2-MAGNETIC CURVES IN EUCLIDEAN 3-SPACE

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Abstract. In this paper, we define the notion of 2-T-magnetic (respectively, 2-N-magnetic and 2-B-magnetic) curve according to Frenet frame in Euclidean 3-space. Also we obtain the 2-magnetic vector field V when the curve is a 2-T-magnetic (respectively, 2-N-magnetic and 2-B-magnetic) trajectory of V according to Frenet frame and give some results and examples for 2-magnetic curves according to Frenet frame.

1. **INTRODUCTION**

The magnetic curves on a Riemannian manifold (M, g) are trajectories of charged particles moving on M under the action of a magnetic field F. A *magnetic field* is a closed 2-form F on M and the *Lorentz force* of the magnetic field F on (M, g) is a (1,1)-tensor field Φ given by $g(\Phi(X), Y) = F(X, Y)$, for any vector fields $X, Y \in \chi(M)$. In dimension 3, the magnetic fields may be defined using divergence-free vector fields. As Killing vector fields have zero divergence, one may define a special class of magnetic fields called *Killing magnetic fields*.

Different approaches in the study of magnetic curves for a certain magnetic field and on the fixed energy level have been rewieved by Munteanu in [8]. He has emphasized them in the case when the magnetic trajectory corresponds to a Killing vector field associated to a screw motion in the Euclidean 3-space. In [9], the authors have investigated the trajectories of charged particles moving in a space modeled by the homogeneous 3-space $S^2 \times \mathbb{R}$ under the action of the Killing magnetic fields.

In [13], the authors have classified all magnetic curves in the 3-dimensional Minkowski space corresponding to the Killing magnetic field $V = a\partial_x + b\partial_y + c\partial_z$, with $a, b, c \in \mathbb{R}$. They have found that, these magnetic curves are helices in E_1^3 and draw the most relevant of them. In 3D semi-Riemannian manifolds, Özdemir et al. have determined the notions of T-magnetic, N-magnetic and B-magnetic curves and give some characterizations for them, where T, N an B are the tangent, normal and binormal vectors of the curve α, respectively [10]. Also in [6], the authors have defined the notions of T-magnetic, N_1-magnetic and N_2-magnetic curves according to Bishop frame $\{T, N_1, N_2\}$ and ξ_1-magnetic, ξ_2-magnetic and B-magnetic curves according to type-2 Bishop frame $\{\xi_1, \xi_2, B\}$ in Euclidean 3-space. They have given some characterizations about these magnetic curves. Furthermore, Kazan and Karadağ have studied the magnetic pseudo null and magnetic null curves in Minkowski 3-space in [7].

In any 3D Riemannian manifold (M, g), magnetic fields of nonzero constant length are one to one correspondence to almost contact structure compatible to the metric g. From this fact, many authors have motivated to study magnetic curves with closed fundamental 2-form in almost contact metric 3-manifolds, Sasakian manifolds, quasi-para-Sasakian manifolds and etc (see [2], [4], [5], [12]).

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On the other hand, the local theory of space curves has been studied by many mathematicians by using Frenet-Serret theorem.

In this study, we define the notion of 2-T-magnetic (respectively, 2-N-magnetic and 2-B-magnetic) curve according to Frenet frame in Euclidean 3-space. Also we obtain the 2-magnetic vector field V when the curve is a 2-T-magnetic (respectively, 2-N-magnetic and 2-B-magnetic) trajectory of V according to Frenet frame and give some results and examples for 2-magnetic curves according to Frenet frame.

2. PRELIMINARIES

Firstly, we will recall Frenet-Serret formulae of a space curve in E^3 Euclidean 3-space.

If T, N and B are unit tangent vector field, unit principal normal vector field and unit binormal vector field of a space curve α, respectively, then $\{T, N, B\}$ is called the moving *Frenet frame* of α and the Frenet-Serret formulae is given by

$$\begin{bmatrix} T' \\ N' \\ B' \end{bmatrix} = \begin{bmatrix} 0 & \kappa & 0 \\ -\kappa & 0 & \tau \\ 0 & -\tau & 0 \end{bmatrix} \begin{bmatrix} T \\ N \\ B \end{bmatrix}, \tag{2.1}$$

where

$$\begin{aligned} g(T,T) &= g(N,N) = g(B,B) = 1, \\ g(T,N) &= g(N,B) = g(B,T) = 0. \end{aligned} \tag{2.2}$$

Here κ and τ are curvature functions which are defined by $\kappa = \kappa(t) = \|T'(t)\|$ and $\tau = \tau(t) = -g(N(t), B'(t))$ [3].

Now, we will give some informations about the magnetic curves in 3-dimensional semi-Riemannian manifolds.

A divergence-free vector field defines a magnetic field in a three-dimensional semi-Riemannian manifold M. It is known that, $V \in \chi(M^n)$ is a Killing vector field if and only if $L_V g = 0$ or, equivalently, $\nabla V(p)$ is a skew-symmetric operator in $T_p(M^n)$, at each point $p \in M^n$. It is clear that, any Killing vector field on (M^n, g) is divergence-free. In particular, if $n = 3$, then every Killing vector field defines a magnetic field that will be called a *Killing magnetic field* [1].

Let (M, g) be an n-dimensional semi-Riemannian manifold. A *magnetic field* is a closed 2-form F on M and the *Lorentz force* Φ of the magnetic field F on (M, g) is defined to be a skew-symmetric operator given by

$$g(\Phi(X), Y) = F(X, Y), \quad \forall X, Y \in \chi(M). \tag{2.3}$$

The *magnetic trajectories* of F are curves α on M that satisfy the *Lorentz equation* (sometimes called the *Newton equation*)

$$\nabla_{\alpha'} \alpha' = \Phi(\alpha'). \tag{2.4}$$

The Lorentz equation generalizes the equation satisfied by the geodesics of M, namely $\nabla_{\alpha'} \alpha' = 0$.

Note that, one can define on M the cross product of two vectors $X, Y \in \chi(M)$ as follows

$$g(X \times Y, Z) = dv_g(X, Y, Z), \quad \forall Z \in \chi(M).$$

If V is a Killing vector field on M, let $F_V = \imath_V dv_g$ be the corresponding Killing magnetic field. By $\imath$ we denote the inner product. Then, the Lorentz force of F_V is

$$\Phi(X) = V \times X.$$

Consequently, the Lorentz force equation (2.4) can be written as

$$\nabla_{\alpha'}\alpha' = V \times \alpha' \tag{2.5}$$

(for detail see [8], [10]).

Now, we will recall the notion of T-magnetic (respectively, N-magnetic and B-magnetic) curve in Euclidean 3-space.

Definition 1. *Let $\alpha : I \subset \mathbb{R} \longrightarrow E^3$ be a curve in Euclidean 3-space and F_V be a magnetic field in E^3. If the tangent vector field T (respectively, the normal vector field N and the binormal field B) of the Frenet frame satisfies the Lorentz force equation $\nabla_{\alpha'}T = \Phi(T) = V \times T$ (respectively $\nabla_{\alpha'}N = \Phi(N) = V \times N$ and $\nabla_{\alpha'}B = \Phi(B) = V \times B$), then the curve α is called a T-**magnetic** (respectively, N-**magnetic and** B-**magnetic) curve*** [11].

Proposition 1. *Let α be a unit speed T-magnetic (respectively, N-magnetic and B-magnetic) curve in Euclidean 3-space. Then, the Lorentz force according to the Frenet frame is obtained as*

$$\begin{bmatrix} \Phi(T) \\ \Phi(N) \\ \Phi(B) \end{bmatrix} = \begin{bmatrix} 0 & \kappa & 0 \\ -\kappa & 0 & \rho \\ 0 & -\rho & 0 \end{bmatrix} \begin{bmatrix} T \\ N \\ B \end{bmatrix}, \tag{2.6}$$

where ρ is a certain function defined by $\rho = g(\Phi N, B)$, (respectively,

$$\begin{bmatrix} \Phi(T) \\ \Phi(N) \\ \Phi(B) \end{bmatrix} = \begin{bmatrix} 0 & \kappa & \mu \\ -\kappa & 0 & \tau \\ -\mu & -\tau & 0 \end{bmatrix} \begin{bmatrix} T \\ N \\ B \end{bmatrix}, \tag{2.7}$$

where μ is a certain function defined by $\mu = g(\Phi T, B)$ and

$$\begin{bmatrix} \Phi(T) \\ \Phi(N) \\ \Phi(B) \end{bmatrix} = \begin{bmatrix} 0 & \gamma & 0 \\ -\gamma & 0 & \tau \\ 0 & -\tau & 0 \end{bmatrix} \begin{bmatrix} T \\ N \\ B \end{bmatrix}, \tag{2.8}$$

where γ is a certain function defined by $\gamma = g(\Phi T, N)$.) [11].

3. 2-MAGNETIC CURVES IN EUCLIDEAN 3-SPACE

In this section, we will investigate the 2-T-magnetic, 2-N-magnetic and 2-B-magnetic curves in Euclidean 3-space (E^3, g). Also, we obtain the magnetic vector field V when the curve is a 2-T-magnetic, 2-N-magnetic and 2-B-magnetic trajectory of V and give some results and examples for these curves.

3.1. 2-T-MAGNETIC CURVES IN EUCLIDEAN 3-SPACE.

Definition 2. *Let $\alpha : I \subset \mathbb{R} \longrightarrow E^3$ be a T-magnetic curve in Euclidean 3-space and F_V be a magnetic field in E^3. If the tangent vector field T of the Frenet frame satisfies the 2-Lorentz force equation $\nabla_{\alpha'}\nabla_{\alpha'}T = \Phi(T') = V \times T'$, then the curve α is called a* 2-T-***magnetic curve.***

Proposition 2. *Let α be a unit speed 2-T-magnetic curve according to Frenet frame in Euclidean 3-space. Then, we have*

$$\begin{bmatrix} \Phi(T') \\ \Phi(N') \\ \Phi(B') \end{bmatrix} = \begin{bmatrix} -\kappa^2 & \kappa' & \kappa\tau \\ 0 & -\kappa^2 - \tau\rho & 0 \\ \kappa\tau & 0 & -\tau\rho \end{bmatrix} \begin{bmatrix} T \\ N \\ B \end{bmatrix}, \tag{3.1}$$

where ρ is a certain function defined by $\rho = g(\Phi(N), B)$.

Proof. Let α be a 2-T-magnetic curve according to Frenet frame in Euclidean 3-space with the Frenet apparatus $\{T, N, B, \kappa, \tau\}$. From the definition of the 2-T-magnetic curve according to Frenet frame and from (2.1), we know that $\Phi(T') = -\kappa^2 T + \kappa' N + \kappa\tau B$. On the other hand, since $\Phi(N') \in Sp\{T, N, B\}$, we have $\Phi(N') = a_1 T + a_2 N + a_3 B$. So, from (2.1), (2.2) and (2.6) we get

$$\begin{aligned} a_1 &= g(\Phi(N'), T) = -g(N', \Phi(T)) = -g(-\kappa T + \tau B, \kappa N) = 0, \\ a_2 &= g(\Phi(N'), N) = -g(N', \Phi(N)) = -g(-\kappa T + \tau B, -\kappa T + \rho B) = -\kappa^2 - \tau\rho, \\ a_3 &= g(\Phi(N'), B) = -g(N', \Phi(B)) = -g(-\kappa T + \tau B, -\rho N) = 0 \end{aligned}$$

and hence we obtain that, $\Phi(N') = (-\kappa^2 - \tau\rho)N$.

Furthermore, from $\Phi(B') = b_1 T + b_2 N + b_3 B$, we have

$$\begin{aligned} b_1 &= g(\Phi(B'), T) = -g(B', \Phi(T)) = -g(-\tau N, \kappa N) = \kappa\tau, \\ b_2 &= g(\Phi(B'), N) = -g(B', \Phi(N)) = -g(-\tau N, -\kappa T + \tau B) = 0, \\ b_3 &= g(\Phi(B'), B) = -g(B', \Phi(B)) = -g(-\tau N, -\rho N) = -\tau\rho \end{aligned}$$

and so, we can write $\Phi(B') = (\kappa\tau)T - (\tau\rho)B$, which completes the proof. □

Proposition 3. *Let α be a unit speed T-magnetic curve according to Frenet frame in Euclidean 3-space. Then, the curve α is a 2–T-magnetic trajectory of a 2-magnetic vector field V if and only if the 2-magnetic vector field V is*

$$V = \tau T + \kappa B \tag{3.2}$$

along the curve α.

Proof. Let α be a 2–T-magnetic trajectory of a 2-magnetic vector field V according to Frenet frame. Using Proposition 2 and taking $V = aT + bN + cB$; from $\Phi(T') = V \times T'$, we get

$$a = \tau,\ c = \kappa,\ \kappa' = 0; \tag{3.3}$$

from $\Phi(N') = V \times N'$, we get

$$a = \rho,\ b = 0,\ c = \kappa \tag{3.4}$$

and from $\Phi(B') = V \times B'$, we get

$$a = \rho,\ c = \kappa \tag{3.5}$$

and so the 2-magnetic vector field V can be written by (3.2). Conversely, if the 2-magnetic vector field V is the form of (3.2), then one can easily see that $V \times T' = \Phi(T')$ holds. So, the curve α is a 2-T-magnetic projectory of the 2-magnetic vector field V according to Frenet frame. □

Corollary 1. *If a curve α is a 2–T-magnetic trajectory of a 2-magnetic vector field V, then the curvature κ of α is constant and we have*

$$\rho = \tau = g(\Phi(N), B). \tag{3.6}$$

Proof. The proof is obvious from (3.3)-(3.5). □

From (2.1), (2.6) and (3.6), we can state the following corollary:

Corollary 2. *If a curve α is a 2–T-magnetic trajectory of a 2-magnetic vector field V, then the Lorentz force Φ corresponds to covariant derivative along α in E^3. Also, we have*

$$\Phi^2(X) = \Phi(X'),$$

for $\forall X \in \{T, N, B\}$.

Corollary 3. *If a curve α is a 2–T-magnetic trajectory of a 2-magnetic vector field V, then we have*

$$g(T, \Phi(T')) + g(B, \Phi(B')) = g(N, \Phi(N')) = -(\kappa^2 + \tau^2).$$

Proof. From (2.1) and Corollary 2, the proof follows. □

Example 1. *Let us consider the curve*

$$\alpha(t) = (\cos t, \sin t, 1), \tag{3.7}$$

which is a unit speed circle in E^3. Here, one can easily calculate its Frenet-Serret trihedra and curvatures as

$$\begin{aligned} T &= (-\sin t, \cos t, 0), \\ N &= (-\cos t, -\sin t, 0), \\ B &= (0, 0, 1), \\ \kappa &= 1, \ \tau = 0, \end{aligned} \tag{3.8}$$

respectively. Here, since the curvature of α is constant and from (3.6) and (3.8), one can easily see that the curve α is a 2-T-magnetic curve for $\sin t \neq 1$. Also from (3.2), the 2-magnetic vector field V when the curve (3.7) is a 2-T-magnetic trajectory of the 2-magnetic vector field V according to Frenet frame (3.8) is

$$V = (0, 0, 1). \tag{3.9}$$

Here, it can be seen that, from (3.8) and (3.9), $\nabla_{\alpha'}\nabla_{\alpha'}\alpha' = V \times T'$ satisfies. So, the curve α is a 2-T-magnetic curve according to Frenet frame with the 2-magnetic vector field (3.9).

When the curve α is 2-T-magnetic according to Frenet frame, the figure of α and V can be drawn as Figure 1.

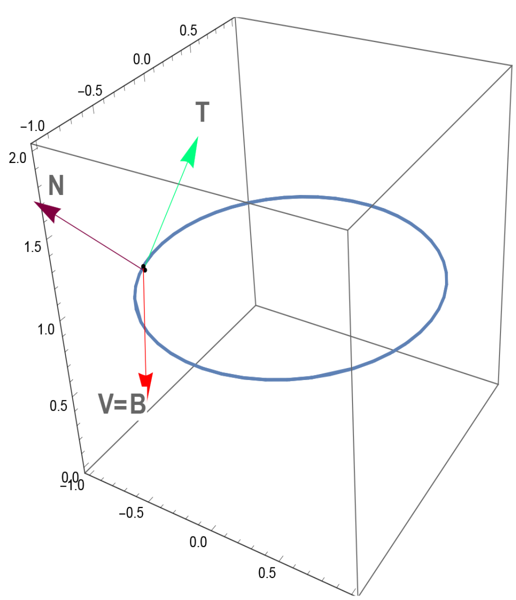

Figure 1: 2-T-magnetic curve α according to Frenet frame and the 2-magnetic vector field V

3.2. 2-N-MAGNETIC CURVES IN EUCLIDEAN 3-SPACE.

Definition 3. *Let $\alpha : I \subset \mathbb{R} \longrightarrow E^3$ be an N-magnetic curve in Euclidean 3-space and F_V be a magnetic field in E^3. If the normal vector field N of the Frenet frame satisfies the 2-Lorentz force equation $\nabla_{\alpha'}\nabla_{\alpha'}N = \Phi(N') = V \times N'$, then the curve α is called a* 2-N***-magnetic curve.***

Proposition 4. *Let α be a unit speed 2-N-magnetic curve according to Frenet frame in Euclidean 3-space. Then, we have*

$$\begin{bmatrix} \Phi(T') \\ \Phi(N') \\ \Phi(B') \end{bmatrix} = \begin{bmatrix} -\kappa^2 & 0 & \kappa\tau \\ -\kappa' & -\kappa^2-\tau^2 & \tau' \\ \kappa\tau & 0 & -\tau^2 \end{bmatrix} \begin{bmatrix} T \\ N \\ B \end{bmatrix}. \tag{3.10}$$

Proof. Let α be a 2-N-magnetic curve according to Frenet frame in Euclidean 3-space with the Frenet apparatus $\{T, N, B, \kappa, \tau\}$. From the definition of the 2-N-magnetic curve according to Frenet frame and from (2.1), we know that $\Phi(N') = -\kappa' T - (\kappa^2+\tau^2)N + \tau' B$. On the other hand, since $\Phi(T') \in Sp\{T, N, B\}$, we have $\Phi(T') = a_1T + a_2N + a_3B$. So, from (2.1), (2.2) and (2.7) we get, $\Phi(T') = (-\kappa^2)T + (\kappa\tau)B$.

Furthermore, from $\Phi(B') = b_1T + b_2N + b_3B$, we have $\Phi(B') = (\kappa\tau)T - (\tau^2)B$, which completes the proof. □

Proposition 5. *Let α be a unit speed N-magnetic curve according to Frenet frame in Euclidean 3-space. Then, the curve α is a 2–N-magnetic trajectory of a 2-magnetic vector field V if and only if the 2-magnetic vector field V is*

$$V = \tau T - \frac{\kappa'}{\tau}N + \kappa B = \tau T + \frac{\tau'}{\kappa}N + \kappa B \tag{3.11}$$

along the curve α.

Proof. Let α be a 2–N-magnetic trajectory of a 2-magnetic vector field V according to Frenet frame. Using Proposition 4 and taking $V = aT + bN + cB$; from $\Phi(T') = V \times T'$, we get

$$a = \tau,\ c = \kappa; \tag{3.12}$$

from $\Phi(N') = V \times N'$, we get

$$a = \tau,\ b = -\frac{\kappa'}{\tau} = \frac{\tau'}{\kappa},\ c = \kappa \tag{3.13}$$

and from $\Phi(B') = V \times B'$, we get

$$a = \tau,\ c = \kappa \tag{3.14}$$

and so the 2-magnetic vector field V can be written by (3.11). Conversely, if the 2-magnetic vector field V is the form of (3.11), then one can easily see that $V \times N' = \Phi(N')$ holds. So, the curve α is a 2-N-magnetic projectory of the 2-magnetic vector field V according to Frenet frame. □

Corollary 4. *If the curve α is a 2–N-magnetic trajectory of a 2-magnetic vector field V, then we have*

$$\kappa^2 + \tau^2 = constant. \tag{3.15}$$

Proof. The proof is obvious from (3.13). □

Example 2. *Let us consider the curve*

$$\alpha(t) = \left(\cos\frac{t}{\sqrt{2}}, \sin\frac{t}{\sqrt{2}}, \frac{t}{\sqrt{2}}\right), \tag{3.16}$$

which is a unit speed circular helix in E^3. Here, one can easily calculate its Frenet-Serret trihedra and curvatures as

$$\begin{aligned} T &= \frac{1}{\sqrt{2}}\left(-\sin\frac{t}{\sqrt{2}}, \cos\frac{t}{\sqrt{2}}, 1\right), \\ N &= \left(-\cos\frac{t}{\sqrt{2}}, -\sin\frac{t}{\sqrt{2}}, 0\right), \\ B &= \frac{1}{\sqrt{2}}\left(\sin\frac{t}{\sqrt{2}}, -\cos\frac{t}{\sqrt{2}}, 1\right), \\ \kappa &= \tau = \frac{1}{2}, \end{aligned} \tag{3.17}$$

respectively. Here, from (3.15), the curve α is a 2-N-magnetic curve. Also from (3.11), the 2-magnetic vector field V when the curve (3.16) is a 2-N-magnetic trajectory of the 2-magnetic vector field V according to Frenet frame (3.17) is

$$V = \left(0, 0, \frac{1}{\sqrt{2}}\right). \tag{3.18}$$

Here, it can be seen that, from (3.17) and (3.18), $\nabla_{\alpha'}\nabla_{\alpha'}N = V \times N'$ satisfies. So, the curve α is a 2-N-magnetic curve according to Frenet frame with the 2-magnetic vector field (3.18).

When the curve α is 2-N-magnetic according to Frenet frame, the figure of α and V can be drawn as Figure 2.

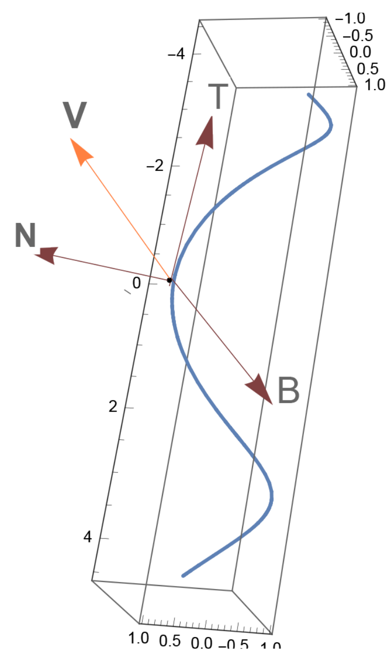

Figure 2: 2-N-magnetic curve α according to Frenet frame and the 2-magnetic vector field V

3.3. 2-B-MAGNETIC CURVES IN EUCLIDEAN 3-SPACE.

Definition 4. *Let $\alpha : I \subset \mathbb{R} \longrightarrow E^3$ be a B-magnetic curve in Euclidean 3-space and F_V be a magnetic field in E^3. If the binormal vector field B of the Frenet frame satisfies the 2-Lorentz force equation $\nabla_{\alpha'}\nabla_{\alpha'}B = \Phi(B') = V \times B'$, then the curve α is called a* ***2-B-magnetic curve.***

Proposition 6. *Let α be a unit speed 2-B-magnetic curve according to Frenet frame in Euclidean 3-space. Then, we have*

$$\begin{bmatrix} \Phi(T') \\ \Phi(N') \\ \Phi(B') \end{bmatrix} = \begin{bmatrix} -\kappa\gamma & 0 & \kappa\tau \\ 0 & -\kappa\gamma-\tau^2 & 0 \\ \kappa\tau & -\tau' & -\tau^2 \end{bmatrix} \begin{bmatrix} T \\ N \\ B \end{bmatrix}, \tag{3.19}$$

where γ is a certain function defined by $\gamma = g(\Phi(T), N)$.

Proof. Let α be a 2-B-magnetic curve according to Frenet frame in Euclidean 3-space with the Frenet apparatus $\{T, N, B, \kappa, \tau\}$. From the definition of the 2-B-magnetic curve according to Frenet frame and from (2.1), we know that $\Phi(B') = \kappa\tau T - \tau' N - \tau^2 B$. On the other hand, since $\Phi(T') \in Sp\{T, N, B\}$, we have $\Phi(T') = a_1T + a_2N + a_3B$. So, from (2.1), (2.2) and (2.8) we get, $\Phi(T') = (-\kappa\gamma)T + (\kappa\tau)B$.

Furthermore, from $\Phi(N') = b_1T + b_2N + b_3B$, we have $\Phi(B') = (-\kappa\gamma - \tau^2)N$, which completes the proof. □

Proposition 7. *Let α be a unit speed B-magnetic curve according to Frenet frame in Euclidean 3-space. Then, the curve α is a 2–B-magnetic trajectory of a 2-magnetic vector field V if and only if the 2-magnetic vector field V is*

$$V = \tau T + \kappa B \tag{3.20}$$

along the curve α.

Proof. Let α be a 2–B-magnetic trajectory of a 2-magnetic vector field V according to Frenet frame. Using Proposition 6 and taking $V = aT + bN + cB$; from $\Phi(T') = V \times T'$, we get

$$a = \tau,\ c = \gamma; \tag{3.21}$$

from $\Phi(N') = V \times N'$, we get

$$a = \tau,\ c = \gamma,\ b = 0 \tag{3.22}$$

and from $\Phi(B') = V \times B'$, we get

$$a = \tau,\ c = \kappa,\ \tau' = 0 \tag{3.23}$$

and so the 2-magnetic vector field V can be written by (3.20). Conversely, if the 2-magnetic vector field V is the form of (3.20), then one can easily see that $V \times B' = \Phi(B')$ holds. So, the curve α is a 2-B-magnetic projectory of the 2-magnetic vector field V according to Frenet frame. □

Corollary 5. *If the curve α is a 2-B-magnetic trajectory of a 2-magnetic vector field V, then the torsion τ of α is constant and we have*

$$\gamma = \kappa = g(\Phi(T), N). \tag{3.24}$$

Proof. The proof is obvious from (3.21)-(3.23). □

From (2.1), (2.8) and (3.24), we get

Corollary 6. *If a curve α is a 2–B-magnetic trajectory of a 2-magnetic vector field V, then the Lorentz force Φ corresponds to covariant derivative along α in E^3. Also, we have*

$$\Phi^2(X) = \Phi(X'),$$

for $\forall X \in \{T, N, B\}$.

Corollary 7. *If a curve α is a 2–B-magnetic trajectory of a 2-magnetic vector field V, then we have*

$$g(T, \Phi(T')) + g(B, \Phi(B')) = g(N, \Phi(N')) = -(\kappa^2 + \tau^2).$$

Proof. From (2.1) and Corollary 6, the proof follows. □

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