**Food and Non-Food Inflation Rate Uncertainty of Ethiopia: A comparison of Symmetric, Asymmetric and Component GARCH models**

**Teshome Hailemeskel Abebe**

Teshome Hailemeskel Abebe, Department of Economics, Ambo University, Ambo, Ethiopia

E-mail: [teshome251990@gmail.com](mailto:teshome251990@gmail.com)

**Abstract**

*In this study, we applied various ARMA-GARCH family models in food and non-food inflation rate of Ethiopia using data from January 1971 through June 2020. The ADF test results suggest that food inflation rate is stationary at level, while non-food inflation rate is stationary at first difference. Moreover, an ARMA (1, 2)and ARIMA (0, 1, 1) models are identified as the best mean model for food inflation and non-food inflation rate, respectively, based on minimum AIC, BIC & HQIC and forecasting error criteria. The ARCH-Lm test on the residual of the fitted ARMA (1, 2) model for food inflation rate shows on ARCH effect, while the residual from the fitted ARMA (1, 2) model of non-food inflation rate indicates the existence of remaining ARCH effect. Thus, non-food inflation requires GARCH family models. From the estimation results of volatility model, TGARCH(1,1) model with student’s t- distributional assumptions of the residual is the best model for non-food inflation rate as determined by minimum AIC, BIC & HQIC and forecasting error (RMSE, MSE, MAE and TIC) criteria. Therefore, in order to forecast non-food inflation rate TGARCH(1,1) model with student’s t- distributional assumptions of the residual was used.*

Keywords: Food inflation, Non-food inflation, ARMA, GARCH Family, Ethiopia

1. **Introduction**

Historically, Ethiopia inflationary experience was moderate and not considered as series as the issue of economic growth. Since 2004, however, the country has experienced high and persistent inflation growth. Several macro-economic stabilization measures and policies were implemented over the past and seemed to be completely failed. The booming economy has yet remained principally constrained by dual macroeconomic problems i.e. price inflation and low international reserves (Mwanakatwe & Barrow 2010).

Hence, rising inflation has become one of the major economic challenges facing Ethiopia. When the rate of inflation become increasing, people will lose confidence with state of the currency since the currency depreciates which results in need of high wages in the economy and for companies to overcome the wage increases they will increase the prices of goods and services so as to continue making profits in offering their services. Furthermore, an unanticipated inflation have a distributive effects from creditors to debtors, which increasing uncertainty affecting consumption, savings, and borrowing and investment decisions at all. This rise a question of knowing the pattern of inflation rate by consumers, producers, government and economists to plan well in budgeting. Thus, modeling food inflation and non-food inflation were attracting the attention of macroeconomists and policy makers for many years both at the theoretical and at the empirical level.

The volatility of inflation has broad economic and financial implications, and this has motivated a vast literature on modeling such volatility. In this regard, Engle (1982) first introduces the autoregressive conditional heteroscedasticity (ARCH) model to assess the validity of the conjecture of Friedman (1977) that the unpredictability of inflation was a primary cause of business cycles since uncertainty due to this unpredictability would affect the investors’ behavior. Pursuing this idea required a model in which this uncertainty could change over time (see: T. G. Andersen*,* et al.,2009, page 18). Consequently, Bollerslev and Taylor (1986) proposed independently a more generalize ARCH (GARCH) model which is more parsimonious model of the conditional variance than a higher order ARCH model.

However, the GARCH model cannot account for leverage effect, even though they account for volatility clustering and leptokurtosis in a series. This necessitated the development of new and extended models over GARCH that resulted into new models such as EGARCH, TGARCH and GJRGARCH models because the behavior of inflation volatility is asymmetric rather than symmetric resulting the symmetric GARCH model provide misleading estimates of inflation uncertainty. Moreover, Power GARCH (PGARCH) is model introduced by Ding et al. (1993) able to capture and model the long memory property often observed in the series of volatility and Engle and Lee (1999) component GARCH model decompose conditional variance into a short run and long run volatility, separately.

In Ethiopia case, few studies (like a study by [Yegnanew Shiferaw](https://papers.ssrn.com/sol3/cf_dev/AbsByAuth.cfm?per_id=1879020), 2012; Belayneh and Emmanuel, 2014; Anteneh et al., 2014; Abebe et. al., 2020, and Teshome, 2020) were conducted to evaluate the performance of GARCH models on explaining different agricultural product price volatility. However, to the best of my knowledge, no enough studies were conducted separately to model food and food inflation volatility of Ethiopia using GARCH family models.

Thus, this paper aims to modeling food and non-inflation uncertainty (volatility) in Ethiopia over the period January 1971 through June 2020. Unlike the existing literature where the inflation uncertainty is generally proxied by symmetric GARCH, asymmetric GARCH and component GARCH (CGARCH) models were used to model inflation uncertainty of Ethiopia under the study period.

The remainder of the paper is organized as follows. Section 2 describes literature on GARCH family models. Section 3 discuss on the data and methodology. Section 4 presents the results and discussions and section 5 presents conclusions and policy implications.

1. **Literature Review**
   1. **Theoretical literature review**

According to Poon (2005), volatility refers to the spread of all likely outcomes of an uncertain variable. Volatility may disrupt the normal activity of day-to-day life of each individual and greatly affect economic performance.

To account volatility in the model specification, Engle (1982) introduced the conditional heteroscedasticity model called an autoregressive conditional heteroscedasticity (ARCH) model. He noted that in estimating the parameters of ARCH model, the maximum likelihood is more efficient. practitioners are often confronted with over-parameterization problem in empirical applications of the ARCH model. Bollerslev (1986) introduced the generalized ARCH (GARCH) model to solve the problem of over-parameterization usually associated with ARCH model. Bollerslev (1986) argued that a simple GARCH model provide a better fit than an ARCH model with a relatively long lag.

Consequently, modifications to the original GARCH model were proposed to overcome the perceived problems with standard GARCH (p, q) models. Firstly, the non-negativity constraint may be violated in practical applications. Secondly, GARCH models cannot account for asymmetric effect of volatility. To overcome these shortcomings, the asymmetric GARCH family models such as: Threshold GARCH (TGARCH) proposed by Zakoian (1994), and Exponential (EGARCH) proposed by Nelson (1991) were introduced. Threshold GARCH (TARCH) offers a way to capture asymmetries and leverage that is alternative to EGARCH, but requires positivity-driven restrictions. The exponential GARCH (EGARCH) model by Nelson (1991) permits asymmetric effects between positive and negative series. EGARCH model specifies volatility in the form of logarithmic transformation. Hence, there are no restrictions on the parameters to ensure the non-negativity of the variance and on the sign of the model parameters (see: Ma Jose, 2010). Subsequently, Engle and Lee (1999) generalize GARCH to a component GARCH (CGARCH) model that decomposes the conditional variance into transitory and permanent components.

* 1. **Empirical literature review**

Among few studies conducted on modeling inflation volatility using the standard GARCH family models are discussed as follows.

Alfred Barimah (2014) examined the asymmetric effects of inflation on inflation uncertainty in Ghana for the period 1963:4 to 2014:2. He applied an Exponential Generalized Autoregressive Heteroscedasticity (EGARCH) model on monthly inflation rates to estimate inflation uncertainty. From the result, the variance equation indicates that inflation uncertainty varies directly with the rate of inflation in highly inflationary periods.

Johnson Okeyo1 et al. (2016) investigate inflation rate volatility in Kenya using ARCH type model on the data spanning from January 1985 to April 2016. The result of the study showed that the EGARCH (1, 1) with generalized error distribution (GED) was the best in modeling and forecasting Kenya’s monthly inflation rate. They recommended that governments, policy makers interested in modeling and forecasting monthly rates of inflation should take into consideration heteroscedasticity models since it captures the volatilities in the monthly rates of inflation.

Rizvi et al. (2014) studied inflation volatility in 10 Asian economies (such as China, Hong Kong, India, Indonesia, Malaysia, Pakistan, Philippines, Singapore, South Korea and Thailand) using quarterly data from 1991 to 2012 by applying different GARCH family models. The result showed that the leverage parameter is statistically significant, indicating the existence of an asymmetric GARCH model in the model specifications. Thus, the Glosten-Jagannathan- Runkle GARCH (GJR-GARCH) model were an important model in estimating the existence of inflation stabilization of bidirectional causality running between inflation and inflation volatility.

Given the compatibility of those GARCH family model for inflation series, the researcher try to to compare the performance of different GARCH type models for monthly food and non-food inflation uncertainty of Ethiopia.

1. **Data Source and Methodology**
   1. **Data and Nature of the series**

This study use secondary data. The variables are monthly food and non-food inflation rate, which were compiled from National Bank of Ethiopia. Linear time series models such as ARIMA models are unable to explain a number of important features. Those common features are leptokurtosis, volatility clustering, leverage effects and long memory.

* 1. **Stationary and Unit Root Test**

The foundation of time series analysis is stationary. Stationary series are characterized by a kind of statistical equilibrium around a constant mean level as well as a constant dispersion around that mean level (Box and Jenkins, 1976). If a time series is not stationary, it is necessary to look for possible transformations that might induce stationary.

Several statistical tests may be conducted to determine whether a series is stationary or non-stationary. In this study, the commonly used unit root test, the Augmented Dickey Fuller (ADF) test, which controls higher-order correlation, is used. In ADF test, If the null unit root (non-stationarity) is not rejected, apply differencing to make the series stationary.

* 1. **ARMA Model Specification**

The Box–Jenkins method (ARIMA) requires that the discrete time series data be equally spaced over time and that there be no missing values in the series. The ARMA model states that the current value of the series depends linearly on its own previous values plus a combination of current and previous values of a white noise error term.

The general a stationary process under an ARMA(p, q) process is given by

where are the coefficients of an AR model and are MA coefficients, while and are an integer indicating the lags of AR and MA model, respectively.

* 1. **Model selection criteria**

When we estimate the mean ARMA model, there are various model selection criteria, which are based on the likelihood function and the number of free parameters from the fitted ARMA model. In this study, the Akaike's Information Criterion (AIC), Bayesian Information Criterion (BIC) and Hannan Quin Information Criterion (HQIC) were used in this study.

* 1. **Parameter Estimation ARMA Models**

In order to estimate the parameters of an ARMA (p, q) model, the maximum likelihood estimation method that maximizes the joint probability density function of the innovation terms were applied.

* 1. **Model Diagnostic Checking**

After we have estimated our candidate ARMA model and before we are going to interpret its result, it is mandatory to check whether the model is appropriately specified or whether the model assumptions are satisfied.

**Breusch- Godfrey Lagrange Multiplier (LM) Test for Serial Correlation**

This test was developed by Breusch and Godfrey in 1978, and is used to test for serial correlation in the error terms. The Lagrange Multiplier (LM) test for serial correlation is computed first by estimating the sample residuals by ordinary least squares (OLS) and regress the current residual on the p lagged residuals.

The auxiliary regression model of residuals is given by:

where is the original regressors in the ARMA model and is a white noise process. The null hypothesis of no serial correlation up to lag p is .

The Obs\*R-squared statistic is the Breusch-Godfrey LM test statistic. If the statistic from the auxiliary regression is computed for this model, then the following asymptotic approximation can be used for the distribution of the test statistic, .

**Testing Normality of the Residual**

Normality tests are used to ascertain whether the residuals of the regression are normally distributed or not. The null hypothesis is that the residuals are normally distributed. Several tests for normality are available but the most commonly used test for normality of regression disturbances is due to Jarque and Bera (1981). The Jarque-Bera test statistic is given by

where T is the sample size. Under the null hypothesis of normality, the test statistic is asymptotically distributed as . Thus, if test statistic is greater than ), we reject the null hypothesis.

**Testing for ARCH Effect**

The Lagrange multiplier test of Engle (1982) is equivalent to the usual F test. To test the null hypothesis that there is no ARCH up to order p in the residuals, we run the regression of squared the residuals on m own lags to test for ARCH of order m as given by:

Then obtain from this auxiliary regression. The test statistic is defined as (the number of observations multiplied by the coefficient of multiple correlation) from the last regression, which is Engle’s LM test statistic. The LM test statistic is asymptotically distributed as a under quite general conditions. The null hypothesis given by . The decision rule is to reject the null hypothesis.

* 1. **VOLATILITY MODEL SPECIFICATION**

One the mean features of financial time series is time varying volatility which refers to a tendency of small values being followed by small values and large values being followed by large values (Andersen et. al., 2009).

**The ARCH Model**

As stated in Tsay (2005) the basic idea of ARCH models is that: the shock is serially uncorrelated, but dependent and the dependence of can be described by a simple quadratic function of its lagged values.

Then the ARCH (q) process proposed by Engle (1982) is given by

where is inflation series, is innovation or error term from the mean (ARMA) model. The positivity of is ensured by the following sufficient restrictions: and.

An ARCH (q) model is covariance stationary if and only if

**Generalized ARCH (GARCH) models**

A Generalized ARCH (GARCH) model introduced by Bollerslev (1986) gives parsimonious way of estimating the parameters and successful in predicting conditional variances.

Thus, GARCH(p, q) (generalized ARCH due to Bollerslev (1986) is given by:

where is the constant term,for is the effect of all every period’s error (the ARCH effect), and , for is the effect of the previous periods variance (the GARCH effect). Bollerslev (1986) shows that the necessary and sufficient condition for the second-order stationarity of model (6) is . In this case, conditional variance forecasts converge upon the long-term average value of the variance (unconditional variance) as the prediction horizon increases.

**The EGARCH model**

Nelson (1991) introduced the exponential GARCH (EGARCH) model. GARCH successfully captures thick-tailed returns, and volatility clustering. However, it is not well suited to capture the “leverage effect,” since the conditional variance in GARCH model is only a function of the magnitude of the lagged residual and not their signs. However, in EGARCH model, depends on both the size and the sign of lagged residuals and which accounts for such an asymmetric response to a shock (negative shocks).

The EGARCH (p, q) model specifies conditional variance in logarithmic form, which means that there is no need to impose an estimation constraint in order to avoid negative variance.

where is magnitude effect, is lagged log conditional variance, is the asymmetric response parameter or leverage parameter. We expect , indicating that with appropriate conditioning of the parameters, this specification captures the stylized fact that a negative shock (bad news) leads to a higher conditional variance in the subsequent period than a positive shock(good news). The logarithmic formulation of the model guarantees positive conditional variance, without imposing restrictions on the parameters.

**The GJR GARCH model**

It is model developed by Glosten, Jaganathan and Runkle (1993) expressed the leverage effect in a quadratic form while EGARCH expressed in the exponential form.

The conditional variance is now given by:

where is an indicator variable in which

In this case, indicating negative shocks (bad news) have a deeper impact on future volatility than positive shocks.

**TGARCH**

Zakoian introduced threshold GARCH (TGARCH) model in 1994. The threshold GARCH is similar to the GJR model, different only because of the conditional standard deviation and absolute return instead of the conditional variance.

Threshold GARCH (p, r, q)) process is defined as:

The conditional volatility is positive when and

In TGARCH we expect to be positive, so that bad news would have a more powerful effect on volatility than good news.

**The Power GARCH (PGARCH) Model**

Ding et al. (1993) introduce the power GARCH model that has the advantage of being able to capture and model the long memory property often observed in volatility series. The primary feature of the power GARCH(p, q) model is the presence of a Box-Cox power trans- formation of the conditional variances.

The Power GARCH (PGARCH) is modeled introduced by Ding et al. (1993) is defined as:

where is the power term parameter and should be greater than zero. The asymmetric effect presents if , and .

**The Component GARCH (CGARCH) Model**

Component GARCH model introduced by Engle and Lee (1993) decompose conditional variance into a temporary or a permanent component. In this study, the component GARCH models are employed to decompose inflation uncertainty into short-run and long-run component by permitting transitory deviations of the conditional volatility around a time-varying trend.

The component GARCH (1, 1) model can be expressed as follows:

where and indicates short run memory, while is the time varying long-run volatility (long run memory). The first equation describes the transitory (short-term) component, which converges to zero with power (. The second equation describes the long-run component, which converges to with powers of .

* 1. **Estimation of ARCH/GARCH models**

The ARCH family models are estimated by maximum likelihood estimation method. It can be employed to find parameter values for both linear and non-linear models (see, Brooks, 2008). However, the GARCH type model needs specification of the distribution assumption of the error term: normal (Gaussian), t-distribution and, Generalized Error Distribution (GED).

* **Normal Distribution**

Engle (1982) and Bollerslev (1986 developed the distribution of the innovations has a standardized normal probability function.

where the probability function or density is named standardized, marked by a star because has zero mean and unit variance.

* **Student t Distribution**

Bollerslev [1987] proposed the standardized Student-t distribution with degree of freedom, which better captures the observed kurtosis.

The Standardized Student-t distribution density function expressed as

where (.) is the usual gamma function, is the degree of freedom which represents the parameter to be estimated. Like, the normal distribution, the t distribution is symmetric around zero mean and its variance, and kurtosis respectively. However, for the density of standardized student-t distribution converges to the density function of standardized student normal distribution.

* **Generalized Error Distribution (GED)**

Nelson [1991] suggested considering the family of Generalized Error Distributions, GED. The GED is a symmetric distribution that can be both leptokurtic and platykurtic depending on the degree of freedom.

When assume a GED has the following density function:

where is tail -thickness parameter,

For, the GED is a standard normal distribution whereas the tails are thicker than in the normal case when, and thinner when. The GED becomes a uniform distribution on the interval when.

* 1. **Model selection**

An important practical problem is the determination of the ARCH order p and the GARCH order q for a particular series. Since GARCH models can be treated as ARMA models for squared residuals, traditional model selection criteria such as the Akaike information criterion (AIC), the Bayesian information criterion (BIC) and Hannan-Quinn Criteria (HQC).

* 1. **Model Adequacy Checking**

After a GARCH model has been fit to the data, the adequacy of the fit should be evaluated. In this study, we apply the ARCH-LM Test for standardized residuals of the fitted GARCH type models.

* 1. **Volatility Forecasting**

Tsay (2005) stated again that the forecasts of the GARCH model are obtained similarly as the forecasts of an ARMA model. If we consider a GARCH (1, 1) model, which is one of the GARCH models under study at the forecast origin k, the 1-step ahead forecast of

For the general GARCH (1, 1) step head forecast of, at origin k, is

* 1. **Measuring the Accuracy of Volatility Models forecasting**

Evaluation of univariate volatility forecasts is relatively straightforward and relies on standard forecast evaluation techniques. Among the common statistical methods, which can be used to observe the prediction accuracy of a model, the root mean square error (RMSE), the mean absolute error (MAE), the mean absolute percent error (MAPE), and the Theil inequality coefficient (TIC) are used in this study. The forecasting statistics are given as follows:

where is one-step head volatility forecast, is the actual volatility and T is the number of forecasts or the number of time or year in the out-of-sample period.

The Mean Absolute Deviation (MAD) is interesting since it is very robust to outliers and this criterion actually gives equal weighting to a large deviation of size z as to a sum of several deviations accumulating to z.

The Theil Inequality Coefficient (TIC) is a scale invariant measure that always lies between zero and one, where zero indicates a perfect fit.

The smaller is the error in the first three forecast error statistics, the better the forecasting ability of that model according to that criterion.

1. **Results and Discussions**
   1. **Results of Descriptive Statistics**

The data used in this study were monthly food and non-food inflation rate of Ethiopia from the period January 1971 through June 2020. To analyze the series, ARMA-GARCH family models were used.

**Graphical analysis**

The first step in time series analysis is time plot of the original series in level against time and observes its graphical properties. This help in understanding the trend as well as pattern of movement of the original series. Here we plot the original series of food & non-food inflation rate in Ethiopia as function of time. The time plots are presented in Figure 1 &2. On Figure 1, food inflation rate looks like white noise series and varying about zero, i.e. close to stationary, while non-food inflation rate on Figure 2 shows somehow non-stationary since the fluctuation rate is relatively high. However, the plot of series by itself is not an end, rather we use as a clue.





Figure 1: The time plot of food-inflation rate

Figure 2:Time plot of non-food inflation rate

**Summary statistics**

Table 1 shows the summary statistics of food and non-food inflation rate. The table reveals the positive mean food and non-food inflation rate of 10.77 and 8.03, respectively. It also shown that monthly food inflation falls to lowest level (-52.6) on July 2001 and reaches its maximum level (91.7) on July 2008.

Moreover, a very high Jarque Berra (J-B) value 343.3 for food inflation and 42.7 for non-food inflation rate and a very small corresponding p-value, following the null of normality was rejected for the data. To support the inference on normality, the skewness (0.99) and (0.65) for food and non-food inflation, respectively are greater than 0 (skewness of a normal distribution is 0) and the kurtosis (6.14) and (3.03) are higher than 3 (kurtosis of a normal distribution is 3). The positive skewness is an indication that the upper tail of the distribution is thicker than the lower tail which implies that the rises more often than it drops, reflecting the renewed confidence in the market.

Table 1: Descriptive statistics of food and non-food inflation rate

|  |  |  |
| --- | --- | --- |
| Statistics | Food inflation rate | Non-food inflation rate |
| Mean | 10.77712 | 8.034845 |
| Std. Dev. | 16.7424 | 7.645212 |
| Min | -52.64756 | -8.082834 |
| Max | 91.73248 | 30.2925 |
| Skewness | 0.993686 | 0.656414 |
| Kurtosis | 6.149851 | 3.037737 |
| Jarque-Bera | 343.3124 | 42.69227 |
| Probability | 0.000000 | 0.000000 |
| Obs | 594 | 594 |

Source: Author’s Computation

* 1. **Unit root test results**

The time series, which is considered in the given study, should be checked for stationarity before we fit a suitable model. In this study, an Augmented Dickey-Fuller test (ADF) test is used to check the stationarity of the monthly inflation series. In the case of dickey fuller test, there may be autocorrelation problems. To tackle such autocorrelation problem, Dickey fuller has developed a test called Augmented Dickey Fuller (ADF) test. In ADF test, the null hypothesis stated that the variable is not stationary or have a unit root test.

The results of the ADF test statistic for food and non-food inflation series are shown below. Normally we use 5% critical value to evaluate the stationarity condition of the series. For example, the test statistic for food inflation rate with constant term and constant & linear trend are 4.7067 and 4.7756 in absolute value, which is greater than 5% critical value (2.86 & 3.4175), respectively, indicating rejection of the null hypothesis of non-stationarity. However, the non-food inflation rate is non-stationary at level since the test statistic with constant (2.47) and constant & linear trend (2.6456) in absolute value is less than 5% critical value (2.86 &3.41), respectively, indicating fail to reject the null of non-stationarity. Thus, we need to apply first difference to make it stationary as indicated in the Table 2. From the results, the first difference of non- food inflation rate is stationary since the test statistic with constant (12.37) and constant & linear trend (12.36) in absolute value is greater than 5% critical value (2.86 & 3.13), respectively.

Table 2: Unit root tests for the series at level and difference

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Variables |  | ADF Test Critical values | | | | |
|  | t-statistic | 1% | 5% | 10% | P-value | |
| Food inflation rate (level) | With constant | -4.7067 | -3.441 | -2.866 | -2.5693 | 0.0001 | |
| Constant & linear trend | -4.7756 | -3.973 | -3.4175 | -3.1312 | 0.0005 | |
| Non-Food inflation rate (level)  Non-Food inflation rate(first difference) | Constant | -2.4757 | -3.4413 | -2.8662 | -2.5693 | 0.1220 | |
| Constant & linear trend  Constant  Constant & linear trend | -2.6456  -12.374  -12.363 | -3.9739  -3.4413  -3.9739 | -3.4175  -2.8662  -3.4175 | -3.1312  -2.569  -3.131 | 0.2601  0.0000  0.0000 | |

Source: Author’s Computation

* 1. **ARIMA type model estimation results**

Before we specify volatility model for the given series, we should to specifying a mean equation. In this study, an Autoregressive Moving Average model (ARMA) type models specify the conditional mean equations for the food and non-food inflation rate.

Given the significance of the coefficients and absence of serial correlation in the residuals and smallest value of information criteria, the following models are determined. From the results on Table 2, ARMA(1,2) model was identified the beast mean model for estimating the coefficients of food inflation rate.

Table 2. Estimation Results of ARMA Models for food inflation with Information Criteria

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Model | Parameter | Coefficients | Std. error | t-statistic | P-value | Information criteria | | |
| AIC BIC HQIC | | |
| ARMA(1,0) |  | 10.8766 | 3.6322 | 2.9944 | 0.0029 | 6.543 | 6.565 | 6.551 |
|  | -0.9244 | 0.0099 | 92.6001 | 0.0000 |
| ARMA(2,0) |  | 10.860 | 3.2048 | 3.3887 | 0.0007 | 6.531 | 6.560 | 6.542 |
|  | 1.0395 | 0.0357 | 29.06785 | 0.0000 |
|  | -0.1242 | 0.03691 | -3.36549 | 0.0008 |
| ARMA(1,1)) |  | 10.8624 | 3.1966 | 3.39806 | 0.0007 | 6.524 | 6.554 | 6.536 |
|  | 0.8965 | 0.0119 | 75.0989 | 0.0000 |
|  | 0.1942 | 0.0266 | 7.2963 | 0.0000 |
| **ARMA(1,2)** |  | 10.896 | 3.6709 | 2.9683 | 0.0031 | 6.512 | 6.549 | 6.526 |
|  | 0.9198 | 0.0170 | 53.9630 | 0.0000 |
|  | 0.1647 | 0.0404 | 4.0706 | 0.0001 |
|  | -0.1217 | 0.0256 | -4.7477 | 0.0000 |

Source: Author’s Computation

Note: Models with no serial correlation in the residuals are considered.

From the results of Table 2, ARIMA(0,1,1) model was identified as the beast mean model for estimating the coefficients of non-food inflation rate using the AIC, BIC and HQIC.

Table 3. Estimation Results of ARIMA Models for non-food inflation with Information Criteria

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Model | Parameter | Coefficients | Std.error | t-statistic | P-value | Information criteria | | |
| AIC BIC HQIC | | |
| ARIMA(1,1,0) |  | 0.0343 | 0.1023 | 0.3354 | 0.7374 | 5.111 | 5.133 | 5.119 |
|  | -0.2616 | 0.0275 | -9.5036 | 0.0000 |
| ARIMA(2,1,0) |  | 0.0342 | 0.0970 | 0.3523 | 0.7247 | 5.111 | 5.140 | 5.122 |
|  | -0.2771 | 0.0299 | -9.2520 | 0.0000 |
|  | -0.0592 | 0.0299 | -1.9768 | 0.0485 |
| **ARIMA(0,1,1)** |  | 0.0340 | 0.0956 | 0.3563 | 0.7217 | 5.110 | 5.132 | 5.119 |
|  | -0.2605 | 0.0301 | -8.6345 | 0.0000 |

Source: Author’s Computation

Note: Models with no serial correlation in the residuals are considered.

* 1. **Model adequacy checking**

Before we consider the fitted model as the best fit and interpret its results, it is mandatory to check whether the model assumptions are satisfied. If the basic model assumptions are violated, then a new model should be specified until it provides an adequate fit to the data.

**Test of serial correlation in the residuals**

In this case, serial correlation in the residuals was tested using the Breusch-Godfrey Serial Correlation LM Test for each of the tentatively selected ARMA models: ARMA (1, 2) and ARIMA (0, 1, 1) models for the conditional mean of food inflation and non-food inflation rate, respectively. The null hypothesis asserts that there is no serial correlation in the residual series. As we observe from Table 4, the serial correlation LM test results for this equation with 1 lags in the test equation strongly reject the null of no serial correlation.

Table 4: Results of Breusch-Godfrey Serial Correlation LM Test of the fitted model

|  |  |  |
| --- | --- | --- |
| Test statistic | Food inflation rate | Non-food inflation rate |
| F-statistic | 2.247 | 0.9405 |
| (0.106) | (0.391) |
| Obs\*R-squared | 14.571 | 1.897 |
| (0.104) | (0.387) |

Source: Author’s Computation

Note: Values inside the bracket are p-values

**Normality test of residuals from the mean equation**

To investigate whether the residuals of the fitted model (mean equation) are normally distributed, the Jarque-Bera test was applied. The residuals normality from ARMA (1, 2) for food inflation and ARIMA(0,1,1) for non-food inflation rate were conducted and reported in Table 5. We can see from Table 5 that the Jarque-Bera statistic is not significant, and hence, there is no significant evidence to reject the null hypothesis of normality. This indicates that the residuals of the fitted models are normally distributed for both of the series under consideration.

Table 5: Normality test of the residuals from the fitted mean model

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Variables | Skweness | Kurtosis | Jarque-Bera Statistic | P-value |
| Food inflation rate | -0.035 | 3.528 | 2.711 | 0.257 |
| Non-food inflation rate | -0.1085 | 3.722 | 4.008 | 0.134 |

Source: Author’s Computation

**Test of ARCH Effect Results**

Before we estimate ARCH type models, there should be volatility clustering and ARCH effect in the residuals of the estimated ARMA(1,2) for food inflation and ARIMA(0,1,1) for non-food inflation rate.

**Table 6.** Result for ARCH LM Test for the fitted models

|  |  |  |
| --- | --- | --- |
| Test statistic | Food inflation rate | Non-food inflation rate |
| F-statistic | 0.2627 | 6.183054 |
| (0.6084) | (0.01325) |
| Obs\*R-squared | 0.2635 | 6.139671 |
| (0.6077) | (0.0132) |

Source: Author’s Computation

**Note**: Values inside parenthesis are p-values.

From the Table 6, we observe that the p-value for food inflation rate is greater than 5% which indicates fail to reject the null of homoscedastic variance in the error term of ARMA(1,2) model. However, the p-value on ARMA(0,1,1) model of non-food inflation rate is less than 5 % indicating to reject the null of homoscedastic variance. Therefore, food inflation rate has a constant variance while non-food inflation rate has a non-constant variance (heteroscedasticity), which requires an application of GARCH type model for non-food inflation rate.

* 1. **Estimation result of ARAM model for food inflation rate**

In order to identify the appropriate ARMA model, the minimum information criteria, absence of serial correlation on the residual, and the most significant coefficients were used. The AR slope coefficients of the model are statistically significant at the 1% marginal significant levels. Thus, the first and second lags of non-food inflation rate have positively predicted the future value of non-food inflation rate. That is the past realization of non-food inflation rate will influence non-food inflation rate at a 1% level. The moving average coefficient is negative and statistically significant at the 1% level, which means the residuals of the first lag will negatively predict non-food inflation rate at the 1% level. Table 7 summarizes the results as below.

Table 7: Interpretation of ARAM (1, 2) model for food inflation rate

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Fitted Model | Parameters | Coefficients | Std.error | t-statistic | P-value |
| ARMA(1,2) |  | 10.89654 | 3.670936 | 2.968327 | 0.0031 |
|  | 0.919852 | 0.017046 | 53.96304 | 0.0000 |
|  | 0.164757 | 0.040475 | 4.070606 | 0.0001 |
|  | -0.121749 | 0.025643 | -4.747798 | 0.0000 |

Source: Author’s Computation

Note: Models with no serial correlation in the residuals are considered.

* 1. **Forecasting**

Before we use the fitted model to forecast the value of the of food inflation rate, we should to compare the forecasting performance of the candidate model using different error criteria, such as RMSE, MAE, MAPE and Theil’s inequality coefficient. From the results in Table 8, the fitted ARMA (1, 2) model has minimum error as compared to other fitted ARMA models which are determined based on minimum information criteria and absence of serial correlation on the residuals.

Table 8: Forecasting evaluation of different ARMA type model for food inflation rate

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Model | Forecasting accuracy Measure | | | |
| RMSE MAE MAPE Theil | | | |
| ARMA(1,0) | 16.697 | 11.8568 | 709.577 | \*0.5441 |
| ARMA(2,0) | 16.651 | 11.8207 | 689.742 | 0.5439 |
| ARMA(1,1) | 16.7104 | 11.8663 | 708.995 | 0.5445 |
| **ARMA(1,2)** | 16.6380 | 11.8120 | 691.244 | 0.5433 |

Source: Author’s Computation

Note: Models with no serial correlation in the residuals are considered.

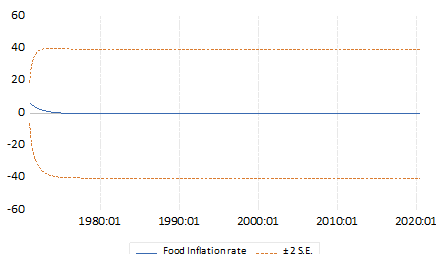


Figure 3: Forecasting the food inflation rate using ARMA (1, 2) model

* 1. **Estimation Results of GARCH type models**

Once the presence of ARCH effects on the residuals of the fitted mean model is confirmed, then we need to estimate the series using GARCH type models. However, before we defined the final model, the optimal lag for GARCH family models has to be determined. In this case, the parameters of the models are estimated using maximum likelihood method under the assumption of different error distributions.

**Model Selection of GARCH Family Model**

In order to determine the order of GARCH type models, the Akaikian information criterion (AIC), Bayesian information criterion (BIC) and Hannan-Quinn Information Criteria (HQIC) were used for selecting the symmetric, asymmetric, and component fitted GARCH models. From Table 9, we observed that TGARCH (1,1) and PGARCH(1,1) models under normal distribution, EGARCH (1,1), TGARCH (1,1), PGARCH(1,1) and CGARCH(1, 1) models under Student’s t-distribution, and GARCH (1, 1), EGARCH (1,1), TGARCH (1,1) and PGARCH(1,1) models under Generalized error distributional assumption of the residuals were selected as candidate models using minimum AIC, BIC and HQIC. Thus, based on the minimum information criteria, TGARCH (1,1) with student’s t-distributional assumption for residuals was identified as the best performing model selected candidate model.

Table 9:Optimal lag selection based AIC, BIC and HQIC under different error distribution

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Model | Error distribution | AIC | BIC | HQIC | Asymmetric effect |
| GARCH(1,1) | Generalized error distribution | 4.6513 | 4.6956 | 4.6686 | \* |
| EGARCH (1,1) | Student’s t-distribution | 4.6354 | 4.6871 | 4.6555 | Significant |
| EGARCH (1,1) | Generalized error distribution (GED) | 4.6488 | 4.7006 | 4.6690 | Significant |
| TGARCH (1,1) | Normal distribution | 4.7120 | 4.7564 | 4.7293 | Significant |
| TGARCH (1,1) | Student's t distribution | 4.6293 | 4.6811 | 4.6495 | Significant |
| TGARCH (1,1) | Generalized error distribution (GED) | 4.6437 | 4.6954 | 4.6638 | Significant |
| PGARCH(1,1) | Normal distribution | 4.7161 | 4.7604 | 4.7333 | Significant |
| PGARCH(1,1) | Student’s t-distribution | 4.6357 | 4.6875 | 4.6559 | Significant |
| PGARCH(1,1) | Generalized error distribution | 4.6498 | 4.7016 | 4.6700 | Significant |
| CGARCH(1, 1) | Student's t distribution | 4.6386 | 4.6977 | 4.6616 | \* |

Source: Author’s Computation

In addition to information criteria, forecasting performance of the candidate GARCH type models were used to identify an appropriate conditional volatility model. The basic accuracy statistics are RMSE, MAE, MAPE and Theil inequality coefficient as shown in Table 10 below. The models with the smallest statistics are used as the best fit for modeling the conditional volatility of of non-food inflation rate.

Table 10: Forecast accuracy statistics for GARCH type model for non-food inflation

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Model | Error distribution | Forecasting accuracy Measure | | | |
| RMSE MAE MAPE Theil | | | |
| GARCH(1,1) | Generalized error distribution | 3.2130 | 2.1538 | 104.696 | \*0.9802 |
| EGARCH (1,1) | Student’s t-distribution | 3.2141 | 2.1550 | 112.481 | 0.9637 |
| EGARCH (1,1) | Generalized error distribution (GED) | 3.2135 | 2.1543 | 109.192 | 0.9702 |
| TGARCH (1,1) | Student's t distribution | 3.2131 | 2.1542 | 111.165 | 0.9662 |
| TGARCH (1,1) | Generalized error distribution (GED) | 3.2134 | 2.1542 | 108.461 | 0.9717 |
| PGARCH(1,1) | Student’s t-distribution | 3.2138 | 2.1547 | 111.035 | 0.9664 |
| PGARCH(1,1) | Generalized error distribution | 3.2134 | 2.1542 | 108.289 | 0.9721 |
| CGARCH(1, 1) | Student's t distribution | 3.2133 | 2.1541 | 107.833 | 0.9730 |

Source: Author’s Computation

From the results on Table 10, TGARCH (1, 1) with Student's t-distributional assumption for residuals perform better to describe volatility since they possess the smallest forecast error measures in the majority of the statistics considered for non-food inflation rate.

**Parameter Estimation Results**

Once the TGARCH (1, 1)model with student’s t-distributional assumption for residuals was selected, as better fit based on information criteria and forecast accuracy measures, then the next step is to interpret the result and forecasting future value of the series. The parameters in the TGARCH (1, 1)model are estimated using the maximum likelihood (ML) method, which are presented on Table 11.

Table 11: Estimation results of TGARCH (1, 1) model for non-food inflation rate

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Variables | Coefficients | Std.error | t-statistic | P-value |
|  | 0.0304 | 0.0259 | 1.1755 | 0.2398 |
|  | 0.1590 | 0.0450 | 3.5329 | 0.0004 |
|  | 0.1376 | 0.0489 | 2.8101 | 0.005 |
|  | 0.9199 | 0.0210 | 43.7219 | 0.0000 |

Source: Author’s Computation

The result on Table 11 indicates that a one month lagged shocks (i.e. ARCH (-1)) of the monthly non-food inflation rate is statistically significant at the 1% level. This indicates that the current month non-food inflation volatility was affected by its 1-month lagged shocks. This may be an indication that current non-food inflation volatility is sensitive to past inflation movements. Similarly, GARCH (-1) terms is which indicates volatility persistence is statistically significant at the 1% level. This indicates that current month inflation volatility affected by its 1-month lagged inflation volatility.

Moreover, the coefficient of the asymmetric term is positive (0.1376) and statistically significant at the 1% level, indicates that bad news (unexpected increase in monthly non-food inflation) has larger impact on the non-food inflation volatility than good news (unexpected decrease in monthly food-inflation volatility). Thus, modeling of information, news of events is very significant determinants of volatility.

* 1. **Model Checking**

In order to check whether the fitted models are good fit to the data ARCH-LM Test for standardized residuals of the fitted TGARCH(1,1) model was performed. As can be seen in Table 12, the ARCH-LM test indicates that the standardized residuals of the fitted model did not exhibit any additional ARCH effect. Therefore, the selection of TGARCH(1,1) model with student’s t distributional assumption of residuals to investigate non-food inflation rate volatility was well justified.

Table 12: ARCH-LM Test for Standardized Residuals of the Fitted TGARCH(1,1) model

|  |  |
| --- | --- |
| Test statistic | Estimates |
| F-statistic | 0.3756 |
| (0.5501) |
| Obs\*R-squared | 0.3585 |
| (0.5493) |

Source: Author’s Computation

Note: Values in parenthesis are p-values.

* 1. **Forecasting**

One of the fundamental uses of developing GARCH model is forecasting. In this section, we examine the forecasting accuracy of the fitted models and then we make in-sample forecasts.

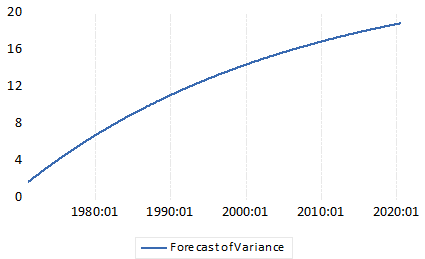


Figure 4: In-sample forecast of non-food inflation volatility using TGARCH(1,1) model.

As we observe from Figure 4, a continuous rise in the volatility of non-food inflation rate was observed.

1. **Conclusion and Recommendations**

An increase in inflation volatility implies higher uncertainty about future prices. As a result, producers and consumers can be affected by the increased inflation volatility, because it increases the uncertainty and the risk in the market. Thus, inflation volatility attracts the attention of researchers to find a suitable model, which can predict the future conditions of the market. This study aims to fit appropriate ARMA-GARCH family models for food and non-food inflation rate of from the period January 1971 through June 2020.

In the preliminary analysis, food inflation rate shows a white noise property, while non-food inflation rate has somehow fluctuation having the characteristics of financial time series such as leptokurtic distributions, which leads to an adequate ground to apply GARCH family models. The result of unit root test shows that food inflation is stationary at level, while non-food inflation rate is stationary at first difference.

On the estimation results of the mean equation, an ARMA type model is appropriate for food inflation rate since ARCH-LM test on the squared residuals of the best fitted ARMA (1, 2) model confirmed the absence of remaining ARC H effect. Thus, we apply an ARMA (1, 2) model for food inflation rate to estimate the coefficients and forecast the future series. However, the ARCH–LM test on the residual of ARIMA (0, 1, 1) model on non-food inflation rate shows the existence of remaining ARCH effect which needs to requires the application of GARCH family models.

In the estimation of volatility family models for non-food inflation rate, TGARCH (1,1) model with Student’s t-distributional assumption of residual was selected as the best fitted model among different kind of candidate models using information criteria (AIC, BIC & HQIC) and forecast error criteria (such us: MAE, MAPE, RMSE and Theil inequality coefficient).

The result of TGARCH (1, 1) model shows that, a one month lagged shocks (i.e. ARCH (-1)) of the monthly non-food inflation rate is statistically significant at the 1% level indicates that the current month non-food inflation volatility was affected by its 1-month lagged shocks. This may be an indication that current non-food inflation volatility is sensitive to past inflation movements. Similarly, GARCH (-1) terms is which indicates volatility persistence is statistically significant at the 1% level. This indicates that current month inflation volatility affected by its 1-month lagged inflation volatility. Moreover, the coefficient of the asymmetric term is positive and statistically significant at the 1% level, indicates that bad news (unexpected increase in monthly non-food inflation) has larger impact on the non-food inflation volatility than good news (unexpected decrease in monthly food-inflation volatility). Thus, modeling of information, news of events is very significant determinants of volatility and GARCH family models are appropriate for the given series (monthly food-inflation volatility) of Ethiopia under the study period considered.

Therefore, inflation volatility which brings risks to consumers especially fixed income earners than producers the concerned stakeholders particularly the government pays careful attention because attempting to avoid such volatility costs the economy far more than its direct costs and leads to inefficiencies and benefits only some part of society. This is due to a direct government interventions to curb inflation volatility can distort markets and lead to resource misallocation if markets are not regulated properly.

**List of Abbreviations**

AIC Akaike's Information Criterion

ARCH-Lm Autoregressive conditional heteroscedasticity Lagrange Multiplier

ARMA Autoregressive Moving Average

BIC Bayesian Information Criterion

CGARCH Component GARCH

EGARCH Exponential Generalized Autoregressive conditional heteroscedasticity

GARCH Generalized Autoregressive conditional heteroscedasticity

GED Generalized error distribution

GJR-GARCH Glosten-Jagannathan- Runkle GARCH

HQIC Hannan Quin Information Criterion

MAE Mean Absolute Error

MSE Mean Squared Error

PGARCH Power GARCH (

RMSE Root Mean Squared Error

TGARCH Threshold Generalized Autoregressive conditional heteroscedasticity

TIC Theil's Inequality Coefficient

**Competing interests**

The author declares that he has no competing interests.

**Author’s contributions**

Teshome Hailemeskel Abebe is the only author of this paper. He collects the data, analysis, and interprets the results, and draft manuscript preparation. He also prepare and approved the final version of the manuscript for publication.

**Availability of supporting data**

The data will be available upon reasonable request.

**Funding**

The author did not receive any financial support for the research, authorship, and/or publication of this article.

**Acknowledgements**

First, I would like to thank the Almighty God, for being with me in all aspects during my study. My thank goes to the staff member of the National Bank of Ethiopia, Mr. Bizuayehu Samuel for his cooperation during the time of data collection. At last, my deepest and heartfelt gratitude goes to my wife Mrs. Misa Getu for her encouragements and support throughout my study.

**Reference**

Anderson, G. T. (2009). *Hand book financial time series*. Department of Finance, Kellogg School of Management, Northwestern University, USA.

Anteneh Asmare Godana, Yibeltal Arega Ashebir, Tewodros Getinet Yirtaw. (2014). Statistical Analysis of Domestic Price Volatility of Sugar in Ethiopia. *American Journal of Theoretical and Applied Statistics.* Vol. 3, No. 6, 2014, pp. 177-183. doi: 10.11648/j.ajtas.20140306.12.

Barimah, A. (2014). Exponential GARCH Modelling of the Inflation-Inflation Uncertainty Relationship for Ghana. Modern Economy, 5, 506-519. <http://dx.doi.org/10.4236/me.2014.55048>.

Belayneh Debasu Kelkay and Emmanuel G/Yohannes. (2014). The Application of Garch Family Models for Agricultural Crop Products in Amhara Region, Ethiopia. Science, Technology and Arts Research Journal, 3(4): 49-58, DOI: http://dx.doi.org/10.4314/star.v3i4.7.

Bollerslev, T. (1986). Generalized Autoregressive conditional heteroscedasticity. *Journal of Econometrics,* 31:307-327.

Bollerslev, T. (1987) A conditionally heteroskedastic time series model for speculative prices and rates of return, *Review of Economics and Statistics*, 69, 3, 542–547.

Box, G.E.P. & Jenkins, G.M. (1976). *Time Series Analysis: Forecasting and Control*, Revised Edition, San Francisco: Holden Day.

Breusch, T. S. (1978). Testing for autocorrelation in dynamic linear models. Australian Economic Papers., 17:334 – 355.

Brooks, C. (2008). *Introductory Econometrics for Finance. (2nd edition.).* Cambridge: Cambridge University Press.

Dickey, D. A. and Fuller, W. A. (1979). Distributions of the Estimators for Autoregressive Time Series with a Unit Root. *Journal of the American Statistical Association,* ***74****, 427-* *431.*

Ding, Z., Granger, C.W.J. and Engle, R.F. (1993) “A long memory property of stock market returns and a new model” *Journal of Empirical Finance* (1) pp. 83 - 106.

Engle, R. F. (1982). Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation. *Journal of Econometrics*, 50; 987-1007.

Engle, R. F., Lee, G. G. J. (1999) A Long-Run and Short-Run Component Model of Stock Return Volatility, in Cointegration, Causality, and Forecasting (Engle R. F. and White H., eds.), *Oxford University Press*.

Friedman, M. (1977), Nobel Lecture: Inflation and Unemployment. *Journal of Political Economy,* 85 (3): 451–472.

Glosten, L. R., R. Jagannathan, and D. Runkle. (1993). Relationship between the expected value and the volatility of the nominal excess return on stocks. *Journal of Finance;* 48:1779-1801.

Godfrey, L. G. (1978). Testing against general autoregressive and moving average error models when the regres- sors include lagged dependent variables. Econometrica, 46(6):1293 – 1302.

Jarque, C. M.; Bera, A. K. (1981). Efficient tests for normality, homoscedasticity and serial independence of regression residuals: Monte Carlo evidence. *Economics Letters,* **7** (4): 313–318.

Johnson Okeyo1, Mwaniki Ivivi and Philip Ngare. (2016). Modelling Inflation Rate Volatility in Kenya Using Arch –Type Model Family. *Research Journal of Finance and Accounting*, ISSN 2222-1697 (Paper) ISSN 2222-2847 (Online) Vol.7, No.23.

MaJose R. V. (2010). Volatility Models with Leverage Effect. Doctoral Thesis. Universidad Carlos III De Mardrid.

Mwanakatwe, P. and L. Barrow. (2010). 'Ethiopia’s Economic Growth Performance: Current Situation and Challenges', The African Development Bank Group, *Economic Brief,* 1(5).

Nelson, Daniel B. (1991). “Conditional Heteroscedasticity in Asset Returns: A New Approach.” *Econometrica.* 59:2, pp. 347–70.

Poon, Ser-Huang. (2005). A Practical Guide to Forecasting Financial Market Volatility. John Wiley & Sons Ltd, The Atrium, Southern Gate, Chichester, West Sussex PO19 8SQ, England.

Shiferaw, Yegnanew. (2012). Modeling Price Volatility for Some Selected Agricultural Products in Ethiopia: ARIMA-GARCH Applications. Available at SSRN: <https://ssrn.com/abstract=2125712> or [http://dx.doi.org/10.2139/ssrn.2125712](https://dx.doi.org/10.2139/ssrn.2125712)

Syed Kumail Abbas Rizvi, Bushra Naqvi, Christian Bordes & Nawazish Mirza. (2014). Inflation volatility: an Asian perspective. *Economic Research-Ekonomska Istraživanja*, 27:1, 280-303, DOI: 10.1080/1331677X.2014.952090.

Teshome Hailemeskel Abebe, Emmanuel Gabreyohannes and Belaineh Legesse. (2020). Statistical Analysis of Price Volatility of Agricultural Commodities Traded at the Ethiopian Commodity Exchange (ECX) Using Multiplicative GARCHMIDAS Two-component Model. *Global Business Review*, 21(1). DOI: 10.1177/0972150919895628.

Teshome Hailemeskel Abebe. (2020). Modeling time-varying coffee price volatility in Ethiopia, *Journal of Applied Economics*, 23:1, 497-518, DOI: 10.1080/15140326.2020.1804304.

Tsay, R. S. (2005). *Analysis of Financial time series*, 2nd edition, John Willey, ISBN: 978-0-471-74618-8

Zakoian, J. (1994). Threshold heteroscedasticity models. *Journal of Economic Dynamics and Control, 18*, 931–944.