Predictive Dynamic Linear Models for External Reserves-Economic Growth Nexus-The Case of Nigeria

Olushina Olawale Awe

[1]Department of Mathematics (Statistics Unit), Obafemi Awolowo University, Ile-Ife, Nigeria.
[1]

Abstract

Over the years, there has been a debate whether there is a need to beef up the level of foreign reserves or trim them down, and this debate is becoming more interesting especially in a developing country like Nigeria. It is usual for countries in the world to hold external reserves in order to have a favourable level of exchange rate especially with a view of stabilizing and establishing a robust economy. Most previous studies had concentrated on modeling External Reserves-Economic Growth Nexus with classical econometric models with static parameters.

In this paper, we propose a Bayesian time-varying parameter dynamic linear model for econometric modeling of external reserves-economic growth nexus using the Nigerian economy as a case study. We assess the predictive performance of external reserves on economic growth in comparison with some selected macroeconomic variables. Our empirical findings reveal that external reserve has the least Mean Squared Prediction Error (MSPE) among the several one-regressor models considered over the years, while the model involving the combination of external reserves and capital expenditure has the least MSPE among the two regressor models considered in our econometric analysis.The economic implications of these results were discussed and used to make policy recommendations.

Key Words: Dynamic Model, Bayesian Inference, Kalman Filter, External Reserves, , MCMC.

1 Introduction

External Reserve is a major economic indicator that has been variously described as International Reserves (IR), Foreign Reserves (FR) or Foreign Exchange Reserves (FER). While

[1]Corresponding email: oau.lisa@gmail.com or ooawe@oauife.edu.ng

there are several definitions of external reserves, the most widely accepted is the one proposed by the International Monetary Fund (IMF) in its balance of payments manual and guidelines on foreign exchange reserve management (2001) which defined external reserves as consisting of official public sector foreign assets that are readily available to, and controlled by the monetary authorities, for direct financing of payment imbalances through intervention in the exchange markets (Kyereboah-Coleman, 2009). There has been a debate among researchers on whether there is a need to beef up the level of foreign reserves or trim them down, and this debate is becoming more interesting especially in the context of a developing economy like Nigeria. Some researchers are of the opinion that keeping scarce resources in reserve when there is a series of burning issues to be attended to domestically may not be a very wise decision (Osabuohien and Egwakhe, 2008).

However, some other researchers have argued that the foreign reserve position determines the rating in the global competitive market of a country and will make the country appear financially responsible and creditworthy. Aluko (2007) opined that external reserves has, in recent times, played a significant role in growing the Nigerian economy by increasing the level of money supply and therefore impacting positively on the level of economic activities as more funds became available for investment in productive activities. Employment was in turn generated and output was increased. Over the years, Nigeria has taken numerous policy initiatives and measures in the management of her external reserves. The phenomenal rise in the level of Nigerian external reserves, especially since the beginning of 2004 has generated a lot of interest and debate among policy makers and members of the public on how reserves should be managed (Chinaemerem and Ebiringa, 2012). Since the early 1970s, Nigerian economy has persistently depended on oil as the main source of foreign exchange earnings with the attendant cycles of economic booms and bursts. Therefore, we are motivated to investigate the predictive contribution of external reserves to Nigeria's economic growth over the years using a specified time-varying parameter Bayesian Dynamic linear model. Basically, there are many ways and methods that have been used for analyzing economic indicators, ranging from simple to very complicated statistical techniques and methods.To the best of our knowledge, our paper is probably the first empirical research where influence of the Nigerian external reserve accumulation policy on the economic growth over time is systematically studied using a dynamic time-varying parameter approach.

Essentially, the main objective of this paper is to investigate the predictive performance of external reserves and other leading economic indicators with respect to economic growth (proxied by GDP). To achieve this objective, we consider a case when the observational variance in the Bayesian dynamic linear model of West and Harrison (1997) is constant and the evolution variance is represented as a fraction of the filtering variance. The rest of this paper is organized as follows: In section 2, we present a brief review of literature on application of dynamic linear model in econometric time series analysis. Section 3 borders on our model specification and Markov chain Monte Carlo(MCMC) approach, while section 4 is on empirical analyses and discussion of results. Finally, section 5 is the conclusion of the paper.

2 Brief Review of Application of Dynamic Linear Models in Econometric Time Series Analysis

Early applications of dynamic linear models to economic time series data include the works of Fama and Gibbons (1982) who modeled the unobserved ex-ante real interest rate as a state variable that follows an AR(1) process. Clark (1987) used an unobserved-components model to decompose quarterly real GNP data into the two independent components of a stochastic trend component and a cyclical component. Another important contribution is the work of Stock and Watson (1991) who defined an unobserved variable, which represents the state of the business cycle, to measure the common element of co-movements in various macroeconomic variables. The dynamic linear model with state space approach offers attractive features with respect to their generality, flexibility and transparency. More detailed treatments of state space models are given by Harvey (1989), Harvey and Shephard (1993) and Hamilton (1994a), among others. Recently, Petris et al. (2009) published one of the most succesful methods for analyzing dynamic linear models in the journal of statistical software.

The frequentist approach to time series analysis and forecasting originated from regression methods which involves specifying a linear parametric relationship between a set of explanatory variables (or exogenous variables) and the dependent (or endogenous variable). The parameters of the model can be estimated in a variety of ways which includes the least squares and maximum likelihood estimation method, but the approach always culminates in striving for some form of statistical orthogonality between the explanatory variables and the residuals of the regression. The most prominent Time-Varying Parameter (TVP) regression model in econometrics has the form

$$y_t = \beta_{0,t} + \beta_{1,t}x_{1t} + \beta_{2,t}x_{2t}... + \beta_{kt}x_{kt} + v_t, v_t \sim N(0, \sigma_v^2) \tag{1}$$

with the TVP β following a random walk specification

$$\beta_{i,t+1} = \beta_{i,t} + \epsilon_{i,t}, \epsilon_{i,t} \sim N(0, \sigma_i^2), i = 0, 1, ..., k \tag{2}$$

. This random walk specification captures a variety of parameter variations and is most conveniently estimated in literature using the state space approach (Durbin and Koopman,2001;Koopman et al,2001;Zivot and Wang ,2002).

Over the past two decades dynamic time series models have become a standard econometric tool for measuring co-movement in macroeconomic time series data. The popularity of these models have risen as methods have been developed to perform factor analysis on large datasets, such as the time-domain approach of Stock and Watson (2002) and the frequency-domain approach of Forni and Reichlin (1998) and Forni, Hallin, Lippi, Reichlin (2001, 2005). Dynamic time series regression can in very general terms be formulated using state space representation of the observations and the state of the system. AutoRegressive (AR) models falls into the class of dynamic time series regressions. They were first introduced by Yule in 1926 and susequently, Slutsky, in 1937 presented Moving Average (MA) schemes. It was Wold (1938), however, who combined both AR and MA schemes and showed that ARMA processes can be used to model all stationary time series as long as the appropriate order of the number of AR terms, and the number of MA terms, was specified. The approach proposed by Box and Jenkins came to be known as the Box-Jenkins methodology to ARIMA models, where the letter "I", between AR and MA, stood for the word "Integrated". Time series and econometric literature in the 1970's were dominated by time-domain analysis techniques advocated by Box and Jenkins (1970) due to so many reasons. The main reason perhaps was that Box and Jenkins provided a complete methodology that resolved many practical issues like non-stationarity, forecasting and optimal control, and did so in a way that was easy for the analyst to implement.

Box and Jenkins provided a way around the problem of nonstationarity by means of a methodology focused on differencing the data. Despite the success of this approach to forecasting in the 1970's there were still some who chose to work within a structural time series framework. For instance, Harrison and Stevens (1976) were successful in formulating the linear Gaussian Markovian state-space model within a Bayesian context. Working with the Kalman filter, they were able to specify their form of dynamic linear models based on time-varying parameters in order to account for nonstationarity. Moreover, the Bayesian approach allows one to specify prior distributions on not only parameters, but also the initial conditions, facilitating convergence of the Kalman gain matrix. Over the past two decades, dynamic linear models have become a standard econometric tool for measuring both co-movement and forecasting macroeconomic time series. The popularity of these models have risen as methods have been developed to perform factor analysis on large datasets, such as the time-domain approach of Stock and Watson (2002) and the frequency-domain approach of Forni and Reichlin (1998) .The works of Otrok and Whiteman (1998), Cui and Dunson (2014),Fuquene et al(2013) and Kim and Nelson (1999) provides a Bayesian alternative to the classical Box and Jenkins approaches. In this paper, we apply a variant of the time-varying parameter dynamic linear model of Harvey (1989), West and Harrison (1997) to the econometric modeling of external reserves-economic growth nexus in Nigeria with the aim of assessing the predictive performance of external reserve in the presence of some selected economic variables in Nigeria.

3 Model Specification and Econometric Methodology

In this section, we propose and specify a Bayesian dynamic linear regression model to assess the predictive relationship between economic growth (proxied by GDP), external reserve and some other key economic indicators of the Nigerian economy. Our model specification takes the following form.

$$
\begin{aligned}
y_t &= X_t\theta_t + v_t & v_t &\sim N(0, V) & (3)\\
\theta_{t+1} &= G_t\theta_t + w_t & w_t &\sim N_p(0, W_t) & (4)\\
\theta_0 &\sim N_p(m_0, C_0)
\end{aligned}
$$

Equation (1) is known as the observation equation while equation (2) is the evolution equation. G_t is a known matrix of order $p * p$ that determines how the observation and state equations evolve in time (see West and Harrison (1997). We assume that all v_t's are independent from the w_t's. Since each parameter at time t only depends on results from time $t-1$, the state parameters are time-varying and constitute a Markov chain.By explicitly allowing for variability in the state regression parameters, we let the system properties change in time in the spirit of (Nakajima et al., 2011, Doh and Connolly, 2013).

In our model, the response y_t is the annual GDP of Nigeria from 1960 to 2009. The matrix X consists of economic indicators measured concurrently with the GDP and includes a column of 1's representing a dynamic intercept term. θ_t are time -varying regression coefficients which model the relationship between the regressors and the response at each time t. G_t is a known state evolution matrix.

3.1 Bayesian Estimation of the Model Parameters

Parameters of interest which are to be estimated are the matrix θ, the error variances V and W_t, and the one-step-ahead forecasts f_t. Since normality is assumed, we estimate θ and f_t by using the Kalman filter (Kalman, 1960). V is assumed to be distributed inverse-gamma a priori and is estimated using a Gibbs sampler in the spirit of (Awe et al., 2015).

For the Kalman filter to run, it is necessary to know V and W_t. Estimation of V is done using the Gibbs sampler described below. Here, we propose the use of discount factors to estimate W_t in the spirit of Awe et al(2015) .

In order to estimate V we use Gibbs sampling. This requires us to draw samples from $V|\theta$ as well as from $\theta|V$. The latter draw is performed using the Forward Filtering Backwards Sampling (FFBS) algorithm (Carter and Kohn, 1994).This algorithm allows for the implementation of Markov chain Monte Carlo(MCMC) approach to dynamic linear models.The forward filtering step is the standard normal linear analysis to give $P(\theta_t|D_t)$ at each t for $t = 1, ..., n$.

We begin by initializing $V^{(0)}$ and running the Kalman filter on the data using these initial values for V.

1. We denote $p(\theta_0, ..., \theta_T|D_T) = \prod_{t=0}^{T} p(\theta_t|\theta_{t+1}, ..., \theta_T, D_T)$

2. We then sample from $p(\theta_T|D_T)$ using the filtering density above.

3. By the Markov property,

$$p(\theta_t|\theta_{t+1}, ..., \theta_T, D_T) = p(\theta_t|\theta_{t+1}, D_T)$$

   It can be shown that this distribution is $N(h_t, H_t)$ where:

$$h_t = m_t + C_t G' R_{t+1}^{-1}(\theta_{t+1} - a_{t+1})$$
$$H_t = C_t - C - tG' R_{t+1}^{-1} G C_t$$

4. We then proceed inductively until we have a complete sample from $p(\theta_0, ..., \theta_T|D_T)$. Since we proceed from $t = T$ to $t = 0$, this is called backwards sampling.

A sample from the posterior state parameter is then generated.

To sample from $V|\theta$ we impose a gamma prior on V^{-1} and derive the posterior hyperparameters. Let $V^{-1} \sim Gamma(a_0, b_0)$, then

$$V^{-1}|\theta \sim Gamma(a_0 + \frac{T}{2}, b_0 + \frac{1}{2}\sum_{t=1}^{T}(y_t - X_t\theta_t)^2)$$

The Gibbs sampler proceeds as follows. 1.First, initialize $V^{(0)} \sim Gamma(a_0, b_0)$. Then, for i = 1, . . . , M,

1. Sample $\theta^{(i)}$ using FFBS.

2. Sample $V^{-1(i)}|\theta^{(i)} \sim Gamma(a_0 + \frac{T}{2}, b_0 + \frac{1}{2}\sum_{t=1}^{T}(y_t - X_t\theta_t^{(i)})^2)$

This Gibbs sampler is run for a given set W_t determined from a given value of δ as mentioned above. We used M = 12,000 with a burn-in period of 2,000. Convergence was quite quick, happening in a relatively few iterations.

4 Empirical Analyses

4.1 Data Presentation

The data used in this study are Nigerian economic indicators sourced from the Central Bank of Nigeria (CBN). The data includes annual money supply (MS), lending rate (LR), gross domestic product (GDP), exchange rate (EXRT), capital expenditure (CE), external debt (ED), and treasury bill rate (TR) for the period between 1960-2009.

In order to investigate other possible macroeconmic variables that might have contemporaneous prediction effect on GDP alongside external reserve, we include in our model each of the other variables aforementioned in turns.

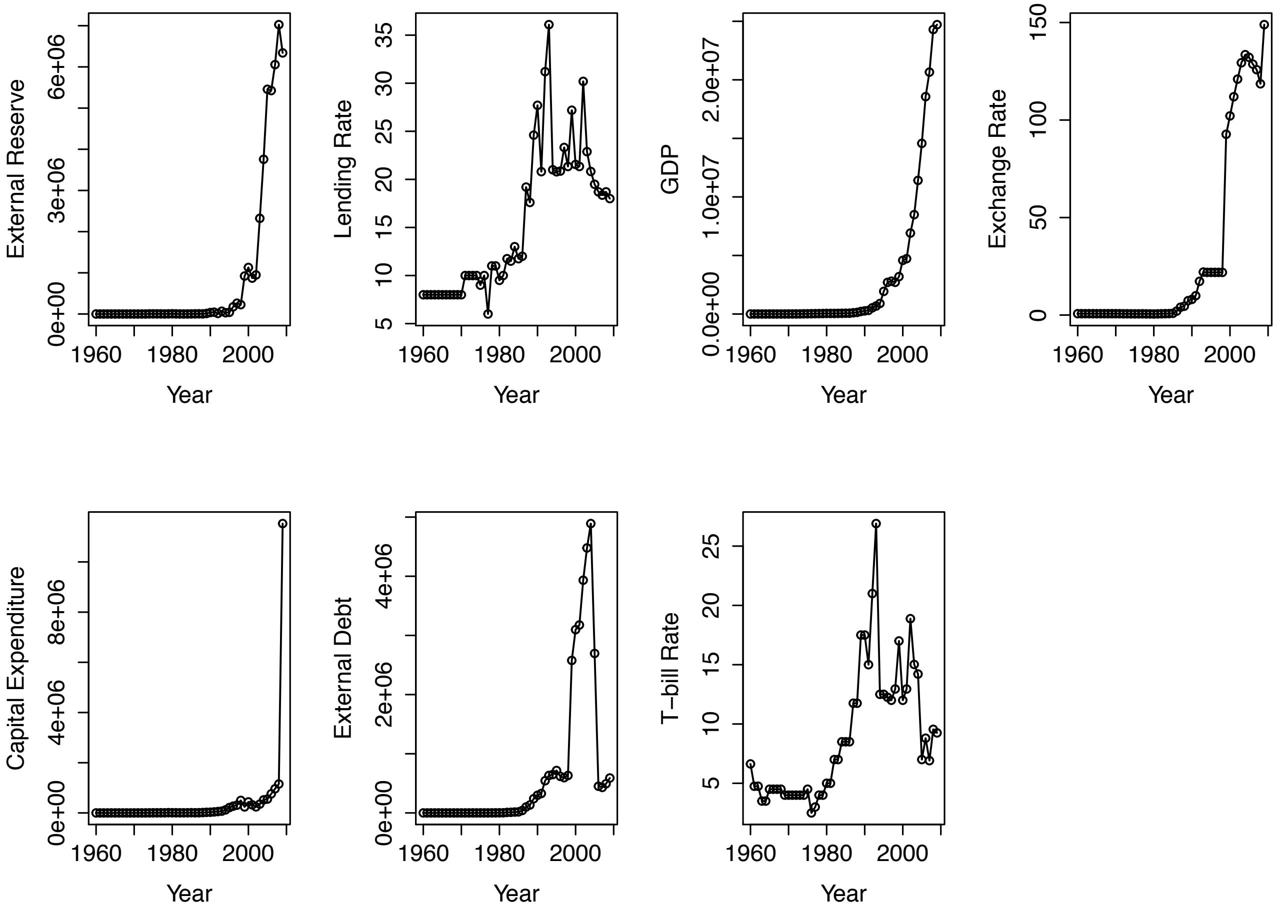

Figure 1: Annual time-series data on money supply (MS), lending rate (LR), gross domestic product (GDP), exchange rate (EXRT), capital expenditure (CE), external debt (ED) and treasury bill rate (TR)

To avoid spurious regressions, we adjust all the monetary economic variables in our data for inflation before taking logarithms of each.This is because inflation adjustment, or ”deflation”, is an important tool in the toolkit for analyzing economic data. This is accomplished by dividing all the monetary time series by a price index, such as the Consumer Price Index (CPI). Adjusting for inflation enables us to uncover the real growth in the variables, if any. It also helps us to stabilize the variance of random or seasonal fluctuations and highlight cyclical patterns in the data.

4.2 Parsimonious Model Selection and Discussion of Results

Predictive performance of the variables was assessed using one-step-ahead Mean Squared Prediction Error (MSPE). Our Gibbs sampler was run, using the range of values of discount factors $\delta \in \{.01, .02, \ldots, .99\}$ while the δ with the lowest MSPE was chosen in each model in Tables 1 to 11. After the Gibbs sampler was run, we assessed convergence by examining trace plots and corroborating the plots with the Geweke test (Geweke, 1991, Nakajima et al., 2011). The tables below contain the estimates of the observation variance(V), the Effective Sample Size(ESS) to ensure we had sufficient replications to estimate V. All of the (absolute) Geweke z statistics (CD) are below the 1.96 threshold, indicating a failure to reject the null hypothesis of stationary means in each time series. Also,the traces of the simulated variances (not shown) do not show any particular sign of non-convergence.

We perform the analysis for varying periods, considering ten years at a time.Table 1 shows the analysis performed for the period 1960-1969.During this period, the model involving lending rate has the lowest MSPE of 5.954 and therefore has the best predictive performance. Table 2 shows the results of analysis perfomed from 1960-1979. Model 1 involving external reserves as predictor has the best performance for this period in terms of MSPE value of 3.022. Table 3 contains results of analysis done for the period 1960-1989. For this period, external reserve still predicts GDP better than other variables.It has the lowest MSPE value of 2.019. In Table 4, thesame analysis was performed for the period 1960-1999.The model containing Capital Expenditure (CE) performs best in predicting GDP for this period with MSPE value of 1.520 but the model containing external reserves has the least observational variance (V) of 0.010. In Table 5, the analysis was done for the the period covering 1960-2009.The model involving external reserve as predictor also performs best in terms of the lowest value of MSPE but surprisingly with a high observational variance.

As we can see from the results in the Tables 1-5, out of all the economic indicators considered, external reserve best predicts economic growth (GDP) of Nigeria for most of the period under study. Tables 6-10 reveals the result of the models with two regressors involving external reserves and all other variables in order to check for their contemporaneous effects on economic growth(GDP). Table 6 shows that the models involving external reserves and external debt has the lowest MSPE for the period 1960-1969.The result is similar in Table 7 for the analysis for period 1960-1979 . In Tables 8 and 9 which covers the periods 1960-1989 and 1960-1999 respectively, the model involving variable combination of external reserves and capital expenditure performs best in terms of lowest MSPE. Finally in Table 10, the result is reverted to reveal the model with external reserve and external debt as predictors having the best performance for the most current period, 1960-2009.

5 Concluding Remarks

In this paper, we proposed a Bayesian time-varying parameter dynamic linear regression model with application to external reserves-economic growth dynamics in Nigeria. The model was estimated via the Markov chain Monte Carlo method to simulate the predictive posterior estimates of model parameters. In our analysis, we find that external reserves has a higher nexus with economic growth than the other macroeconomic variables considered in terms of predictive performance and this result is consistent over the years. Further more, the economic indicator that best predicts economic growth (GDP) when combined with external reserve is external debt for most of the period under consideration. This is not unconnected

Model	Regressors	MSPE	V	CD	ESS	
1	ERES	6.116	0.047	-0.103	2702	
2	CE	6.019	0.016	1.131	8841	
3	ED	6.036	0.024	0.833	2314	
4	TR	6.632	0.286	0.835	4486	
5	ERT	5.957	0.054	0.936	1316	
6	LR	5.954	0.018	1.011	5944	

Table 1: Mean Squared Prediction Error (MSPE), Observation Variance, Geweke Statistic, Effective Sample Size for Various One-Regressor Models (1960-1969) at 12,000 Iterations

Model	Regressors	MSPE	V	CD	ESS	
1	ERES	3.022	0.010	1.250	3674	
2	CE	3.024	0.017	0.964	4584	
3	ED	3.055	0.022	-0.465	1954	
4	TR	3.227	0.047	1.066	5708	
5	ERT	3.017	0.025	-0.207	6659	
6	LR	3.030	0.754	1.011	1112	

Table 2: Mean Squared Prediction Error (MSPE), Observation Variance, Geweke Statistic, Effective Sample Size for Various One-Regressor Models(1960-1979) at 12,000 Iterations

Model	Regressors	MSPE	V	CD	ESS	
1	ERES	2.019	1906.775	1.030	11536	
2	CE	2.021	0.015	1.053	9426	
3	ED	2.043	0.014	0.464	2369	
4	TR	2.148	0.031	1.142	5353	
5	ERT	2.040	0.020	1.025	3512	
6	LR	2.037	0.594	1.027	2667	

Table 3: Mean Squared Prediction Error (MSPE), Observation Variance, Geweke statistic, Effective Sample Size for Various One-Regressor Models (1960-1989) at 12, 000 Iterations

with the fact that foreign exchange reserves are necessary to pay debt and to support cer-

Model	Regressors	MSPE	V	CD	ESS	
1	ERES	1.521	0.010	0.979	5227	
2	CE	1.520	0.036	0.993	8153	
3	ED	1.538	0.011	0.925	2016	
4	TR	1.612	11.000	0.999	8768	
5	ERT	1.535	0.015	1.149	5736	
6	LR	1.537	0.012	-0.881	1901	

Table 4: Mean Squared Prediction Error (MSPE), Observation Variance, Geweke statistic, Effective Sample Size for Various One-Regressor Models (1960-1999) at 12,000 Iterations

Model	Regressors	MSPE	V	CD	ESS	
1	ERES	1.222	1252291.137	1.002	10879	
2	CE	1.264	0.024	1.105	9965	
3	ED	1.236	5.087	1.018	3421	
4	TR	1.309	0.024	-0.401	6018	
5	ERT	1.239	111.536	1.008	10882	
6	LR	1.240	0.010	1.224	1603	

Table 5: Mean Squared Prediction Error (MSPE), Observation Variance, Geweke statistic, Effective Sample Size for Various One-Regressor Models (1960-2009) at 12,000 Iterations

Model	Regressors	MSPE	V	CD	ESS	
1	ERES + CE	6.014	0.019	1.163	4731	
2	ERES + ED	5.957	0.022	0.978	5793	
3	ERES + TR	6.081	0.042	0.941	8054	
4	ERES + ERT	6.103	0.054	0.967	7599	
5	ERES + LR	5.988	0.016	0.999	7375	

Table 6: Mean Squared Prediction Error (MSPE), Observation Variance, Geweke statistic, Effective Sample Size for Various Two-Regressor Models (1960-1969) at 12,000 Iterations

Model	Regressors	MSPE	V	CD	ESS	
1	ERES + CE	3.029	0.019	-0.683	5457	
2	ERES + ED	3.025	0.040	1.011	9764	
3	ERES + TR	3.045	0.012	-1.155	2659	
4	ERES + ERT	3.070	0.020	0.775	6532	
5	ERES + LR	3.043	0.009	0.907	2410	

Table 7: Mean Squared Prediction Error (MSPE), Observation Variance, Geweke statistic, Effective Sample Size for Various Two-Regressor Models (1960-1979) at 12,000 Iterations

tain exchange rate regimes among other factors. It is generally believed that countries with rapidly growing FER/GDP ratios, ceteris paribus, exhibit higher capital productivity and higher rates of economic growth (Aluko, 2007). Moreso, a countrys usable foreign exchange reserve is an important index in the risk models used by credit rating agencies and international financial institutions. Hence, we recommend that the new regime in Nigeria should continue to formulate appropriate monetary policies, maintain adequate reserves while still expending on capital expenditures that are capable of enhancing good living conditions.

Model	Regressors	MSPE	V	CD	E SS	
1	ERES + CE	2.024	0.015	-0.771	6599	
2	ERES + ED	2.029	0.540	0.995	9013	
3	ERES + TR	2.036	0.008	2.242	2427	
4	ERES + ERT	2.059	0.014	-0.056	4666	
5	ERES + LR	2.051	4.876	1.010	10113	

Table 8: Mean Squared Prediction Error (MSPE), Observation Variance, Geweke statistic, Effective Sample Size for Various Two-Regressor Models (1960-1989) at 12,000 Iterations

Model	Regressors	MSPE	V	CD	ESS	
1	ERES + CE	1.523	0.015	-1.259	5968	
2	ERES + ED	1.528	0.026	1.005	7810	
3	ERES + TR	1.537	0.009	0.983	3557	
4	ERES + ERT	1.551	11.600	1.007	10363	
5	ERES + LR	1.548	0.008	0.194	2550	

Table 9: Mean Squared Prediction Error (MSPE), Observation Variance, Geweke statistic, Effective Sample Size for Various Two-Regressor Models (1960-1999) at 12,000 Iterations

Model	Regressors	MSPE	V	CD	ESS	
1	ERES + CE	1.233	2.019	0.987	9049	
2	ERES + ED	1.228	0.023	1.020	7455	
3	ERES + TR	1.240	0.010	1.021	3881	
4	ERES + ERT	1.249	0.022	0.990	6175	
5	ERES + LR	1.248	0.010	1.004	4448	

Table 10: Mean Squared Prediction Error (MSPE), Observation Variance, Geweke Convergence Diagnostic(CD), Effective Sample Size for Various Two-Regressor Models (1960-2009) at 12,000 Iterations

References

J.J Aluko. The monetization of nigeria's foreign exchange inflows. *CBN Bullion*, 31(2), 2007.

Olushina Olawale Awe, Ian Crandell, and A Adedayo Adepoju. A time varying parameter state-space model for analyzing money supply-economic growth nexus. *Journal of Statistical and Econometric Methods*, 4(1):73–95, 2015.

Chris K Carter and Robert Kohn. On gibbs sampling for state space models. *Biometrika*, 81(3):541–553, 1994.

Osuji Casmir Chinaemerem and O.T Ebiringa. Analysis of effect of external reserves management on macroeconomic stability of nigeria. *International Journal of Business Management and Economic Research*, 3(6), 2012.

Kai Cui and David B Dunson. Generalized dynamic factor models for mixed-measurement time series. *Journal of Computational and Graphical Statistics*, 23(1):169–191, 2014.

Taeyoung Doh and Michael Connolly. *The state space representation and estimation of a time-varying parameter VAR with stochastic volatility*. Springer, 2013.

John Geweke. Efficient simulation from the multivariate normal and student-t distributions subject to linear constraints and the evaluation of constraint probabilities. In *Computing science and statistics: Proceedings of the 23rd symposium on the interface*, pages 571–578. Citeseer, 1991.

Andrew C Harvey. The kalman filter and forecasting structural time series models, 1989.

Rudolph Emil Kalman. A new approach to linear filtering and prediction problems. *Journal of Fluids Engineering*, 82(1):35–45, 1960.

Anthony Kyereboah-Coleman. Reserves and foreign exchange in the macroeconomic context. *Presentation at Regional Course on Optimising Reserves and Foreign Exchange Management for Income Generation organised by WAIFEM*, pages 11–15, 2009.

Jouchi Nakajima, Munehisa Kasuya, and Toshiaki Watanabe. Bayesian analysis of time-varying parameter vector autoregressive model for the japanese economy and monetary policy. *Journal of the Japanese and International Economies*, 25(3):225–245, 2011.

Evans S.C. Osabuohien and A.J. Egwakhe. External reserve and the nigerian economy: the dual folded debate. *African Journal of Business and Economic Research*, 3(2 & 3):28–41, 2008.

Giovanni Petris, Sonia Petrone, and Patrizia Campagnoli. *Dynamic linear models with R*. Springer, 2009.

James H Stock and Mark W Watson. Forecasting using principal components from a large number of predictors. *Journal of the American statistical association*, 97(460):1167–1179, 2002.

M. West and P.J. Harrison. *Bayesian Forecasting and Dynamic Models*. Springer-Verlag, New York, 2nd edition, 1997.