Analysis of ore transportation: A financial approach using the Black & Scholes method in real options.

ABSTRACT

The cost of transportation has a great weight in the decision-making of business agents. The correct choice may provide greater returns in the medium and long term since most of the contribution is focused in physical goods. In the research we tried to approach the Black & Scholes model with twin security procedure (comparison of two models with similar methods). This method of pricing financial assets is more used at the moment, since it absorbs the market oscillations in *ex-nunc*, that is, going forward. Three companies were used: MRS - Logistics S.A. and National Steel Company - CSN for railroad, in relation to the pipeline the company was Anglo Ferrous Brazil subsidiary of Anglo American. The real options model was applied to distinguish the investment in ore pipeline or railroad for the ore transportation in the Brazilian scenario, which was very inflated by the central bank interest rate and on any hypothesis, rail transportation was the best in the period analyzed (2008 and 2016).

Keyword: Real options, Investment analysis, Ore transportation, Railroad, ore Pipeline

JEL classification
R42; M21; L92; L95

1. INTRODUCTION

The freight transportation is one of the parameters that make up the logistic cost in a distribution terminal. Techniques and management tools guide the manager in decision-making in order to mitigate the risk inherent in the transportation market. An efficient transportation system provides a more conscious society with the preservation of the environment, the integration of people and a distribution of goods at affordable prices to citizens and companies. Second D’agosto (2015) the transportation system is the integration of resource materials, human, financial and intellectual necessary for the displacement, such as people and cargo.

Companies present in the freight transportation market have their inputs affected by several variables. The main factors that influence the cost of transportation are the volume of cargo, periodicity of transportation and the costs of maintenance and operation. The costs are very sensibility to external shocks and market volatility, some examples are: variations in the price of the inputs, value of the goods, demand, inflation, environmental impact among others. Shafiee, Topal and Nehring (2009) describe the method of real options valuation - ROV as a way to maximize mine production and emphasizes that the most important variable of price and size the reserve. Factors that require the manager constant skills at making decisions regarding projects that can add value reduce risks and deal with the volatility and uncertainty present in both investment projects and the market. Venables et al (2014) emphasizes that the impacts of the transportation system are broad, considering three dimensions: benefits to the user, productivity increase and effects on investment and employment.The research data were compiled in two periods after the initial investment, between the years 2014 to 2016 projected the risk free rate of 6.13% per year. The investment in the railroad corresponds to two distinct segments, the first belonging to National Steel Company - CSN, and the second to MRS - Logistics S / A. The stretch comprising 266 miles between the cities of Brumadinho and Itaguaí in Rio de Janeiro, plus 64 miles comprising Casa da Pedra to Congonhas owned by CSN, both in the state of Minas Gerais count 330 miles. The pipeline works refer to Anglo Ferrous Brazil's largest pipeline in the world (Anglo American), where investments were estimated at US $ 3.6 billion with a 3-year execution term. The works began in 2008 and were finalized in 6 years later for 15 billion dollars. The survey includes approximate values of the planned real, since the data were not requested by the companies mentioned, but rather researched in a social report released to the press and regulatory agencies.

In order to make financial decisions on projects, the investor uses several investment analysis tools to obtain a clearer and broader view of the project, while at the same time providing a more critical view, allowing for more assertive decisions. The importance of the use of several tools, due to the fact that, no tool is able to provide all the answers, however, a combination can complement and in many cases is enough to decide between different projects, analyzing their characteristics, measuring their potential to add higher value companies, return on investment, project execution period, among others.

The research compared the transportation of established and operational railway and pipeline cargo in order to verify in which models are most efficient, which variables affect project contribution decisions more in the light of uncertain investment analysis tools. Both modes have the same product and purpose. The transportation of iron ore in mines, located in the state of Minas Gerais, is being transported to the state of Rio de Janeiro. If it is impossible to obtain results from a pipeline alongside a railway line, the same product with similar mileage is used in the research. The research proposal is to demonstrate the importance of financial tools under uncertainty in order to obtain greater results and decision power. Second Minardi (2004), flexibility in a financial project is only a possibility, not an obligation, but increases the possibilities for decision making. It applies the option real theory - OR as a process of comparing twin security. The method is chosen by increasing Brownian geometric motion, since this method derives from the Markov process. Still, Lee (2011) says that the option is linked to project uncertainty, because instead discounted cash flow, where the value of investment depreciates with increasing volatility, the OR method values managerial flexibility as an option Incorporated. As the proposal is to analyze possible companies established or in implantation, the process is made a probabilistic analysis in *ex-nunc* that is, going forward.

The analysis comprises two ore transportation projects, one through the investment in the pipeline, and the other through the investment in railroad, both between the states of Minas Gerais and Rio de Janeiro with approximately 330 miles away with the capacity to transport on average 2,500 m^3 / h or 15,000,000 tons per year. After the introduction, the second session aims to justify the motivation to carry out this research, the third session presents the methods and models used, in the fourth session of this article we have the application and results obtained through previously introduced mathematical methods and models and finally, the fifth session contemplates the conclusion and the results obtained.

2. FORMULATION OF THE PROBLEM

The research aims to present the best proposal according to the real options theory method - ROT, using the comparison between the models of rail and pipeline transportation, since both have the lowest cost per product transported in long Term, but with a high cost of investment.

Traditional methods of investment analysis such as Discounted Cash Flow (DCF) do not satisfy long-term research, since over time, the economy and financial indicators suffer from external disturbances and temporal oscillations. According to Dixit and Pindyck (1994), the Net Present Value (NPV) method or the discount cash flow (FCD) cannot absorb the value of flexibility because it cannot anticipate the expected risk, mainly because occurs mainly because it uses discount a constant rate, so this method is used for risk-free analysis, since the discount rate is constant and will not change over time. According to Sharpe (1964), Lintner (1965), Black (1972), the risk-free rate is used to reduce risk for portfolio investment, even though it was criticized by Fama and French (1992) for not providing a positive correlation between the return and the beta market. In order to determine the best estimation of the data, Copeland, Koller and Murrin (1995) proposed that the best risk-free rate would be the return of a portfolio with beta equal to zero, as proposed by Black (1972).

The DCF is oriented to economics with low or no oscillation, starting from this premise, we opted for the application of the Real Options Theory (ROT), due to the capacity to incorporate uncertainties into the value of the project, besides pricing the value of the option, aiding in the decision-making process. According to Santos and Pamplona (2001) and Dixit and Pindyck (1994) the traditional cash flow does not absorb probabilistic changes and are irreversible, however the use of ROT mitigates the risk of the project, since this method has reversible characteristics anticipating movements in a probabilistic way . The DCF has irreversible characteristics, assuming a position of now or never with the stationary rate of discount over time.

In the managerial process it is necessary to observe how the information affects the decision making, the impacts and its relation with the internal and external environment of the company. The asymmetric information distorts the interface and the perception that the manager has in relation to environmental data. According to Minardi (2000) and Trigeorges (1993), the resulting asymmetry created by adaptability requires a rule for an expanded NPV that reflects the two component values: the traditional NPV (static or passive) and the value of the operation option and strategic adaptability. In this way, we have:

$$NPV = - Investment + \sum_{t=0}^{\infty} \frac{CF}{(1+i)^n} \qquad [1]$$

and,

Expanded NPV = Static NPV (passive) + Option Value (Administrative Flexibility) [2]

Administrative flexibility is related to the value of the option, in this case we will use European call and put options throughout the research. According to Santos and Pamplona (2005) the call option is the value of administrative flexibility because the investor has the right, but not the obligation to exercise it. Among the types of pricing we use the model of Black & Scholes (1973) to establish an approximation of reality according to market oscillation and exogenous shocks, since the model has variables that measure the impacts of the market on the value of the company.

3. APPLIED METHODOLOGY

The use of the methodological procedure gives us a more enlightening view of the research process, since, using the twin security process where we try to replicate a cash flow of a similar asset. For both transportation, rail or pipeline models, a comparison is made using the Black & Scholes model to determine which asset in a call or put situation is most profitable.

Black & Scholes Model

- Call option

$$C = S_0 N(d1) - E_c e^{-rt} N(d2) \qquad [3]$$

On what:

C = Call option price.

S = Price of the real asset at the present time.

E_c = Value of investment in productivity.

r = Risk free rate.

T = Time to maturity of the option, in years.

σ = Volatility of the asset price, expressed per year.

N (..) = Accumulated normal distribution function.

- Put Option

$$P = E_v e^{-rt} N(-d2) - S_0 N(-d1) \qquad [4]$$

On what:

P = Put option Price.

S = Price of the real asset at the present time.

Ev = Sale price of project E (market value).

R = Risk free rate.

T = Time to maturity of the option, in years.

Where;

$$d1 = \frac{ln(S/E) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \qquad [5]$$

and;

$$d2 = d1 - \sigma\sqrt{T} \qquad [6]$$

4. OBSERVATION OF THE LONG-TERM EFFECTS OF THE BROWNIAN MODEL IN RELATION TO VOLATILITY

The Black & Scholes model follows the geometric brownian motion (MGB)[1] with drift and constant volatility according to the Wiener Process However, it is intended for use in environments with volatility up to 30%, since above this value the model does not present adequate projection reliability, in countries with large fluctuations of risk of financial assets, the Black & Scholes model is not recommended.

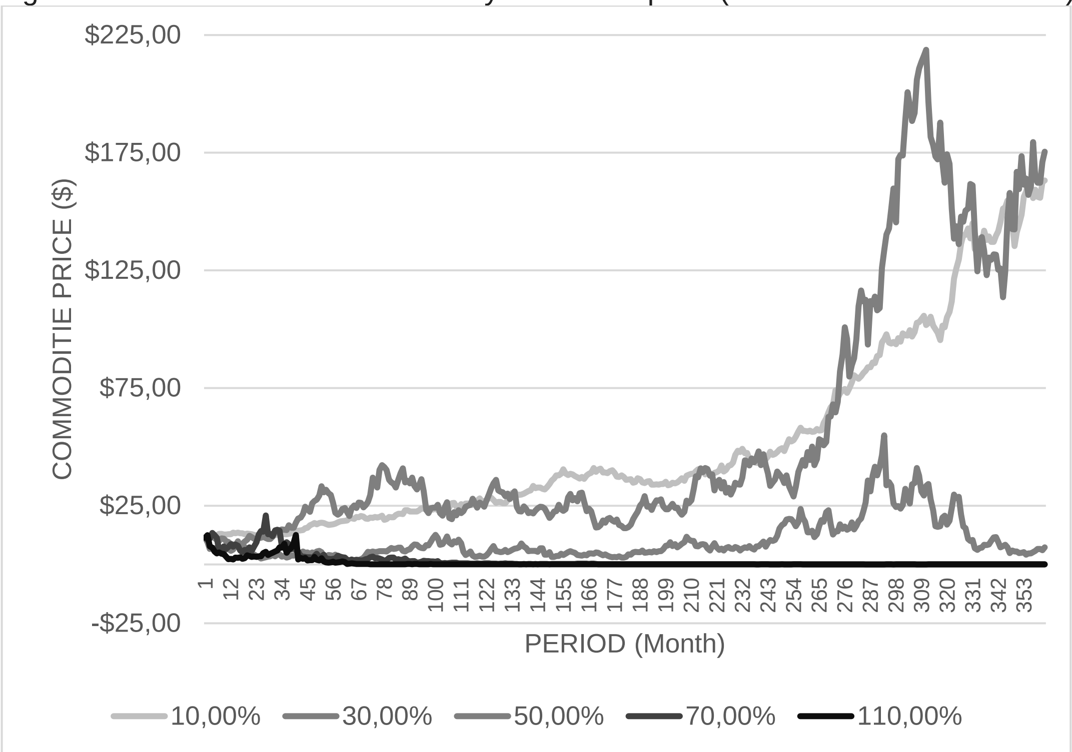
Some products, in this research demonstrated in commodities, suffer interference from exogenous variations, as the model has stochastic variables, the longer the term of use, the greater the external shock in relation to the trend. Evatt et al. (2010) and Haque et al. (2014) the use of the stochastic approach has time limitations, this problem is associated with the Hamilton-Jacobi-Bellman system of partial differential equations, so it is usual to use numerical methods. The MGB follows a log-normal probability distribution, not admitting negative values.

In computational simulations, the increase in volatility showed a sensibility to values up to 30%, respecting a certain long-term trend with increased entropy. The increase in volatility increases the entropy in the model, since the graph line has a tendency to zero losing its effectiveness. As investors are hopeful of getting higher returns for their projects, in high volatility environments, they are "always" expecting the rate to increase to obtain a higher return, where in the first situation, as in the second, the values have increased very accentuated.

This model is oriented to medium-term situations in commodity assets, since the extension of execution time, as well as the fluctuation of the risk rate may induce a wrong analysis of the project. According to Jovanovic (2014) commodity prices are commonly impacted by the fluctuation and uncertainty about the future price, since these products are negotiations in advance and may impact on the sale price in the future. Kulatilaka and Marcus (1992), Laughton and Jacoby (1993), Pickles and Smith (1993), Kulatilaka and Trigeorgis (1994), Mauer and Ott (1995), Palm and Pearson (1986), Cavender (1992) use in the methods of analysis the option of abandonment specifically in the oil and gas market as a way to verify other business opportunities or project exhaustion, due to the constant oscillations of commodity prices.

In tests, the use of random values, measuring the sensibility of the model to the risk of the asset, the model loses its effectiveness with volatility above 30%. Figure 1 exemplifies the model, because variations above 50%, volatility tends to zero. When the volatility approaches 30%, the increase in the evaluation period tends to increase the oscillation of the data. The increase in volatility causes a state of entropy up to 30%, but still acceptable, and transposing the 50% range, the volatility tends to zero losing its effectiveness

Figure 1 - Simulated historical volatility of iron ore price (March / 86 - March / 2016)



Source: Own production

5. COMPARATIVE APPLICATION USING THE BLACK & SCHOLES MODEL IN TWIN SECURITY

The choice of the comparative model between cash flows alone does not correspond to which investment may offer the greatest return over time, since simply the choice of the largest monetary volume is not a criterion, since the initial investments are different one of the others and the return in time are different in their application. The railway model has a more linear structure, however, the pipeline model has a greater investment need in the beginning, being amortized over the years. The space of two periods of analysis is used as a criterion, since the Black & Scholes model pricing the options in the medium term starting from an ex-nunc observation or "going forward". According to Copeland and Antikarov (2003) An option is the right, but not the obligation, to take action (eg defer, expand, contract or abandon) at a predetermined cost called exercise price, during a period of time. The survey collected data from companies involved in the ore transportation from the extraction to the port for export, where both transportation models are used for ship export.

Table 1 - Expectations of values of the modal rail 2014-2016

Railway Modal	(per year.)
Current cash flow (2015)	$536,427,939.88
Present Value (PV) (2015)	$3,831,628,142.00
Present Value (PV) (2016)	$4,315,783,822.96
Long-term market value	$21,543,193,662.16
Current market value (2016)	$3,016,047,112.70
Initial Investment (2014)	$2,592,010,543.39
Initial investment (2016)	$2,919,531,008.11
central bank interest rate on 10/2016	14%
Risk free rate (per year.)	6.13%
Asset Volatility	30.00%
Maturity (years)	2.00

* Market value updated to inflation of 7.87% per year.

Replacing the data of Table 1 in items [5] and [6];

Call option – Railway

$d1 = [ln(3,831,628,142.00 / 2,919,531,008.11) + (0.0613+0.3^{2}*0.5)*2] / 0.3*2^{1/2} = 1.141899259$ [7]

$N(d1) = 0.373252052$ [8]

$d2 = 1.141899259 - 0.3*2^{1/2} = 0.71763519$ [9]

$N(d2) = 0.263508873$ [10]

Put option – Railway

$d1 = [ln(3{,}831{,}628{,}142.00 / 3{,}016{,}047{,}112.70) + (0.0613+0.3^2*0.5)*2] / 0.3*2^{1/2} = 1.065239264$ [11]

$N(d1) = 0.143383835$ [12]

$d2 = 1.06239264 – 0.3*2^{1/2} = 0.640975195$ [13]

$N(d2) = 0.260769399$ [14]

Table 2 – Expectations of values of the modal pipeline 2014-2016

Pipeline Modal	(per year.)
Current cash flow (2015)	$346,568,667.85
Present Value (PV) (2015)	$2,475,490,484.66
Present Flow (PV) (2016)	$2,788,287,743.92
Long-term market value	$29,920,980,600.00
Current market value (2016)	$4,188,937,284.00
Initial Investment (2014)	$3,600,000,000.00
Initial investment (2016)	$4,054,887,684.00
central bank interest rate 10/2016	14%
Risk free rate (per year.)	6.13%
Asset Volatility	30.00%
Maturity (years)	2.00

* Market value updated to inflation of 7.87% per year.

Replacing the data in Table 2 in items [5] and [6];

Call option – Pipeline

$d1 = [ln(2{,}475{,}490{,}484.66 / 4{,}054{,}887{,}684.00) + (0.0613+0.3^2*0.5)*2] / 0.3*2^{1/2} = -0.662050967$ [15]

$N(d1) = -0.246030721$ [16]

$d2 = 0.662050967 – 0.3*2^{1/2} = -1.086315036$ [17]

$N(d2) = -0.361330181$ [18]

Put option – Pipeline

$d1 = [\ln(2,475,490,484.66 / 4,188,937,284.00) + (0.0613+0.3^2*0.5)*2] / 0.3*2^{1/2} = -0.738710962$ [19]

$N(d1) = 0.769958736$ [20]

$d2 = 0.738710962 – 0.3*2^{1/2} = -1.162975031$ [21]

$N(d2) = 0.877580183$ [22]

In this way, it can obtain the data of the call and put options for rail and pipeline modes, replacing items [3] and [4];

Rail Mode - Call option

$C = 3,831,628,142.00 * 0.373252052 – 2,919,531,008.11e^{-0.0613*2}\ 0.263508873$

$= \$ 749,607,119,57$ [23]

Rail Mode - Put option

$P = 3,016,047,112.70\ e^{-0.0613*2} * 0.260769399 – 3,831,628,142.00 * 0.143383835 = \$ 146,351,704.61$ [24]

Pipeline Mode - Call option

$C = 2,475,490,484.66 * -0.246030721 – 4,054,887,684.00\ e^{-0.0613*2}\ -0.361330181 = \$ 687,053,453.83$ [25]

Pipeline Mode - Put option

$P = 4{,}188{,}937{,}284.00\ e^{-0.0613*2} * 0.877580183 - 2{,}475{,}490{,}484.66 * 0.769958736$

$= \$\ 1{,}345{,}941{,}720.74$ [26]

6. RESULTS

According to Minardi (2000) and Trigeorges (1993) must calculate the expanded NPV to have the value of the project with the administrative flexibility. Using the data of items [1], [2] and [3] for a call and put option successively:

Replacing the data of Table 1 in item [2];

Options for Rail

NPV_{ROT} - *call* = 1.239.617.598,61 + 749.607.119,57 = $ 1.989.224.718,18 [27]

NPV_{ROT} – *put* = 424,036,569.31 + $ 146,351,704.61 = $ 570,388,273.93 [28]

Replacing the data in Table 2 in item [3];

Options for pipeline

NPV_{ROT} - *call* = -1.124.509.515,34 + 687.053.453,83 = $ - 437.456.061,51 [29]

NPV_{ROT} - *put* = 588.937.284,00 + 1.345.941.720,74 = $ 1.934.879.004,74 [30]

Table 3 - Search Results

Method	Outcome
$NPV_{(ROT)}$ rail - call	$ 1,989,224,718.18
$NPV_{(ROT)}$ rail - put	$ 570,388,273.93
$NPV_{(ROT)}$ pipeline - call	$ -437,456,061.51

NPV $_{(ROT)}$ pipeline - put	$ 1,934,879,004.74

The research data indicate that the economic scenario of 2016, the best option for investment in ore transportation is railway, since the initial investment of $ 3.6 billion in pipeline mode was not favorable in relation to the annual cash flow of $ 346,568,667.85. The present value in perpetuity of $ 2,475,490,484.66 does not cover the initial investment in 2016. Therefore, there should be no investment in the pipeline model in this scenario. Table 3 shows that the call option for the rail model is the best-selling value if the company decides to sell the assets.

Table 4 - Simulation risk free rate of 7.1% (per year.) and central bank interest rate of 13%

Method	Outcome
NPV $_{(ROT)}$ rail - call	$ 2,400,013,626.40
NPV $_{(ROT)}$ rail - put	$ 523,358,253.79
NPV $_{(ROT)}$ pipeline - call	$ -289,662,807.32
NPV $_{(ROT)}$ pipeline - put	$ 1,734,294,507.14

Table 4 simulates the survey data with the risk free rate of 7.1% per year and central bank interest rate 13% per year, this scenario occurred in August 2008, when Anglo American buys the rights to explore and implement the pipeline. Looking at the proposed period, however, it is not advantageous to acquire the rights of the pipeline, because the call option is uncovered in relation to the investment of 3.6 billion dollars. However, the annual cash flow of $ 346,568,667.85, or current present value of $ 2,788,287,743.92, does not cover the initial investment.

Table 5 - Simulation with investment in the pipeline in 15 billion (2014)

Method	Outcome
NPV $_{(ROT)}$ rail - call	$ 1,989,224,718.18
NPV $_{(ROT)}$ rail - put	$ 570,388,273.93
NPV $_{(ROT)}$ pipeline - call	$ -6,289,281,215.99
NPV $_{(ROT)}$ pipeline - put	$ 15,418,448,262.51

Table 5 shows that even with a $ 15 billion investment, the railroad model would still be the best option because the call option would be overdrawn. The

NPV (ROT) of both options does not cover the initial investment in the current period. The projection of a maximum of two periods is oriented, because the increase in the periodicity in the model with data of the present reduces the long-term profit.

The main contributions of the research were: the simulation of the volatility of the brownian geometric model - MGB, where it confirms only the use of a maximum of 30%, above this value, the model has a tendency to remain constant.

The importance of the Black & Scholes model for environments with high volatility and intensity of external variables becomes an interesting option in the calculation of a net present value - NPV, being more appropriate to the volatile business environment precisely to present a value close to reality since this model absorb the market oscillations.

7. FINAL CONSIDERATIONS

The final result confers the value of the option or the administrative flexibility for decision making, since the use of the Black & Scholes model absorb the market oscillations, reduces the impacts in relation to the differences of the financial return and assists in the management decision making.

The comparison of the call option must be made in relation to the investment, as well as the put option in Black & Scholes, then the call option for railway $C_{railway} < I_{railway}$, the project is discovered or not purchased, investors will not be able to opt for Investment in productivity. The railway option $P_{railway} < I_{railway}$, the operation is discovered, or is not sold. In this case, the only option would be to invest to produce, since regardless of call option does not cover the initial investment, it is still greater than the put option or exercise the right to sell (abandonment option). Regarding the investment in the pipeline, the call option $C_{pipeline} < I_{pipeline}$, the operation is discovered or not bought, investors cannot choose to invest to produce. The put option $P_{pipeline} < I_{pipeline}$, the option is discovered or not sold, there is no better option, exercise the right to sell (abandonment option) or purchase, it is not an option. The investment in pipeline transportation should not be accomplished in accordance with the 2016 scenario with a risk-free rate of 6.13% per year and central bank interest rate at

14% per year. Contributions to new research would be modeling with a rate of variation greater than 30% to understand economies with high rates of oscillation of foreign currencies e new simulations are orientated in relation to the risk-free rate, because in countries that suffer exchange rate shock, inflation. The central bank interest rate can suffer great interference.

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