Fixed Point Theorem and Absorbing Maps in Fuzzy Metric Space

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Abstract

In this paper, the concept of sub-compatibility and sub-sequential continuity in fuzzy metric space has been applied to prove a common fixed point theorem for six self maps using implicit relation. Our result generalizes and extends the result of Ranadive and Chouhan [13].

Keywords: Fuzzy metric space, common fixed point, absorbing maps, sub-compatibility, and sub-sequential continuity.

1. Introduction.

As the theory of fuzzy sets, introduced by Zadeh [18] appeared in 1965 it has been used in a variety of areas of mathematics.. Zadeh [19] estimated that medical diagnosis would be the most liable application domain of Fuzzy set theory. Following Zadeh's idea, Atanassov [1] introduced the concept of intuitionistic fuzzy set to permit grouping elements according to degrees of closeness and isolation. Fuzzy topology is another example of use of Zadeh's theory. George and Veeramani [4] and Kramosil and Michalek [7] have introduced the concept of fuzzy metric spaces which can be regarded as a simplification of the statistical (probabilistic) metric space. Afterwards, Grabiec [5] defined the completeness of the fuzzy metric space. Following Grabiec's work, Fang [3] further established some new fixed point theorems for contractive type mappings in G-complete fuzzy metric spaces. Soon after, Mishra et. al. [8] also obtained numerous common fixed point theorems for asymptotically commuting maps in the same space, which generalize a number of fixed point theorems in metric, Menger, fuzzy and uniform spaces.

The concepts of semi-compatibility and weak-compatibility in fuzzy metric space were given by Singh and Jain [15] which was simplification of commuting and compatible maps. Popa [10, 11] introduced the idea of implicit function to prove a common fixed point theorem in metric spaces. Singh and Jain [16] further extended the result of Popa [10-11] in fuzzy metric spaces. Using the concept of R-weak commutative mappings, Vasuki [17] proved the fixed point theorems for fuzzy metric space. In 2009, using the concept of sub-compatible maps, Bouhadjera et. al. [2] proved common fixed point theorems. In 2010 and 2011, Singh et. al. [14, 16] proved fixed point theorems in fuzzy metric space using the concept of semi-compatibility, weak compatibility and compatibility of type (β) respectively. Ranadive et.al. [13] introduced the concept of absorbing mapping in fuzzy metric space and proved the common fixed point theorem in this space. Moreover, Ranadive et.al. [13] observed that the new notion of absorbing map is neither a sub class of compatible maps nor a subclass of non compatible maps . Afterwards, Mishra et. al. [9] proved fixed point theorems using absorbing mappings in fuzzy metric space.

2. Preliminaries.

Definition 2.1. [7] A binary operation $*: [0, 1] \times [0, 1] \rightarrow [0, 1]$ is continuous t-norm if it satisfies the following conditions:

(1) $*$ is associative and commutative,

(2) $*$ is continuous,

(3) $a * 1 = a$ for all $a \in [0,1]$,

(4) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$, for each $a, b, c, d \in [0,1]$.

Two typical examples of continuous t-norm are $a * b = ab$ and $a * b = \min(a, b)$

Definition 2.2. [7] The three tuple $(X, M,*)$ is called a fuzzy metric space if X is an arbitrary set,* is a continuous t-norm and M is a fuzzy set in $X^2\times[0,\infty)$ satisfying the following conditions:

for all x, y, z $\in$ X and s,t > 0,

(FM-1) $M(x, y, 0) = 0$,

(FM-2) $M(x, y, t) = 1$, for all t > 0 if and only if x = y

(FM-3) $M(x, y, t) = M(y, x, t)$,

(FM-4) $M(x, y, t) * M(y, z, s) \geq M(x, z, t+s)$

(FM-5) $M(x, y, .) : [0, \infty) \to [0, 1]$ is left continuous.

(FM-6) $\lim_{t\to\infty} M(x, y, t) = 1$.

Example 2.1.[7] Let (X,d) be a metric space. Define a*b = min{a,b} and $M(x, y, t) = \frac{t}{t + d(x, y)}$ for all x, y $\in$ X and all t > 0. Then (X, M, *) is a fuzzy metric space. It is called the fuzzy metric space induced by d.

Definition 2.3. [7] A sequence $\{x_n\}$ in a Fuzzy metric space (X,M,*) is said to be a Cauchy sequence if and only if for each $\varepsilon > 0$, $t > 0$ there exists $n_0 \in N$ such that $M(x_n,x_m,t) > 1 - \varepsilon$ for all $n, m \geq n_0$.

The sequence $\{x_n\}$ is said to converge to a point x in X if and only if for each $\varepsilon > 0$, $t > 0$ there exists $n_0 \in N$ such that $M(x_n, x, t) > 1 - \varepsilon$ for all $n \geq n_0$.

A fuzzy metric space (X,M,*) is said to be complete if every Cauchy sequence in it converges to a point in it.

Definition 2.4. A pair (A, B) of self maps of a fuzzy metric space (X, M, *) is said to be reciprocal continuous if $\lim_{n\to\infty} ABx_n = Ax$ and $\lim_{n\to\infty} BAx_n = Bx$ whenever there exists a sequence $\{x_n\} \in X$ such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = x \in X$. If A and B are both continuous then they are obviously reciprocally continuous but the converse need not be true.

Definition 2.5. [15] Let A and B be mappings from fuzzy metric space (X, M, *) into itself. The mappings A and B are said to be compatible if and only if $M(ASx_n, SAx_n, t) \to 1$, for all $t > 0$ whenever $\{x_n\}$ is a sequence in X such that $Sx_n, Ax_n \to p$ for some p in X as $n \to \infty$.

Definition 2.6. [15] Let A and S be mappings from fuzzy metric space (X,M,*) in to itself. Then the mappings A and S are said to be semi-compatible if

$$\lim_{n\to\infty} ASx_n = Sx,$$

whenever $\{x_n\}$is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = x \in X$.

It follows that if (A,S) is semi compatible and Ay = Sy, then ASy = SAy by taking $\{x_n\} = y$ and x = Ay = Sy.

Definition 2.7. [9]. A pair of maps A and B is called weakly compatible pair if they commute at their coincidence points i.e. Ax = Bx if and only if ABx = BAx.

Definition 2.8. [13]. Let A and B be two self maps on a fuzzy metric space (X, M, *) then A is called B-absorbing if there exists a positive integer R > 0 such that $M(Bx, BAx, t) \geq M(Bx, Ax, t/R)$ for all $x \in X$.

Similarly B is called A-absorbing if there exists a positive integer R > 0 such that $M(Ax, ABx, t) \geq M(Ax, Bx, t/R)$ for all $x \in X$.

Preposition 2.1. In a fuzzy metric space (X, M, *) limit of a sequence is unique.

Preposition 2.2. [9] If (A,S) is a semi compatible pair of self maps of a fuzzy metric space (X, M, *) and S is continuous, then (A,S) is compatible.

Lemma 2.1. [8] Let (X, M, *) be a fuzzy metric space. Then for all $x, y \in X$, M(x, y, .) is a non-decreasing function.

Lemma 2.2. [8] Let (X, M, *) be a fuzzy metric space. If there exists $k \in (0, 1)$ such that for all $x, y \in X$, $M(x, y, kt) \geq M(x, y, t)$ for all $t > 0$, then x = y.

Lemma 2.3. [8] Let $\{x_n\}$ be a sequence in a fuzzy metric space (X, M, *). If there exists a number $k \in (0, 1)$ such that $M(x_{n+2}, x_{n+1}, kt) \geq M(x_{n+1}, x_n, t)$, for all $t > 0$ and $n \in N$. Then $\{x_n\}$ is a Cauchy sequence in X.

Preposition 2.3. [6] Let A and B be mappings from a fuzzy metric space (X, M, *) into itself. Assume that (A, B) is reciprocal continuous then (A, B) is semi-compatible if and only if (A, B) is compatible.

Definition 2.9. [6] Self mappings A and S of a fuzzy metric space (X, M, *) are said to be sub-compatible if there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z,\ z \in X \quad \text{and satisfy} \quad \lim_{n\to\infty} M(ASx_n, SAx_n, t) = 1.$$

Clearly, semi-compatible maps are sub-compatible maps but converse is not true.

Example 2.2. Let X = [0,∞) with usual metric d and define $M(x,y,t)=\dfrac{t}{t+d(x,y)}$ for all

x, y∈ X , t > 0 define the self maps A, S as

$$Ax=\begin{cases}2+x, & 0\le x\le 2\\ 3x-1, & 2<x<\infty\end{cases} \quad \text{and} \quad Sx=\begin{cases}2-x, & 0\le x\le 2\\ 3x-2, & 2<x<\infty\end{cases}.$$

Define a sequence $\{x_n\}=\dfrac{2}{n}$ in X. Then

$$Ax_n = 2+\frac{2}{n} \quad \text{and} \quad Sx_n = 2-\frac{1}{n}.$$

Also, $\lim_{n\to\infty} M(ASx_n, SAx_n, t) = \lim_{n\to\infty} M(4, 4, t) = 1$.

Now, $\lim_{n\to\infty} Ax_n = 2$ and $\lim_{n\to\infty} Sx_n = 2$

This implies $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = 2$. But $\lim_{n\to\infty} ASx_n \neq Sx$.

Thus, A and S are sub-compatible but not semi-compatible.

Definition 2.10. Self mappings A and S of a fuzzy metric space (X, M, *) are said to be sub-sequentially continuous if and only if there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z,\ z \in X \text{ and satisfy}$$

$$\lim_{n\to\infty} ASx_n = Az \text{ and } \lim_{n\to\infty} SAx_n = Sz.$$

Clearly, if A and S are continuous or reciprocally continuous then they are obviously sub-sequentially continuous. However, the converse is not true in general.

Example 2.3. Let X = R, endowed with metric d and $M_d(x,y,t) = M(x,y,t) = \dfrac{t}{t + d(x,y)}$

for all x, y$\in$ X , t > 0. Define the self maps A, S as

$$Ax = \begin{cases} 2, & x < 3 \\ x, & x \geq 3 \end{cases} \quad \text{and} \quad Sx = \begin{cases} 2x - 4, & x \leq 3 \\ 3, & x > 3 \end{cases}.$$

Consider a sequence $\{x_n\} = 3 + \dfrac{1}{n}$ then

$$Ax_n = \left(3 + \frac{1}{n}\right) \to 3 \text{ and } SAx_n = S\left(3 + \frac{1}{n}\right) = 3 \neq S(3) = 2 \text{ as } n \to \infty.$$

Thus A and S are not reciprocally continuous but, if we consider a sequence $\{x_n\} = \left(3 - \dfrac{1}{n}\right)$, then $Ax_n = 2$, $Sx_n = 2$, $ASx_n = 2 = A(2)$, $SAx_n = 0 = S(2)$ as $n \to \infty$.

Therefore , A and S are sub-sequentially continuous.

Definition 2.11. [13] A class of implicit relation

Let Φ be the set of all real continuous functions $F : (R^+)^5 \to R$ non-decreasing in first argument satisfying the following conditions :

(i) For u, v $\geq$ 0, F(u, v, v, u, 1) $\geq$ 0 implies that u $\geq$ v.

(ii) F(u, 1, 1, u, 1) $\geq$ 0 or F(u, 1, u, 1, u) $\geq$ 0, or F(u, u, 1, 1, u) $\geq$ 0 implies that u $\geq$ 1.

Example 2.4. Define $F(t_1, t_2, t_3, t_4, t_5) = 16t_1 - 12t_2 - 8t_3 + 4t_4 + t_5 - 1$. Then $F \in \Phi$.

(i) $F(u, v, v, u, 1) = 20(u - v) \geq 0 \Rightarrow u \geq v$.

(ii) $F(u, 1, 1, u, 1) = 20(u - 1) \geq 0 \Rightarrow u \geq 1$ or

$F(u, 1, u, 1, u) = 9(u - 1) \geq 0 \Rightarrow u \geq 1$

or $F(u, u, 1, 1, u) = 5(u - 1) \geq 0 \Rightarrow u \geq 1$.

3. Main Result

Theorem 3.1. Let A, B, S, T, P and Q be self mappings of a complete fuzzy metric space (X, M, *) with t-norm defined by $a * b = \min\{a, b\}$, satisfying :

(3.1) $P(X) \subseteq ST(X)$, $Q(X) \subseteq AB(X)$;

(3.2) Q is ST-absorbing;

(3.3) for some $F \in \Phi$ there exists $q \in (0,1)$ such that for all $x, y \in X$ and $t > 0$

$$F\{M(Px, Qy, qt), M(ABx, STy, t), M(Px, ABx, t), M(Qy, STy, qt), M(Px, STy, t)\} \geq 0.$$

(3.4) AB = BA, ST = TS, PB = BP, QT = TQ.

If the pair of maps (P, AB) is sub-sequential continuous and sub-compatible then P, Q, S, T, A and B have a unique common fixed point in X.

Proof. Let $x_0 \in X$ be any arbitrary point. From (3.1), there exist $x_1, x_2 \in X$ such that

$$Px_0 = STx_1 \quad \text{and} \quad Qx_1 = ABx_2 .$$

Inductively, we can construct sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$Px_{2n-2} = STx_{2n-1} = y_{2n-1} \quad \text{and}$$

$$Qx_{2n-1} = ABx_{2n} = y_{2n} \quad \text{for } n = 1, 2, 3, \dots .$$

Step 1. Putting $x = x_{2n}$ and $y = x_{2n+1}$ for $t > 0$ in (3.3), we get

$$F\{M(Px_{2n}, Qx_{2n+1}, qt), M(ABx_{2n}, STx_{2n+1}, t), M(Px_{2n}, ABx_{2n}, t), M(Qx_{2n+1}, STx_{2n+1}, qt), M(Px_{2n}, STx_{2n+1}, t)\} \geq 0,$$

i.e., $F\{M(y_{2n+1}, y_{2n+2}, qt), M(y_{2n}, y_{2n+1}, t), M(y_{2n+1}, y_{2n}, t), M(y_{2n+2}, y_{2n+1}, qt), M(y_{2n+1}, y_{2n+1}, t)\} \geq 0.$

Using lemmas 2.1 and 2.2, we have

$$M(y_{2n+1}, y_{2n+2}, qt) \geq M(y_{2n}, y_{2n+1}, t).$$

Again substituting $x = x_{2n+2}$ and $y = x_{2n+3}$ in (3.3), we get

$M(y_{2n+2}, y_{2n+3}, qt) \geq M(y_{2n+1}, y_{2n+2}, t)$.

Hence by lemma 2.3, $\{y_n\}$ is a Cauchy sequence in X. Since X is complete, therefore, $\{y_n\} \to z$ in X and also its subsequences converges to the same point i.e. $z \in X$,

i.e. $\{Qx_{2n+1}\} \to z$ and $\{STx_{2n+1}\} \to z$ (1)

$\{Px_{2n}\} \to z$ $\{ABx_{2n}\} \to z$ (2)

Step 2. (P, AB) is sub-compatible and sub-sequentially continuous then there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} Px_n = \lim_{n\to\infty} ABx_n = z, \; z \in X \quad \text{and satisfy}$$

$$\lim_{n\to\infty} M(P(AB)x_n, (AB)Px_n, t) = M(Pz, ABz, t) = 1.$$

Therefore , $Pz = ABz$. (3)

Step 3. Putting $x = Px_{2n}$ and $y = x_{2n+1}$ in condition (3.3), we have

$$F\{M(PPx_{2n}, Qx_{2n+1}, qt), M(ABPx_{2n}, STx_{2n+1}, t), M(PPx_{2n}, ABx_{2n}, t),$$
$$M(Qx_{2n+1}, STx_{2n+1}, qt), M(PPx_{2n}, STx_{2n+1}, t)\} \geq 0$$

Taking $n\to\infty$ and using (1), (2), (3), we get

$$F\{M(Pz, z, qt), M(Pz, z, t), M(Pz, Pz, t), M(z, z, qt), M(Pz, z, t)\} \geq 0$$

$$F\{M(Pz, z, qt), M(Pz, z, t)\} \geq 0$$

i.e. $M(Pz, z, qt) \geq M(Pz, z, t)$

Therefore by using lemma 2.2, we have

$z = Pz = ABz$

Step 4. Putting $x = Bz$ and $y = x_{2n+1}$ in condition (3.3), we get,

$$F\{M(PBz, Qx_{2n+1}, qt), M(ABBz, STx_{2n+1}, t), M(PBz, ABBz, t),$$
$$M(Qx_{2n+1}, STx_{2n+1}, qt), M(PBz, STx_{2n+1}, t)\} \geq 0$$

As $BP = PB$, $AB = BA$, so we have

$P(Bz) = B(Pz) = Bz$ and $(AB)(Bz) = (BA)(Bz) = B(ABz) = Bz$.

Taking $n \to \infty$ and using (1), we get

$$F\{M(Bz, z, qt), M(Bz, z, t), M(Bz, Bz, t), M(z, z, qt), M(Bz, z, t)\} \geq 0$$

$$F\{M(Bz, z, qt), M(Bz, z, t)\} \geq 0$$

i.e., $M(Bz, z, qt) \geq M(Bz, z, t)$.

Therefore by using lemma 2.2, we have

$Bz = z$ and also we have $ABz = Z$

This implies $Az = z$

Therefore $Az = Bz = Pz = z$. (4)

Step 5. As $P(X) \subseteq ST(X)$, there exist $u \in X$ such that

$z = Pz = STu$. (5)

Putting $x = x_{2n}$ and $y = u$ in condition (3.3), we get

$$F\{M(Px_{2n}, Qu, qt), M(ABx_{2n}, STu, t), M(Px_{2n}, ABx_{2n}, t),$$
$$M(Qu, STu, qt), M(Px_{2n}, STu, t)\} \geq 0.$$

Letting $n \to \infty$ and using (2) and (5), we get

$$F\{M(z, Qu, qt), M(z, z, t), M(z, Pz, t), M(Qu, z, qt), M(z, z, t)\} \geq 0$$

As F is non-decreasing in the first argument, we have

$$F\{M(z, Qu, qt), 1, 1, M(Qu, z, qt), 1\} \geq 0$$

i.e., $M(z, Qu, qt) \geq 1$.

Therefore, $z = Qu = STu$.

Since Q is ST absorbing, we have

$$M(STu, STQu, t) \geq M(STu, Qu, t/R) \geq 1$$

i.e., $STu = STQu$ which implies $z = STz$.

Putting x = z and y = z in (3.3) , we get

$$F\{M(Pz, Qz, qt), M(ABz, STz, t), M(Pz, ABz, t), M(Qz, STz, qt), M(Pz, STz, t)\} \geq 0$$

or, $F\{M(z, Qz, qt), M(z, z, t), M(z, z, t), M(Qz, z, qt), M(z, z, t)\} \geq 0$.

As F is non-decreasing in the first argument, we have

$$F\{M(z, Qz, qt), 1, 1, M(Qz, z, qt), 1\} \geq 0,$$

i.e., $M(z, Qz, qt) \geq 1$.

Therefore, z = Qz

Hence, z = Qz = STz.

Step 6. Putting $x = x_{2n}$ and y = Tz in condition (3.3), we get

$$F\{M(Px_{2n}, QTz, qt), M(ABx_{2n}, STTz, t), M(Px_{2n}, ABx_{2n}, t), M(QTz, STTz, qt), M(Px_{2n}, STTz, t)\} \geq 0$$

As QT = TQ and ST = TS, we have

$$QTz = TQz = Tz \quad \text{and} \quad ST(Tz) = T(STz) = TQz = Tz.$$

Letting $n \to \infty$ and using (2) we get

$$F\{M(z, Tz, qt), M(z, Tz, t), M(z, z, t), M(Tz, Tz, qt), M(z, Tz, t)\} \geq 0$$

$$F\{M(z, Tz, qt), M(z, Tz, t)\} \geq 0$$

i.e., $M(z, Tz, qt) \geq M(z, Tz, t)$.

Therefore, by lemma 2.2, we get

$$Tz = z$$

Now, STz = Tz = z implies Sz = z.

Hence, Sz = Tz = Qz = z. (7)

Hence, z is the common fixed point of A, B, S, T, P and Q.

Uniqueness: Let w be another fixed point of A, B, P, Q, S and T. Then putting $x = z$ and $y = u$ in (3.3), we get

$$F\{M(Pz, Qu, qt), M(ABz, STu, t), M(Pz, ABz, t), M(Qu, STu, qt), M(Pz, STu, t)\} \geq 0$$

As F is non-decreasing in the first argument, we have

$$F\{M(z, u, qt), M(z, u, t), M(z, z, t), M(u, u, qt), M(z, u, t)\} \geq 0$$

or, $F\{M(z, u, qt), M(z, u, t), 1, 1, M(z, u, t)\} \geq 0$

i.e. $z = u$.

Hence z is unique fixed point in X.

Remark 3.1. If we take B = T = I (the identity map) in theorem 3.1, we get the following corollary.

Corollary 3.1. Let A, B, S, T, P and Q be self mappings of a complete fuzzy metric space (X, M, *) with t-norm defined by $a * b = \min\{a, b\}$, satisfying :

(3.1) $P(X) \subseteq S(X)$, $Q(X) \subseteq A(X)$;

(3.2) Q is S-absorbing;

(3.3) for some $F \in \Phi$ there exists $k \in (0,1)$ such that for all $x, y \in X$ and $t > 0$

$$F\{M(Px, Qy, kt), M(Ax, Sy, t), M(Px, Ax, t), M(Qy, Sy, kt), M(Px, Sy, t)\} \geq 0.$$

If the pair of maps (P, A) is sub-sequential continuous and sub-compatible then P, Q, S and A have a unique common fixed point in X.

Remark 3.2. In view of Remark 3.1, Corollary 3.1 is a generalization of the result of Ranadive and Chouhan [13] in the sense that condition of reciprocal continuous and semi-compatible maps has been replaced by sub-sequential continuous and sub-compatible maps.

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