**DETERMINATION OF LAG THRESHOLD ON THE MEASURE OF COLLINEARITY**

**BY**

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**Abstract**

To date, the implementation of lag selection procedures within the context of unit root tests is largely based on application of standard information criteria which are well-known to have lag selection problem. In this paper, we propose a new lag selection procedure based on collinearity measures for an autoregression of differenced lagged values. In empirical analysis, the conventional coefficient of multiple determination denoted by  has been widely used in statistical literature for the computation of standard collinearity measures such as variance inflation factor(VIF) and tolerance index(ToL) but given the well-known difference between the underlying assumptions of standard multiple linear regression and a linear autoregression, the conventional coefficient of multiple determination which is used to compute VIF and ToL under the framework of standard multiple linear regression may not provide an appropriate collinearity measures under a linear autoregression setting. Hence, the realization of this obvious inadequacy motivated the introduction of our newly proposed statistic called the coefficient of multiple determination for autoregression of differenced lagged values of time series data. Unit root tests are implemented on autoregression made up of differenced lagged values which requires an appropriate choice of optimal truncation lag. However, in the current literature there is dearth of information on possibility of using collinearity statistics as criteria for lag truncation in unit root tests. Using the newly proposed coefficient of multiple determination for autoregression of differenced lagged series denoted by , we generate a class of collinearity statistics called tolerance index and variance inflation factor for differenced lagged series denoted by  and  respectively. We used these collinearity statistics as criteria for determining lag threshold for unit root tests. We employed augmented Dickey-Fuller(ADF) and generalized Dickey Fuller (DF-GLS) unit root tests to illustrate empirically the application of these collinearity criteria for lag selection and we demonstrated that these criteria select lag-length that is parsimonious thereby avoiding the problem of over-specification of truncation lag-length that is commonly associated with classical lag selection techniques.

**Keywords:** Coefficient of determination for multiple regression, coefficient of determination for autoregression, variance inflation factor, tolerance index, collinearity, unit root test, truncation lag

**1. INTRODUCTION**

To date the challenge of determining optimal autoregressive lag length for unit root test regressions is crucial in the analysis of time series, since the empirical power property of unit root tests is known to be highly sensitive to the choice of truncation lag parameter. Different choices of optimal truncation lag parameters suggested by different classical lag selection criteria constitute a big challenge to applied practitioners as regards the choice of a particular truncation lag among the array of competing optimal truncation lag parameters suggested by different information criteria. Hence, the choice of correct lag length is an important specification decision. Among popular methods used for determination of truncation lag-length in unit root tests, the standard information criteria such as Akaike Information Criterion (AIC) proposed by , Final prediction Error (FPE) due to , Schwarz’s Bayesian Information Criterion (BIC) introduced by and Hannan-Quinn Information Criteria suggested by  are the most widely used lag selection criteria in empirical analysis . However, the major problem with standard information criteria according to  is that these widely used lag selection criteria select optimal truncation lag parameter that leads to poor empirical size and power properties. Poor empirical power property of unit root tests is a consequence of over-specification of optimal truncation lag parameter. In this paper, we explore the collinearity property of time series data to provide an alternative procedure for determination of lag threshold for unit root tests based on two important measures of collinearity: variance inflation factor (VIF) and tolerance (ToL). Conventionally,represents the proportion of variance in the  independent variable that is associated with the other independent variables in the model. According to ,gives an excellent measure of the collinearity of the  independent variable with other independent variables in the model .The tolerance(ToL) for the  independent variable is 1 minus the proportion of variance it shares with the other independent variables in the analysis. This represents the proportion of variance in the  independent variable that is not related to other independent variables in the model. This could serve as a good measure of cut-off for the lag with respect to the variables in the model. The reciprocal of this tolerance may give an important measurement of lag specification for all ,where is optimal truncation is lag and  is the sample size. In this chapter, our focus is to establish an empirical connection between measures of collinearity and procedure for lag truncation in unit root test regressions using VIF and tolerance index as criteria. Following a study by ,it is evident that computation of collinearity statistics with conventional coefficient of determination is theoretically and empirically unjustifiable in the context of linear autoregression given the well-known differences between the underlying assumptions of a standard multiple regression model and a linear autoregression. In view of this, we propose a new coefficient of multiple determination for differenced lagged values of time series denoted by  as a more credible alternative to conventional coefficient of determination denoted by .Our empirical study shall include estimation of variance inflation factor (VIF) and tolerance (ToL) using the new proposed statistic as well as investigation of the performance and robustness of the proposed statistic using a simulation study. One particular advantage of using this proposed statistic for the computation of VIF and ToL is that the proposed statistic is a function of the conventional coefficient of multiple determination. Since the VIF is a measure of how the variance of a regression coefficient for a particular predictor is inflated, it can be used as a criterion for determining the cut-off point for lag truncation within the framework of linear autoregressive unit root tests.

**2. Specification of Tolerance Index and Variance Inflation Factor in Unit Root Testing**

Consider the distributed lag representation of ADF and DF-GLS regression models as follows:

 (1)

We consider the latter part of equation (1) with  regressors and we have

 (2)

Where  ,  ,  and 

Collinearity is related to the existence of near dependences among the columns of the  matrix. The tolerance index on each regressor serves as measure of lag truncation since it relates the association of the  regressor to other lagged regressors in the right-hand side of equation (2).The tolerance is given by:

 (3)

Where  is the conventional coefficient of determination obtained in each of the auxiliary regressions of the form:

 (4)

The shows the proportion of variance of  that is not accounted for by the remaining  regressors and the higher the index , the less the contribution of other  regressors and this can be used as an index of measuring the truncation lag .The variance inflation factor(VIF) can be obtained as the reciprocal of .This is a measure of the increment of sampling variance of the estimated regression coefficient of due to linear relationship among the 

The value of  (5)

Equation (5) produces a measure of truncation of the lag using the coefficients of regression. To measure the point of truncation of lag, the lag L for which VIF is close to 10 or above and  will be the point to consider as the best lag for testing the unit root. We shall later examine the range of values for collinearity statistics which we call variance inflation factor and tolerance index for autoregression of differenced lagged series denoted by and  respectively within the framework of unit root tests. The analogous relationship that exist between the regression model described in equation (4) and conventional autoregressive model used in time series analysis (see ) will allow us to specify the model as:

 (6)

where is a white noise process and  is the dependent variable for the auxiliary autoregression. Equation (6) follows all properties of autoregressive model and could be used to ascertain the optimal lag with respect to  and  defined in equations (3) and (5) above.

 **3. Derivation of Coefficient of Multiple Determination for a Differenced Autoregressive Model**

In standard multiple linear regression model, the collinearity statistics such as tolerance index (ToL) and variance inflation factor (VIF) are usually computed using the conventional coefficient of multiple determination denoted by. Let us the denote these collinearity statistics computed with  as  and  respectively. This study is based on time series regression involving differenced lagged values. We consider an autoregressive model of the form:

 (7)

Where  is the differenced series of the observed time series , is a multiple regression constant,  are regression coefficients to be estimated, are the differenced lagged values of and  is a white noise process.

We introduce a new coefficient of multiple determination for differenced time series within the framework of linear autoregression which we define as  where

  (8)

Where  and  ,Put  into (8), we have

  (9)

From conventional notations, we can have the following representations

 and  ,using these two representations in (9) above, we can factor out the conventional coefficient of multiple determination denoted by  from  as follows:

 (10)

After some algebraic manipulations of equation (10) we have

 (11)

Dividing both sides of (11) by  we have

  (12)

If we let  and in (12) then we have the following:

 (13)

Factorizing LHS of (13) gives

 (14)

 This reduces to

 (15)

Since  takes on values between 0 and 1, we consider the two extreme values of  when  and .We begin with the first case when  in (15), then  is given by

 (16)

Similarly, for the second case when  in (16), we have

 (17)

If we let  and  in equations (16) and (17) respectively, then for the case  , becomes

  (18)

Similarly, for the case  , becomes

 (19)

Following equation (19) , it is obvious that as approaches 1 ,  also approaches 1 but this is not the case when  , as clearly demonstrated in equation (18).The immediate consequences of the preceding results lead to the following properties of  which are easily discernable:

1. The values of  is non-negative
2. When , , this implies that the value of  can never be zero unlike the conventional coefficient of multiple determination that can be zero
3. As the conventional coefficient of multiple determination  approaches 1, the newly proposed coefficient of multiple determination for autoregression of differenced lagged series denoted by  also approaches 1
4. If  this implies that  and  .Hence, when , then and this gives  and .This means that for a differenced series, to measure the point of truncation of lag, the lag L for which is closed to 4 or above and will be the point to consider as the best lag for testing the unit root.

**4. Empirical Analysis of Auxiliary Autoregression**

Auxiliary regression is essentially a descriptive regression of each explanatory variable expressed as a function of other explanatory variables. One of the most popular methods to detect collinearity in time series regression is implementation of auxiliary regression. In standard multiple linear regression, the usual convention for determining the seriousness of multicollinearity problem is to compare the coefficient of multiple determination for an auxiliary regression with the coefficient of multiple determination for the main regression. Multicollinearity is a problem if the former is greater than the latter. By the same token, the seriousness of multicollinearity problem within the framework of linear autoregression is also confirmed if the coefficient of multiple determination for a linear autoregression denoted by  for an auxiliary autoregression is greater than the coefficient of multiple determination for the main autoregression. We shall examine the auxiliary autoregression for model (6) which is defined as follows:

 (20)

Where maximum lag is pre-set at 12 for monthly time series data. The associated twelve auxiliary autoregressions are given as follows:





For  we repeat the same process

Where  are regression coefficients for the explanatory variables in the auxiliary regressions. The error term  is a white noise process satisfying the following properties:

i.

ii.

iii.

**5. Data Analysis**

For empirical illustration, we used both simulated and real datasets in order to evaluate the performance of the newly proposed statistic called coefficient of multiple determination for differenced autoregression denoted by .This new statistic is used for the computation of tolerance index () and variance inflation factor () for the differenced autoregression. For empirical illustration based on real dataset, we employed US inflation data retrieved from the IMF database. The sample contains 624 monthly observations from January, 1962 to December, 2013.For simulated data, the random number generator module of Excel was used to simulate normally distributed series. The simulated series has mean 30 and variance 10. Prior to unit root testing, we examine the both simulated and real datasets for trends and stationarity using the time plots for simulated data and the US inflation rates respectively. Figure 1 and Figure 2 show that the simulated data and the US inflation rates are both stationary.

 FIGURE 1: SIMULATED DATA AT LEVEL





We now proceed to fit an autoregression of order 12 to our datasets and we apply the collinearity criteria to determine the appropriate lag threshold for unit root testing. Using both simulated and real datasets, the empirical results are presented in table 1 and 2 below:

**TABLE 1 : EMPIRICAL RESULT FOR SIMULATED DATA**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Model(DependentVariable) | R | R-Square | Adjusted R-Square | Std. Error of the Estimate | Regression F | Sig | ToL | VIF |
|  | 0.650 | 0.422 | 0.416 | 10.58568 | 64.76 | 0.000 | 0.578 | 1.730 |
|  | 0.812 | 0.659 | 0.655 | 8.12220 |  171.27 | 0.000 | 0.341 | 2.933 |
|  |  0.862 | 0.743 |  0.740 |  7.04066 |  256.68 | 0.000 | 0.225 | 3.891 |
|  |  0.880 | 0.775 |  0.772 |  6.59181 |  279.87 | 0.000 | 0.206 | 4.444 |
|  |  0.891 | 0.794 |  0.791 |  6.31008 |  312.67 | 0.000 | 0.197 | 4.854 |
|  |  0.896 | 0.803 |  0.800 |  6.14481 |  330.46 | 0.000 | 0.197 | 5.076 |
|  |  0.896 | 0.803 |  0.800 |  6.14327 |  329.91 | 0.000 | 0.207 | 5.076 |
|  |  0.890 | 0.793 |  0.790 |  6.29442 |  310.40 | 0.000 | 0.226 | 4.831 |
|  |  0.880 | 0.774 |  0.771 |  6.56866 |  278.14 | 0.000 | 0.258 | 4.425 |
|  |  0.861 | 0.742 |  0.739 |  7.02235 |  233.22 | 0.000 | 0.344 | 3.876 |
|  |  0.810 | 0.656 |  0.652 |  8.08860 |  155.02 | 0.000 | 0.659 | 2.907 |
|  |  0.891 | 0.794 |  0.791 |  6.31008 |  312.67 | 0.000 | 0.197 | 4.854 |

In table 1 above, the collinearity statistics as well as other relevant statistics obtained for the fitted auxiliary autoregressions of the simulated data were presented. The first column of table 1 above describes dependent variable of the particular auxiliary autoregression being analyzed. The second column through the fourth column presents the values for,  and adjusted  respectively. The fifth column through the seventh column presents the values for standard error of the estimates, regression F statistic and significance respectively. The eighth and ninth columns describe the collinearity statistics, tolerance index  and variance inflation factor respectively. Overall, it is clearly evident that all the auxiliary regressions are statistically significant as shown by the values of regression F statistic in the sixth column of the table. The model fit for each auxiliary autoregression is above 0.4.The highest tolerance of 0.578 and the lowest variance inflation factor of 1.730 are achieved at lag 1. This value of tolerance indicates that 57.8% of the variance of the fitted auxiliary autoregression was not accounted for by other regressors and this implies that other regressors have less contribution in the fitted autoregression. Hence, the best point for lag truncation is at lag 1

**TABLE 2: EMPIRICAL RESULTS FOR US INFLATION RATE SERIES**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Model(DependentVariable) | R | R-Square | Adjusted R-Square | Std. Error of the Estimate | Regression F | Sig | ToL | VIF |
|  | 0.416 | 0.173 | 0.158 | 0.27020 | 11.393 | 0.000 | 0.827 | 1.209 |
|  | 0.532 | 0.283 | 0.269 | 0.25157 |  21.477 | 0.000 | 0.717 | 1.395 |
|  |  0.567 | 0.322 |  0.309 |  0.24457 |  25.878 | 0.000 | 0.678 | 1.475 |
|  |  0.573 | 0.329 |  0.316 |  0.24323 |  26.714 | 0.000 | 0.671 | 1.490 |
|  |  0.573 | 0.328 |  0.316 |  0.24333 |  26.640 | 0.000 | 0.672 | 1.488 |
|  |  0.576 | 0.332 |  0.320 |  0.24258 |  27.103 | 0.000 | 0.668 | 1.497 |
|  |  0.575 | 0.331 |  0.319 |  0.24258 |  26.998 | 0.000 | 0.669 | 1.495 |
|  |  0.571 | 0.325 |  0.313 |  0.24327 |  26.322 | 0.000 | 0.675 | 1.481 |
|  |  0.571 | 0.327 |  0.314 |  0.24301 |  26.448 | 0.000 | 0.673 | 1.486 |
|  |  0.577 | 0.335 |  0.308 |  0.24411 |  25.675 | 0.000 | 0.680 | 1.471 |
|  |  0.580 | 0.341 |  0.268 |  0.25101 |  21.340 | 0.000 | 0.659 | 1.517 |
|  |  0.585 | 0.352 |  0.157 |  0.26940 |  11.330 | 0.000 | 0.648 | 1.543 |

 The table 2 above presents the collinearity statistics as well as other relevant statistics obtained for the fitted auxiliary autoregressions for the US inflation rates dataset. The first column of table 2 above describes dependent variable of the particular auxiliary autoregression being analyzed. The second column through the fourth column presents the values for,  and adjusted  respectively. Similarly, the fifth column through the seventh column presents the values for standard error of the estimates, regression F statistic and significance respectively. The eighth and ninth columns describe the collinearity statistics, tolerance index  and variance inflation factor respectively. Overall, it is clearly evident that all the auxiliary regressions are statistically significant as shown by the values of regression F statistic in the sixth column of the table. The model fit for each auxiliary autoregression is less than 0.5.The highest tolerance of 0.827 and the lowest variance inflation factor of 1.209 are achieved at lag 1. This value of tolerance indicates that 82.7% of the variance of the fitted auxiliary autoregression was not accounted for by other regressors and this implies that other regressors have less contribution in the fitted autoregression. Hence, the best point for lag truncation is at lag 1.

**6. CONCLUSION**

This study has employed collinearity statistics as criteria for lag selection in unit root tests. We introduced a new statistic called coefficient of multiple determination for autoregression of differenced lagged series. Using both  and  the lag threshold for unit root tests was found to be 1.This empirical result shows that our lag selection procedure select parsimonious lag structure that guides against over-specification of truncation lag within the context of unit root tests and these collinearity statistics criteria could be used as an alternative lag selection procedure to cross-check the lag structure that may be suggested by conventional lag selection criteria as a way of safe-guarding against problem of over-specification of truncation lag-length that is commonly associated with classical lag selection criteria such as standard information criteria.

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