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A structural vector autoregression model for the study of the Japanese GDP and of the Japanese inflation

Abstract

In this paper is presented an historical decomposition of the Japanese GDP and inflation, make, on quarterly data included between the first quarter of 2000 and the fourth quarter of 2016, through a structural VAR of order 1, with the aim of understanding the contribution of the monetary and fiscal policy to the development of these two variables. In the paper is also studied a dynamic forecast of the growth rate of the Japanese real GDP with an ARIMA model, a model belonging to the family of stochastic processes. The results obtained are in line with the forecasts of the economic theory and do not reveal substantial differences compared to those present in the literature. The authors pause also to discuss some limits related to the techniques used in these analyzes and hope that the paper is a very useful for stimulating research and for bridging economics and mathematics.

Keywords : *Structural vector autoregression model, autoregressive integrated moving average, historical decomposition, Cholesky model, Sims models.*

1. Introduction

After more than forty years of strong growth, the Japanese economy has experienced, in the last decade of the last century, a phase of slow and inexorable decline. The causes of this long period of recession and deflation, better known as the "Crisis of the lost decade", have been and are at the center of the discussion in the academic and political world (not only Japanese).

Continual joint efforts have been carried out by the practitioners and by the academic researchers to understand and to analyze a phenomenon for many respects still unclear which originates from the speculative "bubble" broke out in 1990. According to Shigenori Shiratsuka [2013], Director General of the Institute for Economic and Monetary Studies of the Bank of Japan, the responsibility for the explosion of the bubble is to be attributed own to the Bank of Japan (B0J), which from March 1989 to September 1990 had decided to increase abruptly the interest rate from 2.5% to 6%. The governments, which since that date are alternated, have not been able to revive the economy because they have not taken any significant measures; their interventions, in fact, were uncoordinated, not very effective and tending only at increasing international cooperation with Asian countries and with the USA. Only the government driven by Jun'ichro Koizumi had, unsuccessfully, tried, since 2002, to reduce the public debt with a program of privatization of public assets and had undertaken a timid reform of the tax, destined, however, not to take off. In reality, Koizumi had developed an interesting plan of structural reforms but his program contained some unpopular measures (such as the dismissal of 10.7% of public employees) and so, due to the numerous political contrasts, the plan never saw the light .

The change of direction happened in 2013 when the Japanese government, driven by Prime Minister Shinzo Abe, helped by the Central Bank, undertook a complex macroeconomic policies plan finalized to revive the national economy, an intervention that can be considered, for scope and objectives, perhaps even more significant than those put in act by the Federal Reserve and the Obama administration in the USA, or by the BCE and the EU institutions to counter the recession, began in December 2007. Abe, already premier between 2005 and 2007, had won political elections in December 2012 and, after twenty years of stagnant economy or of recession in the country of the Rising Sun, had placed, among the priorities of its program, the review of the powers of the Bank of Japan (BOJ) and the weakening of a too strong yen, which penalized the country's exports. In previous years, the Bank of Japan had, in fact, used only the weapon of interest rates near to zero; the newly elected prime minister Shinzo Abe, instead, proposed a less prudent solution, an expansionist monetary policy, of public investments and of growth measures, namely a program of long-term structural reform that would allow an increase of investments in private sector, greater competitiveness and an increase of the rate of active population, especially with the inclusion of women's work. According to Abe, what more counted was to get the country out of the

crisis and not the problems of public debt: expected in two years the doubling of the Japanese monetary base and of the amount of securities of public debt in circulation, with an increase of the monetary base of 60-70 billions of yen a year and an annual increase in the amount of public debt of 50 billions of yen, with the total goal of breaking down the deflation and bringing the inflation to 2% in two years.
The complex economic policy plan put into practice by Prime Minister Shinzo Abe, in the spring of 2013, in order to raise Japan from the decennial economic depression, has been called *Abenomics* (the name is a a sincrasi of Abe (his supporter) and economics) and represented a turning point in the "Crisis of the Lost Decade". Indeed, placing itself as a third option between the European rigor and the USA economic plans, Abenomics aimed to encourage the central banks to increase liquidity to invest, subsequently, in public works. In practice, putting aside the question of huge public debt (close to 250% of GDP), Abe, with a series of macroeconomic initiatives, proposed to adopt a policy of stimulus to growth rather than pursuing the line of rigor and spending cuts, as happens in the EU. Unlike Italy, for example, the Japanese public debt is essentially internal (there is not, at least for now, the problem of the spread) and Tokyo has a national coin.

2. Research aims

After a long period of stagnation began with the outbreak of a frightening speculative bubble at the beginning of the years ninety, the Japanese seemed finally to have regained their enthusiasm when, in 2013, the Prime Minister Shinzo Abe launched a courageous economic policy program, baptized "Abenomics", with the aim of getting Japan out of deflation.
The aim of this paper is to understand the effectiveness of the government measures adopted, evaluating their empirical results and future effects or, more precisely, to study, through the estimation of a self regressive vector model (VAR, acronym of Vector Autoregression Model), the effects of financial shocks and monetary policy shocks on the Japanese economic cycle. In other words,in the paper are analyzed the effects of monetary policy and of fiscal policy on the main macroeconomic variables, in particular on GDP and on inflation, in order to describe and evaluate the degree of effectiveness with which the BOJ and the Japanese government pursue their goals. The VAR models, in fact, widely used to measure and understand the effects of monetary policy changes on the entire economy, have given test over time of a remarkable forecasting capacity: their main use is, in fact, precisely the forecast of economic variables in the time. These models (VAR), first used by Sims [(1980)] and subsequently developed, among others, by Bernanke [1986], Blanchard and Quah [1989] and Leeper, Sims and Zha [1996], take the name of structural VAR (or better SVAR, acronym of Structural Vector Autoregression Model), when they are used to model the structure of the economic phenomenon that must be to investigate. In reality, the structural VARs estimate empirically the coefficients that bind the variables of the economic system and, to explain the fluctuations observed historically in the variables everytime and caused by a specific structural shock, try to translate the economic relations, based on the theory, into statistical equations and therefore stochastic. The great advantage of using the SVAR models lies in the fact that, being dynamic models, to obtain identification it is necessary to impose a minimum number of theoretical constraints, without having to resort to a complete model of the economy, without, namely, being constrained, in an excessively stringent way, to the indications of economic theory.
The procedure adopted in the paper consists own in using the structural vector regression models (SVAR) by means of which the authors have a tried to expose in the paper a specific econometric analysis, to analyze the macroeconomic fluctuations of the Japanese economy.The analysis carried out is a Historical decomposition, one of the most interesting analyzes that can be conducted with the models for multivariate analysis and which allows to observe the contribution of structural shocks to the series considered.
In this paper, the authors present, in particular, a Historical decomposition of Japanese quarterly GDP and of Japanese quarterly inflation, carried out through a structural VAR, namely a SVAR model (acronym of Structural VAR), from the first quarter of 2000 to the fourth quarter of 2016, with the objective of understanding the contribution of the monetary policy and of the fiscal policy on the performance of these two variables. The variables considered to build the model are six, all on a quarterly basis; in order: public expenditure, GDP, tax revenue deriving from the sum between revenue from corporate taxes and consumption tax revenue, inflation at CPI, Bank of Japan's official discount rate, nominal interest rate on USA Federal funds. These variables were referred to as: GSPEND, GDP, TAX REVENUE, INFLATION, NIR and SFFR, respectively. The only exogenous variable to the system among those considered is SFFR, which depends on the monetary

policy decisions of the Federal Reserve and not of the BOJ. The first three variables, originally nominal, were transformed into real variables through the implicit deflator of GDP and, subsequently, converted into a logarithmic form. All the others have not undergone any kind of processing. In order to avoid the problem of setting the restrictions to be made to the matrix of disturbances and following the instructions of Jack Lucchetti [2015] for the implementation of the SVAR package in the Gretl software, the authors used, among the various models of SVAR, a particular recursive model, that Lucchetti himself, like other econometricians, defines also as "Cholesky model". On the Gretl software, instead, this model is called "plain Cholesky". It was decided to impose only one delay as this value is the one most appropriate on the base of the Bayesian information criterion (BIC).

In the paper is also carried out a dynamic forecast of the real growth rate of the Japanese GDP , carried out with an ARIMA model (acronym of Auto Regressive Integrated Moving Average), namely a particular type of models able to investigate historical series that present certain characteristics and used, in particular, for predicting the future trend of the series. The investigation was conducted on quarterly aggregate data from the first quarter of 2000 to the fourth quarter of 2016. The sources, from which it was possible to find the data are, are: the website of the Federal Reserve (USA central bank) and the Bank World. The program used for the research is Thomson Reuters DataStream software, suitable for managing and for analyzing time series, both economic and financial.

That one the authors want to highlight are the limits of Abenomics which, although until today, has not allowed to reach the objectives in terms of growth and of inflation prefixed by policy makers and although, in the immediate future, it seems not to offer any prospect of development to the Japanese economy, it has, however, achieved interesting results in its initial phases and can be up to now considered the best economic policy plan carried out to counter the crisis of the lost decade. The authors hope, furthermore, that the paper is a very useful for stimulating research at the interface of mathematics and economics (useful, namely, in bridging economics and mathematics, in particular) and that it builds a link that can become beneficial for both disciplines involved.

The paper is organized as follows. In section 2, after the introduction, are analysed the research aims while section 3 is dedicated to the years of the "economic miracle" and to the formation of the speculative bubble that determined the crisis of the lost decade. In section 4 is introduced the Abenomics and some critical evaluations on it are examined; subsequently, in section 5 is carried out, through an ARIMA model, a dynamic forecast of the GDP growth rate. Section 6 is devoted to Historical decomposition of the Japanese quarterly GDP and of the Japanese quarterly inflation, carried out with an alternative approach, namely with the SVAR methodology. Section 7, finally, provides a conclusion and contains a general discussion on directions for future research.

3. The years of the"economic miracle"and formation of the speculative bubble

In the second postwar period, Japan's GDP experienced a period of robust and lasting growth. According to data reported by the World Bank, it rose from a value of 44.307 billion USA dollars in 1960 to 211.514 billion in 1970, (+ 377%). The reasons for the "Japanese economic miracle" must be found in the disruptive structural changes that invested the country from the immediate second postwar period, transforming it into a modern nation and significantly innovating its economic system, still backward under many aspects.The main driver of development was export.

A key role in industrial and commercial development was played by the Ministry of Trade and Industry (MITI). This organism (still existing even if under different name), to took care of supporting exports and, more generally,of accessing foreign markets of national companies, of guiding energy policy and of promoting technological development. For this purpose were promoted policies of exchange rate devaluation and of moderate inflation.

Parallel to the expansion of their business turnover, the Japanese companies strengthened their ties with the banking world through the "*keiretsu*", industrial corporations born from the ashes from the "*zaibatsu*" (the pre-war corporations), aimed at fostering synergistic cooperations between companies and financial institutions. The energy crises of 1973 and 1979 did not have a particularly dramatic impact on the Japanese economy. The two shocks caused, in 1974 and 1980 respectively, slight recessions and a large increase in inflation. In both cases, however, already in the two following years the GDP returned to grow and the prices normalized. The 80s were years of further development.

However, since own 1985, began to swell the double speculative bubble that affected both the financial market and real estate markets ("baburu keiki"). The climate of general trust in fact had pushed banks linked to keiretsu to grant credit against in front of insufficient guarantees to companies, which in turn invested the capital received in the construction sector. This had led to a rapid increase prices of land, for residential use, and of the buildings. The phenomenon was well evidenced by a study by Noguchi and Poterba [1991]. The table below shows the data of the "Residential Land Price Index", set equal to 100 in 1983 for the different regions of the country

Table 1.2 **Residential Land Price Index (1983 = 100)**

	1984	1985	1986	1987	1988	1989	1990	1991
Greater Tokyo	102.2	103.9	107.0	130.1	219.3	220.2	234.7	250.2
Tokyo	102.9	105.9	112.7	169.6	283.2	265.3	264.5	264.8
Special wards	103.2	107.2	117.9	208.5	300.5	284.9	286.0	286.9
Kanagawa	102.0	103.6	106.0	118.8	220.7	203.9	205.3	211.3
Saitama	101.5	102.0	102.3	104.7	167.3	181.5	202.0	226.4
Chiba	101.8	102.6	103.4	109.8	179.3	210.3	261.6	312.6
Greater Osaka	103.6	106.7	109.5	113.2	134.3	178.2	278.1	296.2
Osaka	103.5	106.9	110.4	115.2	138.9	188.2	298.5	304.8
Kyoto	103.9	107.1	110.5	114.3	124.5	164.1	274.2	315.1
Hyogo	103.6	106.0	107.1	109.7	140.1	182.6	269.3	292.2
Nara	103.2	105.8	107.7	109.5	112.7	143.0	214.8	235.2
Greater Nagoya	102.4	104.0	105.5	107.2	115.0	139.9	160.9	191.1
Aichi	102.2	103.7	105.1	106.7	115.1	135.6	163.9	192.7
Mie	103.7	106.0	108.1	109.9	112.2	118.0	136.0	173.8

In 1991 bursted the real estate bubble. In 1980 the price per square meter of land for residential use was 21,200 yen, in 1990 it ascended to 34,000 yen. In 1991 it ascended to 36,600 yen. In the following years there were constant drops.

The financial bubble bursted instead in 1990. The Nikkei 225 index reached its all-time high just in December 1989. About the causes of the formation of the two bubbles several hypotheses have been formulated

Several hypotheses have been formulated about the causes of the formation of the two bubbles. Some, like Carlo Filippini [2008], support the phenomenon would be the consequence of the agreement of the Plaza of 1985. In fact, while from a political point of view for Japan the agreement was significant, because it involved the strengthening of its position in the international monetary system, on the economic level its consequences were disastrous: in the three-year period 1985-1987 the value of exports to the main trading partners (especially the US) decreased from 41.955 billion yen to 33.315 billion, causing so a deterioration of the trade surplus, the main driver of the national economy.

Others, instead, support the thesis of Shigenori Shiratsuka [2013], director general of the Institute for Economic and Monetary Studies, who in a paper on the topic, published in 2003, stated that the responsibility of the explosion of the bubble was to be attributed to a series of incorrect monetary policy decisions adopted by the BOJ, such as that of raising the interest rate abruptly from 2.5% to 6% ,from March 1989 to September 1990. Anyway, the burst of bubble in 1991 conventionally represents the start year of the so-called "lost decade", a long period of recession and deflation. In 1990 the annual growth rate of GDP was equal to + 5.57%. After three years there was a growth rate of + 0.17%. In the following three years, there was an appreciable recovery in GDP, reaching + 3.1% in 1996 and then a new traumatic fall up to -1.12% in 1998. This led to the sadly known GDP " W effect ". Again in 2000 it returned to a positive rate of + 2.78%. In 2002, in front of a growth rate of + 0.11% , the end of the recession was officially announced. The trend continued to decline until 2013, (just think - 5.41% in 2009). With regard to price trends, phases of weak inflation followed to prolonged periods of more or less marked deflation. In 2002 it registered a - 0.92%. In 2009, the historical minimum peak of -1.35 was reached. In 2013, it returned to a weak + 0.34%. What must be noted is that from 1991 to 2013 inflation was always well below the optimal level of 2%. One of the negative effects of deflation was obviously an increase in the unemployment rate, which until 1991 had always remained below 3% and that from that year increased progressively, reaching its historical maximum of 5.4% in 2002. There was also a worsening of public finances.

The ratio gross debt / GDP exploded from 63.49% in 1991 to 232.46% in 2013.

This strong increase was caused not only by a reduction of the denominator but also by the exponential increase in public spending, started, in truth, as early as 1980.

4.The complex economic policy plan: the Abenomics. Some critical evaluations.

A clear breakthrough came only after the parliamentary elections of 2012 which marked the affirmation of the Liberal Democratic Party (LDP) and the return to the government of Shinzo Abe, (who had already been Prime Minister in 2006-2007) who won the elections promising an plan of economic policy to lift the country from the situation in which it lay**,** for twenty-one years. The program, known as "Abenomics", is fundamentally based on three arrows: expansive monetary policy, flexible fiscal policy and structural reforms.

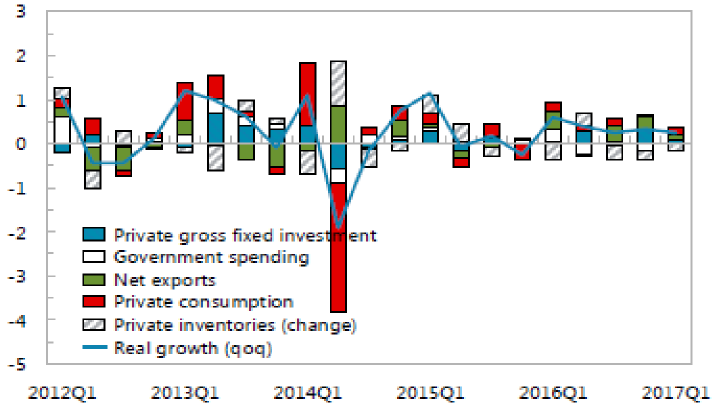
The "first arrow" is precisely that of expansive monetary policy and competes, naturally, at the BOJ, whose current governor is Haruhiko Kuroda, who has been in charge since 2013 and intends to support the government in achieving its objective. Initially Kuroda did not change the official discount rate and interest rate, leaving it at 0.30%, level that was set by his predecessor Masaaki Shirakawa in December 2008. The main action of policy monetary undertaken by the current governor is known as "Quantitative and qualitative easing" (QQE), began in March 2013 and is still ongoing. This is an unconventional operation of open market consisting mainly of the acquisition on the secondary market of government securities but also of other assets, such as real estate trust and ETF shares, for an annual amount of about 80 trillion (namely, 80 billion billion) of yen. The innovative element (the "qualitative" element) of the QQE is forward guidance, namely the conditioning of operators' behavior, keeping the interest rate at a certain level for a long period of time. Kuroda also inaugurated the Negative Interest Rate Policy (NIRP), bringing in negative territory and government bond long-term yields. The most significant example is that of the Tresaury Bill, which have a rate below zero since 2015. Up to now the BoJ has not provided precise indications regarding the conclusion of the QQE. It is important also to underline how the BoJ has never wanted to carry out public debt monetization operations.

The "second arrow" is that of "flexible" fiscal policy. In January 2013, the government announced a plan to increase public spending for a total value of 23.700 trillion yen (also taking into account the additional budget), divided equally between prefectures and Tokyo government. The "third arrow", that of structural reforms, mainly because of political contrasts and trade union categories, is the most problematic for the government and in fact in large part still under realization. Regarding the assessments of the Abenomics some observers show a cautious optimism, (while highlighting the inertia on structural reforms and the increase of public debt), and others, (mainly of Keynesian orientation), in at first favorable to the increase in public spending and subsequently disappointed by the decision to ease the expansionary fiscal policy. In July 2017, the International Monetary Fund (IMF) published its latest report on the conditions of the Japanese economy. From the pages drawn up by the Washington Institute, a moderate appreciation of the Abenomics emerges**.** In the introduction it reads: "Abenomics has improved economic conditions and started structural reforms but has not yet reached a definitive exit from deflation. The economy has expanded at a rate higher than the last five quarters and the unemployment has fallen to a new low level. The key drivers were the fiscal stimulus of short-term and the growth of aggregate demand. However, the inflation, the sustainability of the public debt, the growth objectives, must still be achieved. The risk of the outlook becoming negative over the medium term is particularly significant."

The IMF then explained that the aging of the population and the contraction of the workforce brake the growth and the productivity**.** The merit of Abenomics is to have guaranteed more favorable conditions of access to credit, to have increased the profits of great companies and to have supported employment, especially employment of women. The reforms that have most contributed to achieving this result are those of the energy market, of the agricultural sector, of the promotion of trade and investment and of corporate governance. The data also show that with these measures the real GDP from 2013 to 2015 has grown more than estimated.

Contributions to QoQ Real Growth (SA)

(In percent)

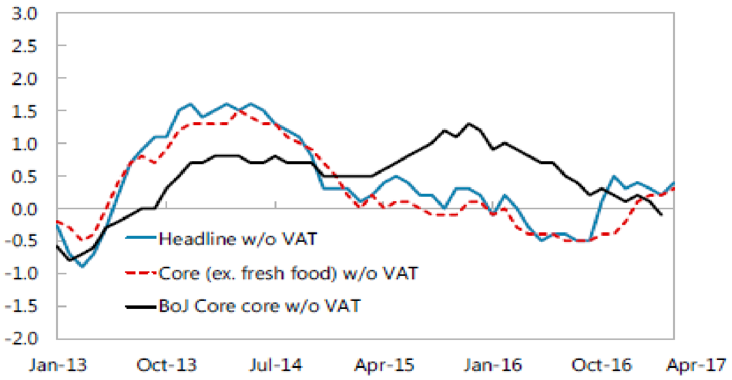


Source: Haver Analytics.

Disaggregating the data on the real GDP, it is clear that from the first quarter of 2013 up to the first quarter of 2017, public expenditure and net exports drove the growth. The latter also became more stable and sustained in 2016, more precisely in the second quarter of the year, when net exports reached their peak in the last five years, coinciding with the increase in aggregate demand and with the fall in imports, thanks to the reduction of costs for energy supply. Obviously this had beneficial effects on the payments balance. There is more concern for inflation, which is stubborn to remain low and has contracted through 2016, through to the strengthening of the yen and the fall in the cost of energy.

Inflation Indicators

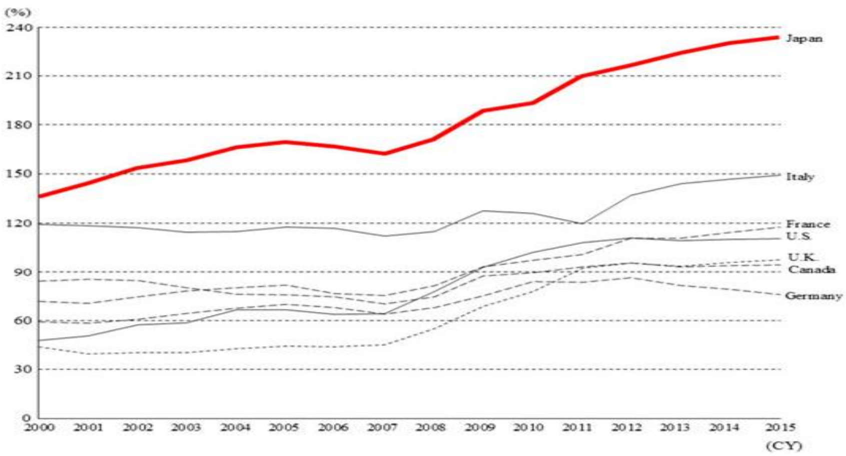
(YoY; in percent)



Sources: Haver Analytics; IMF staff estimates.

The position of the neo-keynesian economists on Abenomics distinguishes a first phase, (between II and III four months of 2013), in which the government has adopted a policy of fiscal flexibility, which has had the effect of a substantial increase of the GDP, and a second phase (II four-month 2014) of austerity which, with the increase in the consumption tax, brought the country back into recession.

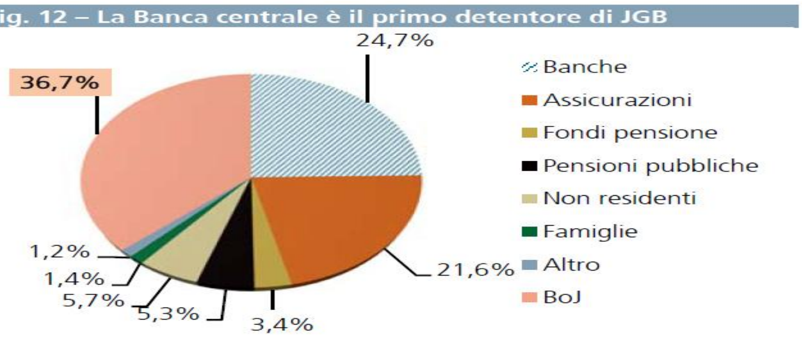
Therefore, in their view, the real pivot of Abenomics is not the QQE, but the increase in public spending initially desired and then failed due to fears related to debt sustainability.
In a study on the sustainability of Japanese fiscal policy, Shiro Armstrong and Tatsuyoshi Okimoto [2016] wrote: "Japanese public debt is at an unprecedented level with a gross debt / GDP ratio of over 230% and a net debt / GDP over 150%. There are three major factors that have influenced the sustainability of fiscal policy: the enormous increase in extraordinary government bond issues; the continuous budget deficit; the growth of long-term public spending. The debt in general will be sustainable as long as the markets trust. The path of fiscal consolidation requires the increase in tax rates, the reduction of public spending, the increase in the tax base and the overcoming of economic difficulties."
Below is the performance of the ratio gross debt / GDP for the countries of the G7:



As can be seen, the Japanese debt has seen a much larger increase than that of the other major world powers. In 2000, this value was not particularly alarming (58.8%) if compared with that of the country with the highest rate (Italy with 96.4%). The overtaking occurred in 2008 (95.3% of Japan against 92.1% of Italy). In 2015 it came to 146.8%.
The authors also noted how the problem of sustainability had worsened in 2014 with the increase in interest expenditure. This phenomenon would have been caused by the extraordinary emissions of government bonds that would have put discussion the credibility of the government at the eyes of the financial markets.Others, however, wanted to emphasize that the sustainability of public debt is not a serious problem, or rather it is not a problem that currently appears to be binding. In 2015, Japanese debt has exceeded the 245% of GDP, level almost four times above the average of the old continent. However, in the country up until now there has never been a default risk. In Japan there is a heterogeneous set of factors that give a perception of general reliability of the country-system, such as the stability of the yen, which is in fact considered a refuge currency, the rarity of strikes and the tendency of families to invest their savings in government bonds, for which the majority of borrowers are national operators and the debt is almost completely immune to speculative attacks.

Below is the composition of the public debt recorded by the Ministry of Finance in September 2016.

Fig. 12 – La Banca centrale è il primo detentore di JGB

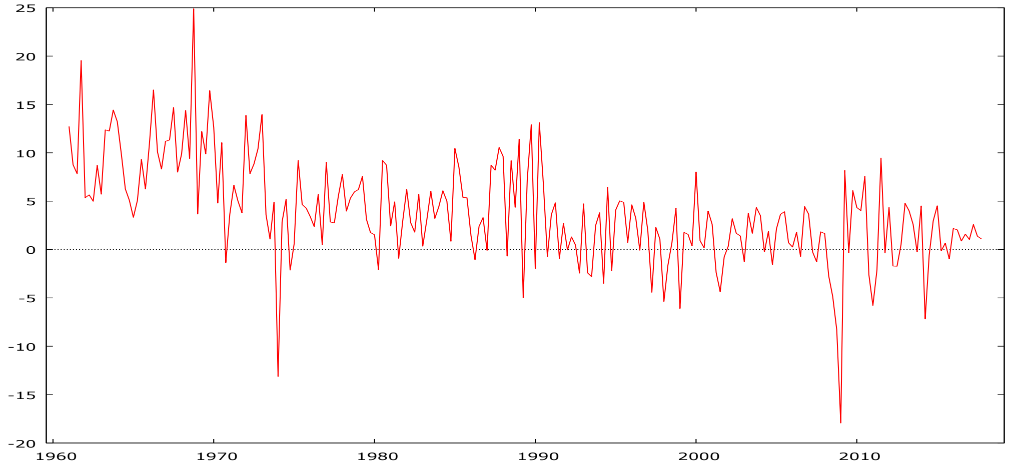


Nota: dati a fine settembre 2016; totale JGB: 971.1 tln di yen
Fonte: Ministry of Finance

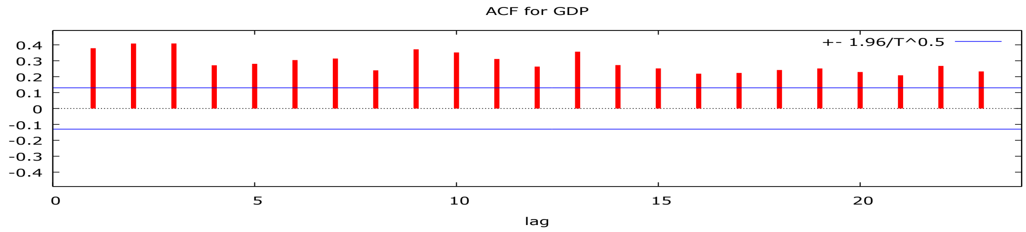
As can be seen, non-resident borrowers represent just 5.3% of the total. The significant share currently held by the BOJ, (36.7%), has grown, starting from 2013, thanks to the QQE.

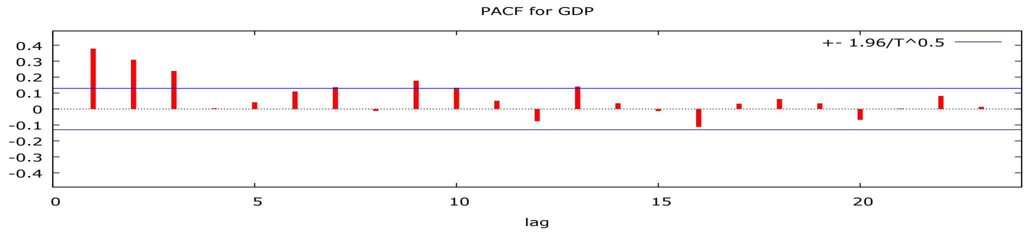
5. A dynamic forecast of the growth rate of real GDP

The first analysis carried out is a dynamic forecast of the real growth rate of GDP through a model ARIMA (p, d, q). This model was introduced in 1951 by Peter Whittle [1951] and perfected by George Box and Gwilyn Jenkins [1976], which in 1976 proposed a relative estimation method known as the "Box-Jenkins method". Through the DataStream software of Thomson Reuters, which extracted data from the Economic Outlook of the OECD of 28 November 2017, the authors arrived at the historical serie with quarterly frequency real growth rate of the GDP at 2005 prices, in the sample period Q1 1961 - Q1 2018, (the last value represents an estimate of the OECD and not an actual number). The total number of observations is T = 229. The series has been given the name of GDP. The graph relating to its time trend is as follows:

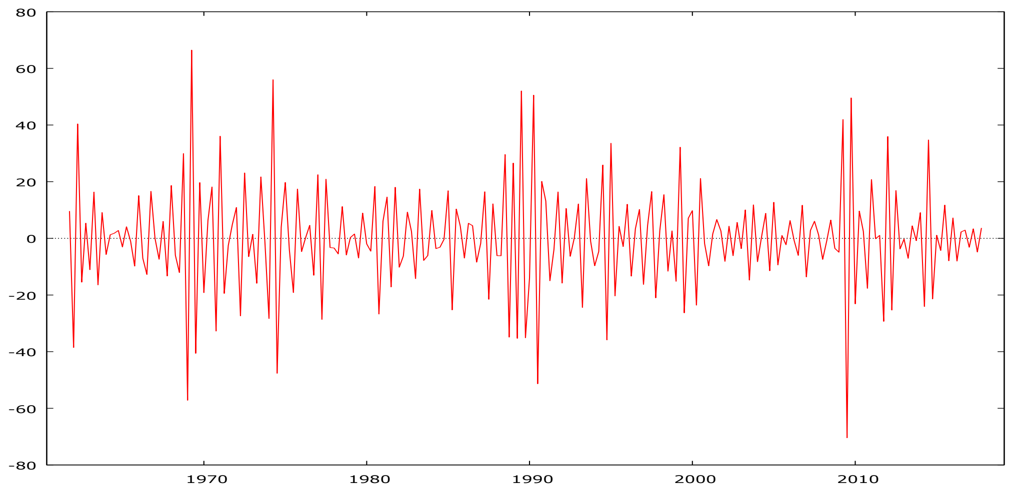


As can be seen, the series does not appear stationary in average and in variance. Furthermore, as evidenced by the correlogram, the variable has a seasonal trend.

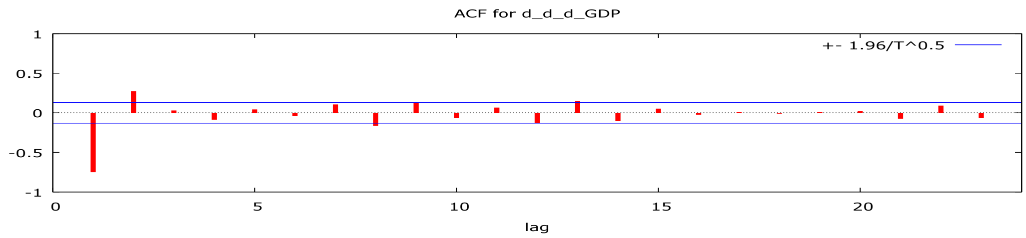


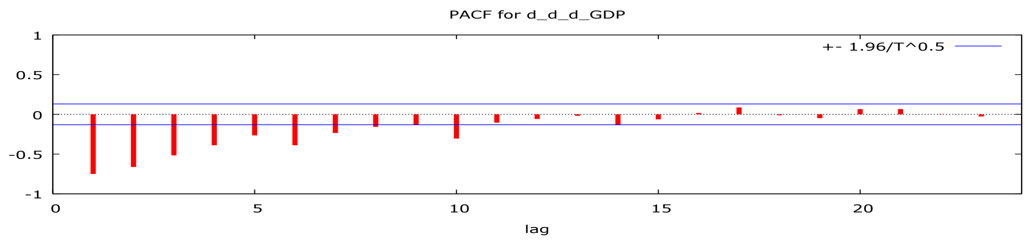


It became therefore necessary to calculate a difference third of the variable, (indicated by d_d_d_GDP). The timeplot below indicates that the series has become stationary both in the mean and in the variance:



From the observation of the underlying correlogram of d_d_d_GDP it results that the first two global autocorrelations (ACF) and the first ten partial autocorrelations (PACF) are statistically significant:





These are the significance levels for each lag compared to the p-value:
Autocorrelation Function for d_d_d_GDP
***, **, * indicate respectively significance at the level of 1%, 5%, 10%
Standard errors 1/T^0,5

LAG	ACF		PACF		Q-stat. [p-value]	
1	-0,7501	***	-0,7501	***	128,8542	[0,000]
2	0,2730	***	-0,6623	***	145,9977	[0,000]
3	0,0297		-0,5162	***	146,2011	[0,000]
4	-0,0866		-0,3893	***	147,9404	[0,000]
5	0,0428		-0,2652	***	148,3676	[0,000]
6	-0,0382		-0,3902	***	148,7095	[0,000]
7	0,1059		-0,2358	***	151,3465	[0,000]
8	-0,1627	**	-0,1592	**	157,6019	[0,000]
9	0,1316	**	-0,1361	**	161,7120	[0,000]
10	-0,0639		-0,3076	***	162,6853	[0,000]
11	0,0663		-0,1082		163,7379	[0,000]
12	-0,1297	*	-0,0632		167,7901	[0,000]
13	0,1521	**	-0,0224		173,3877	[0,000]
14	-0,1063		-0,1390	**	176,1331	[0,000]
15	0,0530		-0,0680		176,8178	[0,000]
16	-0,0234		0,0136		176,9521	[0,000]
17	0,0089		0,0851		176,9715	[0,000]
18	-0,0107		-0,0118		176,9999	[0,000]
19	0,0125		-0,0485		177,0389	[0,000]
20	0,0210		0,0645		177,1491	[0,000]
21	-0,0738		0,0665		178,5183	[0,000]
22	0,0911		0,0046		180,6123	[0,000]
23	-0,0676		-0,0251		181,7706	[0,000]

Given this result, being statistically significant the first two gobal autocorrelations and the first 10 partial autocorrelations , it is advisable to estimate on GDP a ARIMA model (10; 3; 2); the result is the following:

Model 10: ARMA, using the observations 1961:4-2018:1 (T = 226)
Dependent variable: d_d_d_GDP
Standard errors based on Hessiana

	Coefficient	*Std. Error*	*z*	*p-value*	
Const	6,65258e-06	2,80878e-05	0,2368	0,8128	
Phi_1	−0,850672	0,0674114	−12,62	<0,0001	***
Phi_2	−0,631292	0,0885488	−7,129	<0,0001	***
Phi_3	−0,439435	0,0954612	−4,603	<0,0001	***
Phi_4	−0,482247	0,0984445	−4,899	<0,0001	***
Phi_5	−0,463816	0,100474	−4,616	<0,0001	***
Phi_6	−0,387531	0,100238	−3,866	0,0001	***
Phi_7	−0,306064	0,0977551	−3,131	0,0017	***
Phi_8	−0,380389	0,0947778	−4,013	<0,0001	***
Phi_9	−0,222241	0,0881766	−2,520	0,0117	**
Phi_10	−0,0741073	0,0672973	−1,101	0,2708	
Theta_1	−1,99957	0,000849009	−2355,	<0,0001	***
Theta_2	0,999999	0,000630226	1587,	<0,0001	***

Mean dependent var.	−0,011670	S.D. dependent var.	18,83006
Mean of innovations	0,002063	S.D. of innovations	4,312137
Log-likelihood	−663,6372	Akaike criterion	1355,274

Schwarz criterion	1403,162	Hannan-Quinn	1374,600

		Real	*Immaginary*	*Modulus*	*Frequency*
AR					
	Root 1	-1,1566	-0,4790	1,2518	-0,4375
	Root 2	-1,1566	0,4790	1,2518	0,4375
	Root 3	0,4199	-1,1256	1,2014	-0,1932
	Root 4	0,4199	1,1256	1,2014	0,1932
	Root 5	0,9446	-0,6296	1,1352	-0,0936
	Root 6	0,9446	0,6296	1,1352	0,0936
	Root 7	-0,4699	-1,1048	1,2005	-0,3140
	Root 8	-0,4699	1,1048	1,2005	0,3140
	Root 9	-1,2374	-1,2966	1,7923	-0,3713
	Root 10	-1,2374	1,2966	1,7923	0,3713
MA					
	Root 1	0,9998	-0,0207	1,0000	-0,0033
	Root 2	0,9998	0,0207	1,0000	0,0033

The model presents a parameter, phi 10, (indicated with phi_10), statistically not significant. Furthermore, how is it possible to notice the two roots of the MA component are in module equal (and not superior) to 1. For this reason, this model must be discarded.
The number of AR and MA components has been progressively reduced until reaching the conclusion that the best estimable forecasting model is an ARIMA (5; 3; 1) on PIL1 excluding a constant:

Model 17: ARMA, using the observations 1961:4-2018:1 (T = 226)
Dependent variable: d_d_d_GDP
Standard errors based on Hessiana

	Coefficient	*Std. Error*	*z*	*p-value*	
Phi_1	−1,40826	0,0649957	−21,67	<0,0001	***
Phi_2	−1,32097	0,108957	−12,12	<0,0001	***
Phi_3	−0,913723	0,127021	−7,193	<0,0001	***
Phi_4	−0,527913	0,109585	−4,817	<0,0001	***
Phi_5	−0,210842	0,0654616	−3,221	0,0013	***
theta_1	−1,00000	0,0111472	−89,71	<0,0001	***

Mean dependent var.	−0,011670	S.D. dependent var.	18,83006
Mean of innovations	−0,081880	S.D. of innovations	5,441961
Log-likelihood	−709,0844	Akaike criterion	1432,169
Schwarz criterion	1456,112	Hannan-Quinn	1441,831

		Real	*Immaginary*	*Modulus*	*Frequency*
AR					
	Root 1	-1,4032	0,0000	1,4032	0,5000
	Root 2	-0,8234	-1,0099	1,3030	-0,3589
	Root 3	-0,8234	1,0099	1,3030	0,3589
	Root 4	0,2731	-1,3843	1,4110	-0,2190
	Root 5	0,2731	1,3843	1,4110	0,2190
MA					
	Root 1	1,0000	0,0000	1,0000	0,0000

In this case the model parameters are all statistically significant, although the only MA root is in module not greater than 1.
Based on the estimated coefficients, the model equation can be specified. The general equation of the models ARIMA (p, d, q) is:

$$\Phi_p(B)\nabla^d Z_t = \Theta_q(B)\alpha_t$$

from which:

$$Z_t = \varphi_1 Z_{t-1} + \varphi_p Z_{t-p} + \alpha_t - \theta_1\alpha_{t-1} - ..... - \theta_q\alpha_{t-q}$$

where Z_t is a linear stationary stochastic process defined as an infinite linear combination of white noise processes $(\nabla^d Z_t)$ differenziated d times, B is the translation operator, $\alpha_t \approx WN(0,\sigma^2)$, $(\varphi_1,....,\varphi_p)$ and $(\theta_1,....,\theta_q)$ are the coefficient referred respectively to the Auto Regressive component $(AR(p))$ and to Moving Average component $(MA(q))$

In the case in question:

$$Z_t = \varphi_1 Z_{t-1} + \varphi_2 Z_{t-2} + \varphi_3 Z_{t-3} + \varphi_4 Z_{t-4} + \varphi_5 Z_{t-5} + \alpha_t - \theta_1\alpha_{t-1}$$

$$Z_t = -1{,}40826 Z_{t-1} - 1{,}32097 Z_{t-2} - 0{,}913723 Z_{t-3} - 0{,}527913 Z_{t-4} - 0{,}210842 Z_{t-5} + \alpha_1 + 1{,}00000\alpha_{t-1}$$

where Z_t, as already specified, is a process differentiated 3 times.

The correlogram of residues shows how, on a maximum delay of 23, the global autocorrelations statistically significant are 5, (ie 21% of the total):

Autocorrelation function of residues
***, **, * indicate respectively significance at the levels of 1%, 5%, 10%
Standard errors 1/T^0,5

LAG	ACF	PACF	Q-stat. [p-value]
1	-0,0793	-0,0793	
2	-0,0727	-0,0795	
3	-0,1414 **	-0,1558 **	
4	-0,1560 **	-0,1961 ***	
5	-0,1529 **	-0,2341 ***	
6	-0,0525	-0,1900 ***	
7	0,0812	-0,0805	20,5678 [0,000]
8	-0,1135 *	-0,2979 ***	23,6107 [0,000]
9	0,1161 *	-0,1208 *	26,8134 [0,000]
10	0,1003	-0,0793	29,2116 [0,000]
11	0,0223	-0,1275 *	29,3303 [0,000]
12	-0,0528	-0,1942 ***	30,0003 [0,000]
13	0,1097	-0,0053	32,9108 [0,000]
14	-0,0089	-0,0401	32,9300 [0,000]
15	-0,0208	0,0012	33,0359 [0,000]
16	-0,0764	-0,1011	34,4668 [0,000]
17	-0,0569	-0,0691	35,2656 [0,000]
18	0,0143	0,0244	35,3161 [0,000]
19	0,0457	0,0533	35,8349 [0,001]
20	-0,0080	-0,0575	35,8510 [0,001]
21	-0,0601	-0,0717	36,7579 [0,001]
22	0,0375	-0,0117	37,1125 [0,002]
23	0,0202	-0,0125	37,2162 [0,003]

For completeness, a Ljung - Box test was performed to verify that all estimated autocorrelations of residues are equal to zero. Selecting as number of degrees of freedom 7, namely the difference between the number of statistically significant autocorrelations and the number of parameters of the model, is obtained as follows:

Autocorrelation test up to order 7

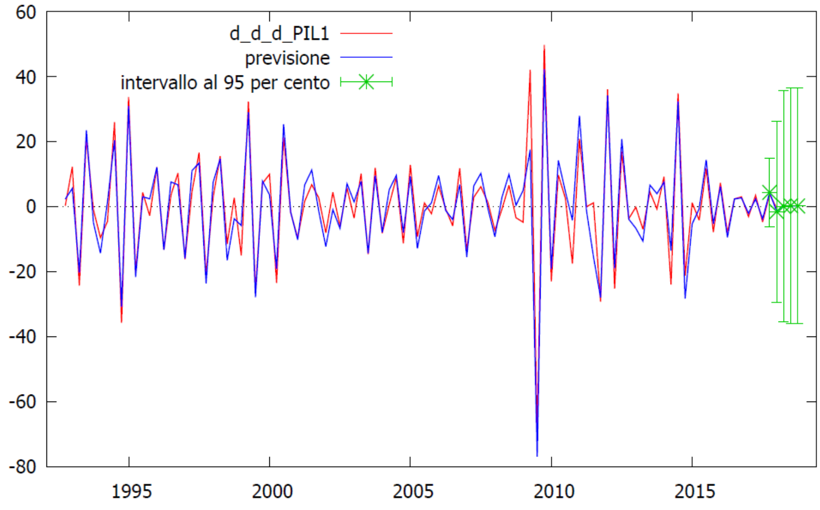
Ljung-Box Q' = 20,4809, with p-value = P(Chi-quadro(1) > 20,4809) = 6,023e-006

The value of the random variable $\chi^2_{(7)}$ is $\chi^2_{(7)} = 20,28$ with $\alpha = 0,05$. Then $Q > \chi^2_{(7)}$

In any case, he authors proceed to generate a dynamic forecast of GDP in the interval Q4 2017 - Q4 2018. The software suggests drawing the confidence interval using error bars and a value of 1 - α = 0.95:

For confidence interval at 95%, z(0,025) = 1,96

Oss	d_d_d_GDP	Forecast	Std. Error	95% interval
2017:4	3,66430	4,24296	5,44196	(-6,42308, 14,9090)
2018:1	-0,540300	-1,61707	14,1906	(-29,4302, 26,1960)
2018:2	Undefined	-0,0268249	18,1199	(-35,5412, 35,4875)
2018:3	Undefined	0,137192	18,4601	(-36,0440, 36,3184)
2018:4	Undefined	0,0970213	18,4781	(-36,1193, 36,3134)



Mean error	0,24906
Root of Mean Squared Error	0,86438
Mean Absolute Error	0,82772
Mean Percentage Error	-107,54
Mean Absolute Percentage Error	107,54
Theil’s U	0,25609
Bias proportion, UM	0,083021
Regression proportion, UR	0,91698
Disturbance proportion, UD	0

The confidance interval is plotted by error bars (in the graph these bars are green).

As can be seen, the mean square error and the Theil coefficient, both equal to about 0.25, are at a low level. The proportion of the disturbance is 0, that of the bias is almost nil, while the proportion of the regression is near to 1.

The results show a very slight contraction in the second quarter of the current year, (- 0.02%) and an equally weak growth in the third (+ 0.31%) and in the fourth quarter (+ 0.09%). This would lead to argue that Abenomics does not offer, in the short term, particularly exciting results in terms of product growth. These data must naturally be considered with great caution, since the model, as already mentioned, has good predictive capacity, but is still affected by the imperfection relative to the root of the MA component and by the structure of the non-optimal residues.

Observations on possible criticalities

From the observation of the graph of the third difference it is possible notice a persistence of a not negligible dispersion around the mean. Moreover, there is the difficulty in making the variable perfectly stationary in variance.

The fact that the test statistic of Ljung Box is slightly higher than the percentile of random variable $\chi^2(7)$ with $\alpha = 0.05$ indicates a non-optimal structure of the residues and this naturally has negative effects on the efficacy of the model which must be taken into account. The modulus not higher than one of the coefficient relative to the MA component constitutes a second element of criticalities of the estimated model.

Although the coefficient of Theil and the mean error are at a low level, all other measures of predictive goodness are at non-optimal levels. Moreover it is necessary to consider by the decomposition of the mean error that disturbance proportion (UD) is near to 0, (the recommended value is 1) and the regression proportion (UR) is near to 1, (the ideal value is 0). Only the bias proportion (UM) is at acceptable level (about 0).

Despite the presence of such imperfections it was impossible to estimate a better model on the variable taken into consideration.

6. A Historical decomposition of the GDP and of the inflation

Sometimes it is interesting to quantify how much a given structural shock explains of the historically observed fluctuations in the VAR variables or better it is interesting to know the cumulative effect of a given structural shock on each variable at every given point in time.

For example, may be interesting the shocks to the variability of the growth of real GDP over the last decades, in the average contribution of monetary policy, but mostly in the question of whether monetary policy shocks countered the recession.

The cumulative effect of the individual shocks on each single variable of the dataset, in each moment of the sample period is quantified by the historical decomposition. Such historical decompositions may be computed from covariance stationary VAR models. Infact, the technique of historical decomposition is most easily explained by reference to the VAR's moving average representation. Structural forecast error variance decompositions and structural impulse response functions describe the average movements in the data. They represent unconditional expectations.

In this paragraph, the authors just present a Historical decomposition of the Japanese quarterly GDP and of the quarterly Japanese inflation, carried out through a structural VAR of order 1 (SVAR (1)), from the first quarter of 2000 to the fourth quarter of 2016.

Following the indication of the guide of Jack Lucchetti [2015] for the implementation of the SVAR package in the Gretl software, in order to avoid the problem of setting the restrictions to be made to the structural shocks matrix, the authors used, among the various models of SVAR, a particular recursive model, that Lucchetti himself, like other econometricians, defines also as "Cholesky model". On the Gretl software, instead, this model is called "plain Cholesky".

The generic equation of the SVAR is :

$$B_0 y_t = B_1 y_{t-1} + B_2 y_{t-2} + ... + B_p y_{t-p} + w_t$$

where y_t is a vector of $k \times 1$ economic variables and is presumed to be zero mean for expository purposes, B_0 and B_i ($i = 1...... p$) are matrices of $k \times k$ dimensions, while w_t is a $k \times 1$ vector and is assumed to be white noise. The model is structural in that the elements of w_t are mutually uncorrelated and have clear interpretations in terms of an underlying economic model. The *k* variables of the model are driven by *k* distinct shocks such that their variance-covariance matrix $\sum_w$ is of full rank (the variance-covariance matrix $\sum_w$ is diagonal and its components are also mutually uncorrelated). In this way it is excluded that the structural model includes equations that are simply identity, as is common in traditional simultaneous equations models, rather than being subject to stochastic errors. So that for the model to be adequate, the number of shocks must necessarily be equal to *k*, namely equal to the number of variables considered. This assumption excludes, for example, the standard real business cycle model, in which all macroeconomic aggregates are driven by a technology shock only such that the covariance structure of the data is singular.

Let be B_0^{-1} the inverse matrix of B_0; because $B_0^{-1}B = A$ the SVAR model can be exespressed in reduced form as:

$$y_t = B_0^{-1}B_1 y_{t-1} + ... + B_0^{-1}B_p y_{t-p} + B_0^{-1}w_t = A_1 y_{t-1} + ... + A_p y_{t-p} + u_t$$

Going back and forth between the structural and thereduced-form representation requires knowledge of the matrix B_0 that governs the instantaneous relationships among the model variables or of its inverse, the structural impact multiplier matrix B_0^{-1}.

Given that $u_t = B_0^{-1}w_t$ this matrix allows to express the typically mutually correlated reduced-form innovations u_t as weighted averages of the mutually uncorrelated structural innovations w_t, with the elements of B_0^{-1} serving as the weights.

The matrix B_o^{-1} is also defined as a multiplier of the structural impact since its elements act as weights to calculate the u_t as the weighted average of the w_t in the following way:

$$u_t = B_0^{-1}w_t$$

Multiplying both members for B, follows:

$$Bu_t = Aw_t$$

In doing so, starting from the reduced form, it obtained the relationship that allows to calculate the elements of the structural shock vector.

A useful starting point for determining the structural impulse responses are the responses of y_{t+1} to the reduced-form errors u_t. They can be obtained by considering the $VAR(1)$ representation of the $VAR(p)$ process:

$$Y_t = A\,Y_{t-1} + U_t$$

where

$$Y_t = \begin{pmatrix} y_t \\ . \\ . \\ y_{t-p+1} \end{pmatrix} \qquad A = \begin{bmatrix} A_1 & A_2 ....A_{p-1} & A_p \\ I_k & 0.................0 & 0 \\ 0 & I_k................0 & 0 \\ \vdots & & \\ 0 & 0..............I_k & 0 \end{bmatrix} \quad \text{and} \quad U_t = \begin{pmatrix} u_t \\ 0 \\ . \\ . \\ 0 \end{pmatrix}$$

The SVAR in form is called C-model if it is $A = I$. A particular type of C-model, called recursive model, estimated precisely here, is obtained by placing B as a lower triangular matrix. One of the main properties of the recursive model concerns the estimator of maximum likelihood of A and is the decomposition of Cholesky of $\sum_u$, decomposition that allows to easily solve the problem of the identification of structural shocks.

As already mentioned, in this section the authors just present a Historical decomposition of the Japanese quarterly GDP and of the quarterly Japanese inflation, carried out through a structural VAR of order 1 (SVAR (1)), from the first quarter of 2000 to the fourth quarter of 2016.

The Historical decomposition can be defined synthetically starting from a stationary VAR model in covariance. Assuming to have data ranging from 1 to t, the variables y_t of the dataset can be expressed as functions of the shocks that can be estimated and of those that can not be estimated, because they are earlier than the starting moment of the sample period $t = 1$

$$y_t = \sum_{s=0}^{t-1}\Theta_s\, w_{t-s} + \sum_{s=t}^{\infty}\Theta_s\, w_{t-s}$$

In other words, the value of y_t depends on shocks w_1 ….. w_t that can be estimated and on shocks that predate the start of the sample at $t=1$ and hence cannot be estimated.
Then, y_t can be write as sum of two components. The first represent that part of the historical time series attributable to innovations since T and can be further esamine to establish the role of the innovations of each variable separately. The second component is formed solely from information available at time T.
The historical decomposition partitions responsability for the difference between the base projection and the actual series among the innovations of the variables in the VAR.
From the previous sum, it is clear that the introduction of innovations since T in all variables yields the actual series; hence the importance of any one variable, or set of variables, can be determined by examining the extent to which the introduction of the innovations since T in that variable or set of variables closes the gap between the base projection and the actual series.
To the growth of t the MA components are canceled (die out) and the second term (which corresponds to the period preceding the sample), has an increasingly negligible effect on y_t (will have a steadily diminishing effect on y_t as t increases) and can be omitted:

$$y_t \approx \sum_{s=0}^{t-1} \Theta_s \, w_{t-s}$$

This last approximation allows to denote the y_t by:

$$\hat{y}_t \approx \sum_{s=0}^{t-1} \Theta_s \, w_{t-s}$$

Now the authors proceed by plotting the estimated y_t and the (suitably demeaned and detrended) actual data *yt* in the same plot and also discarding the initial observations (also known "transient observations"), namely, those initial observations for which the two series do not coincide. These observations form the "transition period". The length of the transition period and the persistence of the effects of the shocks depend on the dominant root of the VAR process, that is the root whose value is near to one. More the root takes on value near to one, the greater the length of the transition period and the greater is the persistence of shocks on the y_t. With regard to the remaining sample period it is necessary to break down the sum in the previous equation to isolate the cumulative contribution of each shock to each element of the estimated y_t. This is done with three simple steps: the definition of the matrix coefficients of the structural MA $(\Theta_0, ..., \Theta_{T-1},)$ the calculatation of the structural shocks as $wt = B_o u_t$ for $t = 1,....T$; the coincidence between each structural shock and the appropriate impulse-response weighting factor, as required by a structural MA type representation.
In this paper, the variables considered to build the model are six, all on a quarterly basis: public spending, GDP, tax revenue deriving from the sum between corporate income and consumption tax revenue, CPI inflation, the National Interest Rate of the Bank of Japan, nominal interest rate on USA Federal funds. These variables have been indicated respectively as: GSPEND, GDP, TAXREV, INFLATION, NIR and SFFR. The only variable exogenous to the system, among those considered, is SFFR, which depends on the monetary policy decisions of the Federal Reserve and not of the BOJ. The data were searched through Thomson Reuters DataStream software, more precisely GSPEND and TAXREV were extracted from the database of the data analysis company Oxford Economics, GDP comes from the database of OECD, INFLATION was calculated starting from the historical series of the Consumer Price Index (CPI) reported on the website of the Federal Reserve of Saint Louis (FRED), NIR and SFFR were found on the database of the International Monetary Fund.
The first three variables, originally nominal, were transformed into real variables through the implicit deflator of GDP and subsequently converted into a logarithmic form. All the others have not undergone any kind of processing.

The SVAR in form is called C-model if it is $A = I$. A particular type of C-model, called recursive model, estimated precisely here, is obtained by placing B as a lower triangular matrix. One of the main properties of the recursive model concerns the estimator of maximum likelihood of A and is the decomposition of Cholesky of $\sum_u$, decomposition that allows to easily solve the problem of the identification of structural shocks.
In practice, the construction of historical decompositions involves three simple steps.
Step 1 is to compute the structural MA coefficient matrices $\Theta_0......\Theta_{T-1}$. Step 2 is to compute the structural shocks $w_t = B_0 u_t \quad t = 1,........T$. Step 3 is to match up each structural shock, said shock j, with the appropriate impulse response weight, as required by the structural moving average representation, to form $T \times 1$ vectors of fitted values for variable k, denoted $\hat{y}_t$ for $j = 1......K$
In other words, the historical decompositions are computed by replacing the unknown quantities by the usual estimates. It is important to remember that historical decompositions involve an approximation error. This approximation error arises because is truncated the moving average representation. For example, y_{k1} depends on the structural shocks at date 1 as well as the infinite history of structural shocks. With much of the history of shocks unobserved, the approximation is bound to be poor initially. As we recursively update $\hat{y}_{kt}$, however, more and more of the recent structural shocks that receive high impulse response weights are captured, and the weights of earlier unobserved shocks decline. Thus, $\hat{y}_{kt}$ approaches y_{kt}. How fast this convergence takes place depends on the persistence of y_{kt}.
Considering as maximum order 4, it was decided to impose only one delay because this value is the most appropriate one based on the Bayesian Information Criterion (BIC), as shown here:

VAR system, maximum order delays 4
The asterisks below indicate the best (that is, minimized) values of the respective information criteria, AIC = Akaike criterion,
BIC = Schwarz Bayesian criterion and HQC = Hannan-Quinn criterion.

lag	loglik	p(LR)	AIC	BIC	HQC
1	-263,30598		9,322062	10,502701*	9,787175
2	-220,10329	0,00000	8,753228*	10,777181	9,550565*
3	-205,43366	0,24998	9,076052	11,943319	10,205613
4	-177,44778	0,00037	8,982743	12,693324	10,444528

Was thus estimated an SVAR (1) , including a constant and incorporating a time trend in the variables of the set to improve the readability of the output.

Optimization method = Scoring algorithm
Unconstrained Sigma:

0,64215	-0,07830	-0,05813	0,02438	0,00756
-0,07830	0,97609	0,58259	0,06937	0,02002
-0,05813	0,58259	6,35204	0,03911	0,01844
0,02438	0,06937	0,03911	0,24585	0,00742
0,00756	0,02002	0,01844	0,00742	0,00371

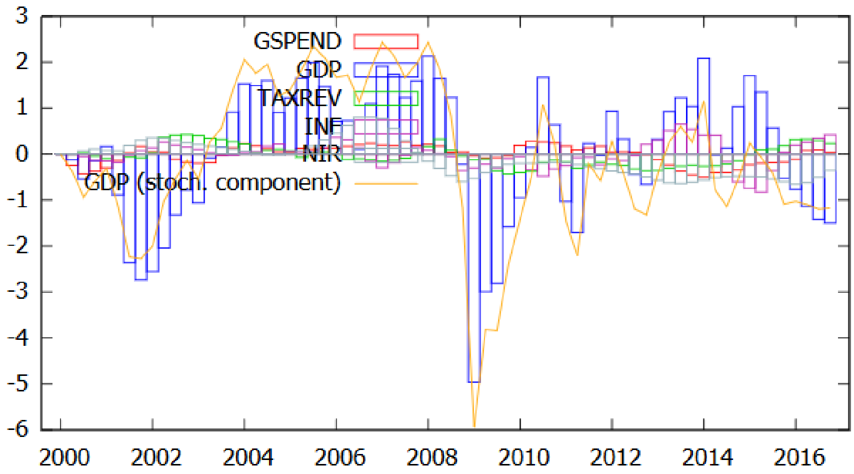
	coefficient	Std. Error	z	p-value	
C[1; 1]	0,801343	0,0687146	11,66	2,00e-031	***
C[2; 1]	-0,0977081	0,119516	-0,8175	0,4136	
C[3; 1]	-0,0725463	0,305571	-0,2374	0,8123	
C[4; 1]	0,0304177	0,0600721	0,5064	0,6126	
C[5; 1]	0,00943898	0,00734084	1,286	0,1985	

C[1; 2]	0,00000	0,00000	NA	NA	
C[2; 2]	0,983131	0,0843028	11,66	2,00e-031	***
C[3; 2]	0,585381	0,301356	1,942	0,0521	*
C[4; 2]	0,0735859	0,0596828	1,233	0,2176	
C[5; 2]	0,0213010	0,00706374	3,016	0,0026	***
C[1; 3]	0,00000	0,00000	NA	NA	
C[2; 3]	0,00000	0,00000	NA	NA	
C[3; 3]	2,45033	0,210114	11,66	2,00e-031	***
C[4; 3]	-0,000716634	0,0593483	-0,01208	0,9904	
C[5; 3]	0,00271787	0,00681952	0,3985	0,6902	
C[1; 4]	0,00000	0,00000	NA	NA	
C[2; 4]	0,00000	0,00000	NA	NA	
C[3; 4]	0,00000	0,00000	NA	NA	
C[4; 4]	0,489398	0,0419655	11,66	2,00e-031	***
C[5; 4]	0,0113691	0,00674546	1,685	0,0919	*
C[1; 5]	0,00000	0,00000	NA	NA	
C[2; 5]	0,00000	0,00000	NA	NA	
C[3; 5]	0,00000	0,00000	NA	NA	
C[4; 5]	0,00000	0,00000	NA	NA	
C[5; 5]	0,0550404	0,00471968	11,66	2,00e-031	***

Log-likelihood = -281,396

The decomposition of GDP is therefore analyzed :

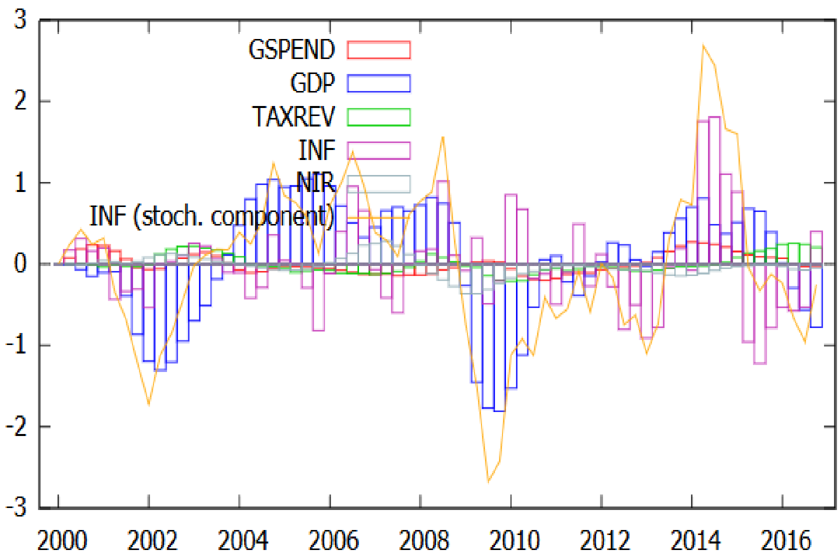
Historical decomposition of the Japanese GDP (2000-2016). Source: authors's selaboration.



From 2000 to 2003 there is a phase of recession in which the lowering of the interest rate with the quantitative easing proves to be ineffective. Public spending makes a contribution initially slightly positive and then slightly negative. Only between the second half of 2002 and 2003 the tax revenue variable seems to buffer the product's fall along with the interest rate. Between 2004 and 2008 there is a growth phase, probably driven by demand, in which the fiscal lever and the interest rate seem to play a marginal role. In the two-year period 2006-2008 there is a slightly negative contribution from inflation. Between 2008 and 2010 the demand crisis opens a strong phase of recession in which all the others variables have a slightly negative impact. In the two-year period 2010-2012, phases of contraction and expansion of aggregate demand alternate, in which government spending seems to play a positive role, while taxes have recessionary effects. In 2013, with Abenomics, GDP has risen thanks to a recovery in consumer confidence. Starting from the second half of 2014, with the increase in the consumption tax rate, demand is deteriorating, as evidenced by the new lowering of inflation, and with it also the product. Here it also seems that monetary policy contributes negatively, (albeit slightly). During this period, consumer expectations worsened, as shown by the fact that domestic consumption, which in 2013 was worth 3039.81 billion of dollars, fell down to

2479.11 billion in 2015[1]. In 2016, they returned to growth, reaching 2757.67 billion dollar. However, since 2014 they have steadily decreased in relation to GDP, rising from 58.4% this year to 56.56% in 2015 and to 55.83% in 2016[2].
Specularly, the gross national saving decreased from 2012 to 2013, going from 21.1% to 20.8% of GDP and has started growing again from 2014 to 24.3% in 2016[3].
More analytically, the role that exports have had on growth must also be considered. In the first quarter of 2014, real exports of goods and services calculated at Chained yen 2011 amounted to 78,651.5 billion, and grew continuously up to 84369.4 billion until the first quarter of 2015 and remained fairly constant until the end of the year[4]. In 2016 the growth of the variable recovered more strongly, reaching 8 86678.0 billion yen in the fourth quarter. This growth also results in relative terms. In 2013, exports accounted for 15.91%, 17.54% in 2014 and 17.63% in 2015. In 2016, however, the ratio fell to 16.14%[5].
In this context, investments played a very marginal role. In 2013, total investments constituted 23.19% of GDP, 23.88% in 2014, 23.90% in 2015 and 23.35% in 2016[6].
Starting from 2015, the analysis shows a positive contribution from the TAXREV variable. This is due to the fact that in that year the revenue from corporate income tax is reduced by 1% compared to the previous year (178 billion yen). If the TAXREV variable had considered the total revenue, there would have been a negative contribution, given that the latter variable had an overall increase of 6659 billion compared to 2014.
Now is analyzed the decomposition of the inflation:

Historical decomposition of the Japanese inflation (2000-2016). Source: authors's elaboration.



Observing the years immediately after 2001, it is noted that the first Quantitative easing of BOJ resolved itself in complete failure. Until the end of the year there was a negative trend of the variable due to the contraction in demand, only partially offset by public spending. There was a recovery in 2002-2003, despite the drop in demand continuing and becoming more marked. Between the end of 2003 and of 2007 there is a fluctuating trend. In this period the demand contributes positively, fiscal policy first seems to support the price growth (2003-2005), but then seems that the obstacles it (2005-2007). Always in 2007, the monetary policy of the BOJ, which up until this point had no significant effect, began to favor inflation. In the years following 2008, (with a partial exception in 2011), the sharp contraction in demand and public spending and the parallel increase in taxes conducted the country into considerable deflation. In 2013, Abenomics proved to

[1] Sorce: World Bank
[2] Sorce: World Bank
[3] Sorce: World Bank
[4] Source: Federal Reserv of Saint Louise
[5] Sorce: World Bank
[6] Source: Cabinet Office of Japan

be very effective in increasing inflation, primarily thanks to to the recovery in demand, making it go from + 0.34% of that year to + 2.76% of the following year. Starting from the second half of 2014, the latter, in conjunction with the increase in the consumption tax rate, experiences a new slowdown due to the progressive weakening of the demand component and follows a new period of deflation (+ 0.79% in 2015 and - 0.11% in 2016). In reality, in the three-year period 2013-2016, the fiscal policy apparently contributes negatively to price growth. From 2015 instead it is possible note a positive contribution from the same, perhaps due to the fact that the reduction in the corporate tax has stimulated a contained increase in the aggregate supply.

Observations and possible criticalities

The estimate of the structural VAR model has produced some results that seem to confirm partially what has been known up to now by the literature on the effects of monetary policy on the prices and on the GDP, namely, in particular, that in general an increase (a decrease) in the interest rate by the central bank involves, ceteris paribus, an increase (a decrease) of the inflation and of the GDP. Moreover, from these studies it is evident that the effect on prices is manifested in the medium-long term but is lasting, while the effect on the GDP is manifested in the short term but tends to wane rapidly. More precisely, in the case of the use of the VAR models, an increase in the interest rate leads to a reduction of the GDP after about two quarters and a return to its initial level after about two years.

In our case, in the fourth quarter of 2001, the lowering of the interest rate in the QE, (started in March of that year), seems to encourage the gradual recovery of the GDP. This positive contribution seems to gradually decline until it disappears completely in the second quarter of 2004, although the GDP continues to grow. The positive effects of the monetary policy on inflation started to manifest in the fourth quarter of 2001 and vanish in the first quarter of 2004. Also in this case, however seems that the variable does not return to its initial level after two years again.

The effects of monetary policy detected by our model are, however, marginal throughout the sample period considered**.**

In the paper, furthermore, it appears that from the comparison with the p-value it follows that unfortunately some coefficients are not statistically significant. However, with the data object of interest it is not possible to estimate a better model.

Due to the lack of data, the tax revenue variable includes the revenues deriving only from two single taxes and not from the sum of the total of direct and indirect taxes. Because of this, it does not take into account of the effects of tax policy of the Abe government on household income and tends instead to overestimate the effects of the corporate income tax reduction and the increase in consumption tax implemented by the government.

7. Conclusions and future perspectives

In light of the analysis reported in the paper it can be said that the effectiveness of Abenomics' policies was, until 2016, certainly very limited, especially considering the wide range of measures undertaken, The Historical decomposition, carried out in the paper, shows, in fact, that neither monetary policy neither fiscal policy turned out really effective on GDP. The recovery between 2013 and 2014 was stimulated by the improvement in consumers confidence that pushed domestic demand . After this year, the increase of the tax on the consumptions made return the Japan into recession. In 2016, furthermore, also the foreign component also slowed down. Also, the forward guidance and the NIRP have had very limited effectiveness, as shown by the trend of total investments. This is significant if you take into account the fact that the Bank of Japan is the only central institution, in addition to the Federal Reserve, to practice an explicit guidance.

But the biggest failure of the BOJ (the Bank of Japan) is the missed achieving of the objective in terms of inflation: in fact, the deflation, to today, has not been effectively removed.

Before 2014, the results on inflation were very satisfactory, because the prices had returned to growth. Evidently, the economic agents had trusted in the credibility of the monetary policy plan launched by the Central Bank to counter deflation and had to elaborated, in a rational way, the expectations on the price growth. The increase in the consumption tax in 2014, however, discouraged the consumptions making reappear then the specter of deflation. In other words, the choice to increase the rate of an indirect tax to reorganize public accounts following the increase of the expense in previous years turned out,by the government, to be particularly wrong in a country in which the economic agents are characterized by having a high marginal propensity to save.

The dynamic forecast of the growth rate, exposed in the paper, shows, then, that also taking into account the limitations of the estimated model, even the next quarters will still be characterized by

flat growth. After the 2017 parliamentary elections, the new minister of the finance announced that he wants to bring the legal age for access to the pension retirement cheque at the age of 85 year, (the highest in the world!) to rearrange the fragile national pension system (currently salary).
In conclusion, it will be interesting to observe, in the coming years, how the executive will strive to combine economic growth and fiscal rigor. In other words, the future of the Japanese economy will depend on how the government decides to combine growth and budget balance.
An important finish line is represented by the Tokyo Olympics in 2020, 56 years after the Games of 1964, which determined the entry of Japan among the world economic powers.
The Abe's Japan has a dream: to regain the lost power and the lost prestige relaunching the economy, strengthening the maritime defense capabilities with the abolition of the self-imposed ceiling of 1% of GDP for the expenses destined for defense and eventually rewriting the pacifist Constitution. The problem is that these two perspectives and the geopolitical identities that derive from them are not compatible: the China wants to become a "regional hegemon" power, while Japan opposes it, supported by Washington. The United States, aware that they can no longer be a global hegemon in a world multicenter beyond that multipolar , aim therefore to prevent that other regional hegemons subjects affirming themselves, especially in Asia-Pacific where they intend to continue to play the role of "external balancer".

Compliance with Ethical Standards:

Conflict of Interest

Author A declares that she has no conflict of interest. Author B declares that he has no conflict of interest.

Ethical approval

This article does not contain any studies with human participants performed by any of the authors.

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