies, i.e. which have not in isolation any influence on global and average variables. The monetary union is made only of ‘small’ economies, whose policy decisions have no consequences on global variables in the monetary union. Besides, financial markets are assumed to be complete both at the national and international level in this monetary union (risks are fully shared among households), and countries share the same common interest rate. This common interest rate is defined by the monetary policy of the common central bank, whereas each government defines autonomously its fiscal policy (public expenditure and tax revenues). Public expenditure is, therefore, an endogenous variable in our model. Productivity or taxation rates shocks can differ between countries; however, for simplicity, all countries share the same preferences.

In the current paper, all economic variables are expressed as deviations from their non-stochastic and long run steady state values, which could be observed with a growth rate corresponding to the constant trend or potential output growth rate. Logarithms are always expressed with lower case letters. In comparison with a given country (i), variables with an asterisk (*) refer to the monetary union as a whole.

2.1 Households’ behaviour

Aggregate demand for the country (i) results from the log-linearization of the Euler equation, which describes the representative household’s expenditure decisions. We suppose that the economy (i) is populated by a unit measure of households indexed by (i). In our model, the representative household provides labour and it consumes goods. In a given period (T), the representative household/consumer in country (i) maximizes an inter-temporal utility function:

$$max \sum_{t=T}^{\infty} \beta^{t-T} E_T[U_{i,t}] \quad (1)$$

Where: $E_t()$ is the rational expectation operator conditional on information available at date (t), and (β) is the time discount factor. Prices of goods, interest rates, taxation rates and wages are then taken as given by the representative household. This maximization is subject to the life time and inter-temporal nominal budget constraint, for whatever date (T) considered at which the actualization is realized:

$$P_{i,T}C_{i,T} + E_T[\sum_{t=T}^{\infty}\frac{P_{i,t+1}C_{i,t+1}}{(1+i_t)\dots(1+i_T)}] = W_{i,T}L_{i,T}(1-t_{i,T}) + E_T[\sum_{t=T}^{\infty}\frac{W_{i,t+1}L_{i,t+1}(1-t_{i,t+1})}{(1+i_t)\dots(1+i_T)}] < \infty \quad (2)$$

With, in the country (i) in period (t): ($C_{i,t}$): real consumption; ($P_{i,t}$): level of consumer prices; ($W_{i,t}$): nominal hourly wage; ($t_{i,t}$): taxation rate on personal income; ($L_{i,t}$): hours worked by the representative household; (i_t): common nominal interest rate in all the monetary union.

Current consumption and anticipated consumption levels after the current period (T) mustn't exceed current real activity revenues and anticipated revenues for all future periods. Therefore, in this model, we allow for the possibility to borrow from one period to another, but we limit anticipated future revenues in order to avoid the possibility of Ponzi schemes.

We suppose that the utility function of a representative household has the form:

$$U_{i,t} = \alpha_c\frac{\theta}{(\theta-1)}(C_{i,t})^{\frac{(\theta-1)}{\theta}} + \alpha_g\frac{\theta}{(\theta-1)}(G_{i,t})^{\frac{(\theta-1)}{\theta}} - \alpha_l\frac{1}{(1+\varphi)}L_{i,t}^{(1+\varphi)} \quad (3)$$

The indices ($0<\alpha_c<1$), ($0<\alpha_g<1$) and ($0<\alpha_l<1$) are the respective weights given to consumption of private goods, public goods and leisure in the utility function.

Utility is an increasing and concave function of ($C_{i,t}$), an index of the household's consumption of all national or foreign goods that are supplied; (θ) is the elasticity of intertemporal substitution. Utility is also an increasing and concave function of public goods and services provided in the home country ($G_{i,t}$). One of the main contribution of the current paper is thus not to neglect this possibility of utility-enhancing public spending. Indeed, without including public expenditure in the utility function as substitute with private consumption, a budgetary policy of increase in public spending could not rationally be justified[2]. So, we augment the traditional New-Keynesian models by considering the fact that public expenditure can increase the utility of consumers. Utility is also a decreasing and convex function of the hours worked ($L_{i,t}$), with ($\varphi\geq0$).

In this context, the result of the maximization of equation (1) under the constraint (2) implies the following first order Euler condition, regarding timing of expenditure decisions and inter-temporal substitution, for whatever period (T):

$$\frac{1}{P_{i,T}}\frac{\partial U_{i,T}}{\partial C_{i,T}} = \frac{\beta(1+i_T)}{P_{i,T+1}}\frac{\partial E_T(U_{i,T+1})}{\partial C_{i,T+1}} = \frac{\beta^k(1+i_{T+k-1})\dots(1+i_T)}{P_{i,T+k}}\frac{\partial E_T(U_{i,T+k})}{\partial C_{i,T+k}} \quad (4)$$

Furthermore, we can mention that according to equation (3), $\frac{\partial U_{i,T}}{\partial C_{i,T}} = \alpha_c(C_{i,T})^{-\frac{1}{\theta}}$, and therefore: equation (4) implies, ($\forall$T):

$$C_{i,T} = [\frac{E_T(P_{i,T+1})}{\beta(1+i_T)P_{i,T}}]^{\theta}E_T(C_{i,T+1}) \quad (5)$$

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[2] Ganelli (2003) considers public and private consumption as perfect substitutes, and therefore, he introduces them in a non-separable way in the utility function. Then, he shows that a national or foreign fiscal expansion tends to crowd-out private national consumption and to depress output.

So, in logarithms, with log(1+i_t)~(i_t) provided (i_t) is sufficiently small; with [$\pi_{i,t} = p_{i,t} - p_{i,t-1} = \log(P_{i,t}) - \log(P_{i,t-1})$]: inflation rate for consumption prices; we have:

$$c_{i,T} = E_T(c_{i,T+1}) - \theta[i_T - E_T(\pi_{i,T+1}) + log\,\beta] \qquad (6)$$

In the same way, for the whole monetary union, we have:

$$c_T^* = E_T(c_{T+1}^*) - \theta[i_T - E_T(\pi_{T+1}^*) + log\,\beta] \qquad (7)$$

where (c_T^*) is the variation of consumption for the whole monetary union.

Besides, we suppose that in a given country (i), public expenditure is financed by current public resources and taxes [$\partial(P_{i,T}G_{i,T}) \sim \partial(t_{i,T}W_{i,T}L_{i,T})$], to the limit of the public indebtedness that we will mention below. So, for the representative agent in the country (i), we obtain the following optimal substitution between private, public consumption and working time:

$$\frac{\partial U_{i,T}}{\partial C_{i,T}} = -\frac{P_{i,T}}{W_{i,T}(1-t_{i,T})}\frac{\partial U_{i,T}}{\partial L_{i,T}} = \frac{\partial U_{i,T}}{\partial G_{i,T}} \qquad (8)$$

Therefore, a higher real wage net of taxes reduces the marginal utility of leisure and increases the one of labour. Besides, regarding the labour demand, we have:

$$\frac{\partial U_{i,T}}{\partial L_{i,T}} = -\alpha_l L_{i,T}^{\varphi} = -\frac{W_{i,T}(1-t_{i,T})}{P_{i,T}}\frac{\partial U_{i,T}}{\partial C_{i,T}} = -\frac{W_{i,T}(1-t_{i,T})}{P_{i,T}}\alpha_c(C_{i,T})^{-\frac{1}{\theta}} \qquad (9)$$

So, in differentials and in logarithms, we obtain:

$$l_{i,T} = \frac{1}{\varphi}(w_{i,T} - p_{i,T}) - \frac{1}{\varphi}t_{i,T} + \frac{1}{\varphi}\log\left(\frac{\alpha_c}{\alpha_l}\right) - \frac{1}{\varphi\theta}c_{i,T} \qquad (10)$$

Therefore, labour supply increases with the real wage, but it decreases with the taxation rate ($t_{i,T}$) and with the disutility of working time (φ).

Besides, in this paper, we suppose that the 'law of one price' prevails: 'pricing to market' and discrimination between national or foreign producers is not possible[3]. Prices of goods consumed are the same in a given country, whatever the place where they have been produced: there is a perfect substitutability between national and foreign goods. Furthermore, as suggested by Gali and Monacelli (2005, 2008), we can decompose the consumption of households in the country (i) ($C_{i,t}$) between the share (1-η) of consumption of goods produced in the home country ($C_{i,t}^i$), (1- η) being a measure of home bias in the consumption decisions, and the share (η) of consumption of imported goods produced in the foreign countries ($C_{i,t}^*$), (η) being an indicator of openness of the national country. So, we have:

$$C_{i,t} = \frac{(C_{i,t}^i)^{(1-\eta)}(C_{i,t}^*)^{\eta}}{(1-\eta)^{(1-\eta)}\eta^{\eta}} \qquad (11)$$

In the same way, the consumer price index ($P_{i,t}$) can be divided between prices of nationally produced goods ($P_{i,t}^i$) and prices of goods produced in the foreign countries (P_t^*).

$$P_{i,t} = (P_{i,t}^i)^{(1-\eta)}(P_t^*)^{\eta}$$

[3] See for example Monacelli (2005) for a study of the consequences of deviations from the law of one price and an analysis of incomplete pass-through. In particular, this has the effect of generating endogenously a short run trade-off between stabilizing output and inflation in response to efficient productivity shocks.

$$p_{i,t} = p^i_{i,t} + \eta\left(p^*_t - p^i_{i,t}\right) \qquad (12)$$

So, we have the following relation between national inflation measured in terms of consumer price index ($\pi_{i,t}$) or in terms of producer price index ($\pi^i_{i,t}$):

$$\pi_{i,t} = p_{i,t} - p_{i,t-1} = \left(p^i_{i,t} - p^i_{i,t-1}\right) + \eta\left[\left(p^*_t - p^i_{i,t}\right) - \left(p^*_{t-1} - p^i_{i,t-1}\right)\right]$$
$$= \pi^i_{i,t} + \eta\left[\left(p^*_t - p^i_{i,t}\right) - \left(p^*_{t-1} - p^i_{i,t-1}\right)\right] \qquad (13)$$

Therefore, consumer prices increase more than producer prices if the competitiveness and the terms of trade improve, if the country (i) is very open (η is high) with a faster increase in prices of imported goods in comparison with nationally produced goods.

Furthermore, for the representative household in the country (i), the optimal allocation of expenditure between the national country (i) and the foreign countries implies:

$$P^i_{i,t}C^i_{i,t} = (1-\eta)P_{i,t}C_{i,t}$$
$$P^*_tC^*_{i,t} = \eta P_{i,t}C_{i,t} \qquad (14)$$

In the same way, regarding consumption of goods produced in the home country by foreigners ($C^i_{*,t}$), we have: $P^i_{i,t}C^i_{*,t} = \eta P^*_t C^*_t$

Finally, by combining equations (6) and (7), and using equation (12), we obtain[4]:

$$c_{i,T} = c^*_T + \theta(p^*_T - p_{i,T}) = c^*_T + \theta(1-\eta)(p^*_T - p^i_{i,T}) \qquad (15)$$

This equation also implies: $C_{i,T} = C^*_T(\frac{P^*_T}{P_{i,T}})^\theta \qquad (16)$

Therefore, consumption in the national country increases more than in the foreign countries if inflation is higher in the foreign countries, all the more as the substitutability between goods (θ) is high and as countries are less open (η is small).

2.2 Equilibrium on the goods market and demand equation

The level of public expenditure and taxation rates are fixed at the national level by the budgetary authorities. In this paper, we consider only the income taxation rate for households[5]. For simplicity, we suppose that all government debt is held nationally. For the government of the country (i), the budgetary constraint is then the following:

$$B_{i,t} = (1 + i_{t-1})B_{i,t-1} + \left(P_{i,t}G_{i,t}\right) - \left(t_{i,t}W_{i,t}L_{i,t}\right) \qquad (17)$$

With ($B_{i,t}$): nominal public debt of the government (i) at the end of period (t).
However, as taxes are available, government debt dynamics is irrelevant for the non-fiscal variables and the determination of the private sector equilibrium. We can thus abstract from it for the rest of the paper.

We are now going to derive the equilibrium on the goods market regarding the global demand. Using equations (12), (14) and (16), clearing on the goods market in the country (i) requires:

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[4] $c_{i,T} - E_T\left(c_{i,T+1}\right) = -\theta\log\beta - \theta i_T + \theta E_T\left(\pi_{i,T+1}\right)$
$= c^*_T - E_T(c^*_{T+1}) - \theta[E_T(p^*_{T+1}) - p^*_T] + \theta\left[E_T\left(p_{i,T+1}\right) - p_{i,T}\right].$

[5] See for example Eggertsson (2009) for a precise study of various fiscal policies: variations of public expenditure, which are either substitutable or not substitutable with private consumption, of the sales tax on consumption, of the payroll tax on labor, of the tax on financial assets, of the tax on profits, or of the tax on households' revenues.

$$Y_{i,T} = C_{i,T}^{i} + C_{*,T}^{i} + G_{i,T} = (1-\eta)\frac{P_{i,T}}{P_{i,T}^{i}}C_{i,T} + \eta\frac{P_{T}^{*}}{P_{i,T}^{i}}C_{T}^{*} + G_{i,t}$$
$$= \frac{P_{i,T}}{P_{i,T}^{i}}C_{i,T}\left[(1-\eta)+\eta\left(\frac{P_{i,T}}{P_{T}^{*}}\right)^{\theta-1}\right] + G_{i,t}$$
$$= \left(\frac{P_{T}^{*}}{P_{i,T}^{i}}\right)^{\eta}C_{i,T}[(1-\eta)+\eta\left(\frac{P_{i,T}^{i}}{P_{T}^{*}}\right)^{(1-\eta)(\theta-1)}] + G_{i,t} \quad (18)$$

Therefore, in logarithms and in variations, if we suppose that the weight of the public sector is (γ=G/Y) in the monetary union, we obtain:

$$y_{i,T} = (1-\gamma)c_{i,T} + (1-\gamma)\eta[1-(\theta-1)(1-\eta)](p_{T}^{*}-p_{i,T}^{i}) + \gamma g_{i,T} \quad (19)$$

And at the level of the whole monetary union, using equations (15) and (19), we have:

$$y_{T}^{*} = (1-\gamma)c_{T}^{*} + \gamma g_{T}^{*} = y_{i,T} - (1-\gamma)[\theta-\eta(\theta-1)(2-\eta)](p_{T}^{*}-p_{i,T}^{i}) - \gamma(g_{i,T}-g_{T}^{*}) \quad (20)$$

Besides, by combining equations (6) and (19)[6], and afterwards using (13), we obtain:

$$y_{i,T} = E_T(y_{i,T+1}) - (1-\gamma)\theta[i_T - E_T(\pi_{i,T+1}) + log\beta] + \gamma|g_{i,T} - E_T(g_{i,T+1})]$$
$$-(1-\gamma)\eta[1-(\theta-1)(1-\eta)][E_T(p_{T+1}^{*}-p_{i,T+1}^{i}) - (p_{T}^{*}-p_{i,T}^{i})]$$
$$= E_T(y_{i,T+1}) - (1-\gamma)\theta[i_T - E_T(\pi_{i,T+1}^{i}) + log\beta] + \gamma|g_{i,T} - E_T(g_{i,T+1})]$$
$$+(1-\gamma)\eta(\theta-1)(2-\eta)[E_T(p_{T+1}^{*}-p_{i,T+1}^{i}) - (p_{T}^{*}-p_{i,T}^{i})] \quad (21)$$

Finally, combining equations (20) and (21), and with: $log\beta = -\log\left[1+\left(\frac{1-\beta}{\beta}\right)\right] \sim -\frac{(1-\beta)}{\beta}$ as (1-β) is small, we can obtain:

$$x_{i,T} = E_T(x_{i,T+1}) - \sigma[i_T - E_T(\pi_{i,T+1}^{i}) - \overline{r_{i,T}}] \quad (22)$$
$$\overline{r_{i,T}} = \frac{(1-\beta)}{\beta} - \frac{1}{\sigma}[y_{i,T}^{p} - E_T(y_{i,T+1}^{p})] + \frac{\gamma}{\sigma}[g_{i,T} - E_T(g_{i,T+1})]$$
$$+\frac{\eta(\theta-1)(2-\eta)}{\sigma\theta}[(y_{T}^{*}-\gamma g_{T}^{*}) - E_T(y_{T+1}^{*}-\gamma g_{T+1}^{*})]$$

In the same way, for the whole monetary union, we have:

$$x_{T}^{*} = E_T(x_{T+1}^{*}) - \theta(1-\gamma)[i_T - E_T(\pi_{T+1}^{*}) - \overline{r_{T}^{*}}] \quad (23)$$
$$\overline{r_{T}^{*}} = \frac{(1-\beta)}{\beta} - \frac{1}{\theta(1-\gamma)}[y_{T}^{p*} - E_T(y_{T+1}^{p*})] + \frac{\gamma}{\theta(1-\gamma)}[g_{T}^{*} - E_T(g_{T+1}^{*})]$$

- $\sigma = (1-\gamma)[\theta-\eta(\theta-1)(2-\eta)]$: real interest rate elasticity of the output-gap, 'inter-temporal elasticity of substitution' of household expenditure in open economy.
- $\overline{r_{i,T}}$: Equilibrium or natural real interest rate, which corresponds to the steady-state real rate of return if prices and wages were fully flexible. It is the real interest rate

[6] (6) implies: $c_{i,T} = E_T(c_{i,T+1}) - \theta[i_T - E_T(\pi_{i,T+1}) + log\,\beta]$
(19) gives: $(1-\gamma)E_T(c_{i,T+1}) = E_T(y_{i,T+1}) - (1-\gamma)\eta[1-(\theta-1)(1-\eta)]E_T(p_{T+1}^{*} - p_{i,T+1}^{i}) - \gamma E_T(g_{i,T+1})$.

required to keep aggregate demand equal at all times to the natural rate of output. It includes all non-monetary disturbances. It is a decreasing function of shocks on preferences (β), and of the temporary increase in potential output. It is also an increasing function of the current temporary increase in private economic consumption in all the monetary union. Finally, the equilibrium interest rate is an increasing function of the current temporary increase in public expenditure ($g_{i,T}$). Indeed, this temporary increase in public expenditure, holding public spending constant tomorrow, rises the price of output today in a flexible price equilibrium.
So, as the temporary increase in government spending boosts the real equilibrium interest rate, it increases the output-gap. As national output and production increase, more labour is demanded, real wages increase, agents' income increases, enhancing demand for foreign and national goods. However, national firms then raise their output and prices, deteriorating the price competitiveness of national products and reducing national exports, which can then mitigate the previous effect.

Therefore, as mentioned by Clarida *et al*. (2001), the small economy problem is then quite similar to the traditional New-Keynesian framework of a closed economy. According to equations (22) and (23), higher future expected output increases current output and consumption, because households prefer to smooth consumption, and then higher future revenues raise their current consumption and current output. Current output is also a decreasing function of the excess of the real interest rate above its natural level, because of the inter-temporal substitution of consumption.

However, in an open economy framework, the real interest rate elasticity of the output gap (σ) decreases with the openness of the national country[7] (η). Indeed, an increase in the real interest rate implies a real appreciation of the national currency. As the inflation rate is then higher in the rest of the monetary union than in the national country, it can sustain the demand for national goods, and mitigate the negative effect on national demand. The real interest rate elasticity of the output gap also decreases with the weight of the public sector in the economy (γ), whereas it increases with the share of private consumption (1-γ).

2.3 The supply equation

We suppose a continuum of firms in the country (i). The representative firm produces a differentiated good in a monopolistic competition setting. It defines prices in order to maximize its profit, taking other variables as given. Capital is supposed to be fixed in the short run, whereas labour is defined according to the maximization program of households in equation (10). It is the only factor of production which is variable in the short run. So, we abstract from capital accumulation in this paper. Monopolistic competition gives to goods suppliers a market power regarding price-setting, while at the same time fitting the empirical evidence of a large number of firms in the economy. So, the production function has the following form, for the representative firm in the country (i):

$$Y_{i,t} = A_{i,t} L_{i,t}^{1-\nu} \qquad 0 < \mathrm{v} < 1 \qquad (24)$$

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[7] $\frac{\partial \sigma}{\partial \theta} = (1-\gamma)(1-\eta)^2 > 0$; $\frac{\partial \sigma}{\partial \eta} = -2(1-\gamma)(\theta-1)(1-\eta) < 0$.

With $(A_{i,t})$: technology or productivity shock, common to all firms in the same country, and evolving exogenously over time; $(L_{i,t})$: number of hours worked; (ν) represents the decreasing returns of the production function.

Let's consider a Calvo-type framework of staggered priced, where a fraction $(0<\alpha<1)$ of goods prices remain unchanged each period, whereas prices are adjusted for the remaining fraction $(1-\alpha)$ of goods. Monopolistically competitive firms choose their nominal prices to maximize profits subject to constraints on the frequency of future price adjustments, and taking into account that prices may be fixed for many periods. So, they minimize the loss function:

$$Min_{p_{i,t}^{i}} \sum_{k=0}^{\infty} (\alpha\beta)^k E_t(p_{i,t}^{i,r} - \widetilde{p_{i,t+k}^{i,r}})^2 \qquad (25)$$

Where $(\widetilde{p_{i,t}^{i,r}})$ is the logarithm of the optimal price that a firm (i) will set in period (t) if there were no price rigidity.

The firm minimizes expected losses in profit for all future periods (t+k) due to the fact that it will not be able to set a frictionless optimal price in this period (t+k).These losses are subject to the actualization rate (β), as closer profits are given a higher weight than more distant ones. Besides, the probability that the price $(p_{i,t}^{i})$ will be fixed for (k) periods, until the period (t+k), is (α^k). Thus, by deriving in function of the reset price $(p_{i,t}^{i,r})$, we have:

$\sum_{k=0}^{\infty}(\alpha\beta)^k p_{i,t}^{i,r} = \frac{1}{(1-\alpha\beta)} p_{i,t}^{i,r} = \sum_{k=0}^{\infty}(\alpha\beta)^k E_t(\widetilde{p_{i,t+k}^{i,r}})$, which implies:

$$p_{i,t}^{i,r} = (1-\alpha\beta) \sum_{k=0}^{\infty} (\alpha\beta)^k E_t(\widetilde{p_{i,t+k}^{i,r}}) \qquad (26)$$

Therefore, the firm (i) tries to set the optimal reset price $(p_{i,t}^{i,r})$ to the level of a weighted average of the prices that it would have expected to reset in the future if there weren't any price rigidities.

The optimal strategy of the firm (i) is to fix prices at marginal costs: $\widetilde{p_{i,t}^{i,r}} = mc_{i,t}$, where $(mc_{i,t})$ is the nominal marginal production cost of the firm (i). Furthermore, prices in period (t) are an average of past prices and reset prices:

$$p_{i,t}^{i} = \alpha p_{i,t-1}^{i} + (1-\alpha) p_{i,t}^{i,r} \qquad (27)$$

So, by combining equations (26) and (27)[8], we obtain:

$$p_{i,t}^{i,r} = \frac{1}{(1-\alpha)} p_{i,t}^{i} - \frac{\alpha}{(1-\alpha)} p_{i,t-1}^{i} = \frac{\alpha\beta}{(1-\alpha)} E_t(p_{i,t+1}^{i}) - \frac{\alpha^2\beta}{(1-\alpha)} p_{i,t}^{i} + (1-\alpha\beta) mc_{i,t}$$

Therefore, we have the following equation regarding inflation for producer prices:

$$\pi_{i,t}^{i} = \beta E_t(\pi_{i,t+1}^{i}) + \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} (mc_{i,t} - p_{i,t}^{i}) \qquad (28)$$

Inflation then depends on expected future inflation, and on the gap between the frictionless optimal price level and the current price level, i.e.: on the real marginal cost.

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[8] Equation (26) implies: $p_{i,t}^{i,r} = \alpha\beta E_t(p_{i,t+1}^{i,r}) + (1-\alpha\beta)\widetilde{p_{i,t}^{i,r}}$

Equation (24) implies: $p_{i,t}^{i,r} = \frac{1}{(1-\alpha)} p_{i,t}^{i} - \frac{\alpha}{(1-\alpha)} p_{i,t-1}^{i}$.

Indeed, inflationary pressures can be due to the fact that prices which can be reset by firms are increased.

We still have to clarify the expression of the real marginal production cost for the representative firm of the country (i) [see for example: Woodford (2003, p.182) or Gali (2008, pp.43-49)]. According to equation (24), the variable production cost of the quantity ($Y_{i,t}$) is: $W_{i,t}L_{i,t} = W_{i,t}(\frac{Y_{i,t}}{A_{i,t}})^{\frac{1}{1-\nu}}$. So, differentiating this expression, the nominal marginal production cost of the quantity ($Y_{i,t}$) is:

$$\frac{\partial(W_{i,t}L_{i,t})}{\partial Y_{i,t}} = \frac{W_{i,t}}{A_{i,t}^{\frac{1}{1-\nu}}(1-\nu)}(Y_{i,t})^{\frac{\nu}{1-\nu}} \qquad (29)$$

So, in logarithms and in variations, we obtain the following real marginal production cost:

$$(mc_{i,t} - p_{i,t}^i) = (w_{i,t} - p_{i,t}^i) - \frac{1}{1-\nu}a_{i,t} - log(1-\nu) + \frac{\nu}{1-\nu}y_{i,t} \quad (30)$$

Besides, for a given period (T), equations (10) and (24) imply:

$$(w_{i,T} - p_{i,T}) = \frac{\varphi}{1-\nu}y_{i,T} - \frac{\varphi}{1-\nu}\mathrm{a}_{i,T} + t_{i,T} - \log\left(\frac{\alpha_c}{\alpha_l}\right) + \frac{1}{\theta}c_{i,T} \qquad (31)$$

Regarding national consumption, using equations (15) and (20), we have:

$$c_{i,T} = \frac{1}{(1-\gamma)}y_T^* - \frac{\gamma}{(1-\gamma)}g_T^* + \theta(1-\eta)(p_T^* - p_{i,T}^i) \qquad (32)$$

Therefore, by combining equations (30), (31) and (32), and also using equations (12) and (20), we obtain:

$$(mc_{i,T} - p_{i,T}^i) = \left[\frac{(\varphi+\nu)}{(1-\nu)} + \frac{1}{\sigma}\right]y_{i,T} - \frac{\gamma}{\sigma}g_{i,T} - \frac{\eta(\theta-1)(2-\eta)}{\sigma\theta}(y_T^* - \gamma g_T^*)$$
$$- \frac{(1+\varphi)}{(1-\nu)}\mathrm{a}_{i,T} + t_{i,T} - \log\left(\frac{\alpha_c}{\alpha_l}\right) - \log(1-\nu) \qquad (33)$$

Indeed, real marginal production costs increase with national economic activity ($y_{i,t}$), because of the expansionary effect of economic activity on employment. They also increase with foreign private economic activity ($y_t^* - \gamma g_t^*$) and national exports. Indeed, national exports imply a wealth effect on national economic activity and consumption, and then on labour supply. Obviously, real marginal production costs also decrease with positive productivity shocks ($a_{i,t}$), and they increase with taxation rates ($t_{i,t}$). They also decrease with public expenditure ($g_{i,t}$). Indeed, a higher level of public expenditure crowds out private consumption for a given output level, and it also tends to reduce real wages.

Finally, by combining equations (28) and (33), for the country (i), we have:

$$\pi_{i,T}^i = \beta E_T(\pi_{i,T+1}^i) + k_1 k_2 x_{i,T} \qquad (34)$$

$$with: k_1 = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} \qquad k_2 = \left[\frac{1}{\sigma} + \frac{(\varphi+\nu)}{(1-\nu)}\right]$$

$$y_{i,T}^p = \frac{(1+\varphi)}{k_2(1-\nu)}\mathrm{a}_{i,T} - \frac{1}{k_2}t_{i,T} + \frac{\gamma}{k_2\sigma}g_{i,T} + \frac{\eta(\theta-1)(2-\eta)}{\sigma\theta k_2}(y_T^* - \gamma g_T^*)$$
$$+ \frac{1}{k_2}\log(1-\nu) + \frac{1}{k_2}\log\left(\frac{\alpha_c}{\alpha_l}\right)$$

In the same way, for the whole monetary union, we have:

$$\pi_T^* = \beta E_T(\pi_{T+1}^*) + k_1 k_2^* x_T^* \qquad (35)$$
$$with: \qquad k_2^* = \left[\frac{1}{\theta(1-\gamma)} + \frac{(\varphi+\nu)}{(1-\nu)}\right]$$
$$y_T^{p*} = \frac{(1+\varphi)}{k_2^*(1-\nu)} a_T^* - \frac{1}{k_2^*} t_T^* + \frac{\gamma}{\theta k_2^*(1-\gamma)} g_T^* + \frac{1}{k_2^*}\log(1-\nu) + \frac{1}{k_2^*}\log\left(\frac{\alpha_c}{\alpha_l}\right)$$

- ($y_{i,T}^p$), potential output: it is an increasing function of productivity shocks ($a_{i,t}$), but it is negatively correlated with taxation rates ($t_{i,t}$). Potential output also increases with the growth of private economic activity in the rest of the monetary union, increasing national exports and tensions on the utilization of national production capacities. Finally, potential output also increases with public expenditure ($g_{i,t}$). Indeed, a positive government spending gap is inflationary due to the increased demand for home goods, which leads to higher work effort. This, in turn, translates into higher marginal costs, and, thus, higher national prices.

Equation (34) is the simplest form of the New-Keynesian Phillips curve in open economy. In this equation, ($k_1 k_2$) is an indicator of price flexibility. This parameter decreases with the indicator of price-stickiness (α), the longer the average time interval between price changes. It increases with the measure of decreasing returns in the production function (v). Price flexibility increases with the dis-utility, for households, of labour supply (φ), as labour supply is then more elastic to the real wage level. Finally, it also decreases with the inter-temporal elasticity of substitution of household expenditure in open economy (σ), and then it increases with the degree of openness of the national country (η). Indeed, openness of the country implies that national output is more sensitive to the terms of trade ($p_T^* - p_{i,T}^i$), and then that in case of economic growth, the weaker competitiveness of the national country can efficiently contribute to reduce tensions on national economic activity, marginal costs and inflation.

3 Determination of optimal economic policies

In order to define optimal economic policies according to the macroeconomic framework, we first have to detail the preferences of the economic authorities.

3.1 Preferences of the economic authorities

The preferences of the central bank can be given micro-foundations. The objective function is then derived as a second-order Taylor series approximation to the level of expected life-time utility function of the representative household. Therefore, we will consider in this paper that the welfare objective is a decreasing function of quadratic variations in the inflation rate or in economic activity in comparison with their optimal values [Woodford (2003), pp.383-391]. The central bank's inter-temporal loss minimization problem is realized in the framework of a quadratic loss function with 'flexible inflation targeting'. This term was first mentioned by Svensson, in order to define a monetary policy which would not only be concerned with inflation stabilization, but which also attaches a non-negligible weight to output stabilization. The instrument of the central bank is the short term nominal interest rate, and it chooses the path of all current and future nominal interest rates to minimize the following loss function:

$$Min \sum_{t=0}^{\infty} \beta^t E_t[(\pi_t^* - \pi^{opt})^2 + \lambda_{CB}(x_t^* - x^{opt})^2] \qquad (36)$$

Where: $(0<\beta<1)$ is the time discount factor, and $(\lambda_{CB}>0)$ is the weight given by the central bank to the goal of stabilizing the output gap in comparison with the goal of targeting an inflation level normalized to unity.

(π^{opt}) is the optimal inflation target chosen by the monetary authority. We suppose that this target must be above zero, and high enough in order that the central bank can reach this goal with a minimized risk of violating the Zero Lower Bound constraint. The target must be high enough to avoid the risk that a bad shock would push the economy in a deflationary spiral which could exacerbate welfare losses for representative households, instead of making the economy revert to a stable equilibrium. Therefore, there is a kind of 'inflation bias' necessary in the context of the ZLB.

Besides, the central bank tries to stabilize the output-gap, to limit under-utilization of resources and deviations of real economic activity from its natural level, which is the efficient level corresponding to the productive potential of the economy. So, the output-gap target (x^{opt}) can be considered as positive, if the natural rate of output is perceived as inefficiently low, because of the small amount of market power held by producers of differentiated goods, or because of the delays in prices adjustments. The social optimum regarding the output level may thus exceed its natural level, because of distortions such as imperfect competition or taxes.

Another condition to integrate in our modelling is: $(i_t \geq 0)$, the Zero Lower Bound on the nominal interest rate, the non-negativity constraint on short term nominal interest rates, also introduced in Adam and Billi (2006, 2007), Jung *et al.* (2005), Eggertsson and Woodford (2003) or Nakov (2008), for example. Indeed, nobody would choose to hold assets bearing a negative return whereas they can hold money bearing a zero nominal return. This constraint only means that the marginal utility of money holding and of real monetary balances cannot be negative for a representative consumer. It results from the availability of cash as a riskless, perfectly liquid zero-return asset. This ZLB condition is widely admitted in the economic literature, even when theoretically, it is an implication of transaction and storage cost properties of the medium of exchange, as mentioned and theoretically discussed by McCallum (2000). The validity of this condition depends upon the assumption that it is costless at the margin to store money, the economy's medium of exchange. While some central banks, such as the European Central Bank (ECB) and the Swiss National Bank (SNB) have introduced slightly negative nominal rates, there is clearly a limit to how negative the nominal interest rate can be before savers turn to cash. Hence, while the true bound might not be exactly zero, it is likely to be some small negative number. Considering explicitly this constraint implies a major non-linearity in our modelling. We can also mention that according to equation (22), the average real interest rate is positive according to the time preference of the representative household $(\beta<1)$, at least in the long run. So, the nominal interest rate should remain positive with small enough disturbances, and the ZLB should not be binding. However, the real equilibrium interest rate can temporary become negative if public expenditure, investment or exports are temporary very small, or in case of a temporary increase in productivity.

The ZLB constrains the capacity of the central bank to stimulate the economy in case of downturns. So, if economic authorities pursue a very low inflation target, there can be a 'deflationary trap', where conventional open market operations cannot stabilize the economic situation, and with longer lasting recessions. One solution to this problem is the commitment of monetary policy studied by many authors. In this framework, the interest rate should be lowered pre-emptively before the recessionary situation becomes uncontrolled and before reaching the ZLB. In the same way, interest rates should be kept to zero at the beginning of the recovery period, below what would be prescribed by an optimal discretionary monetary policy and for a longer time. Such a policy could mitigate the fall in output and inflation. However, the current paper suggests another efficient solution to the ZLB problem, which is the conduct of a more efficient policy-mix. Indeed, budgetary policies could complement monetary policy in a ZLB framework.

Finally, the loss function of the government of the country (i) is as follows:

$$Min\sum_{t=0}^{\infty}\beta^t E_t[(\pi_{i,t}-\pi^{opt})^2+\lambda_{x,G}(x_{i,t}-x^{opt})^2+\lambda_{g,G}(g_{i,t}-g^{opt})^2] \qquad (37)$$

Where: ($\lambda_{x,G}$) and ($\lambda_{g,G}$) are the respective weights (supposed to be the same for all member countries of the monetary union) given by the governments to the goal of stabilizing the output gap and stabilizing the level of public expenditure, in comparison with the goal of targeting an inflation level normalized to unity.

In order not to make the modelling more complex, we also suppose that the inflation rate and economic activity targets are the same for the common central bank and for the governments of the member countries of the monetary union. We can mention that Gali and Monacelli (2005) show that a Taylor rule targeting a national producer price index gives better results in terms of welfare than targeting a consumer price index. However, consumer prices are more usually considered and observed by public authorities, and the results of our modelling would not be much altered. So, we will retain a goal in terms of consumer prices.

Finally, (g^{opt}) is the targeted level of public expenditure (government consumption), the optimal level that would be chosen if prices were perfectly flexible, and the level which can stabilize the average public debt level in the monetary union.

3.2 Definition of the policy-mix

We suppose that the central bank cannot commit to future policies, and chooses the current interest rate by re-optimizing every period. In choosing its optimal monetary policy, the central bank takes private sector expectations as given. Monetary policy is then 'time consistent', as rational expectations imply that the central bank has no incentive to change its plans in an unexpected way. Future expectations about inflation, output and interest rate cannot be manipulated by the central bank; they are then independent from current actions. So, the discretionary problem is reduced to a sequence of static optimization problems in which the central bank minimizes current period losses.

Using equations (23), (35) and (36), in period (T), the central bank chooses a path for its interest rate minimizing the following loss function, for all member countries of the monetary union:

$$\mathcal{L}_{i,T} = E_T \sum_{t=T}^{\infty} \beta^t \{[(\pi_t^* - \pi^{opt})^2 + \lambda_{CB}(x_t^* - x^{opt})^2] + z_{i2,t}[\pi_t^* - \beta\pi_{t+1}^* - k_1 k_2^* x_t^*] + z_{i1,t}\left[x_t^* - x_{t+1}^* + \theta(1-\gamma)(i_t - \pi_{t+1}^* - \bar{r}_t^*)\right] + z_{i3,t}[i_t - 0]\} \quad (38)$$

The optimal first order conditions of equation (38) for a given period (T) are as follows:

$$\begin{cases} \dfrac{\partial \mathcal{L}_{i,T}}{\partial \pi_T^*} = 2(\pi_T^* - \pi^{opt}) + z_{i2,T} = 0 \\ \dfrac{\partial \mathcal{L}_{i,T}}{\partial x_T^*} = 2\lambda_{CB}(x_T^* - x^{opt}) + z_{i1,T} - k_1 k_2^* z_{i2,T} = 0 \\ \dfrac{\partial \mathcal{L}_{i,T}}{\partial i_T} = z_{i1,T}\theta(1-\gamma) + z_{i3,T} = 0 \end{cases}$$

$$\begin{cases} \{x_T^* - E_T(x_{T+1}^*) + \theta(1-\gamma)[i_T - E_T(\pi_{T+1}^*) - \bar{r}_T^*]\}\, z_{i1,T} = 0 \\ \{\pi_T^* - \beta E_T(\pi_{T+1}^*) - k_1 k_2^* x_T^*\} z_{i2,T} = 0 \\ z_{i3,T}\, i_T = 0 \qquad i_T \geq 0 \end{cases} \quad (39)$$

Regarding fiscal policies, minimizing the loss function (37) taking into account the constraints (22) and (34), the optimal level of public expenditure for the government of the country (i) in period (T) is as follows [see equation (A9) in Appendix A]:

$$g_{i,T} = f\{\overbrace{g_{i,T-1}}^{-}, \overbrace{x_{i,T-1}}^{+}, \sum_{n=T}^{T+N} \overbrace{E_T(i_n)}^{+}, \sum_{n=T+1}^{T+N} \overbrace{E_T(g_{i,n})}^{-}, \sum_{n=T-1}^{T+N} \overbrace{E_T(g_n^*)}^{+}, \sum_{n=T-1}^{T+N} \overbrace{E_T(a_{i,n})}^{+}, \sum_{n=T-1}^{T+N} \overbrace{E_T(a_n^*)}^{-}, \sum_{n=T-1}^{T+N} \overbrace{E_T(t_{i,n})}^{-}, \sum_{n=T-1}^{T+N} \overbrace{E_T(t_n^*)}^{+}, \sum_{n=T-1}^{T+N} \overbrace{E_T(x_n^*)}^{-}, \overbrace{E_T(g_{i,T+N+1})}^{+}, \overbrace{E_T(g_{T+N+1}^*)}^{-}, \overbrace{E_T(a_{i,T+N+1})}^{-}, \overbrace{E_T(a_{T+N+1}^*)}^{+}, \overbrace{E_T(t_{i,T+N+1})}^{+}, \overbrace{E_T(t_{T+N+1}^*)}^{-}, \overbrace{E_T(x_{i,T+N+1})}^{-}, \overbrace{E_T(x_{T+N+1}^*)}^{+}, \overbrace{E_T(\pi_{i,T+N+1}^{i})}^{-}, \overbrace{g^{opt}}^{+}, \overbrace{\pi^{opt}}^{+}, \overbrace{x^{opt}}^{+}\} \quad (40)$$

Where the sign (-) signifies a decreasing and the sign (+) an increasing function.

In the same way, we can obtain the optimal level of global public expenditure in period (T) in all the monetary union [see equation (B8) in Appendix B]:

$$g_T^* = f\{\sum_{n=T}^{T+N} \overbrace{E_T(i_n)}^{+}, \sum_{n=T+1}^{T+N} \overbrace{E_T(g_n^*)}^{-}, \sum_{n=T}^{T+N} \overbrace{E_T(a_n^*)}^{+}, \sum_{n=T}^{T+N} \overbrace{E_T(t_n^*)}^{-}, \overbrace{E_T(x_{T+N+1}^*)}^{-}, \overbrace{E_T(\pi_{T+N+1}^*)}^{-}, \overbrace{E_T(g_{T+N+1}^*)}^{+}, \overbrace{E_T(a_{T+N+1}^*)}^{-}, \overbrace{E_T(t_{T+N+1}^*)}^{+}, \overbrace{\pi^{opt}}^{+}, \overbrace{x^{opt}}^{+}, \overbrace{g^{opt}}^{+}\} \quad (41)$$

Therefore, we can notice that economic policies are conflicting in the monetary union: public expenditure is an increasing function of the nominal interest rate. If the common monetary policy is too contractionary (higher interest rates), budgetary policies are more expansionary and public expenditures increase in order to compensate.

Budgetary expenditure is also an increasing function of the targeted levels of public expenditure, economic activity and inflation, which are common to all the monetary union.

3.3 Calibration of the model

We now need to calibrate our model, in order to obtain precise numerical results.

- All papers consider a discount factor near β=0.99.
- Correia *et al.* (2013) consider the following share of public expenditure in GDP: γ=0.2. Erceg and Linde (2014) or Schmidt (2013) also take: γ=0.2. Beetsma and Jensen (2005) consider γ=0.25: private consumption is about three times as large as public consumption. In this chapter, we will consider: γ=0.2.
- Erceg and Linde (2014) consider that the intertemporal elasticity of substitution is σ^*=1. Burgert and Schmidt (2014) take σ^*=1.6. Ferrero (2009) takes σ^*=4.5, Nakata (2015) considers σ^*=6, whereas Schmidt (2013) takes σ^*=6.25. We will consider a high value of σ^* =6.4 in the current chapter, which implies: $\theta = \frac{\sigma^*}{(1-\gamma)} = \frac{6.4}{0.8} = 8$.
- Like Colciago *et al.* (2008), we can calibrate the home bias at 0.7, which implies a degree of openness: η=0.3. This implies: $\sigma_i = 0.8 * (8 - 0.3 * 7 * 1.7) = 3.54$, which is in conformity with the above mentioned intertemporal elasticity of substitution.
- Gali and Monacelli (2005) consider a labor supply elasticity: φ=3. Eggertsson (2006) takes φ =2. Erceg and Linde (2014) take φ =2.5. Nakata (2015) takes φ =1. In this paper, we will consider φ=2.
- Gali and Monacelli (2005), Nakata (2015) or Beetsma and Jensen (2005) consider a price stickiness parameter: α=0.75. Correia *et al.* (2013) take α=0.85. Ferrero (2009) takes α= 0.75 or 0.8. Mertens and Ravn (2014) take α=0.65. Schmidt (2013) or Burgert and Schmidt (2014) take α=0.66. Leith and Malley (2002) consider that, after introducing open economy factors, and with also foreign intermediate goods as input in production for national firms, price flexibility would be higher in the United-States (α=0.54), in Italy (α=0.56) or in the United-Kingdom (α=0.60) than in France (α=0.75) or in Germany (α=0.87). In this paper, we will consider α=0.8.
- Correia *et al.* (2013) take ν=0.33 as returns in the production function. Erceg and Linde (2014) consider ν=0.3. Ferrero (2009) takes ν=0.47. In this paper, we will consider ν=0.3.
- Regarding the preferences of the economic authorities, we will consider that the common central bank gives a higher weight to price stability: (λ_{CB}=0.1). On the contrary, the governments give a higher weight to output gap stabilization: ($\lambda_{x,G} = 2$) and ($\lambda_{g,G} = 0.05$).

4 Optimal equilibrium when the Zero Lower Bound is not binding

Regarding monetary policy, if the Zero Lower Bound is not binding, in a given period (T), we have: (i_T>0), and ($z_{i3,T} = z_{i1,T} = 0$). Therefore, the system (39) is reduced to:

$$\begin{cases} k_1k_2^*(\pi_T^* - \pi^{opt}) + \lambda_{CB}(x_T^* - x^{opt}) = 0 \\ i_T = \overline{r_T^*} - \dfrac{1}{\theta(1-\gamma)}x_T^* + \dfrac{1}{\theta(1-\gamma)}E_T(x_{T+1}^*) + E_T(\pi_{T+1}^*) \\ \pi_T^* = \beta E_T(\pi_{T+1}^*) + k_1k_2^*x_T^* \end{cases} \quad (42)$$

Therefore, the first and third equations of the system (42) imply:

$$x_T^* = -\frac{k_1k_2^*\beta}{(k_1^2k_2^{*2} + \lambda_{CB})}E_t(\pi_{T+1}^*) + \frac{k_1k_2^*}{(k_1^2k_2^{*2} + \lambda_{CB})}\pi^{opt} + \frac{\lambda_{CB}}{(k_1^2k_2^{*2} + \lambda_{CB})}x^{opt}$$

$$\pi_T^* = \frac{\beta\lambda_{CB}}{(k_1^2k_2^{*2} + \lambda_{CB})}E_T(\pi_{T+1}^*) + \frac{k_1^2k_2^{*2}}{(k_1^2k_2^{*2} + \lambda_{CB})}\pi^{opt} + \frac{\lambda_{CB}k_1k_2^*}{(k_1^2k_2^{*2} + \lambda_{CB})}x^{opt} \quad (43)$$

So, the optimal common monetary policy can stabilize average values in the monetary union as long as the ZLB is not binding. Shocks on the equilibrium real interest rate (demand shocks) and expected variations of the output-gap can be fully stabilized by monetary policy, provided variations of interest rates are costless. The common monetary policy then allows a perfect stabilization of average productivity shocks (a_T^*), average shocks on taxation rates (t_T^*), or shocks on preferences (β) in all the monetary union.

4.1 Optimal economic policies

Replacing the values of ($\overline{r_T^*}$) in equation (B3) (see Appendix B) and (x_T^*) in equation (43) in the value of (i_T) in the system (42), the optimal monetary policy is then as follows:

$$\begin{aligned} i_T = {} & \frac{(1-\beta)}{\beta} + \frac{\gamma(\varphi+\nu)}{[1-\nu+\theta(1-\gamma)(\varphi+\nu)]}[g_T^* - E_T(g_{T+1}^*)] \\ & + [1 + \frac{k_1k_2^*\beta}{\theta(1-\gamma)(k_1^2k_2^{*2}+\lambda_{CB})}]E_t(\pi_{T+1}^*) + \frac{1}{\theta(1-\gamma)}E_T(x_{T+1}^*) \\ & - \frac{1}{[1-\nu+\theta(1-\gamma)(\varphi+\nu)]}\{(1+\varphi)[a_T^* - E_T(a_{T+1}^*)] \\ & \quad - (1-\nu)[t_T^* - E_T(t_{T+1}^*)]\} \\ & - \frac{1}{\theta(1-\gamma)(k_1^2k_2^{*2}+\lambda_{CB})}(\lambda_{CB}x^{opt} + k_1k_2^*\pi^{opt}) > 0 \end{aligned} \quad (44)$$

Therefore, using the expression of (g_T^*) in equation (B8) in Appendix B, for N→∞, we obtain the following optimal monetary policy:

$$\begin{aligned} i_T = {} & \frac{(1-\beta)}{\beta} + \frac{\theta(1-\gamma)\gamma^2(\varphi+\nu)^2k_1k_2^*}{\lambda_{g,G}[1-\nu+\theta(1-\gamma)(\varphi+\nu)]^2}\sum_{n=T+1}^{\infty} z_{n-T}\,[i_n - \frac{(1-\beta)}{\beta}] \\ & + \frac{1}{[1-\nu+\theta(1-\gamma)(\varphi+\nu)]}[\gamma(\varphi+\nu)g^{opt} - (1+\varphi)a_T^* + (1-\nu)t_T^*] \\ & - \frac{1}{[1-\nu+\theta(1-\gamma)(\varphi+\nu)]}\left[1 + \frac{\theta(1-\gamma)\gamma^2(\varphi+\nu)^2k_1k_2^*z_1}{\lambda_{g,G}[1-\nu+\theta(1-\gamma)(\varphi+\nu)]^2}\right] \\ & \quad E_T[\gamma(\varphi+\nu)g_{T+1}^* - (1+\varphi)a_{T+1}^* + (1-\nu)t_{T+1}^*] \\ & - \frac{\theta(1-\gamma)\gamma^2(\varphi+\nu)^2k_1k_2^*}{\lambda_{g,G}[1-\nu+\theta(1-\gamma)(\varphi+\nu)]^3}\sum_{n=T+2}^{\infty}(z_{n-T} - z_{n-T-1})\,E_T[\gamma(\varphi+\nu)g_n^* - (1+\varphi)a_n^* \\ & \quad + (1-\nu)t_n^*] \end{aligned}$$

$$+\left[1+\frac{\theta^2(1-\gamma)^2\gamma^2(\varphi+\nu)^2\left(\lambda_{x,G}+k_1^2k_2^{*2}\right)}{\lambda_{g,G}[1-\nu+\theta(1-\gamma)(\varphi+\nu)]^2}\right]\left[1+\frac{k_1k_2^*\beta}{\theta(1-\gamma)(k_1^2k_2^{*2}+\lambda_{CB})}\right]E_T(\pi_{T+1}^*)$$
$$+\left[\frac{1}{\theta(1-\gamma)}+\frac{\theta(1-\gamma)\gamma^2(\varphi+\nu)^2\left(\lambda_{x,G}+k_1^2k_2^{*2}\right)}{\lambda_{g,G}[1-\nu+\theta(1-\gamma)(\varphi+\nu)]^2}\right]E_T(x_{T+1}^*)$$
$$-\frac{1}{(k_1^2k_2^{*2}+\lambda_{CB})}\left[\frac{\lambda_{CB}}{\theta(1-\gamma)}-\frac{k_1^2k_2^{*2}\left(\lambda_{x,G}-\lambda_{CB}\right)\theta(1-\gamma)\gamma^2(\varphi+\nu)^2}{\lambda_{g,G}[1-\nu+\theta(1-\gamma)(\varphi+\nu)]^2}\right]x^{opt}$$
$$-\frac{k_1k_2^*}{(k_1^2k_2^{*2}+\lambda_{CB})}\left[\frac{1}{\theta(1-\gamma)}+\frac{\theta(1-\gamma)\gamma^2(\varphi+\nu)^2\left(\lambda_{x,G}-\lambda_{CB}\right)}{\lambda_{g,G}[1-\nu+\theta(1-\gamma)(\varphi+\nu)]^2}\right]\pi^{opt} \quad (45)$$

First, there is a smoothing of interest rates variations, as the current interest rate should increase above its ‘natural level’ ($\frac{(1-\beta)}{\beta}$) if future increases in the nominal interest rate above its natural level are expected for the following periods. These shocks on households’ preferences (β) can perfectly be stabilized by monetary policy, and they necessitate no variation of public expenditure [see equations (46) and (47) below].

The common interest rate should also decrease with temporary positive productivity shocks (a_T^*) or weaker average taxation rates (t_T^*) in the monetary union. Besides, we find again a traditional result in the economic literature. In response to a rise in expected future inflation in all the monetary union (π_{T+1}^*), the nominal interest rate should rise more than proportionately in order to increase real interest rates [the coefficient for expected inflation is above unity; ‘Taylor prescription’], and to decrease global demand. This increase of the real interest rate allows a better stabilization of prices and economic activity levels. It avoids the existence of self-fulfilling sunspot equilibriums, with a decline in the real interest rate and an outburst in inflation. The nominal interest rate should also increase with the excess of the expected future economic activity level (x_{T+1}^*).above its target.

Finally, the common interest rate should increase with shocks on the targeted level of public expenditure (expansionary budgetary policy and conflicting economic policies), but it should decrease with the targeted economic activity or inflation levels, in order to sustain economic growth.

Combining equations (44) for (i_T) and the optimal budgetary policies [see equation (A9) in Appendix A and equation (B8) in Appendix B], we obtain the following levels of optimal budgetary expenditures [see Appendix C]:

$$g_T^*=f\{\sum_{n=T+1}^{\infty}\overbrace{E_T(\pi_n^*)}^{+},\sum_{n=T+1}^{\infty}\overbrace{E_T(\mathrm{x}_n^*)}^{+},\overbrace{\pi^{opt}}^{-},\overbrace{x^{opt}}^{-},\overbrace{g^{opt}}^{+}\} \quad (46)$$

$$(g_{i,T}-g_T^*)=f\{\overbrace{(g_{i,T-1}-g_{T-1}^*)}^{-},\sum_{n=T+1}^{\infty}\overbrace{\left[E_T(g_{i,n})-E_T(g_n^*)\right]}^{-},\overbrace{x_{i,T-1}}^{+},\overbrace{x_{T-1}^*}^{-},\overbrace{\pi^{opt}}^{+},\overbrace{x^{opt}}^{+},$$
$$\sum_{n=T-1}^{\infty}\overbrace{\left[E_T(\mathrm{a}_{i,n})-E_T(\mathrm{a}_n^*)\right]}^{+},\sum_{n=T-1}^{\infty}\overbrace{\left[E_T(\mathrm{t}_{i,n})-E_T(\mathrm{t}_n^*)\right]}^{-},\sum_{n=T+1}^{\infty}\overbrace{E_T(x_n^*)}^{-},\sum_{n=T+1}^{\infty}\overbrace{E_T(\pi_n^*)}^{-}\} \quad (47)$$

So, in combination with the common monetary policy, global budgetary expenditure should stabilize average variables in the monetary union. However, there is then a conflict of goals between economic policies. Indeed, the global budgetary policy is expansionary in case of increases in future expected average inflation rates or economic activity levels, in order to compensate for the more contractionary monetary policy. On the contrary, the global budgetary policy is contractionary in case of higher targeted inflation rates or economic activity levels, in order to compensate for the more expansionary monetary policy.

Furthermore, the national budgetary policy in the country (i) should stabilize deviations between national and global economic variables (for $a_{i,T}$ and $t_{i,T}$), and it depends on the openness of the national country for the stabilization of average economic variables (for x_n^* or π_n^*, n≥T+1). We can also mention inertia and smoothing of public expenditure, as the latter depends on past and future differentials with the corresponding values in the rest of the monetary union.

4.2 Stabilization of global demand shocks

The ZLB is not binding regarding the stabilization of positive global demand or inflationary shocks in the monetary union ($x_T^* > 0\ or\ \pi_T^* > 0$). More precisely, with: $x_T^* = \rho_{x*}x_{T-1}^* + \varepsilon_{xT*}$ and $\pi_T^* = \rho_{\pi*}\pi_{T-1}^* + \varepsilon_{\pi*T}$, where (ε_{x*T}) and $(\varepsilon_{\pi*T})$ are white noises, we obtain the following optimal monetary policy [see equation (45)]:

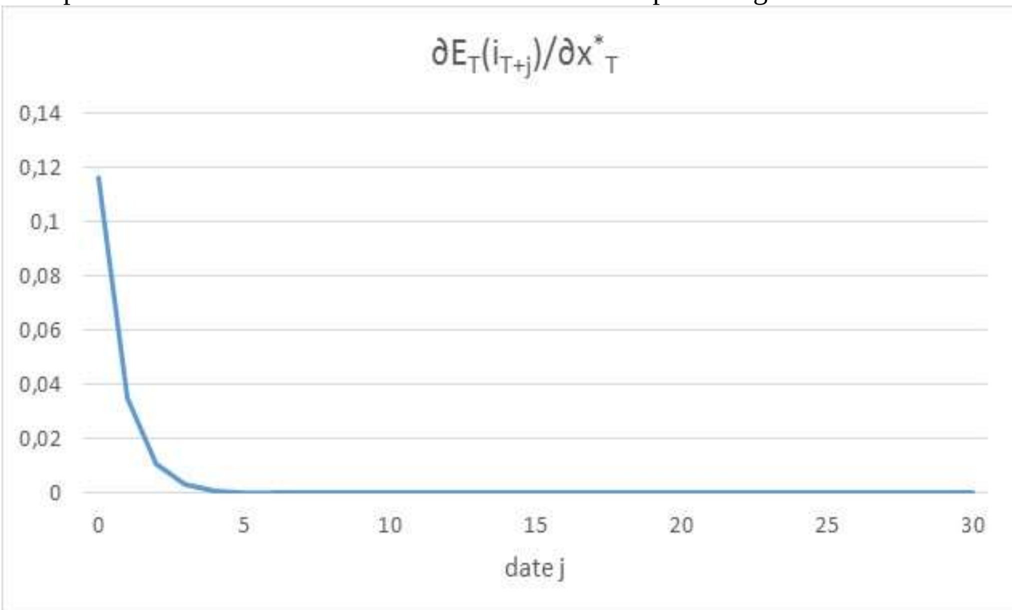
$$\frac{\partial E_T(i_{T+j})}{\partial \pi_T^*} = \frac{\rho_{\pi*}^{j+1}}{\theta(1-\gamma)}\left[\theta(1-\gamma) + \frac{k_1 k_2^* \beta}{(k_1^2 k_2^{*2} + \lambda_{CB})}\right]\left[1 + \frac{\theta^2(1-\gamma)^2\gamma^2(\varphi+\nu)^2\left(\lambda_{x,G} + k_1^2 k_2^{*2}\right)}{\lambda_{g,G}[1-\nu+\theta(1-\gamma)(\varphi+\nu)]^2}\right]$$

$$\frac{\partial E_T(i_{T+j})}{\partial x_T^*} = \frac{\rho_{x*}^{j+1}}{\theta(1-\gamma)}\left[1 + \frac{\theta^2(1-\gamma)^2\gamma^2(\varphi+\nu)^2\left(\lambda_{x,G} + k_1^2 k_2^{*2}\right)}{\lambda_{g,G}[1-\nu+\theta(1-\gamma)(\varphi+\nu)]^2}\right] \quad for\ j \geq 0 \quad (48)$$

Monetary policy stabilizes average shocks in the monetary union, and it is contractionary after a positive global demand shock. The initial increase in interest rates tends towards ($\frac{1}{\theta(1-\gamma)}$), and it is therefore all the more accentuated as the inter-temporal elasticity of substitution of household expenditure is weak, and as monetary policy is then less efficient in stabilizing economic activity. Monetary policy is also more contractionary in order to compensate for more active budgetary policies ($\lambda_{g,G}$ is small and $\lambda_{x,G}$ is high), and if price flexibility is high in the monetary union (α is weak). Besides, the progressive decrease of interest rates is the fastest as the shock persistence (ρ_{x*} or $\rho_{\pi*}$) is weak.

So, with the basic calibration of our model, if the positive global demand shock has a persistence ($\rho_{x*} = 0.3$), we have the following graph:

Graph 1: Variation of the nominal interest rate after a positive global demand shock



As regards budgetary policies, according to equations (C1) and (C3) in Appendix C, we have:

$$\frac{\partial\left(g_{i,T}-g_T^*\right)}{\partial x_T^*}=\frac{\partial g_T^*}{\partial x_T^*}=\frac{\partial\left(g_{i,T}-g_T^*\right)}{\partial \pi_T^*}=\frac{\partial g_T^*}{\partial \pi_T^*}=0$$

$$\frac{\partial E_T(g_{T+j}^*)}{\partial x_T^*}=\frac{\gamma(\varphi+\nu)k_1k_2^*z_{j-1}}{\lambda_{g,G}[1-\nu+\theta(1-\gamma)(\varphi+\nu)]}\rho_{x*}^j \qquad for\ j\geq 1 \qquad (49)$$

$$\frac{\partial E_T\left(g_{i,T+j}-g_{T+j}^*\right)}{\partial x_T^*}=-\frac{(1-\nu+\sigma\varphi+\sigma\nu)k_1k_2(\varphi+\nu)\gamma\sigma}{[\gamma^2(\varphi+\nu)^2\sigma^2\left(\lambda_{x,G}+k_1^2k_2^2\right)+\lambda_{g,G}(1-\nu+\sigma\varphi+\sigma\nu)^2]}$$
$$\{\frac{\eta(v_{j-1}+k_1k_2^*w_{j-1})}{\theta(1-\gamma)}+\frac{\eta(\theta-1)(2-\eta)(\varphi+\nu)e_j}{\theta(1-\nu+\sigma\varphi+\sigma\nu)}$$
$$+\frac{(1-\nu+\sigma\varphi+\sigma\nu)k_1k_2^*z_{j-1}}{\sigma[1-\nu+\theta(1-\gamma)(\varphi+\nu)]k_1k_2}-\frac{[1-\nu+\theta(1-\gamma)(\varphi+\nu)]e_{j-1}}{\theta(1-\gamma)(1-\nu+\sigma\varphi+\sigma\nu)}$$
$$-\frac{\sigma\eta\left(\lambda_{x,G}+k_1^2k_2^2\right)(\theta-1)(2-\eta)(\varphi+\nu)\left(v_{j-1}+k_1k_2^*w_{j-1}\right)}{\theta(1-\nu+\sigma\varphi+\sigma\nu)k_1k_2}\}\rho_{x*}^j \qquad j\geq 1 \quad (50)$$

In the same way, regarding inflationary shocks, we have:

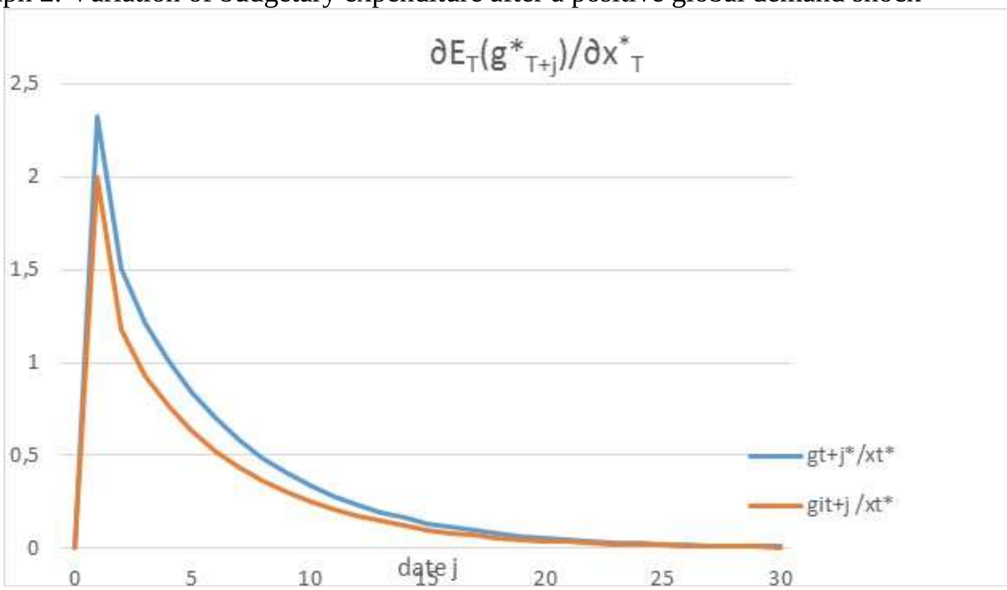
$$\frac{\partial E_T(g_{T+j}^*)}{\partial \pi_T^*}=\left[\theta(1-\gamma)+\frac{k_1k_2^*\beta}{(k_1^2k_2^{*2}+\lambda_{CB})}\right]\frac{\gamma(\varphi+\nu)k_1k_2^*z_{j-1}}{\lambda_{g,G}[1-\nu+\theta(1-\gamma)(\varphi+\nu)]}\rho_{\pi*}^j \quad j\geq 1 \quad (51)$$

$$\frac{\partial E_T\left(g_{i,T+j}-g_{T+j}^*\right)}{\partial \pi_T^*}=\frac{(1-\nu+\sigma\varphi+\sigma\nu)k_1k_2\gamma(\varphi+\nu)\sigma}{\left[\gamma^2(\varphi+\nu)^2\sigma^2\left(\lambda_{x,G}+k_1^2k_2^2\right)+\lambda_{g,G}(1-\nu+\sigma\varphi+\sigma\nu)^2\right]}$$

$$\left[\theta(1-\gamma)+\frac{k_1k_2^*\beta}{(k_1^2k_2^{*2}+\lambda_{CB})}\right]\{\frac{e_{j-1}}{\theta(1-\gamma)}-\frac{(1-\nu+\sigma\varphi+\sigma\nu)k_1k_2^*z_{j-1}}{[1-\nu+\theta(1-\gamma)(\varphi+\nu)]\sigma k_1k_2}-\frac{\eta\left(v_{j-1}+k_1k_2^*w_{j-1}\right)}{\theta(1-\gamma)}+\frac{\sigma\eta\left(\lambda_{x,G}+k_1^2k_2^2\right)(\theta-1)(2-\eta)(\varphi+\nu)\left(v_{j-1}+k_1k_2^*w_{j-1}\right)}{(1-\nu+\sigma\varphi+\sigma\nu)k_1k_2\theta}\}\,\rho_{\pi*}^j \qquad j\geq 1 \qquad (52)$$

Therefore, budgetary policies can have an exponential tendency if the shock persistence is too high (ρ_{x*} or $\rho_{\pi*}$>0.35). Nevertheless, with our basic calibration, after a positive demand shock ($x_T^* > 0$), if we suppose that the shock persistence is (ρ_{x*}=0.3), we have the following graph:

Graph 2: Variation of budgetary expenditure after a positive global demand shock



Budgetary policies are then expansionary after a positive global demand shock, in order to compensate for the more contractionary monetary policy: economic policies are conflicting. Besides, these budgetary policies are, obviously, all the more expansionary as the weight given to stabilizing economic activity ($\lambda_{x,G}$) is high, whereas the budgetary policies are less constrained ($\lambda_{g,G}$ is weak). They are also all the more expansionary as the weight of the public sector in the economy (γ) is high.

Furthermore, the national budgetary policy is slightly less expansionary than the global budgetary policy, and this differential is accentuated if the openness of the monetary union (η) is high. More precisely, the national budgetary policy is exactly as expansionary as the global budgetary policy ($g_{i,T} = g_T^*$) if the countries are quite closed (η→0), but it is less expansionary if the member countries of the monetary union are more open (η→1). Indeed, if the countries are very open, the positive foreign demand shock can already contribute to increase national exports and economic activity, and the national budgetary policy can then be less active.

Average economic activity and inflation are then perfectly stabilized by economic policies in all the monetary union [see equation (43)]. However, according to the previous economic policies, the stabilization of economic variables in a given country (i) depends on the openness of the member countries of the monetary union. Indeed, with equations (C3), (C4) and (C5) in Appendix C, we obtain[9]:

$$\frac{\partial x_{i,T}}{\partial x_T^*}=\frac{\sigma\gamma(\varphi+\nu)}{(1-\nu+\sigma\varphi+\sigma\nu)}\frac{\partial\left(g_{i,T}-g_T^*\right)}{\partial x_T^*}=0 \qquad (53)$$

$$\frac{\partial E_T\left(x_{i,T+j}\right)}{\partial x_T^*}=\frac{\sigma\gamma(\varphi+\nu)}{(1-\nu+\sigma\varphi+\sigma\nu)}\frac{\partial\left(g_{i,T+j}-g_{T+j}^*\right)}{\partial x_T^*}+\frac{\sigma}{(1-\nu+\sigma\varphi+\sigma\nu)\theta}\{(\varphi+\nu)\eta(\theta-1)(2-\eta)\left(a_j+k_1k_2b_j-v_{j-1}-k_1k_2^*w_{j-1}\right)-\frac{[(1-\nu)+\theta(1-\gamma)(\varphi+\nu)]}{(1-\gamma)}\left(a_{j-1}+k_1k_2b_{j-1}\right)\}\rho_{x*}^j \qquad j\geq 1 \qquad (54)$$

$$\frac{\partial \pi_{i,T}^i}{\partial x_T^*}=\frac{\sigma k_1k_2\gamma(\varphi+\nu)}{(1-\nu+\sigma\varphi+\sigma\nu)}\frac{\partial\left(g_{i,T}-g_T^*\right)}{\partial x_T^*}=0 \qquad (55)$$

$$\frac{\partial E_T(\pi_{i,T+j}^i)}{\partial x_T^*}=\frac{\sigma k_1k_2\gamma(\varphi+\nu)}{(1-\nu+\sigma\varphi+\sigma\nu)}\frac{\partial\left(g_{i,T+j}-g_{T+j}^*\right)}{\partial x_T^*}+\frac{k_1k_2}{(1-\nu+\sigma\varphi+\sigma\nu)\theta}\{(\varphi+\nu)\eta(\theta-1)(2-\eta)\left(b_j+\sigma d_j-\sigma v_{j-1}-\sigma k_1k_2^*w_{j-1}\right)-\frac{[(1-\nu)+\theta(1-\gamma)(\varphi+\nu)]}{(1-\gamma)}\left(b_{j-1}+\sigma d_{j-1}\right)\}\rho_{x*}^j \qquad j\geq 1 \qquad (56)$$

So, positive global demand shocks imply recessionary and deflationary tensions because of the more contractionary common monetary policy (increase in interest rates). Nevertheless, these tensions can be reduced by expansionary budgetary policies. So,

[9] In the same way, regarding inflationary shocks, we have:

$$\frac{\partial x_{i,T}}{\partial \pi_T^*}=\frac{\sigma\gamma(\varphi+\nu)}{(1-\nu+\sigma\varphi+\sigma\nu)}\frac{\partial\left(g_{i,T}-g_T^*\right)}{\partial \pi_T^*}=0 \qquad \frac{\partial \pi_{i,T}^i}{\partial \pi_T^*}=\frac{\sigma k_1k_2\gamma(\varphi+\nu)}{(1-\nu+\sigma\varphi+\sigma\nu)}\frac{\partial\left(g_{i,T}-g_T^*\right)}{\partial \pi_T^*}$$

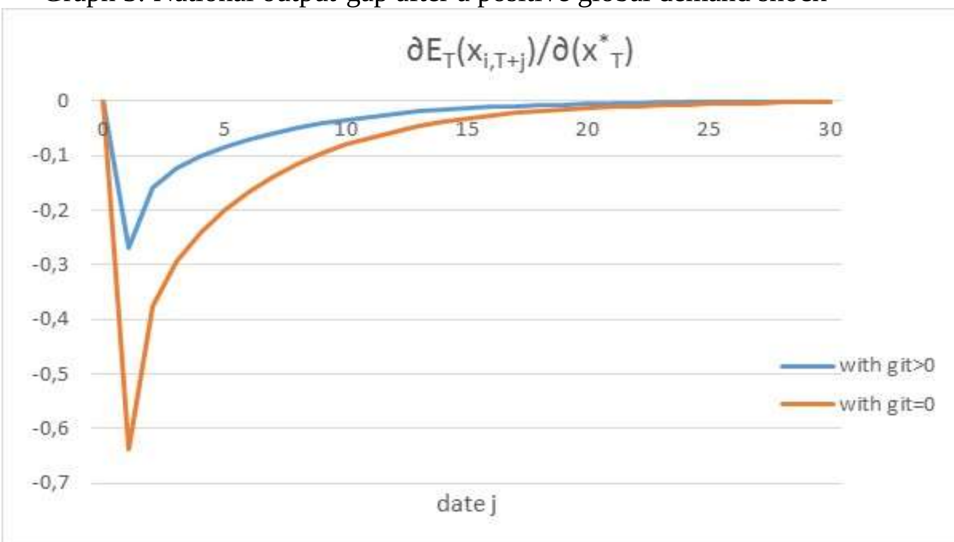
For j≥1: $$\frac{\partial E_T(x_{i,T+j})}{\partial \pi_T^*}=\frac{\sigma\gamma(\varphi+\nu)}{(1-\nu+\sigma\varphi+\sigma\nu)}\frac{\partial\left(g_{i,T+j}-g_{T+j}^*\right)}{\partial \pi_T^*}-\sigma\left[1+\frac{k_1k_2^*\beta}{\theta(1-\gamma)(k_1^2k_2^{*2}+\lambda_{CB})}\right]\left[a_{j-1}+k_1k_2b_{j-1}+\frac{(\varphi+\nu)\eta(\theta-1)(2-\eta)(1-\gamma)\left(v_{j-1}+k_1k_2^*w_{j-1}\right)}{(1-\nu+\sigma\varphi+\sigma\nu)}\right]\rho_{\pi*}^j$$

$$\frac{\partial E_T(\pi_{i,T+j}^i)}{\partial \pi_T^*}=\frac{\sigma k_1k_2\gamma(\varphi+\nu)}{(1-\nu+\sigma\varphi+\sigma\nu)}\frac{\partial\left(g_{i,T+j}-g_{T+j}^*\right)}{\partial \pi_T^*}-k_1k_2\left[1+\frac{k_1k_2^*\beta}{\theta(1-\gamma)(k_1^2k_2^{*2}+\lambda_{CB})}\right]\{b_{j-1}+\sigma d_{j-1}+\frac{\sigma(\varphi+\nu)\eta(\theta-1)(2-\eta)(1-\gamma)\left(v_{j-1}+k_1k_2^*w_{j-1}\right)}{(1-\nu+\sigma\varphi+\sigma\nu)}\}\rho_{\pi*}^j$$

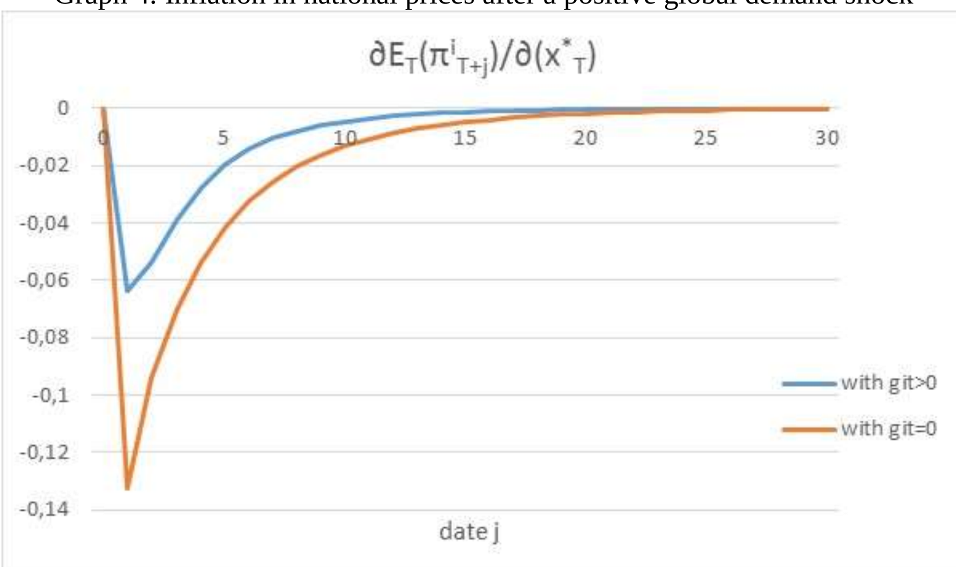
recession and deflation are accentuated if the countries are more closed (η is weak), or if the size of the public sector (γ) in the economy is high. However, obviously, the budgetary policy is all the more efficient in limiting the size of the economic recession and deflation if this policy is less constrained ($\lambda_{g,G}$ is weak) and if stabilizing the output gap ($\lambda_{x,G}$) has the highest weight for the governments. Finally, the persistence of demand shocks (ρ_{x*}) extends the duration of the disequilibrium in economic variables.

Indeed, with our basic calibration, after a positive demand shock ($x_T^* > 0$), if we suppose that the shock persistence is (ρ_{x*}=0.3), we have the following graphs:

Graph 3: National output-gap after a positive global demand shock



Graph 4: Inflation in national prices after a positive global demand shock



4.3 Shocks on productivity or taxation rates

The ZLB is not binding regarding the stabilization of negative productivity ($a_{i,T} < 0$) or positive taxation rates ($t_{i,T} > 0$) shocks in the monetary union. More precisely, if we suppose that productivity or taxation rates shocks in the national country and in all the monetary union are as follows: $a_T^* = \rho_{a*} a_{T-1}^* + \varepsilon_{a*T}$; $a_{i,T} = \rho_{ai} a_{i,T-1} + \varepsilon_{aiT}$; $t_T^* = \rho_{t*} t_{T-1}^* + \varepsilon_{t*T}$; $t_{i,T} = \rho_{ti} t_{i,T-1} + \varepsilon_{tiT}$, where (ε_{a*T}), (ε_{aiT}), (ε_{t*T}) and (ε_{tiT}) are white noises, equation (45) implies the following optimal monetary policy:

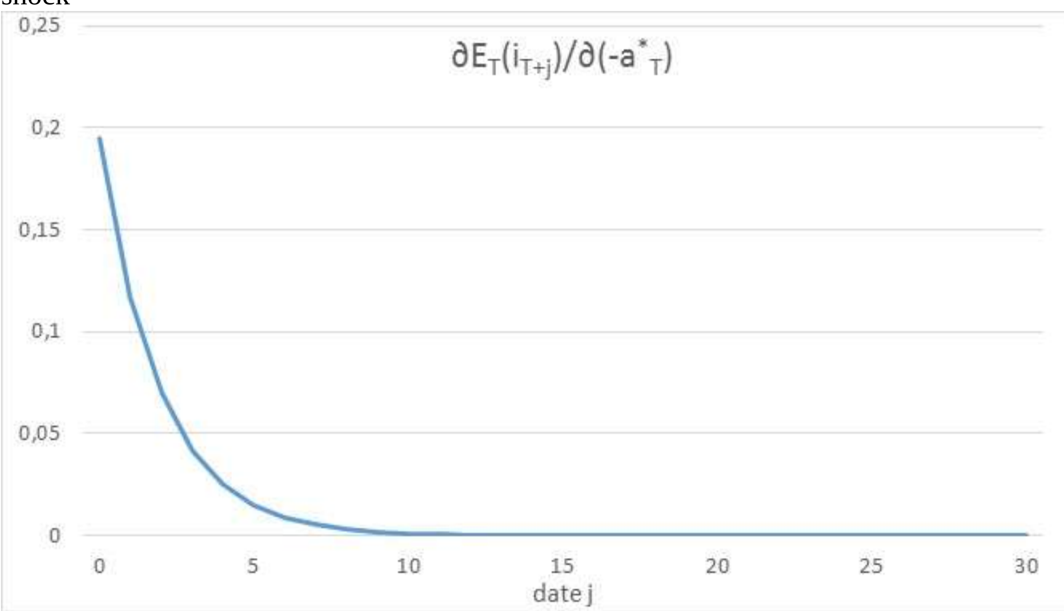
$$\frac{\partial E_T(i_{T+j})}{\partial(-a_T^*)} = \frac{(1+\varphi)\rho_{a*}^j}{[1-\nu+\theta(1-\gamma)(\varphi+\nu)]} \qquad j \geq 0 \qquad (57)$$

$$\frac{\partial E_T(i_{T+j})}{\partial t_T^*} = \frac{(1-\nu)\rho_{t*}^j}{[1-\nu+\theta(1-\gamma)(\varphi+\nu)]} \qquad j \geq 0 \qquad (58)$$

So, monetary policy should be contractionary (increase in interest rates) in order to compensate for a decrease in average productivity or for higher average taxation rates in all the monetary union. Indeed, both imply a temporary decrease of potential output (y_T^{p*}) according to equation (35), which contributes to increase the real average equilibrium interest rate ($\overline{r_T^*}$) and the output-gap according to equation (23). Besides, monetary policy is all the more active as the inter-temporal elasticity of households' expenditure [$\theta(1-\gamma)$] is weak, limiting the efficiency of monetary policy. The persistence of the shocks (ρ_{a*} or ρ_{t*}) mainly contributes to increase the duration of the monetary activism.

So, according to our basic calibration, with a persistence of productivity shocks (ρ_{a*}=0.6), we have the following graph:

Graph 5: Variation of the nominal interest rate after an average negative productivity shock



Budgetary policy is then globally neutral and inactive in the stabilization of average productivity or taxation rates shocks. Equation (C1) in Appendix C implies:

$$\frac{\partial g^*_{T+j}}{\partial a^*_T} = \frac{\partial g^*_{T+j}}{\partial t^*_T} = 0 \quad for\ j \geq 0.$$

Indeed, the common monetary policy can perfectly stabilize these average shocks.

Nevertheless, according to equation (C3) in Appendix C, the national budgetary policy in a given country (i) has then to stabilize idiosyncratic shocks and the asymmetry of its own shocks in comparison with their average values in all the monetary union. In the same way, Gali and Monacelli (2008) find that the policy-mix that is optimal from the viewpoint of the monetary union as a whole requires that inflation be stabilized at the union level by the common central bank, whereas fiscal policies have a neutral fiscal stance in the aggregate, without inflationary pressure at the level of the monetary union. Besides, beyond their role in providing public services, fiscal policies should have a country-specific stabilization role and limit the size of the output gap and inflation differentials resulting from country-specific shocks. So, we obtain:

$$\frac{\partial E_T(g_{i,T+j} - g^*_{T+j})}{\partial(a_{i,T} - \mathrm{a}^*_T)} = \frac{\gamma(\varphi + \nu)\sigma^2\left(\lambda_{x,G} + k_1^2k_2^2\right)(1+\varphi)\rho_{ai}^j}{[\gamma^2(\varphi+\nu)^2\sigma^2\left(\lambda_{x,G} + k_1^2k_2^2\right) + \lambda_{g,G}(1-\nu+\sigma\varphi+\sigma\nu)^2]} \quad j \geq 0 \quad (59)$$

$$\frac{\partial E_T(g_{i,T+j} - g^*_{T+j})}{\partial a^*_T} = \frac{\gamma(\varphi + \nu)\sigma^2\left(\lambda_{x,G} + k_1^2k_2^2\right)(1+\varphi)(\rho_{ai}^j - \rho_{a*}^j)}{[\gamma^2(\varphi+\nu)^2\sigma^2\left(\lambda_{x,G} + k_1^2k_2^2\right) + \lambda_{g,G}(1-\nu+\sigma\varphi+\sigma\nu)^2]} \quad j \geq 1 \quad (60)$$

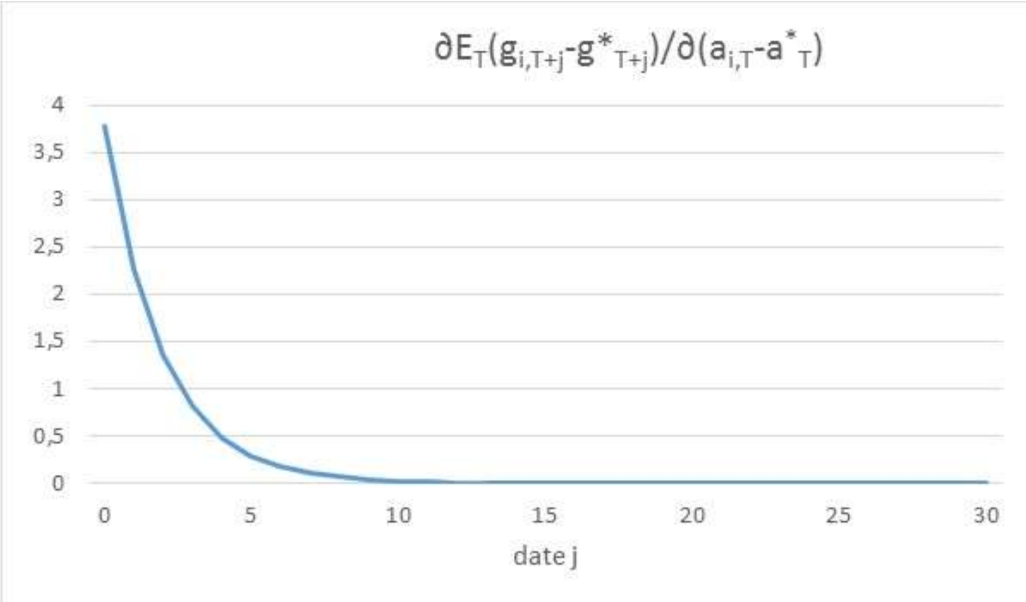
$$\frac{\partial E_T(g_{i,T+j} - g^*_{T+j})}{\partial(\mathrm{t}^*_T - t_{i,T})} = \frac{\gamma(\varphi + \nu)\sigma^2\left(\lambda_{x,G} + k_1^2k_2^2\right)(1-\nu)\rho_{ti}^j}{[\gamma^2(\varphi+\nu)^2\sigma^2\left(\lambda_{x,G} + k_1^2k_2^2\right) + \lambda_{g,G}(1-\nu+\sigma\varphi+\sigma\nu)^2]} \quad j \geq 0 \quad (61)$$

$$\frac{\partial E_T(g_{i,T+j} - g^*_{T+j})}{\partial t^*_T} = \frac{\gamma(\varphi + \nu)\sigma^2\left(\lambda_{x,G} + k_1^2k_2^2\right)(1-\nu)\left(\rho_{t*}^j - \rho_{ti}^j\right)}{[\gamma^2(\varphi+\nu)^2\sigma^2\left(\lambda_{x,G} + k_1^2k_2^2\right) + \lambda_{g,G}(1-\nu+\sigma\varphi+\sigma\nu)^2]} \quad j \geq 1 \quad (62)$$

Therefore, the national budgetary policy should be expansionary if there is a weaker decrease in productivity or if this shock is more persistent than in the rest of the monetary union. Indeed, negative productivity shocks tend to increase the equilibrium real interest rate and the output gap according to equations (22) and (34). In the same way, the national budgetary policy should be expansionary if there is a weaker increase in taxation rates or if this shock is less persistent than in the rest of the monetary union. Besides, obviously, the national budgetary policy is then all the more active as the weight given by the governments to the stabilization of economic activity ($\lambda_{x,G}$) is high, whereas the weight given to the stabilization of budgetary expenditure ($\lambda_{g,G}$) is weak. The national budgetary policy is also all the more active as price flexibility is high (α is small). In case of a productivity shock, like Beetsma and Jensen (2005), we also find that a low elasticity of the labor supply (φ) calls for a more active fiscal policy, because the fluctuations in production effort associated with relative price movements are more costly.

So, according to our basic calibration, with a persistence of productivity shocks (ρ_{ai}=0.6), we have the following graph:

Graph 6: Variation of the national public expenditure after an asymmetric productivity shock



With the previously mentioned economic policies, according to equations (C3), (C4) and (C5) in Appendix C, average productivity shocks[10] are perfectly stabilized in the monetary union, whereas they imply the following levels of economic activity and inflation in a given country (i):

$$\frac{\partial E_T(x_{i,T+j})}{\partial(\mathrm{a}_{i,T}-\mathrm{a}_T^*)} = -\frac{\sigma}{(1-\nu+\sigma\varphi+\sigma\nu)}\left[(1+\varphi)\rho_{ai}^{j} - \gamma(\varphi+\nu)\frac{\partial\left(g_{i,T+j}-g_{T+j}^*\right)}{\partial\left(\mathrm{a}_{i,T}-\mathrm{a}_T^*\right)}\right]$$

$$= -\frac{\sigma\lambda_{g,G}(1-\nu+\sigma\varphi+\sigma\nu)(1+\varphi)\rho_{ai}^{j}}{[\gamma^2(\varphi+\nu)^2\sigma^2\left(\lambda_{x,G}+k_1^2k_2^2\right)+\lambda_{g,G}(1-\nu+\sigma\varphi+\sigma\nu)^2]} \quad j\geq 0 \quad (63)$$

$$\frac{\partial E_T(x_{i,T+j})}{\partial \mathrm{a}_T^*} = -\frac{\sigma}{(1-\nu+\sigma\varphi+\sigma\nu)}\left[(1+\varphi)\left(\rho_{ai}^{j}-\rho_{a*}^{j}\right) - \gamma(\varphi+\nu)\frac{\partial\left(g_{i,T+j}-g_{T+j}^*\right)}{\partial \mathrm{a}_T^*}\right]$$

$$= -\frac{\sigma\lambda_{g,G}(1-\nu+\sigma\varphi+\sigma\nu)(1+\varphi)\left(\rho_{ai}^{j}-\rho_{a*}^{j}\right)}{[\gamma^2(\varphi+\nu)^2\sigma^2\left(\lambda_{x,G}+k_1^2k_2^2\right)+\lambda_{g,G}(1-\nu+\sigma\varphi+\sigma\nu)^2]} \quad j\geq 1 \quad (64)$$

$$\frac{\partial E_T(\pi_{i,T+j}^{i})}{\partial\left(\mathrm{a}_{i,T}-\mathrm{a}_T^*\right)} = k_1k_2\frac{\partial E_T(x_{i,T+j})}{\partial(\mathrm{a}_{i,T}-\mathrm{a}_T^*)} \quad j\geq 0 \qquad \frac{\partial E_T(\pi_{i,T+j}^{i})}{\partial \mathrm{a}_T^*} = k_1k_2\frac{\partial E_T(x_{i,T+j})}{\partial \mathrm{a}_T^*} \quad j\geq 1 \quad (65)$$

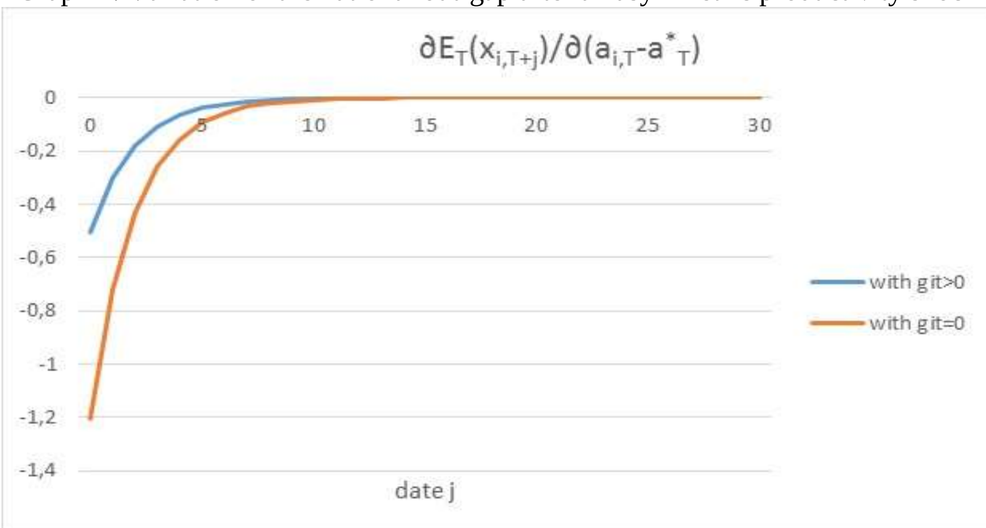
[10] In the same way, regarding taxation rates shocks, we have:

$$k_2\frac{\partial E_T(x_{i,T+j})}{\partial \mathrm{t}_T^*} \quad j\geq 1.$$

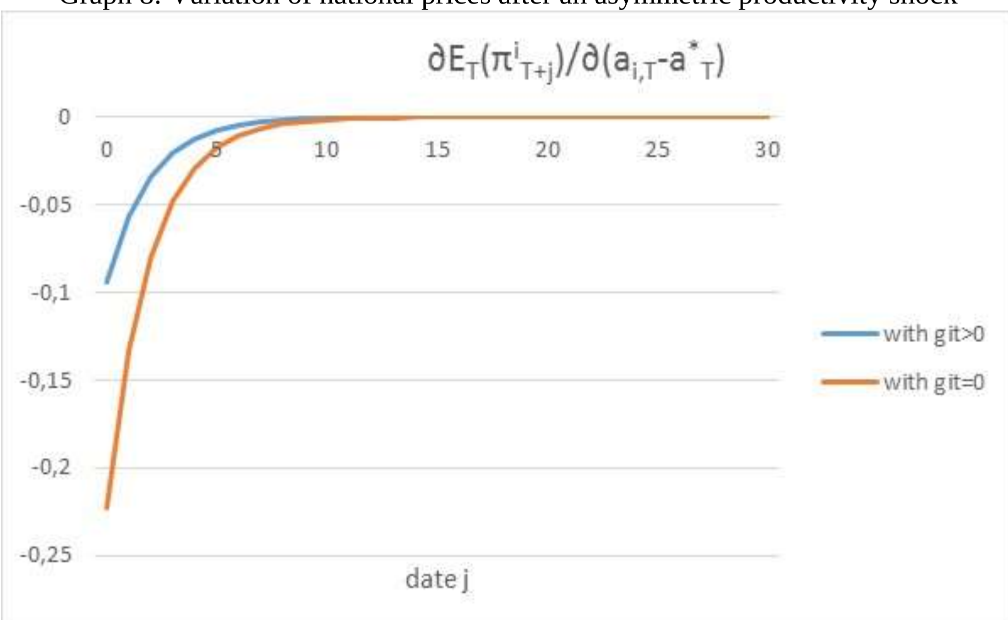
The optimal national budgetary policy can then perfectly stabilize productivity or taxation rates shocks, provided this policy can be active and $(\lambda_{g,G})$ is nearly null. However, a constraint on the use of the budgetary policy, for example in case of an excessive indebtedness level, avoids a perfect stabilization of economic variables. There are then recessionary and deflationary tensions due to the weaker negative productivity shock or weaker increase in taxation rates in the national country.

So, according to our basic calibration, with a persistence of productivity shocks (ρ_{ai}=0.6), we have the following graphs:

Graph 7: Variation of the national out-gap after an asymmetric productivity shock



Graph 8: Variation of national prices after an asymmetric productivity shock



Therefore, economic activity and inflation are better stabilized by an active budgetary policy ($g_{i,t}>0$), but a constraint on the use of this budgetary policy ($\lambda_{g,G} > 0$) avoids a perfect stabilization of national economic variables. Besides, when the budgetary policy is active, economic variables are better stabilized when prices are flexible (α is small), and when the weight of the public sector in the economy (γ) is high.

5 Optimal equilibrium when the ZLB is binding

The aim of the current paper is now to study, in a Zero Lower Bound framework, if budgetary policies can be more useful and are expected to be more active than if the ZLB is not binding. Regarding monetary policy, if the ZLB is binding in a given period (T), and is supposed to be binding for N periods until a period (T+N), in the period (T), we have: ($i_T=0$), ($z_{i1,T} > 0$) as monetary policy is not optimal regarding demand, and therefore, according to equation (39), we have to solve the following system:

$$\begin{cases} z_{i2,T} = -2(\pi_T^* - \pi^{opt}) \\ z_{i1,T} = -2k_1k_2^*(\pi_T^* - \pi^{opt}) - 2\lambda_{CB}(x_T^* - x^{opt}) \\ z_{i3,T} = 2\theta(1-\gamma)[k_1k_2^*(\pi_T^* - \pi^{opt}) + \lambda_{CB}(x_T^* - x^{opt})] \\ x_T^* = E_t(x_{T+1}^*) + \theta(1-\gamma)E_t(\pi_{T+1}^*) + \theta(1-\gamma)\overline{r_T^*} \\ \pi_T^* = \beta E_t(\pi_{T+1}^*) + k_1k_2^*x_T^* \end{cases} \quad (66)$$

Cook and Devereux (2011) construct a model of the international transmission of liquidity trap shocks. Then, they find that in a global environment, fiscal policy may be effective in raising economic activity when the economy is stuck in a liquidity trap, but it does so in a 'beggar thy neighbor' fashion: the cross country spillover effect of fiscal policy is negative and potentially large. Indeed, the national fiscal expansion in a liquidity trap implies a depreciation of the national currency, and a higher demand of national goods to the detriment of foreign goods. Therefore, there is little case for a coordinated global fiscal expansion. For the most part, the country worst hit by a liquidity trap shock should use its own policy to respond, without much help from foreign policies. But what is this optimal budgetary policy?

5.1 Optimal budgetary policies

When the ZLB is binding, we can obtain the following optimal budgetary policy in the country (i) (see Appendix D):

$$g_{i,T} = f\{\overbrace{\pi^{opt}}^{+}, \overbrace{x^{opt}}^{+}, \overbrace{g^{opt}}^{+}, \overbrace{(g_{i,T-1} - g_{T-1}^*)}^{-}, \overbrace{a_{t-1}^*}^{-}, \overbrace{t_{t-1}^*}^{+}, \overbrace{x_{i,T-1}}^{+}, \overbrace{x_{T-1}^*}^{-}, \sum_{n=T+1}^{T+N} \overbrace{E_T(g_{i,n})}^{-},$$

$$\sum_{n=T+1}^{T+N} \overbrace{E_T(g_n^*)}^{+}, \sum_{n=T-1}^{T+N} \overbrace{E_T(a_{i,n})}^{+}, \sum_{n=T+1}^{T+N} \overbrace{E_T(a_n^*)}^{-}, \sum_{n=T-1}^{T+N} \overbrace{E_T(t_{i,n})}^{-}, \sum_{n=T+1}^{T+N} \overbrace{E_T(t_n^*)}^{+},$$

$$\sum_{n=T+1}^{T+N+1} \overbrace{E_T(x_n^*)}^{-}, \overbrace{E_T(\pi_{i,T+N+1}^i)}^{-}, \overbrace{E_T(\pi_{T+N+1}^*)}^{-}, \overbrace{E_T(x_{i,T+N+1})}^{-}, \overbrace{E_T(x_{T+N+1}^*)}^{+}, \overbrace{E_T(g_{i,T+N+1})}^{+},$$

$$\overbrace{E_T(g^*_{T+N+1})}^{-}, \overbrace{E_T\left(a_{i,T+N+1}\right)}^{-}, \overbrace{E_T(a^*_{T+N+1})}^{+}, \overbrace{E_T\left(t_{i,T+N+1}\right)}^{+}, \overbrace{E_T(t^*_{T+N+1})}^{-}\} \quad (67)$$

Therefore, the government of the country (i) smoothes variations of its national public expenditure, as the latter depends on past and future public expenditure. However, when the ZLB is binding, there is no direct dependence of the national policy on the foreign budgetary policy (g^*_T). We can also mention that shocks on households' preferences (β) can no longer be stabilized by the economic authorities.

Furthermore, the values of economic variables at date (T+N+1) of exit of the ZLB in the national country as well as in the rest of the monetary union also influence the current inflation rate and economic activity at date (T); these expectations can sustain current economic growth and inflation (see the precise values in Appendix D). In the same way, postponing the date of exit of the ZLB (increasing N) also sustains current economic activity and inflation. In these conditions, the budgetary policy can then be more contractionary in the country (i). We are now going to analyse the stabilization of various shocks, and the importance of active budgetary policies in this stabilization, when the ZLB is binding.

Eggertsson (2009) believes that the principal goal of policy at zero interest rates should not be to increase aggregate supply by manipulating aggregate supply incentives. Indeed, policies aimed at increasing aggregate supply (for example: reducing labor taxes) would then be counterproductive, because they can create deflationary expectations. Instead, the goal of policy should be to increase aggregate demand: the overall level of spending in the economy. What are the results of our modelling?

5.2 Stabilization of global demand shocks

The ZLB can be binding regarding the stabilization of negative demand shocks reducing the output-gap in all the monetary union ($x^*_T < 0$). In this case, active budgetary policies can be efficient and improve economic stabilization. Indeed, monetary policy is inefficient if the ZLB is binding, because it can only be expansionary by promising future real interest rates smaller than the natural rates and influencing private expectations, which is only possible in case of commitment. On the contrary, the budgetary policy can still be active with its current public expenditure when the ZLB is binding.

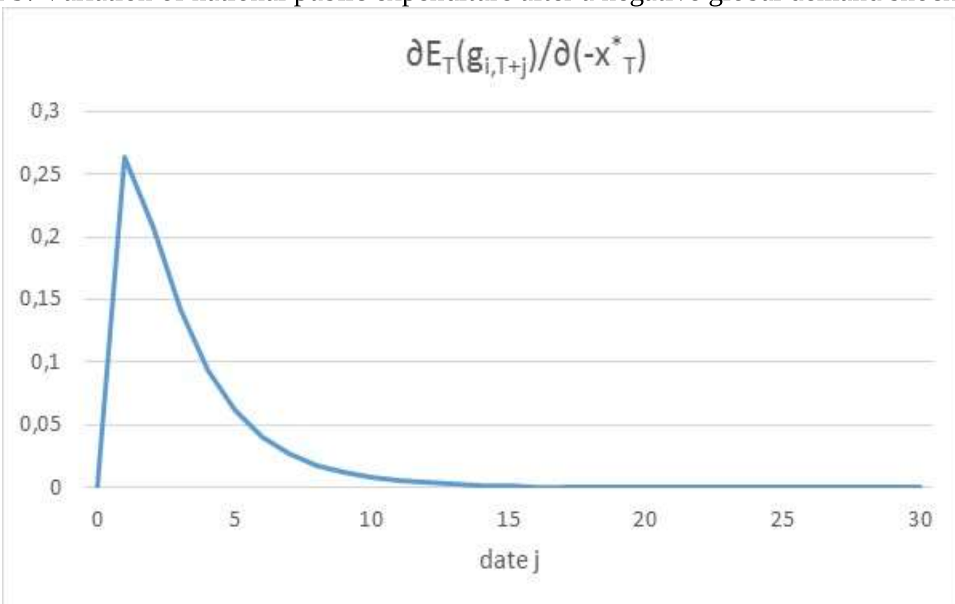
In case of negative demand shocks in all the monetary union, with: $x^*_T = \rho_{x*} x^*_{T-1} + \varepsilon_{xT*}$, we obtain the following relations [see equation (B8) in Appendix B and equation (D2) in Appendix D]:

$$\begin{cases} \dfrac{\partial g_{i,T}}{\partial(-x^*_T)} = \dfrac{\partial g^*_T}{\partial(-x^*_T)} = \dfrac{\partial g^*_{T+j}}{\partial(-x^*_T)} = 0 \qquad \textit{and for } 1 \le j \le N: \\ \dfrac{\partial E_T(g_{i,T+j})}{\partial(-x^*_T)} = \dfrac{\gamma(\varphi+\nu)^2\sigma k_1 k_2 \eta(\theta-1)(2-\eta)\left(e_j - e_{j-1}\right)\rho^j_{x*}}{\theta[\lambda_{g,G}(1-\nu+\sigma\varphi+\sigma\nu)^2 + \gamma^2(\varphi+\nu)^2\sigma^2(\lambda_{x,G}+k_1^2k_2^2)]} > 0 \end{cases} \quad (68)$$

Therefore, the national budgetary policy shouldn't be as active as without ZLB constraint, where it had to compensate for the monetary policy in a framework where economic policies were conflicting. Nevertheless, the optimal national budgetary policy can become expansionary after the first period when the ZLB is binding. This expansionary budgetary policy aims at increasing global demand and at creating future inflationary expectations in the national country. Indeed, public expenditure can rise global demand for national goods, inducing firms to increase prices and their production levels. Besides, as monetary policy cannot reduce its nominal interest rate, inflationary expectations can reduce the real interest rate and contribute to reduce the recessionary consequences of the shock. These expectations can then create a multiplicative effect contributing to sustain output and inflation.

Budgetary policies can have an exponential tendency if the shock persistence is too high (ρ_{x*}>0.45). Nevertheless, with our basic calibration, after a negative demand shock ($x_T^* < 0$), if we suppose (ρ_{x*}=0.3), we have the following graph:

Graph 9: Variation of national public expenditure after a negative global demand shock



Obviously, the budgetary activism depends on the openness of the member countries of the monetary union (η), and the budgetary policy is all the more expansionary and efficient in order to create inflationary expectations as openness is high [at least for η<0.4]. Indeed, the national budgetary policy is inactive and it doesn't vary in case of negative global demand shocks if the member countries of the monetary union are closed (η=0). However, if the countries are open, budgetary authorities should make a fiscal expansion in order to increase the demand for home goods, to boost national inflation and to depreciate the real exchange rate. Nevertheless, we can also mention that the multiplier decreases and is smaller if the openness of the national country is really high (η>0.4). Indeed, an increase in government spending, besides its direct effect on national demand, also appreciates the real exchange rate, if the inflation rate is smaller in the national country than in the rest of the monetary union. Goods produced abroad then become more attractive, which is detrimental to national economic growth, and which should moderate the intensity of the budgetary activism.

The national budgetary policy is also all the more expansionary as the weight given to stabilizing economic activity ($\lambda_{x,G}$) is high, whereas this budgetary policy is less constrained ($\lambda_{g,G}$ is weak). It is all the more expansionary as the weight of the public sector (γ) in the economy is weak, and as the labour supply elasticity (φ), the intertemporal elasticity of private demand (θ) or the returns in the production function (ν) are high. Finally, the persistence in negative demand shocks (ρ_{x*}) increases the duration of the expansionary budgetary policy.

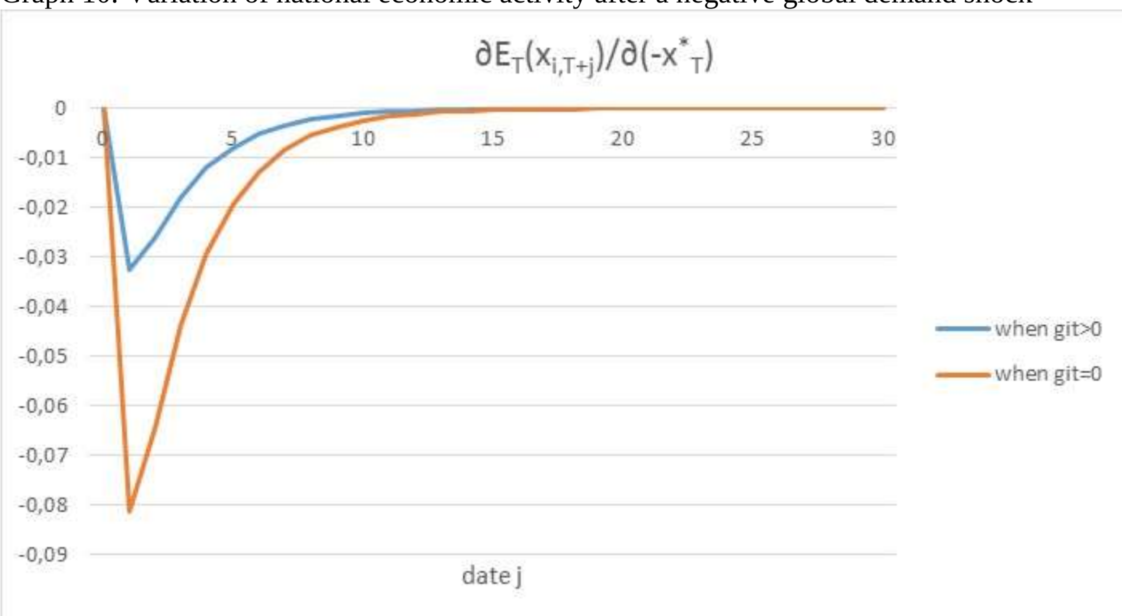
According to the values of optimal output-gap and inflation rate in equations (D3) and (D4) in Appendix D, we obtain, for 1⩽j⩽N:

$$\frac{\partial E_T\left(x_{i,T+j}\right)}{\partial(-x_T^*)} = \frac{\sigma(\varphi+\nu)}{(1-\nu+\sigma\varphi+\sigma\nu)}[\gamma\frac{\partial g_{i,T+j}}{\partial(-x_T^*)} - \frac{\eta(\theta-1)(2-\eta)}{\theta}\left(a_j-a_{j-1}+k_1k_2b_j-k_1k_2b_{j-1}\right)\rho_{x*}^j] \quad (69)$$

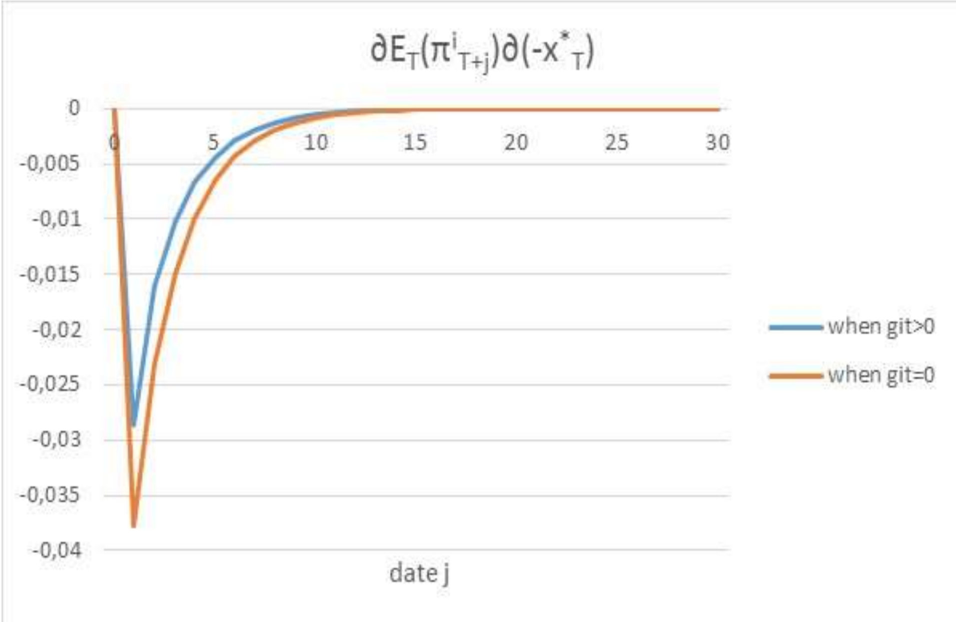
$$\frac{\partial E_T\left(\pi_{i,T+j}^i\right)}{\partial(-x_T^*)} = \frac{k_1k_2(\varphi+\nu)}{(1-\nu+\sigma\varphi+\sigma\nu)}[\sigma\gamma\frac{\partial g_{i,T+j}}{\partial(-x_T^*)} - \frac{\eta(\theta-1)(2-\eta)}{\theta}\left(b_j-b_{j-1}+\sigma d_j-\sigma d_{j-1}\right)\rho_{x*}^j] \quad (70)$$

With the basic calibration of our model, after a negative global demand shock ($x_T^* < 0$), if we suppose (ρ_{x*}=0.3), we have therefore the following graphs:

Graph 10: Variation of national economic activity after a negative global demand shock



Graph 11: Variation of national prices after a negative global demand shock



Therefore, a negative global demand shock implies a decrease of the current economic activity level and deflationary tensions, which cannot be avoided by monetary policy, as the nominal interest rate can no longer decrease if the ZLB is binding. The deflationary situation then becomes independent of current policy actions. If private agents continue to anticipate this decrease of activity and prices, nothing can avoid this deflationary spiral. Besides, recession and deflation are accentuated if the intertemporal elasticity of substitution of household expenditure [$\theta(1-\gamma)$] is high, if the labour supply elasticity (φ) or if the returns in the production function (v) are high.

For likely values of duration of price contracts, the recessionary tensions also increase with the strongest price flexibility (α is weak). Werning (2011) also mentions this counter-intuitive result that price flexibility exacerbates the depression in case of a liquidity trap. Indeed, flexible prices lead to more vigorous deflation, raising the real interest rate, increasing the desire for saving, lowering consumption and demand, reinforcing the recessionary and deflationary pressures, and creating a vicious cycle.

In case of negative global demand shocks, if the monetary policy is constrained by the ZLB and inefficient, an expansionary national budgetary policy can then improve the stabilization of economic variables, in comparison with the case where this budgetary policy cannot be active. Besides, this budgetary policy is more efficient in order to reduce the recessionary tensions if the budgetary policy is active ($\lambda_{x,G}$ is high and $\lambda_{g,G}$ is weak), and if the weight of the public sector in the economy (γ) is high.

Furthermore, whereas the stabilization of economic variables could be perfect if the countries were closed ($\eta \rightarrow 0$) whatever the budgetary policy, the disequilibria in economic variables increase with the openness of the member countries of the monetary union. Indeed, in case of negative global demand shocks, if monetary policy is constrained by the ZLB, the limited decrease of the nominal interest rate cannot avoid the appreciation of the real exchange rate due to the weaker inflation rate in the national country. This effect is accentuated if the countries are very open (high trade elasticity), as the demand for home goods is then stronger and the recessionary tensions accentuated.

5.3 Shocks on productivity or taxation rates

We suppose that productivity or taxation rates shocks in the national country and in all the monetary union are AR(1) processes, as mentioned in section 4.3. The ZLB can be binding and monetary policy inefficient (the interest rate can no longer decrease) in case of a positive average productivity shock ($a_T^* > 0$), or of a negative average taxation rates shock ($t_T^* < 0$) in all the monetary union.

In this context, the stabilization of asymmetric shocks, and of the differential between national productivity and taxation rates shocks and their average values in all the monetary union, by the national country is exactly the same as the one mentioned in section 4.3 when the ZLB is not binding. However, the difference is that when the ZLB is binding, monetary policy can no longer be efficient in stabilizing average productivity or taxation rates shocks in all the monetary union. Therefore, beyond the stabilization of asymmetric shocks, the national budgetary policy must also stabilize these average productivity or taxation rates shocks. Indeed, we have (see Appendix D):

$$\frac{\partial g_T^*}{\partial a_T^*} = \frac{\gamma(\varphi+\nu)\theta^2(1-\gamma)^2\left(\lambda_{x,G}+k_1^2k_2^{*2}\right)(1+\varphi)\rho_{a*}^j}{\left\{\theta^2(1-\gamma)^2\gamma^2(\varphi+\nu)^2\left(\lambda_{x,G}+k_1^2k_2^{*2}\right)+\lambda_{g,G}[1-\nu+\theta(1-\gamma)(\varphi+\nu)]^2\right\}} \qquad j \geq 0 \quad (71)$$

$$\frac{\partial g_{i,T+j}}{\partial(a_{i,T}-a_T^*)} = \frac{\gamma(\varphi+\nu)\sigma^2(\lambda_{x,G}+k_1^2k_2^2)(1+\varphi)\rho_{ai}^j}{[\lambda_{g,G}(1-\nu+\sigma\varphi+\sigma\nu)^2+\gamma^2(\varphi+\nu)^2\sigma^2(\lambda_{x,G}+k_1^2k_2^2)]} = \frac{\partial g_{i,T+j}}{\partial a_T^*} \qquad j \geq 0 \quad (72)$$

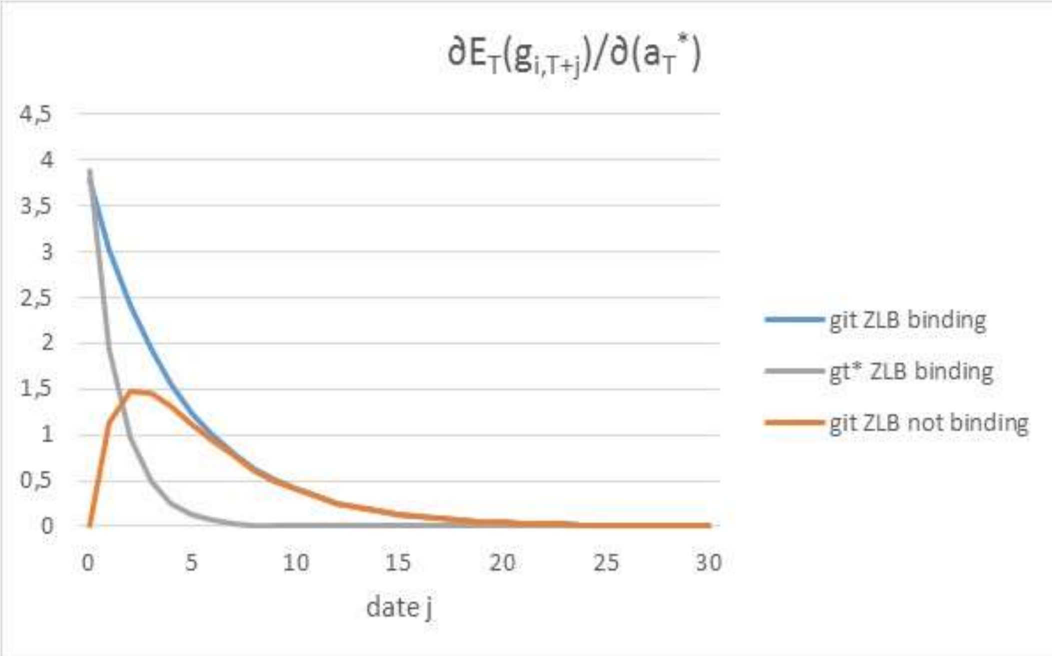
$$\frac{\partial g_T^*}{\partial(-t_T^*)} = \frac{\gamma(\varphi+\nu)\theta^2(1-\gamma)^2\left(\lambda_{x,G}+k_1^2k_2^{*2}\right)(1-\nu)\rho_{t*}^j}{\left\{\theta^2(1-\gamma)^2\gamma^2(\varphi+\nu)^2\left(\lambda_{x,G}+k_1^2k_2^{*2}\right)+\lambda_{g,G}[1-\nu+\theta(1-\gamma)(\varphi+\nu)]^2\right\}} \qquad j \geq 0 \quad (73)$$

$$\frac{\partial g_{i,T+j}}{\partial\left(t_T^*-t_{i,T}\right)} = \frac{\gamma(\varphi+\nu)\sigma^2\left(\lambda_{x,G}+k_1^2k_2^2\right)(1-\nu)\rho_{ti}^j}{\left[\lambda_{g,G}(1-\nu+\sigma\varphi+\sigma\nu)^2+\gamma^2(\varphi+\nu)^2\sigma^2\left(\lambda_{x,G}+k_1^2k_2^2\right)\right]} = \frac{\partial g_{i,T+j}}{\partial(-t_T^*)} \qquad j \geq 0 \quad (74)$$

The national and global budgetary policies should then be expansionary in case of a positive average productivity shock ($a_T^* > 0$), or in case of a decrease of average taxation rates ($t_T^* < 0$) in all the monetary union. This is the case even if the shock persistence is identical in all member countries of the monetary union, contrary to the framework where the ZLB isn't binding (see section 4.3). Besides, obviously, budgetary policies are more active if the weight given by the governments to the stabilization of economic activity ($\lambda_{x,G}$) is high whereas the weight given to the stabilization of budgetary expenditure ($\lambda_{g,G}$) is weak. Budgetary policies are also all more active as the intertemporal elasticity of substitution (θ) or as the weight of the public sector (γ) are high, or as price flexibility is high (α is weak). Finally, the budgetary expenditure remains highest for a longer time if the shock persistence increases.

So, according to our basic calibration, with persistence of productivity shocks (ρ_{a*}=0.5 and ρ_{ai} =0.8), we have the following graph:

Graph 12: Variation of public expenditure after an average productivity shock



The stabilization of economic variables in then exactly the same as the one obtained when the ZLB is not binding (see section 4.3) in case of asymmetric productivity $(\mathrm{a}_{i,T}-\mathrm{a}_T^*)$ or taxation rates $(\mathrm{t}_{i,T}-\mathrm{t}_T^*)$ shocks. These shocks can perfectly be stabilized by the national budgetary policy provided the latter can be active and is not constrained. However, regarding average shocks in all the monetary union, we have the following situation (see Appendix D)[11]:

$$\frac{\partial x_{i,T+j}}{\partial \mathrm{a}_T^*}=-\frac{\sigma}{(1-\nu+\sigma\varphi+\sigma\nu)}\left[(1+\varphi)\rho_{ai}^j-\gamma(\varphi+\nu)\frac{\partial g_{i,T+j}}{\partial \mathrm{a}_T^*}\right]$$
$$=-\frac{\sigma(1+\varphi)\lambda_{g,G}(1-\nu+\sigma\varphi+\sigma\nu)\rho_{ai}^j}{\left[\lambda_{g,G}(1-\nu+\sigma\varphi+\sigma\nu)^2+\gamma^2(\varphi+\nu)^2\sigma^2\left(\lambda_{x,G}+k_1^2k_2^2\right)\right]}\quad j\geq 0\quad (75)$$

$$\frac{\partial x_{i,T+j}}{\partial(\mathrm{a}_{i,T}-\mathrm{a}_T^*)}=-\frac{\sigma}{(1-\nu+\sigma\varphi+\sigma\nu)}\left[(1+\varphi)\rho_{ai}^j-\gamma(\varphi+\nu)\frac{\partial g_{i,T+j}}{\partial\left(\mathrm{a}_{i,T}-\mathrm{a}_T^*\right)}\right]$$
$$=-\frac{\sigma(1+\varphi)\lambda_{g,G}(1-\nu+\sigma\varphi+\sigma\nu)\rho_{ai}^j}{\left[\lambda_{g,G}(1-\nu+\sigma\varphi+\sigma\nu)^2+\gamma^2(\varphi+\nu)^2\sigma^2\left(\lambda_{x,G}+k_1^2k_2^2\right)\right]}\quad j\geq 0\quad (76)$$

[11] In the same way, regarding taxation rates shocks, we have:

$$\frac{\partial x_{i,T+j}}{\partial \mathrm{t}_T^*}=\frac{\sigma}{(1-\nu+\sigma\varphi+\sigma\nu)}\left[(1-\nu)\rho_{ti}^j+\gamma(\varphi+\nu)\frac{\partial g_{i,T+j}}{\partial \mathrm{t}_T^*}\right]\quad j\geq 0$$

$$\frac{\partial x_{i,T+j}}{\partial(\mathrm{t}_{i,T}-\mathrm{t}_T^*)}=\frac{\sigma}{(1-\nu+\sigma\varphi+\sigma\nu)}\left[(1-\nu)\rho_{ti}^j+\gamma(\varphi+\nu)\frac{\partial g_{i,T+j}}{\partial\left(\mathrm{t}_{i,T}-\mathrm{t}_T^*\right)}\right]\quad j\geq 0$$

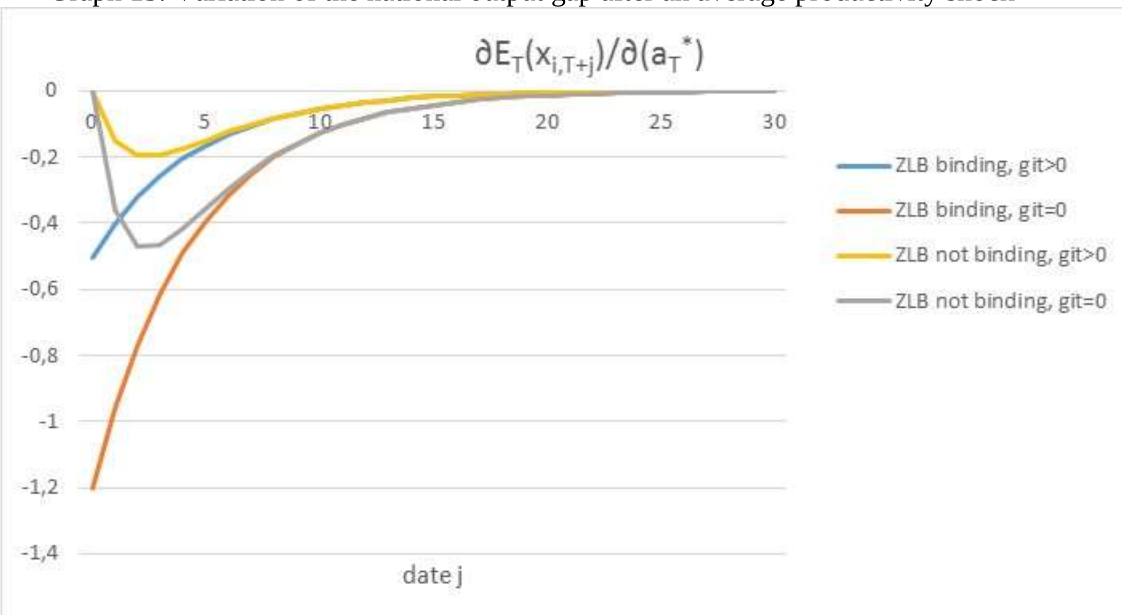
$$\frac{\partial \pi_{i,T+j}^i}{\partial \mathrm{t}_T^*}=k_1k_2\frac{\partial x_{i,T+j}}{\partial \mathrm{t}_T^*}\quad j\geq 0\qquad \frac{\partial \pi_{i,T+j}^i}{\partial\left(\mathrm{t}_{i,T}-\mathrm{t}_T^*\right)}\quad j\geq 0$$

$$\frac{\partial\pi_{i,T+j}^{i}}{\partial a_{T}^{*}}=k_1k_2\frac{\partial x_{i,T+j}}{\partial a_{T}^{*}} \quad j\geq 0 \qquad \frac{\partial\pi_{i,T+j}^{i}}{\partial(a_{i,T}-a_{T}^{*})}=k_1k_2\frac{\partial x_{i,T+j}}{\partial(a_{i,T}-a_{T}^{*})} \quad j\geq 0 \quad (77)$$

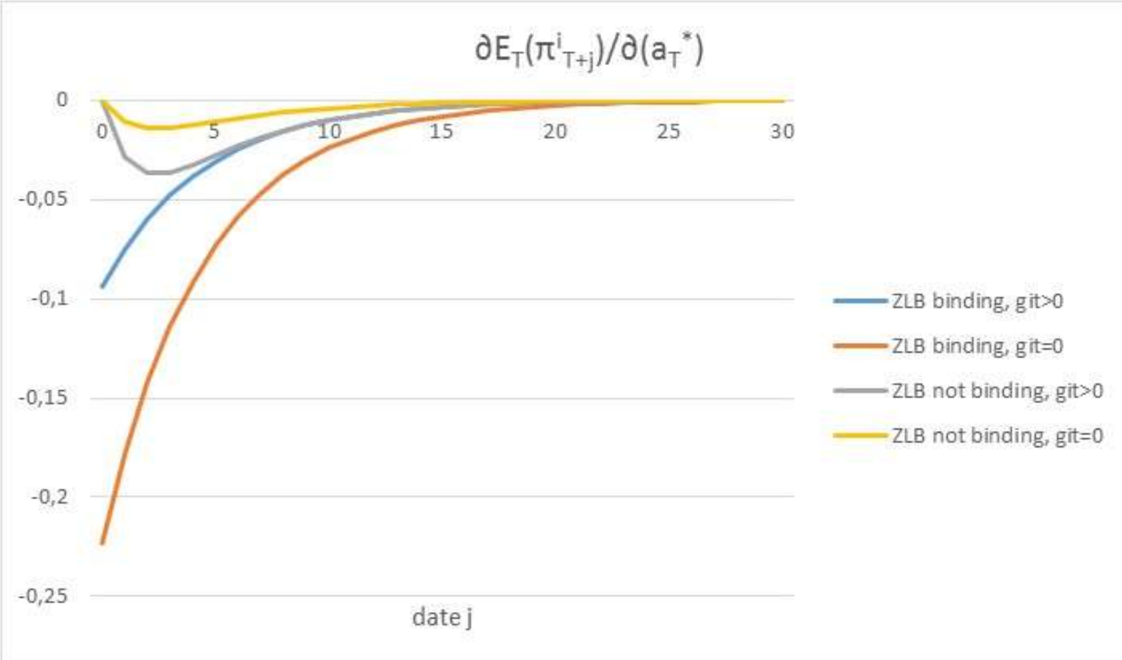
Therefore, when the budgetary policies are constrained ($\lambda_{g,G}>0$), and when the ZLB is binding, a positive average productivity shock or a decrease of average taxation rates in all the monetary union imply recessionary and deflationary tensions. The latter exist even if the shock persistence is the same in the national country and in the rest of the monetary union, contrary to the framework where the ZLB isn't binding.

So, according to our basic calibration, with persistence of productivity shocks (ρ_{a*}=0.5 and ρ_{ai} =0.8), we have the following graphs:

Graph 13: Variation of the national output gap after an average productivity shock



Graph 14: Variation of national prices after an average productivity shock



Besides, recession and deflation are more limited if the weight given by the governments to the stabilization of economic activity ($\lambda_{x,G}$) is high, whereas the weight given to the stabilization of budgetary expenditure ($\lambda_{g,G}$) is weak. So, when the ZLB is binding, budgetary policy can considerably reduce the deflationary and recessionary tensions. Nevertheless, the cost of this budgetary policy is that public expenditure deviates from its optimal level (g^{opt}), which can worsen the problems of indebtedness of the national country. For example, Benigno and Woodford (2003) consider a model where the only sources of fiscal revenue are distortionary taxes, and a framework with price stickiness, where the ZLB never binds. Then, they show that the monetary authority should take into account the consequences of its actions for the government budget: the implications of inflation and interest rates paths for the real burden on the public debt.

Furthermore, economic stabilization is also improved if the weight of the public sector (γ) in the economy is high, or if price flexibility is high (α is weak). Finally, the disequilibrium remains highest for a longer time if the shock persistence increases.

6 Conclusion

In conclusion, the results of our paper underline the usefulness of active budgetary policies and of an efficient policy-mix in order to stabilize economic activity and inflation, in particular when the Zero Lower Bound is binding.

In case of global demand shocks, when the ZLB is not binding, economic policies can be conflicting, and the budgetary policies then have to be active in order to compensate for the monetary policy. On the contrary, when the ZLB is binding, budgetary policies have another usefulness: in case of negative global demand shocks, when the countries are open, they aim at creating higher inflationary expectations sustaining demand in the national country, and at limiting the recessionary and deflationary consequences of the shock.

Besides, in case of productivity or taxation rates shocks, monetary policy can perfectly stabilize average variables in all the monetary union when the Zero Lower Bound is not binding. So, the national budgetary policy should only stabilize asymmetric shocks and the differential of national productivity or taxation rates shocks with their average values in all the monetary union. Therefore, the national budgetary policy should be expansionary if there is a weaker negative productivity shock or if the latter is more persistent than in the rest of the monetary union; it should also be expansionary if there is a smaller increase in taxation rates or if the latter shock is less persistent than in the rest of the monetary union. The national budgetary policy could then perfectly stabilize asymmetric shocks, provided it can be sufficiently active and it is not constrained by an excessive indebtedness level, for example.

On the contrary, when the ZLB is binding, monetary policy becomes inefficient to stabilize positive average productivity shocks or declines in average taxation rates in all the monetary union. Indeed, the common nominal interest rate should be reduced, whereas it is already nearly null. Budgetary policies should then be expansionary, in order to reduce the recessionary and deflationary tensions due to the shock. Therefore, the national budgetary policy should be more active, in order to stabilize not only differentials in the persistence of shocks between the national country and the rest of the monetary union, but also in order to stabilize average global shocks.

So, the budgetary policy can be useful in order to stabilize productivity or taxation rates shocks if the ZLB is binding and if the monetary policy becomes less efficient in order to stabilize average variables in all the monetary union. Nevertheless, future studies should also introduce more precisely the importance of the indebtedness level of the national country in the modelling. Indeed, if the ZLB is binding, an excessive indebtedness level could limit the usefulness and the efficiency of the budgetary policy in order to stabilize economic variables.

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Appendix A: Optimal national budgetary policy

In the country (i) and in period (T), using equations (22), (34) and (37), the government chooses a path for its public expenditure minimizing the following loss function:

$$\mathcal{L}_{i,T} = E_T \sum_{t=T}^{\infty} \beta^t \{[(\pi_{i,t} - \pi^{opt})^2 + \lambda_{x,G}(x_{i,t} - x^{opt})^2 + \lambda_{g,G}(g_{i,t} - g^{opt})^2] + z_{i4,t}\left[x_{i,t} - x_{i,t+1} + \sigma\left(i_t - \pi^i_{i,t+1} - \overline{r_{i,t}}\right)\right] + z_{i5,t}\left[\pi^i_{i,t} - \beta\pi^i_{i,t+1} - k_1 k_2 x_{i,t}\right] \quad (A1)$$

Moreover, equations (13), (20), (34) for ($y^p_{i,t}$) and (k_2), and (35) for (y^{p*}_t) and (k^*_2) imply:

$$\pi_{i,t} = \pi^i_{i,t} + \frac{\eta}{\sigma}\left(x_{i,t} - x^*_t - x_{i,t-1} + x^*_{t-1}\right) + \frac{\eta^2(\theta-1)(2-\eta)(1-\nu)}{\sigma\theta(1-\nu+\sigma\varphi+\sigma\nu)}(x^*_t - x^*_{t-1}) + \frac{\eta}{(1-\nu+\sigma\varphi+\sigma\nu)}[(1+\varphi)\left(a_{i,t} - a^*_t - a_{i,t-1} + a^*_{t-1}\right) - (1-\nu)\left(t_{i,t} - t^*_t - t_{i,t-1} + t^*_{t-1}\right) - \gamma(\varphi+\nu)\left(g_{i,t} - g^*_t - g_{i,t-1} + g^*_{t-1}\right)] \qquad (A2)$$

Regarding the equilibrium interest rate, equations (22) and (34) imply:

$$\overline{r_{i,t}} = \frac{(1-\beta)}{\beta} + \frac{\gamma(\varphi+\nu)}{(1-\nu+\sigma\varphi+\sigma\nu)}\left[g_{i,t} - E_T\left(g_{i,t+1}\right)\right] - \frac{1}{(1-\nu+\sigma\varphi+\sigma\nu)}\{(1+\varphi)\left[a_{i,t} - E_t\left(a_{i,t+1}\right)\right] - (1-\nu)\left[t_{i,t} - E_t\left(t_{i,t+1}\right)\right]\} + \frac{(\varphi+\nu)\eta(\theta-1)(2-\eta)(1-\gamma)}{(1-\nu+\sigma\varphi+\sigma\nu)[1-\nu+\theta(1-\gamma)(\varphi+\nu)]}\{(1+\varphi)[a^*_t - E_t(a^*_{t+1})] - (1-\nu)[t^*_t - E_t(t^*_{t+1})] - \gamma(\varphi+\nu)[g^*_t - E_t(g^*_{t+1})]\} + \frac{(\varphi+\nu)\eta(\theta-1)(2-\eta)}{(1-\nu+\sigma\varphi+\sigma\nu)\theta}[x^*_t - E_t(x^*_{t+1})] \qquad (A3)$$

So, optimal first order conditions of equation (A1) for a given period (T) are as follows:

$$\begin{cases} \dfrac{\partial \mathcal{L}_{i,T}}{\partial \pi^i_{i,T}} = 2\left(\pi_{i,T} - \pi^{opt}\right) + z_{i5,T} = 0 \\ \dfrac{\partial \mathcal{L}_{i,T}}{\partial x_{i,T}} = 2\left(\pi_{i,T} - \pi^{opt}\right)\dfrac{\eta}{\sigma} + 2\lambda_{x,G}\left(x_{i,T} - x^{opt}\right) + z_{i4,T} - k_1 k_2 z_{i5,T} = 0 \\ \dfrac{\partial \mathcal{L}_{i,T}}{\partial g_{i,T}} = 2\left(\pi_{i,T} - \pi^{opt}\right)\dfrac{\partial \pi_{i,T}}{\partial g_{i,T}} + 2\lambda_{g,G}(g_{i,T} - g^{opt}) - z_{i4,T}\sigma\dfrac{\partial \overline{r_{i,T}}}{\partial g_{i,T}} = 0 \\ \left\{x_{i,T} - E_t\left(x_{i,T+1}\right) + \sigma\left[i_T - E_t\left(\pi^i_{i,T+1}\right) - \overline{r_{i,T}}\right]\right\} z_{i4,T} = 0 \\ \left\{\pi^i_{i,T} - \beta E_t\left(\pi^i_{i,T+1}\right) - k_1 k_2 x_{i,T}\right\} z_{i5,T} = 0 \end{cases} \qquad (A4)$$

The two first equations of the system (A4) imply:

$$z_{i4,T} = -2\left(\frac{\eta}{\sigma} + k_1 k_2\right)\left(\pi_{i,T} - \pi^{opt}\right) - 2\lambda_{x,G}\left(x_{i,T} - x^{opt}\right)$$

Besides, according to the values for ($\pi_{i,T}$) in (A2) and for ($\overline{r_{\iota,T}}$) in (A3), the third equation of the system (A4) implies:

$$g_{i,T} = g^{opt} + \frac{\gamma(\varphi+\nu)\eta}{\lambda_{g,G}(1-\nu+\sigma\varphi+\sigma\nu)}\left(\pi_{i,T} - \pi^{opt}\right) + \frac{\sigma\gamma(\varphi+\nu)}{2\lambda_{g,G}(1-\nu+\sigma\varphi+\sigma\nu)} z_{i4,T}$$
$$= g^{opt} - \frac{\gamma(\varphi+\nu)\sigma}{\lambda_{g,G}(1-\nu+\sigma\varphi+\sigma\nu)}\left[k_1 k_2\left(\pi_{i,T} - \pi^{opt}\right) + \lambda_{x,G}\left(x_{i,T} - x^{opt}\right)\right] \qquad (A5)$$

By combining equations (A2) and (A5), we have then the following level of national public expenditure:

$$\left[1 - \frac{\lambda_{g,G}(1-\nu+\sigma\varphi+\sigma\nu)^2}{\eta\gamma^2(\varphi+\nu)^2\sigma k_1 k_2}\right] g_{i,T}$$
$$= g_{i,T-1} - \frac{\lambda_{g,G}(1-\nu+\sigma\varphi+\sigma\nu)^2}{\eta\gamma^2(\varphi+\nu)^2\sigma k_1 k_2} g^{opt} + \left(g_T^* - g_{T-1}^*\right)$$
$$- \frac{(1-\nu+\sigma\varphi+\sigma\nu)}{\eta\gamma(\varphi+\nu)k_1 k_2}\left(k_1 k_2 \pi^{opt} + \lambda_{x,G} x^{opt}\right) + \frac{(1-\nu+\sigma\varphi+\sigma\nu)}{\eta\gamma(\varphi+\nu)} \pi_{i,T}^i$$
$$+ \frac{1}{\gamma(\varphi+\nu)}\left[(1+\varphi)\left(\mathrm{a}_{i,T} - \mathrm{a}_T^* - \mathrm{a}_{i,T-1} + \mathrm{a}_{T-1}^*\right)\right.$$
$$\left. - (1-\nu)\left(t_{i,T} - t_T^* - t_{i,T-1} + t_{T-1}^*\right)\right]$$
$$- \frac{(1-\nu+\sigma\varphi+\sigma\nu)}{\gamma(\varphi+\nu)\sigma} x_{i,T-1} + \frac{(1-\nu+\sigma\varphi+\sigma\nu)(\sigma\lambda_{x,G} + \eta k_1 k_2)}{\eta\gamma(\varphi+\nu)\sigma k_1 k_2} x_{i,T}$$
$$- \frac{[1-\nu+\theta(1-\gamma)(\varphi+\nu)]}{(1-\gamma)\theta\gamma(\varphi+\nu)}\left(x_T^* - x_{T-1}^*\right) \qquad (A6)$$

The values for ($\pi_{i,T}^i$) and ($x_{i,T}$) are defined by the two last equations of the system (A4), which imply: $\pi_{i,T}^i = (\beta + k_1 k_2 \sigma)E_t\left(\pi_{i,T+1}^i\right) + k_1 k_2 E_t\left(x_{i,T+1}\right) - k_1 k_2 \sigma\left(i_T - \overline{r_{\iota,T}}\right)$.

Therefore, the system to solve is the following:

$$\begin{pmatrix} x_{i,T} \\ \pi_{i,T}^i \end{pmatrix} = \begin{pmatrix} 1 & \sigma \\ k_1 k_2 & (\beta + \sigma k_1 k_2) \end{pmatrix} \begin{pmatrix} E_T\left(x_{i,T+1}\right) \\ E_T\left(\pi_{i,T+1}^i\right) \end{pmatrix} - \sigma \begin{pmatrix} 1 \\ k_1 k_2 \end{pmatrix}\left(i_T - \overline{r_{\iota,T}}\right) \qquad (A7)$$
$$\begin{pmatrix} x_{i,T} \\ \pi_{i,T}^i \end{pmatrix} = A^{N+1} \begin{pmatrix} E_T\left(x_{i,T+N+1}\right) \\ E_T\left(\pi_{i,T+N+1}^i\right) \end{pmatrix} - \sigma \sum_{n=T}^{T+N} A^{n-T} \begin{pmatrix} 1 \\ k_1 k_2 \end{pmatrix}\left(i_n - \overline{r_{\iota,n}}\right)$$

We have then to find the solution of the following matrix:

$$A^n = \begin{pmatrix} 1 & \sigma \\ k_1 k_2 & (\beta + \sigma k_1 k_2) \end{pmatrix}^n = \begin{pmatrix} a_n & b_n \\ c_n & d_n \end{pmatrix}$$

with: $u_n = (1 + \beta + \sigma k_1 k_2)u_{n-1} - \beta u_{n-2}$ as characteristic equation of this matrix. Besides, we obtain the following economic variables:

$$x_{i,T} = a_{N+1}E_T(x_{i,T+N+1}) + b_{N+1}E_T(\pi^i_{i,T+N+1}) - \sigma \sum_{n=T}^{T+N} (a_{n-T} + k_1 k_2 b_{n-T})(i_n - \overline{r_{i,n}})$$

$$\pi^i_{i,T} = c_{N+1}E_T(x_{i,T+N+1}) + d_{N+1}E_T(\pi^i_{i,T+N+1}) - \sigma \sum_{n=T}^{T+N} (c_{n-T} + k_1 k_2 d_{n-T})(i_n - \overline{r_{i,n}}) \quad (A8)$$

$$r^2 - (1 + \beta + \sigma k_1 k_2) r + \beta = 0$$

The two solutions of this equation are the following:

$$r_1 = \frac{(1 + \beta + \sigma k_1 k_2)}{2} + \frac{\sqrt{(1 + \beta + \sigma k_1 k_2)^2 - 4\beta}}{2}$$

$$r_2 = \frac{(1 + \beta + \sigma k_1 k_2)}{2} - \frac{\sqrt{(1 + \beta + \sigma k_1 k_2)^2 - 4\beta}}{2}$$

So, solutions of the above recurrent sequence take the following form: $u_n = (x) r_1^n + (y) r_2^n$.

$$a_0 = a_1 = 1$$

$$a_n = \left[\frac{(1 - \beta - \sigma k_1 k_2) + \sqrt{(1 + \beta + \sigma k_1 k_2)^2 - 4\beta}}{2\sqrt{(1 + \beta + \sigma k_1 k_2)^2 - 4\beta}}\right]\left[\frac{(1 + \beta + \sigma k_1 k_2)}{2} + \frac{\sqrt{(1 + \beta + \sigma k_1 k_2)^2 - 4\beta}}{2}\right]^n + \left[\frac{(-1 + \beta + \sigma k_1 k_2) + \sqrt{(1 + \beta + \sigma k_1 k_2)^2 - 4\beta}}{2\sqrt{(1 + \beta + \sigma k_1 k_2)^2 - 4\beta}}\right]\left[\frac{(1 + \beta + \sigma k_1 k_2)}{2} - \frac{\sqrt{(1 + \beta + \sigma k_1 k_2)^2 - 4\beta}}{2}\right]^n$$

$$b_0 = 0 \quad b_1 = \sigma \qquad b_n = \frac{\sigma}{\sqrt{(1 + \beta + \sigma k_1 k_2)^2 - 4\beta}} (r_1^n - r_2^n)$$

$$c_n = \frac{k_1 k_2}{\sigma} b_n$$

$$d_0 = 1 \qquad d_1 = \beta + \sigma k_1 k_2$$

$$d_n = \left[\frac{(-1 + \beta + \sigma k_1 k_2) + \sqrt{(1 + \beta + \sigma k_1 k_2)^2 - 4\beta}}{2\sqrt{(1 + \beta + \sigma k_1 k_2)^2 - 4\beta}}\right]\left[\frac{(1 + \beta + \sigma k_1 k_2)}{2} + \frac{\sqrt{(1 + \beta + \sigma k_1 k_2)^2 - 4\beta}}{2}\right]^n + \left[\frac{(1 - \beta - \sigma k_1 k_2) + \sqrt{(1 + \beta + \sigma k_1 k_2)^2 - 4\beta}}{2\sqrt{(1 + \beta + \sigma k_1 k_2)^2 - 4\beta}}\right]\left[\frac{(1 + \beta + \sigma k_1 k_2)}{2} - \frac{\sqrt{(1 + \beta + \sigma k_1 k_2)^2 - 4\beta}}{2}\right]^n$$

Finally, we can put the expressions of ($x_{i,T}$) and ($\pi^i_{i,T}$) obtained in equations (A8), and the expression of ($\overline{r_{i,n}}$) in equation (A3), in the value of ($g_{i,T}$) in equation (A6). So, we have:

$$\left[1+\frac{\gamma^2(\varphi+\nu)^2\sigma^2(\lambda_{x,G}+k_1^2k_2^2)}{\lambda_{g,G}(1-\nu+\sigma\varphi+\sigma\nu)^2}\right]g_{i,T}$$

$$= g^{opt}-\frac{\eta\gamma^2(\varphi+\nu)^2\sigma k_1k_2}{\lambda_{g,G}(1-\nu+\sigma\varphi+\sigma\nu)^2}\left(g_{i,T-1}-g^*_{T-1}\right)$$

$$+\frac{\eta\gamma(\varphi+\nu)\sigma k_1k_2}{\lambda_{g,G}(1-\nu+\sigma\varphi+\sigma\nu)^2}\left[(1+\varphi)(\mathrm{a}_{i,T-1}-\mathrm{a}^*_{T-1})-(1-\nu)(t_{i,T-1}-\mathrm{t}^*_{T-1})\right]$$

$$+\frac{\gamma(\varphi+\nu)\sigma^2(\lambda_{x,G}+k_1^2k_2^2)}{\lambda_{g,G}(1-\nu+\sigma\varphi+\sigma\nu)^2}\left[(1+\varphi)\mathrm{a}_{i,T}-(1-\nu)t_{i,T}\right]$$

$$+\left[\frac{(\varphi+\nu)(\theta-1)(2-\eta)(1-\gamma)\sigma\left(\lambda_{x,G}+k_1^2k_2^2\right)}{(1-\nu+\sigma\varphi+\sigma\nu)^2k_1k_2}-\frac{1}{(1-\nu+\sigma\varphi+\sigma\nu)}\right]$$

$$\frac{\eta\gamma(\varphi+\nu)\sigma k_1k_2}{\lambda_{g,G}[1-\nu+\theta(1-\gamma)(\varphi+\nu)]}\left[\gamma(\varphi+\nu)g^*_T-(1+\varphi)\mathrm{a}^*_T+(1-\nu)\mathrm{t}^*_T\right]$$

$$-\frac{\gamma(\varphi+\nu)\sigma k_1k_2}{\lambda_{g,G}(1-\nu+\sigma\varphi+\sigma\nu)^2}\sum_{n=T+1}^{T+N}(e_{n-T}-e_{n-T-1})\,E_t[\gamma(\varphi+\nu)g_{i,n}-(1+\varphi)\mathrm{a}_{i,n}$$

$$+(1-\nu)t_{i,n}]$$

$$+\frac{\gamma(\varphi+\nu)^2\sigma k_1k_2\eta(\theta-1)(2-\eta)(1-\gamma)}{\lambda_{g,G}(1-\nu+\sigma\varphi+\sigma\nu)^2[1-\nu+\theta(1-\gamma)(\varphi+\nu)]}\sum_{n=T+1}^{T+N}(e_{n-T}-e_{n-T-1})$$

$$E_T[\gamma(\varphi+\nu)g^*_n-(1+\varphi)\mathrm{a}^*_n+(1-\nu)\mathrm{t}^*_n]$$

$$-\frac{\gamma(\varphi+\nu)k_1k_2}{\lambda_{g,G}(1-\nu+\sigma\varphi+\sigma\nu)}\left[k_1k_2b_{N+1}+\left(\eta+\frac{\sigma\lambda_{x,G}}{k_1k_2}\right)a_{N+1}\right]E_T\left(x_{i,T+N+1}\right)$$

$$+\frac{\gamma(\varphi+\nu)^2\sigma k_1k_2\eta(\theta-1)(2-\eta)}{\lambda_{g,G}\theta(1-\nu+\sigma\varphi+\sigma\nu)^2}e_NE_t(x^*_{T+N+1})$$

$$-\frac{\gamma(\varphi+\nu)\sigma}{\lambda_{g,G}(1-\nu+\sigma\varphi+\sigma\nu)}\left[k_1k_2d_{N+1}+\left(\frac{\eta k_1k_2}{\sigma}+\lambda_{x,G}\right)b_{N+1}\right]E_T\left(\pi^i_{i,T+N+1}\right)$$

$$+\frac{\gamma(\varphi+\nu)\sigma k_1k_2}{\lambda_{g,G}(1-\nu+\sigma\varphi+\sigma\nu)^2}e_NE_T[\gamma(\varphi+\nu)g_{i,T+N+1}-(1+\varphi)\mathrm{a}_{i,T+N+1}$$

$$+(1-\nu)t_{i,T+N+1}]$$

$$-\frac{\gamma(\varphi+\nu)^2\sigma k_1k_2\eta(\theta-1)(2-\eta)(1-\gamma)}{\lambda_{g,G}(1-\nu+\sigma\varphi+\sigma\nu)^2[1-\nu+\theta(1-\gamma)(\varphi+\nu)]}e_N$$

$$E_t[\gamma(\varphi+\nu)g^*_{T+N+1}-(1+\varphi)\mathrm{a}^*_{T+N+1}+(1-\nu)t^*_{T+N+1}]$$

$$+\frac{\gamma(\varphi+\nu)\sigma}{\lambda_{g,G}(1-\nu+\sigma\varphi+\sigma\nu)}\{k_1k_2\sum_{n=T}^{T+N}e_{n-T}\left[i_n-\frac{(1-\beta)}{\beta}\right]+\left(k_1k_2\pi^{opt}+\lambda_{x,G}x^{opt}\right)\}$$

$$-\frac{\eta\gamma(\varphi+\nu)\sigma k_1k_2[1-\nu+\theta(1-\gamma)(\varphi+\nu)]}{(1-\gamma)\theta\lambda_{g,G}(1-\nu+\sigma\varphi+\sigma\nu)^2}x^*_{T-1}+\frac{\eta\gamma(\varphi+\nu)k_1k_2}{\lambda_{g,G}(1-\nu+\sigma\varphi+\sigma\nu)}x_{i,T-1}$$

$$-\left[\sigma\left(\lambda_{x,G}+k_1^2k_2^2\right)(\theta-1)(2-\eta)(1-\gamma)(\varphi+\nu)-k_1k_2(1-\nu+\sigma\varphi+\sigma\nu)\right]$$

$$\frac{\gamma(\varphi+\nu)\sigma\eta}{\lambda_{g,G}\theta(1-\nu+\sigma\varphi+\sigma\nu)^2(1-\gamma)}x_T^*$$
$$-\frac{\gamma(\varphi+\nu)^2\sigma k_1k_2\eta(\theta-1)(2-\eta)}{\lambda_{g,G}\theta(1-\nu+\sigma\varphi+\sigma\nu)^2}\sum_{n=T+1}^{T+N}(e_{n-T}-e_{n-T-1})\,E_t(x_n^*) \qquad (A9)$$

$$e_0=\eta+\frac{\sigma\lambda_{x,G}}{k_1k_2}+\sigma k_1k_2 \qquad e_1=\eta+\frac{\sigma\lambda_{x,G}}{k_1k_2}+\sigma(1+\eta+\beta)k_1k_2+\sigma^2\left(\lambda_{x,G}+k_1^2k_2^2\right)$$
$$e_n=\left(\eta+\frac{\sigma\lambda_{x,G}}{k_1k_2}\right)a_n+[(1+\eta)k_1k_2+\sigma\lambda_{x,G}]b_n+\sigma k_1k_2d_n$$

Appendix B: Optimal economic variables in all the monetary union

For all the monetary union, the optimal global budgetary policy minimizes the following loss function:

$$\mathcal{L}_T^*=E_T\sum_{t=T}^{\infty}\beta^t\{[(\pi_t^*-\pi^{opt})^2+\lambda_{x,G}(x_t^*-x^{opt})^2+\lambda_{g,G}(g_t^*-g^{opt})^2]$$
$$+z_{i5,t}^*[\pi_t^*-\beta\pi_{t+1}^*-k_1k_2^*x_t^*]$$
$$+z_{i4,t}^*\left[x_t^*-x_{t+1}^*+\theta(1-\gamma)\left(i_t-\pi_{t+1}^*-\overline{r_t^*}\right)\right] \qquad (B1)$$

The optimal first order conditions of equation (B1) for a given period (T) are as follows:

$$\begin{cases}\dfrac{\partial\mathcal{L}_T^*}{\partial\pi_T^*}=2(\pi_T^*-\pi^{opt})+z_{i5,T}^*=0\\ \dfrac{\partial\mathcal{L}_T^*}{\partial x_T^*}=2\lambda_{x,G}(x_T^*-x^{opt})+z_{i4,T}^*-k_1k_2^*z_{i5,T}=0\\ \dfrac{\partial\mathcal{L}_T^*}{\partial g_t^*}=2\lambda_{g,G}(g_T^*-g^{opt})-z_{i4,T}^*\theta(1-\gamma)\dfrac{\partial\overline{r_T^*}}{\partial g_T^*}=0\\ \left\{x_T^*-E_t(x_{T+1}^*)+\theta(1-\gamma)\left[i_T-E_T(\pi_{T+1}^*)-\overline{r_T^*}\right]\right\}z_{i4,T}^*=0\\ \{\pi_T^*-\beta E_T(\pi_{T+1}^*)-k_1k_2^*x_T^*\}z_{i5,T}^*=0 \qquad (B2)\end{cases}$$

Regarding the equilibrium interest rate, equations (23) and (35) imply:

$$\overline{r_T^*}=\frac{(1-\beta)}{\beta}+\frac{\gamma(\varphi+\nu)}{[1-\nu+\theta(1-\gamma)(\varphi+\nu)]}[g_T^*-E_T(g_{T+1}^*)]$$
$$-\frac{1}{[1-\nu+\theta(1-\gamma)(\varphi+\nu)]}\{(1+\varphi)[a_T^*-E_T(a_{T+1}^*)]$$
$$-(1-\nu)[t_T^*-E_T(t_{T+1}^*)]\}\ (B3)$$

The two first equations of the system (B2) imply:

$$z_{i4,T}^{*}=-2\lambda_{x,G}(x_T^{*}-x^{opt})-2k_1k_2^{*}(\pi_T^{*}-\pi^{opt})$$

So, the third equation of the system (B2) implies:

$$\begin{aligned} g_T^{*} &= g^{opt}+z_{i4,T}^{*}\frac{\theta(1-\gamma)\gamma(\varphi+\nu)}{2\lambda_{g,G}[1-\nu+\theta(1-\gamma)(\varphi+\nu)]} \\ &= g^{opt}-\frac{\theta(1-\gamma)\gamma(\varphi+\nu)}{\lambda_{g,G}[1-\nu+\theta(1-\gamma)(\varphi+\nu)]}[\lambda_{x,G}(x_T^{*}-x^{opt}) \\ &\quad +k_1k_2^{*}(\pi_T^{*}-\pi^{opt})] \quad (B4) \end{aligned}$$

The two last equations of the system (B2) imply:

$$\pi_T^{*}=[\beta+k_1k_2^{*}\theta(1-\gamma)]E_t(\pi_{T+1}^{*})+k_1k_2^{*}E_t(x_{T+1}^{*})-k_1k_2^{*}\theta(1-\gamma)(i_T-\overline{r_T^{*}})$$

Therefore, the system to solve is the following:

$$\begin{pmatrix} x_T^{*} \\ \pi_T^{*} \end{pmatrix}=\begin{pmatrix} 1 & \theta(1-\gamma) \\ k_1k_2^{*} & \beta+k_1k_2^{*}\theta(1-\gamma) \end{pmatrix}\begin{pmatrix} E_t(x_{T+1}^{*}) \\ E_t(\pi_{T+1}^{*}) \end{pmatrix}-\begin{pmatrix} 1 \\ k_1k_2^{*} \end{pmatrix}\theta(1-\gamma)(i_T-\overline{r_T^{*}}) \quad (B5)$$

$$\begin{pmatrix} x_T^{*} \\ \pi_T^{*} \end{pmatrix}=B^{N+1}\begin{pmatrix} E_T(x_{T+N+1}^{*}) \\ E_T(\pi_{T+N+1}^{*}) \end{pmatrix}-\theta(1-\gamma)\sum_{n=T}^{T+N}B^{n-T}\begin{pmatrix} 1 \\ k_1k_2^{*} \end{pmatrix}(i_n-\overline{r_n^{*}})$$

We have then to find the solution of the following matrix:

$$B^n=\begin{pmatrix} 1 & \theta(1-\gamma) \\ k_1k_2^{*} & \beta+k_1k_2^{*}\theta(1-\gamma) \end{pmatrix}^n=\begin{pmatrix} v_n & w_n \\ x_n & y_n \end{pmatrix}$$

with: $u_n^{*}=[1+\beta+k_1k_2^{*}\theta(1-\gamma)]u_{n-1}^{*}-\beta u_{n-2}^{*}$ as characteristic equation of this matrix.
Besides, we obtain:

$$x_T^{*}=v_{N+1}E_T(x_{t+N+1}^{*})+w_{N+1}E_T(\pi_{T+N+1}^{*})-\theta(1-\gamma)\sum_{n=T}^{T+N}(v_{n-T}+k_1k_2^{*}w_{n-T})(i_n-\overline{r_n^{*}})$$

$$\begin{aligned} \pi_T^{*} &= x_{N+1}E_T(x_{t+N+1}^{*})+y_{N+1}E_T(\pi_{T+N+1}^{*}) \\ &\quad -\theta(1-\gamma)\sum_{n=T}^{T+N}(x_{n-T}+k_1k_2^{*}y_{n-T})(i_n-\overline{r_n^{*}}) \quad (B6) \end{aligned}$$

$$r^{*2}-[1+\beta+k_1k_2^{*}\theta(1-\gamma)]r^{*}+\beta=0$$

The two solutions of this equation are the following:

$$r_1^{*}=\frac{[1+\beta+k_1k_2^{*}\theta(1-\gamma)]}{2}+\frac{\sqrt{[1+\beta+k_1k_2^{*}\theta(1-\gamma)]^2-4\beta}}{2}$$

$$r_2^{*}=\frac{[1+\beta+k_1k_2^{*}\theta(1-\gamma)]}{2}-\frac{\sqrt{[1+\beta+k_1k_2^{*}\theta(1-\gamma)]^2-4\beta}}{2}$$

$$v_0 = v_1 = 1$$

$$v_n = \left\{\frac{1-\beta-k_1k_2^*\theta(1-\gamma)+\sqrt{[1+\beta+k_1k_2^*\theta(1-\gamma)]^2-4\beta}}{2\sqrt{[1+\beta+k_1k_2^*\theta(1-\gamma)]^2-4\beta}}\right\}r_1^{*n} + \left\{\frac{-1+\beta+k_1k_2^*\theta(1-\gamma)+\sqrt{[1+\beta+k_1k_2^*\theta(1-\gamma)]^2-4\beta}}{2\sqrt{[1+\beta+k_1k_2^*\theta(1-\gamma)]^2-4\beta}}\right\}r_2^{*n}$$

$$w_0 = 0 \qquad w_1 = \theta(1-\gamma) \qquad w_n = \frac{\theta(1-\gamma)}{\sqrt{[1+\beta+k_1k_2^*\theta(1-\gamma)]^2-4\beta}}\left(r_1^{*n}-r_2^{*n}\right)$$

$$x_n = \frac{k_1k_2^*}{\theta(1-\gamma)}w_n$$

$$y_0 = 1 \qquad y_1 = \beta + k_1k_2^*\theta(1-\gamma)$$

$$y_n = \left\{\frac{-1+\beta+k_1k_2^*\theta(1-\gamma)+\sqrt{[1+\beta+k_1k_2^*\theta(1-\gamma)]^2-4\beta}}{2\sqrt{[1+\beta+k_1k_2^*\theta(1-\gamma)]^2-4\beta}}\right\}r_1^{*n} + \left\{\frac{1-\beta-k_1k_2^*\theta(1-\gamma)+\sqrt{[1+\beta+k_1k_2^*\theta(1-\gamma)]^2-4\beta}}{2\sqrt{[1+\beta+k_1k_2^*\theta(1-\gamma)]^2-4\beta}}\right\}r_2^{*n}$$

Besides, we can put the expressions of (x_T^*) and (π_T^*) obtained in equations (B6) in the value of (g_T^*) in equation (B4). So, we have:

$$g_T^* = g^{opt} - \frac{\gamma(\varphi+\nu)k_1k_2^*}{\lambda_{g,G}[1-\nu+\theta(1-\gamma)(\varphi+\nu)]}\left[\frac{\theta(1-\gamma)\lambda_{x,G}}{k_1k_2^*}v_{N+1} + k_1k_2^*\,w_{N+1}\right]E_T(x_{T+N+1}^*) - \frac{\theta(1-\gamma)\gamma(\varphi+\nu)}{\lambda_{g,G}[1-\nu+\theta(1-\gamma)(\varphi+\nu)]}\left(\lambda_{x,G}w_{N+1}+k_1k_2^*y_{N+1}\right)E_T(\pi_{T+N+1}^*) + \frac{\theta(1-\gamma)\gamma(\varphi+\nu)k_1k_2^*}{\lambda_{g,G}[1-\nu+\theta(1-\gamma)(\varphi+\nu)]}\sum_{n=T}^{T+N} z_{n-T}\left(i_n-\overline{r_n^*}\right) + \frac{\theta(1-\gamma)\gamma(\varphi+\nu)}{\lambda_{g,G}[1-\nu+\theta(1-\gamma)(\varphi+\nu)]}\left(\lambda_{x,G}x^{opt}+k_1k_2^*\pi^{opt}\right) \qquad (B7)$$

$$z_0 = \left[\frac{\lambda_{x,G}}{k_1k_2^*}+k_1k_2^*\right]\theta(1-\gamma) \qquad z_1 = \left[\frac{\lambda_{x,G}}{k_1k_2^*}+(1+\beta)k_1k_2^*+\left(\lambda_{x,G}+k_1^2k_2^{*2}\right)\theta(1-\gamma)\right]\theta(1-\gamma)$$

$$z_n = \frac{\lambda_{x,G}\theta(1-\gamma)}{k_1k_2^*}v_n + [\lambda_{x,G}\theta(1-\gamma)+k_1k_2^*]w_n + \theta(1-\gamma)k_1k_2^*y_n$$

Finally, we can use the expression of $(\overline{r_T^*})$ in equation (B3) to obtain the following level of global public expenditure in all the monetary union:

$$\left[1+\frac{\lambda_{g,G}[1-\nu+\theta(1-\gamma)(\varphi+\nu)]^2}{\theta^2(1-\gamma)^2\gamma^2(\varphi+\nu)^2\left(\lambda_{x,G}+k_1^2k_2^{*2}\right)}\right]g_T^* = \frac{1}{\gamma(\varphi+\nu)}\left[(1+\varphi)\mathrm{a}_T^*-(1-\nu)t_T^*\right]$$

$$+\frac{[1-\nu+\theta(1-\gamma)(\varphi+\nu)]k_1k_2^*}{\theta(1-\gamma)\gamma(\varphi+\nu)\left(\lambda_{x,G}+k_1^2k_2^{*2}\right)}\sum_{n=T}^{T+N} z_{n-T}\left[i_n-\frac{(1-\beta)}{\beta}\right]$$
$$-\frac{k_1k_2^*}{\theta(1-\gamma)\left(\lambda_{x,G}+k_1^2k_2^{*2}\right)}\sum_{n=T+1}^{T+N}(z_{n-T}-z_{n-T-1})\,E_T[g_n^*-\frac{(1+\varphi)}{\gamma(\varphi+\nu)}a_n^*$$
$$+\frac{(1-\nu)}{\gamma(\varphi+\nu)}t_n^*]$$
$$-\frac{[1-\nu+\theta(1-\gamma)(\varphi+\nu)]}{\theta^2(1-\gamma)^2\gamma(\varphi+\nu)\left(\lambda_{x,G}+k_1^2k_2^{*2}\right)}[\theta(1-\gamma)\lambda_{x,G}v_{N+1}+k_1^2k_2^{*2}\,w_{N+1}]E_T(x_{T+N+1}^*)$$
$$-\frac{[1-\nu+\theta(1-\gamma)(\varphi+\nu)]}{\theta(1-\gamma)\gamma(\varphi+\nu)\left(\lambda_{x,G}+k_1^2k_2^{*2}\right)}(\lambda_{x,G}w_{N+1}+k_1k_2^*y_{N+1})E_T(\pi_{T+N+1}^*)$$
$$+\frac{k_1k_2^*}{\theta(1-\gamma)\left(\lambda_{x,G}+k_1^2k_2^{*2}\right)}\,z_N\left[E_T(g_{T+N+1}^*)-\frac{(1+\varphi)}{\gamma(\varphi+\nu)}E_T(\mathrm{a}_{T+N+1}^*)+\frac{(1-\nu)}{\gamma(\varphi+\nu)}E_T(t_{T+N+1}^*)\right]$$
$$+\frac{\lambda_{g,G}[1-\nu+\theta(1-\gamma)(\varphi+\nu)]^2}{\theta^2(1-\gamma)^2\gamma^2(\varphi+\nu)^2\left(\lambda_{x,G}+k_1^2k_2^{*2}\right)}g^{opt}$$
$$+\frac{[1-\nu+\theta(1-\gamma)(\varphi+\nu)]}{\theta(1-\gamma)\gamma(\varphi+\nu)\left(\lambda_{x,G}+k_1^2k_2^{*2}\right)}\left(\lambda_{x,G}x^{opt}+k_1k_2^*\pi^{opt}\right)\qquad(B8)$$

Appendix C: Optimal variables when the ZLB is not binding

Putting the optimal nominal interest rate in equation (44) in the level of global budgetary expenditure in equation (B8), we obtain the following optimal level of global budgetary expenditure, with N→∞:

$$g_T^*=g^{opt}+\frac{\theta(1-\gamma)\gamma(\varphi+\nu)}{\lambda_{g,G}[1-\nu+\theta(1-\gamma)(\varphi+\nu)]}\left(\lambda_{x,G}x^{opt}+k_1k_2^*\pi^{opt}\right)$$
$$-\frac{\gamma(\varphi+\nu)k_1k_2^*}{\lambda_{g,G}[1-\nu+\theta(1-\gamma)(\varphi+\nu)](k_1^2k_2^{*2}+\lambda_{CB})}\sum_{n=T}^{\infty}z_{n-T}\,(\lambda_{CB}x^{opt}+k_1k_2^*\pi^{opt})$$
$$+\frac{\theta(1-\gamma)\gamma(\varphi+\nu)k_1k_2^*}{\lambda_{g,G}[1-\nu+\theta(1-\gamma)(\varphi+\nu)]}\left[1+\frac{k_1k_2^*\beta}{\theta(1-\gamma)(k_1^2k_2^{*2}+\lambda_{CB})}\right]\sum_{n=T+1}^{\infty}z_{n-T-1}\,E_t(\pi_n^*)$$
$$+\frac{\gamma(\varphi+\nu)k_1k_2^*}{\lambda_{g,G}[1-\nu+\theta(1-\gamma)(\varphi+\nu)]}\sum_{n=T+1}^{\infty}z_{n-T-1}\,E_T(x_n^*)\qquad(C1)$$

Putting equations (B3) and (B6) for (x_T^*) and the optimal nominal interest rate in equation (44) in the optimal level of national budgetary expenditure in equation (A9), for N→∞, we have:

$$\left[1+\frac{\lambda_{g,G}(1-\nu+\sigma\varphi+\sigma\nu)^2}{\gamma^2(\varphi+\nu)^2\sigma^2\left(\lambda_{x,G}+k_1^2k_2^2\right)}\right]g_{i,T}=g_T^*+\frac{\lambda_{g,G}(1-\nu+\sigma\varphi+\sigma\nu)^2}{\gamma^2(\varphi+\nu)^2\sigma^2\left(\lambda_{x,G}+k_1^2k_2^2\right)}g_T^{opt}$$
$$-\frac{\eta k_1k_2}{\sigma(\lambda_{x,G}+k_1^2k_2^2)}\left[\left(g_{i,T-1}-g_{T-1}^*\right)-\frac{(1+\varphi)}{\gamma(\varphi+\nu)}\left(a_{i,T-1}-a_{T-1}^*\right)+\frac{(1-\nu)}{\gamma(\varphi+\nu)}\left(t_{i,T-1}-t_{T-1}^*\right)\right]$$
$$+\frac{1}{\gamma(\varphi+\nu)}\left[(1+\varphi)\left(a_{i,T}-a_T^*\right)-(1-\nu)\left(t_{i,T}-t_T^*\right)\right]$$
$$-\frac{k_1k_2}{\sigma\left(\lambda_{x,G}+k_1^2k_2^2\right)}\sum_{n=T+1}^{\infty}(e_{n-T}-e_{n-T-1})\,E_t\left[\left(g_{i,n}-g_n^*\right)-\frac{(1+\varphi)}{\gamma(\varphi+\nu)}\left(a_{i,n}-a_n^*\right)+\frac{(1-\nu)}{\gamma(\varphi+\nu)}\left(t_{i,n}-t_n^*\right)\right]$$
$$+\frac{(1-\nu+\sigma\varphi+\sigma\nu)}{\gamma(\varphi+\nu)\sigma(\lambda_{x,G}+k_1^2k_2^2)}\left(k_1k_2\pi^{opt}+\lambda_{x,G}x^{opt}\right)$$
$$-\frac{(1-\nu+\sigma\varphi+\sigma\nu)k_1k_2}{\gamma(\varphi+\nu)\sigma\left(\lambda_{x,G}+k_1^2k_2^2\right)\theta(1-\gamma)(k_1^2k_2^{*2}+\lambda_{CB})}$$
$$\sum_{n=T}^{\infty}\Big[\frac{\sigma\eta\left(\lambda_{x,G}+k_1^2k_2^2\right)(\theta-1)(2-\eta)(1-\gamma)(\varphi+\nu)}{(1-\nu+\sigma\varphi+\sigma\nu)k_1k_2}(v_{n-T}+k_1k_2^*w_{n-T})-\eta(v_{n-T}+k_1k_2^*w_{n-T})+e_{n-T}\Big]\left(\lambda_{CB}x^{opt}+k_1k_2^*\pi^{opt}\right)$$
$$+\frac{(1-\nu+\sigma\varphi+\sigma\nu)k_1k_2}{\gamma(\varphi+\nu)\sigma\left(\lambda_{x,G}+k_1^2k_2^2\right)}\left[1+\frac{k_1k_2^*\beta}{\theta(1-\gamma)(k_1^2k_2^{*2}+\lambda_{CB})}\right]$$
$$\sum_{n=T}^{\infty}\left[\frac{\sigma\eta\left(\lambda_{x,G}+k_1^2k_2^2\right)(\theta-1)(2-\eta)(1-\gamma)(\varphi+\nu)}{k_1k_2(1-\nu+\sigma\varphi+\sigma\nu)}(v_{n-T}+k_1k_2^*w_{n-T})-\eta(v_{n-T}+k_1k_2^*w_{n-T})+e_{n-T}\right]E_t(\pi_{n+1}^*)$$
$$+\frac{\eta k_1k_2(1-\nu+\sigma\varphi+\sigma\nu)}{\gamma(\varphi+\nu)\sigma^2(\lambda_{x,G}+k_1^2k_2^2)}x_{i,T-1}-\frac{\eta k_1k_2[1-\nu+\theta(1-\gamma)(\varphi+\nu)]}{\gamma(\varphi+\nu)\sigma(\lambda_{x,G}+k_1^2k_2^2)(1-\gamma)\theta}x_{T-1}^*$$
$$+\frac{k_1k_2}{\gamma\sigma\left(\lambda_{x,G}+k_1^2k_2^2\right)\theta(1-\gamma)(\varphi+\nu)}\sum_{n=T+1}^{\infty}\{[1-\nu+\theta(1-\gamma)(\varphi+\nu)]e_{n-T-1}$$
$$-\eta(1-\nu+\sigma\varphi+\sigma\nu)(v_{n-T-1}+k_1k_2^*w_{n-T-1})-\eta(\theta-1)(2-\eta)(1-\gamma)(\varphi+\nu)e_{n-T}$$
$$+\frac{\sigma\eta\left(\lambda_{x,G}+k_1^2k_2^2\right)(\theta-1)(2-\eta)(1-\gamma)(\varphi+\nu)}{k_1k_2}(v_{n-T-1}+k_1k_2^*w_{n-T-1})\}E_T(x_n^*)\quad(C2)$$

So, putting the optimal global public expenditure (g_T^*) obtained in equation (C1) in the national public expenditure in equation (C2), we have:

$$\left[1+\frac{\lambda_{g,G}(1-\nu+\sigma\varphi+\sigma\nu)^2}{\gamma^2(\varphi+\nu)^2\sigma^2\left(\lambda_{x,G}+k_1^2k_2^2\right)}\right]\left(g_{i,T}-g_T^*\right)$$
$$=-\frac{\eta k_1k_2}{\sigma(\lambda_{x,G}+k_1^2k_2^2)}\left[\left(g_{i,T-1}-g_{T-1}^*\right)-\frac{(1+\varphi)}{\gamma(\varphi+\nu)}\left(a_{i,T-1}-a_{T-1}^*\right)+\frac{(1-\nu)}{\gamma(\varphi+\nu)}\left(t_{i,T-1}-t_{T-1}^*\right)\right]$$
$$+\frac{1}{\gamma(\varphi+\nu)}\left[(1+\varphi)\left(a_{i,T}-a_T^*\right)-(1-\nu)\left(t_{i,T}-t_T^*\right)\right]$$
$$-\frac{k_1k_2}{\sigma\left(\lambda_{x,G}+k_1^2k_2^2\right)}\sum_{n=T+1}^{\infty}(e_{n-T}-e_{n-T-1})\,E_t\left[\left(g_{i,n}-g_n^*\right)-\frac{(1+\varphi)}{\gamma(\varphi+\nu)}\left(a_{i,n}-a_n^*\right)+\frac{(1-\nu)}{\gamma(\varphi+\nu)}\left(t_{i,n}-t_n^*\right)\right]$$
$$+\frac{(1-\nu+\sigma\varphi+\sigma\nu)}{\gamma(\varphi+\nu)\sigma\left(\lambda_{x,G}+k_1^2k_2^2\right)}\{\frac{\lambda_{x,G}(1-\nu)[\sigma-\theta(1-\gamma)]}{[1-\nu+\theta(1-\gamma)(\varphi+\nu)]\sigma}x^{opt}$$
$$+\frac{[k_1k_2\sigma-\theta(1-\gamma)k_1k_2^*](1-\nu)+\sigma\theta(1-\gamma)(\varphi+\nu)(k_1k_2-k_1k_2^*)]}{[1-\nu+\theta(1-\gamma)(\varphi+\nu)]\sigma}\pi^{opt}\}$$
$$+\frac{\sigma(1-\nu+\sigma\varphi+\sigma\nu)}{\theta(1-\gamma)\sigma^2\left(\lambda_{x,G}+k_1^2k_2^2\right)\gamma(\varphi+\nu)(k_1^2k_2^{*2}+\lambda_{CB})}$$
$$\sum_{n=T}^{\infty}\{\frac{\theta(1-\gamma)k_1k_2^*(1-\nu+\sigma\varphi+\sigma\nu)z_{n-T}}{\sigma[1-\nu+\theta(1-\gamma)(\varphi+\nu)]}+\eta k_1k_2(v_{n-T}+k_1k_2^*w_{n-T})-k_1k_2e_{n-T}$$
$$-\frac{\sigma\eta\left(\lambda_{x,G}+k_1^2k_2^2\right)(\theta-1)(2-\eta)(1-\gamma)(\varphi+\nu)}{(1-\nu+\sigma\varphi+\sigma\nu)}(v_{n-T}+k_1k_2^*w_{n-T})\}(\lambda_{CB}x^{opt}+k_1k_2^*\pi^{opt})$$
$$-\frac{(1-\nu+\sigma\varphi+\sigma\nu)}{\gamma(\varphi+\nu)\sigma\left(\lambda_{x,G}+k_1^2k_2^2\right)}\left[1+\frac{k_1k_2^*\beta}{\theta(1-\gamma)(k_1^2k_2^{*2}+\lambda_{CB})}\right]$$
$$\sum_{n=T+1}^{\infty}\{-\frac{\sigma\eta\left(\lambda_{x,G}+k_1^2k_2^2\right)(\theta-1)(2-\eta)(1-\gamma)(\varphi+\nu)}{(1-\nu+\sigma\varphi+\sigma\nu)}(v_{n-T-1}+k_1k_2^*w_{n-T-1})+\eta k_1k_2(v_{n-T-1}+k_1k_2^*w_{n-T-1})-k_1k_2e_{n-T-1}$$
$$+\frac{(1-\nu+\sigma\varphi+\sigma\nu)\theta(1-\gamma)k_1k_2^*}{[1-\nu+\theta(1-\gamma)(\varphi+\nu)]\sigma}z_{n-T-1}\}E_t(\pi_n^*)$$
$$+\frac{\eta k_1k_2(1-\nu+\sigma\varphi+\sigma\nu)}{\gamma(\varphi+\nu)\sigma^2(\lambda_{x,G}+k_1^2k_2^2)}x_{i,T-1}-\frac{\eta k_1k_2[1-\nu+\theta(1-\gamma)(\varphi+\nu)]}{\gamma(\varphi+\nu)\sigma(\lambda_{x,G}+k_1^2k_2^2)(1-\gamma)\theta}x_{T-1}^*$$
$$-\frac{(1-\nu+\sigma\varphi+\sigma\nu)}{\gamma\sigma\left(\lambda_{x,G}+k_1^2k_2^2\right)}\sum_{n=T+1}^{\infty}\{-\frac{\sigma\eta\left(\lambda_{x,G}+k_1^2k_2^2\right)(\theta-1)(2-\eta)}{\theta(1-\nu+\sigma\varphi+\sigma\nu)}(v_{n-T-1}+k_1k_2^*w_{n-T-1})$$

$$+\frac{(1-\nu+\sigma\varphi+\sigma\nu)k_1k_2^*}{\sigma[1-\nu+\theta(1-\gamma)(\varphi+\nu)](\varphi+\nu)}z_{n-T-1}$$
$$-\frac{[1-\nu+\theta(1-\gamma)(\varphi+\nu)]k_1k_2}{\theta(1-\gamma)(1-\nu+\sigma\varphi+\sigma\nu)(\varphi+\nu)}e_{n-T-1}$$
$$+\frac{\eta k_1k_2}{\theta(1-\gamma)(\varphi+\nu)}(v_{n-T-1}+k_1k_2^*w_{n-T-1})$$
$$+\frac{\eta k_1k_2(\theta-1)(2-\eta)}{\theta(1-\nu+\sigma\varphi+\sigma\nu)}e_{n-T}\}E_T(x_n^*) \quad (C3)$$

Using the value of $(\overline{r_{i,T}})$ in (A3), the nominal interest rate (i_T) in (44), equations (B3) and (B6) for (x_T^*), and equations (A8) for optimal economic variables, we have:

$$x_{i,T}=\frac{\sigma}{(1-\nu+\sigma\varphi+\sigma\nu)}[\gamma(\varphi+\nu)(g_{i,T}-g_T^*)-(1+\varphi)(\mathrm{a}_{i,T}-\mathrm{a}_T^*)$$
$$+(1-\nu)(t_{i,T}-\mathrm{t}_T^*)]$$
$$+\frac{\gamma\sigma(\varphi+\nu)}{(1-\nu+\sigma\varphi+\sigma\nu)}\sum_{n=T+1}^{\infty}(a_{n-T}-a_{n-T-1}+k_1k_2b_{n-T}-k_1k_2b_{n-T-1})$$
$$E_T[(g_{i,n}-g_n^*)-\frac{(1+\varphi)}{\gamma(\varphi+\nu)}(\mathrm{a}_{i,n}-a_n^*)+\frac{(1-\nu)}{\gamma(\varphi+\nu)}(\mathrm{t}_{i,n}-t_n^*)]$$
$$-\sigma\left[1+\frac{k_1k_2^*\beta}{\theta(1-\gamma)(k_1^2k_2^{*2}+\lambda_{CB})}\right]\sum_{n=T+1}^{\infty}\{(a_{n-T-1}+k_1k_2b_{n-T-1})$$
$$+\frac{(\varphi+\nu)\eta(\theta-1)(2-\eta)(1-\gamma)}{(1-\nu+\sigma\varphi+\sigma\nu)}(v_{n-T-1}+k_1k_2^*w_{n-T-1})\}E_t(\pi_n^*)$$
$$+\frac{\sigma}{(k_1^2k_2^{*2}+\lambda_{CB})\theta}\sum_{n=T+1}^{\infty}\{\frac{(a_{n-T-1}+k_1k_2b_{n-T-1})}{(1-\gamma)}$$
$$+\frac{(\varphi+\nu)\eta(\theta-1)(2-\eta)}{(1-\nu+\sigma\varphi+\sigma\nu)}(v_{n-T-1}+k_1k_2^*w_{n-T-1})\}(\lambda_{CB}x^{opt}$$
$$+k_1k_2^*\pi^{opt})$$
$$+\sigma\sum_{n=T+1}^{\infty}\{\frac{(\varphi+\nu)\eta(\theta-1)(2-\eta)}{(1-\nu+\sigma\varphi+\sigma\nu)\theta}(a_{n-T}+k_1k_2b_{n-T}-v_{n-T-1}-k_1k_2^*w_{n-T-1})$$
$$-\frac{[(1-\nu)+\theta(1-\gamma)(\varphi+\nu)]}{\theta(1-\gamma)(1-\nu+\sigma\varphi+\sigma\nu)}(a_{n-T-1}+k_1k_2b_{n-T-1})\}E_T(x_n^*) \qquad (C4)$$

$$\pi_{i,T}^i=\frac{\sigma k_1k_2}{(1-\nu+\sigma\varphi+\sigma\nu)}[\gamma(\varphi+\nu)(g_{i,T}-g_T^*)-(1+\varphi)(\mathrm{a}_{i,T}-\mathrm{a}_T^*)$$
$$+(1-\nu)(t_{i,T}-\mathrm{t}_T^*)]$$
$$+\frac{k_1k_2\gamma(\varphi+\nu)}{(1-\nu+\sigma\varphi+\sigma\nu)}\sum_{n=T+1}^{\infty}(b_{n-T}+\sigma d_{n-T}-b_{n-T-1}-\sigma d_{n-T-1})$$
$$E_T[(g_{i,n}-g_n^*)-\frac{(1+\varphi)}{\gamma(\varphi+\nu)}(\mathrm{a}_{i,n}-a_n^*)+\frac{(1-\nu)}{\gamma(\varphi+\nu)}(\mathrm{t}_{i,n}-t_n^*)]$$

$$-k_1 k_2 \left[1 + \frac{k_1 k_2^* \beta}{\theta(1-\gamma)(k_1^2 k_2^{*2} + \lambda_{CB})}\right] \sum_{n=T+1}^{\infty} \{(b_{n-T-1} + \sigma d_{n-T-1}) + \frac{\sigma(\varphi+\nu)\eta(\theta-1)(2-\eta)(1-\gamma)(v_{n-T-1} + k_1 k_2^* w_{n-T-1})}{(1-\nu+\sigma\varphi+\sigma\nu)}\} E_t(\pi_n^*)$$
$$+ \frac{k_1 k_2}{\theta(k_1^2 k_2^{*2} + \lambda_{CB})} \sum_{n=T+1}^{\infty} \{\frac{(b_{n-T-1} + \sigma d_{n-T-1})}{(1-\gamma)} + \frac{\sigma(\varphi+\nu)\eta(\theta-1)(2-\eta)(v_{n-T-1} + k_1 k_2^* w_{n-T-1})}{(1-\nu+\sigma\varphi+\sigma\nu)}\}(\lambda_{CB} x^{opt} + k_1 k_2^* \pi^{opt})$$
$$+ k_1 k_2 \sum_{n=T+1}^{\infty} \{\frac{(\varphi+\nu)\eta(\theta-1)(2-\eta)}{(1-\nu+\sigma\varphi+\sigma\nu)\theta}(b_{n-T} + \sigma d_{n-T} - \sigma v_{n-T-1} - \sigma k_1 k_2^* w_{n-T-1}) - \frac{[(1-\nu) + \theta(1-\gamma)(\varphi+\nu)]}{\theta(1-\gamma)(1-\nu+\sigma\varphi+\sigma\nu)}(b_{n-T-1} + \sigma d_{n-T-1})\} E_T(x_n^*) \quad (C5)$$

Appendix D: Optimal variables when the ZLB is binding

Equation (B6) for the global economic activity and equation (B3) for ($\overline{r_T^*}$) imply:

$$x_T^* = v_{N+1} E_T(x_{T+N+1}^*) + w_{N+1} E_T(\pi_{T+N+1}^*) + \frac{\theta(1-\gamma)(1-\beta)}{\beta} \sum_{n=T}^{T+N} (v_{n-T} + k_1 k_2^* w_{n-T})$$
$$- \frac{\theta(1-\gamma)}{[1-\nu+\theta(1-\gamma)(\varphi+\nu)]}(v_N + k_1 k_2^* w_N)\,[\gamma(\varphi+\nu) E_T(g_{T+N+1}^*) - (1+\varphi) E_T(\mathrm{a}_{T+N+1}^*) + (1-\nu) E_T(t_{T+N+1}^*)]$$
$$+ \frac{\theta(1-\gamma)}{[1-\nu+\theta(1-\gamma)(\varphi+\nu)]}[\gamma(\varphi+\nu) g_T^* - (1+\varphi)\mathrm{a}_T^* + (1-\nu) t_T^*]$$
$$+ \frac{\theta(1-\gamma)}{[1-\nu+\theta(1-\gamma)(\varphi+\nu)]} \sum_{n=T+1}^{T+N} (v_{n-T} + k_1 k_2^* w_{n-T} - v_{n-T-1} - k_1 k_2^* w_{n-T-1}) [\gamma(\varphi+\nu) E_T(g_n^*) - (1+\varphi)\mathrm{a}_n^* + (1-\nu) t_n^*] \qquad (D1)$$

Equation (A9) for the optimal national budgetary policy and equation (D1) imply:

$$\left[1 + \frac{\gamma^2(\varphi+\nu)^2\sigma^2(\lambda_{x,G} + k_1^2 k_2^2)}{\lambda_{g,G}(1-\nu+\sigma\varphi+\sigma\nu)^2}\right] g_{i,T} = g_T^{opt} - \frac{\eta\gamma^2(\varphi+\nu)^2\sigma k_1 k_2}{\lambda_{g,G}(1-\nu+\sigma\varphi+\sigma\nu)^2}(g_{i,T-1} - g_{T-1}^*)$$
$$+ \frac{\eta\gamma(\varphi+\nu)\sigma k_1 k_2}{\lambda_{g,G}(1-\nu+\sigma\varphi+\sigma\nu)^2}\left[(1+\varphi)(\mathrm{a}_{i,T-1} - \mathrm{a}_{T-1}^*) - (1-\nu)(t_{i,T-1} - \mathrm{t}_{T-1}^*)\right]$$

$$+\frac{\gamma(\varphi+\nu)\sigma^2(\lambda_{x,G}+k_1^2k_2^2)}{\lambda_{g,G}(1-\nu+\sigma\varphi+\sigma\nu)^2}[(1+\varphi)\mathrm{a}_{i,T}-(1-\nu)t_{i,T}]$$

$$-\frac{\gamma(\varphi+\nu)\sigma k_1k_2}{\lambda_{g,G}(1-\nu+\sigma\varphi+\sigma\nu)^2}\sum_{n=T+1}^{T+N}(e_{n-T}-e_{n-T-1})\,E_t[\gamma(\varphi+\nu)g_{i,n}-(1+\varphi)\mathrm{a}_{i,n}$$
$$+(1-\nu)t_{i,n}]$$

$$+\frac{\gamma(\varphi+\nu)\sigma\eta}{\lambda_{g,G}(1-\nu+\sigma\varphi+\sigma\nu)^2[1-\nu+\theta(1-\gamma)(\varphi+\nu)]}$$
$$\sum_{n=T+1}^{T+N}\{(\theta-1)(2-\mathrm{\eta})(1-\gamma)(\varphi+\nu)[k_1k_2(e_{n-T}-e_{n-T-1})$$
$$-\sigma\big(\lambda_{x,G}+k_1^2k_2^2\big)(v_{n-T}+k_1k_2^*w_{n-T}-v_{n-T-1}-k_1k_2^*w_{n-T-1})]$$
$$+k_1k_2(1-\nu+\sigma\varphi+\sigma\nu)(v_{n-T}+k_1k_2^*w_{n-T}-v_{n-T-1}-k_1k_2^*w_{n-T-1})\}$$
$$E_T[\gamma(\varphi+\nu)g_n^*-(1+\varphi)\mathrm{a}_n^*+(1-\nu)\mathrm{t}_n^*]$$

$$-\frac{\gamma(\varphi+\nu)k_1k_2}{\lambda_{g,G}(1-\nu+\sigma\varphi+\sigma\nu)}\left[k_1k_2b_{N+1}+\left(\mathrm{\eta}+\frac{\sigma\lambda_{x,G}}{k_1k_2}\right)a_{N+1}\right]E_T\big(x_{i,T+N+1}\big)$$

$$+\frac{\gamma(\varphi+\nu)^2\sigma k_1k_2\eta(\theta-1)(2-\mathrm{\eta})}{\lambda_{g,G}\theta(1-\nu+\sigma\varphi+\sigma\nu)^2}e_NE_t(x_{T+N+1}^*)$$

$$-\frac{\gamma(\varphi+\nu)\sigma\eta}{\lambda_{g,G}\theta(1-\nu+\sigma\varphi+\sigma\nu)^2(1-\gamma)}\big[\sigma\big(\lambda_{x,G}+k_1^2k_2^2\big)(\theta-1)(2-\mathrm{\eta})(1-\gamma)(\varphi+\nu)$$
$$-k_1k_2(1-\nu+\sigma\varphi+\sigma\nu)\big][v_{N+1}E_T(x_{T+N+1}^*)+w_{N+1}E_T(\pi_{T+N+1}^*)]$$

$$-\frac{\gamma(\varphi+\nu)\sigma}{\lambda_{g,G}(1-\nu+\sigma\varphi+\sigma\nu)}\left[k_1k_2d_{N+1}+\left(\frac{\mathrm{\eta}k_1k_2}{\sigma}+\lambda_{x,G}\right)b_{N+1}\right]E_T\big(\pi_{i,T+N+1}^i\big)$$

$$+\frac{\gamma(\varphi+\nu)\sigma k_1k_2}{\lambda_{g,G}(1-\nu+\sigma\varphi+\sigma\nu)^2}e_NE_T[\gamma(\varphi+\nu)g_{i,T+N+1}-(1+\varphi)\mathrm{a}_{i,T+N+1}$$
$$+(1-\nu)t_{i,T+N+1}]$$

$$-\frac{\gamma(\varphi+\nu)\sigma\eta}{\lambda_{g,G}(1-\nu+\sigma\varphi+\sigma\nu)^2[1-\nu+\theta(1-\gamma)(\varphi+\nu)]}$$
$$\{(\theta-1)(2-\mathrm{\eta})(1-\gamma)(\varphi+\nu)[k_1k_2e_N-\sigma\big(\lambda_{x,G}+k_1^2k_2^2\big)(v_N+k_1k_2^*w_N)]$$
$$+k_1k_2(1-\nu+\sigma\varphi+\sigma\nu)(v_N+k_1k_2^*w_N)\}$$
$$E_t[\gamma(\varphi+\nu)g_{T+N+1}^*-(1+\varphi)\mathrm{a}_{T+N+1}^*+(1-\nu)t_{T+N+1}^*]$$

$$+\frac{\gamma(\varphi+\nu)\sigma}{\lambda_{g,G}(1-\nu+\sigma\varphi+\sigma\nu)}\big(k_1k_2\pi^{opt}+\lambda_{x,G}x^{opt}\big)$$

$$-\frac{\gamma(\varphi+\nu)\sigma(1-\beta)}{\lambda_{g,G}(1-\nu+\sigma\varphi+\sigma\nu)^2\beta}\sum_{n=T}^{T+N}[k_1k_2(1-\nu+\sigma\varphi+\sigma\nu)(e_{n-T}-\eta v_{n-T}$$
$$-\eta k_1k_2^*w_{n-T})$$
$$+\sigma\eta\big(\lambda_{x,G}+k_1^2k_2^2\big)(\theta-1)(2-\mathrm{\eta})(1-\gamma)(\varphi+\nu)(v_{n-T}+k_1k_2^*w_{n-T})]$$

$$-\frac{\mathrm{\eta}\gamma(\varphi+\nu)\sigma k_1k_2[1-\nu+\theta(1-\gamma)(\varphi+\nu)]}{(1-\gamma)\theta\lambda_{g,G}(1-\nu+\sigma\varphi+\sigma\nu)^2}x_{T-1}^*+\frac{\mathrm{\eta}\gamma(\varphi+\nu)k_1k_2}{\lambda_{g,G}(1-\nu+\sigma\varphi+\sigma\nu)}x_{i,T-1}$$

$$-\frac{\gamma(\varphi+\nu)^2\sigma k_1k_2\eta(\theta-1)(2-\mathrm{\eta})}{\lambda_{g,G}\theta(1-\nu+\sigma\varphi+\sigma\nu)^2}\sum_{n=T+1}^{T+N}(e_{n-T}-e_{n-T-1})\,E_t(x_n^*)\qquad(D2)$$

By replacing equations (A3) for $(\overline{r_{i,n}})$ and (D1) for (x_T^*) in equations (A8), we obtain the following levels of economic activity and inflation:

$$x_{i,T} = \sigma\frac{(1-\beta)}{\beta}\sum_{n=T}^{T+N}[a_{n-T} + k_1k_2b_{n-T} + \frac{(\varphi+\nu)\eta(\theta-1)(2-\eta)(1-\gamma)(v_{n-T}+k_1k_2^*w_{n-T})}{(1-\nu+\sigma\varphi+\sigma\nu)}] + \frac{\sigma}{(1-\nu+\sigma\varphi+\sigma\nu)}[\gamma(\varphi+\nu)g_{i,T} - (1+\varphi)\mathrm{a}_{i,T} + (1-\nu)t_{i,T}] + \frac{\sigma}{(1-\nu+\sigma\varphi+\sigma\nu)}\sum_{n=T+1}^{T+N}(a_{n-T}+k_1k_2b_{n-T}-a_{n-T-1}-k_1k_2b_{n-T-1}) [\gamma(\varphi+\nu)g_{i,n} - (1+\varphi)\mathrm{a}_{i,n} + (1-\nu)t_{i,n}] - \frac{\sigma(\varphi+\nu)\eta(\theta-1)(2-\eta)(1-\gamma)}{(1-\nu+\sigma\varphi+\sigma\nu)[1-\nu+\theta(1-\gamma)(\varphi+\nu)]}\sum_{n=T+1}^{T+N}[a_{n-T}+k_1k_2b_{n-T}-a_{n-T-1} + k_1k_2b_{n-T-1} - v_{n-T} - k_1k_2^*w_{n-T} + v_{n-T-1} + k_1k_2^*w_{n-T-1}] E_T[\gamma(\varphi+\nu)g_n^* - (1+\varphi)a_n^* + (1-\nu)t_n^*] - \frac{\sigma(a_N+k_1k_2b_N)}{(1-\nu+\sigma\varphi+\sigma\nu)}E_T[\gamma(\varphi+\nu)g_{i,T+N+1} - (1+\varphi)\mathrm{a}_{i,T+N+1} + (1-\nu)t_{i,T+N+1}] + [a_N + k_1k_2b_N - v_N - k_1k_2^*w_N]\frac{\sigma(\varphi+\nu)\eta(\theta-1)(2-\eta)(1-\gamma)}{(1-\nu+\sigma\varphi+\sigma\nu)[1-\nu+\theta(1-\gamma)(\varphi+\nu)]} E_T[\gamma(\varphi+\nu)g_{T+N+1}^* - (1+\varphi)a_{T+N+1}^* + (1-\nu)t_{T+N+1}^*] + \frac{\sigma(\varphi+\nu)\eta(\theta-1)(2-\eta)}{(1-\nu+\sigma\varphi+\sigma\nu)\theta}w_{N+1}E_T(\pi_{T+N+1}^*) + a_{N+1}E_T(x_{i,T+N+1}) + b_{N+1}E_T(\pi_{i,T+N+1}^i) - (a_N + k_1k_2b_N - v_{N+1})\frac{\sigma(\varphi+\nu)\eta(\theta-1)(2-\eta)}{(1-\nu+\sigma\varphi+\sigma\nu)\theta}E_t(x_{T+N+1}^*) + \frac{\sigma(\varphi+\nu)\eta(\theta-1)(2-\eta)}{(1-\nu+\sigma\varphi+\sigma\nu)\theta}\sum_{n=T+1}^{T+N}(a_{n-T}+k_1k_2b_{n-T}-a_{n-T-1} + k_1k_2b_{n-T-1})\,x_n^* \quad (D3)$$

$$\pi_{i,T}^i = \frac{k_1k_2(1-\beta)}{\beta}\sum_{n=T}^{T+N}[b_{n-T}+\sigma d_{n-T} + \frac{\sigma(\varphi+\nu)\eta(\theta-1)(2-\eta)(1-\gamma)(v_{n-T}+k_1k_2^*w_{n-T})}{(1-\nu+\sigma\varphi+\sigma\nu)}] + \frac{\sigma k_1k_2\gamma(\varphi+\nu)}{(1-\nu+\sigma\varphi+\sigma\nu)}g_{i,T} - \frac{k_1k_2\sigma}{(1-\nu+\sigma\varphi+\sigma\nu)}\left[(1+\varphi)\mathrm{a}_{i,T} - (1-\nu)t_{i,T}\right] + \frac{k_1k_2}{(1-\nu+\sigma\varphi+\sigma\nu)}\sum_{n=T+1}^{T+N}(b_{n-T}-b_{n-1-T}+\sigma d_{n-T}-\sigma d_{n-T-1}) E_T[\gamma(\varphi+\nu)g_{i,n} - (1+\varphi)\mathrm{a}_{i,n} + (1-\nu)t_{i,n}]$$

$$-\frac{k_1 k_2 (b_N+\sigma d_N)}{(1-\nu+\sigma\varphi+\sigma\nu)}[\gamma(\varphi+\nu)E_T(g_{i,T+N+1})-(1+\varphi)E_T(\mathrm{a}_{i,T+N+1})$$
$$+(1-\nu)E_T(t_{i,T+N+1})]$$
$$-\frac{k_1 k_2(\varphi+\nu)\eta(\theta-1)(2-\eta)(1-\gamma)}{(1-\nu+\sigma\varphi+\sigma\nu)[1-\nu+\theta(1-\gamma)(\varphi+\nu)]}\sum_{n=T+1}^{T+N}(b_{n-T}-b_{n-T-1}+\sigma d_{n-T}$$
$$-\sigma d_{n-T-1}-\sigma v_{n-T}+\sigma v_{n-T-1}-\sigma k_1 k_2^* w_{n-T}+\sigma k_1 k_2^* w_{n-T-1})$$
$$E_t[\gamma(\varphi+\nu)g_n^*-(1+\varphi)\mathrm{a}_n^*+(1-\nu)\mathrm{t}_n^*]$$
$$+\frac{k_1 k_2(\varphi+\nu)\eta(\theta-1)(2-\eta)(1-\gamma)}{(1-\nu+\sigma\varphi+\sigma\nu)[1-\nu+\theta(1-\gamma)(\varphi+\nu)]}(b_N-\sigma v_N+\sigma d_N-\sigma k_1 k_2^* w_N)$$
$$[\gamma(\varphi+\nu)E_T(\mathrm{g}_{T+N+1}^*)-(1+\varphi)E_T(\mathrm{a}_{T+N+1}^*)+(1-\nu)E_T(\mathrm{t}_{T+N+1}^*)]$$
$$+d_{N+1}E_T(\pi_{i,T+N+1}^i)+\sigma k_1 k_2\frac{(\varphi+\nu)\eta(\theta-1)(2-\eta)w_{N+1}}{(1-\nu+\sigma\varphi+\sigma\nu)\theta}E_T(\pi_{T+N+1}^*)$$
$$+\frac{k_1 k_2}{\sigma}b_{N+1}E_T(x_{i,T+N+1})+\frac{k_1 k_2(\varphi+\nu)\eta(\theta-1)(2-\eta)}{(1-\nu+\sigma\varphi+\sigma\nu)\theta}(\sigma v_{N+1}-b_N$$
$$-\sigma d_N)E_T(x_{t+N+1}^*)$$
$$+\frac{k_1 k_2(\varphi+\nu)\eta(\theta-1)(2-\eta)}{(1-\nu+\sigma\varphi+\sigma\nu)\theta}\sum_{n=T+1}^{T+N}(b_{n-T}-b_{n-T-1}+\sigma d_{n-T}$$
$$-\sigma d_{n-T-1})E_T(x_n^*)\quad (D4)$$