**New Inequalities of The Real Parts of The Zeros of Polynomials**

M.Al-Hawari(1), R. Brahmeh(2)  
Mohammad Al-Hawari,Irbid National University ,Jordan,Irbid(1)

Ragheb R. Brahmeh (2)   
E-mail: [analysis2003@yahoo.com](mailto:analysis2003@yahoo.com)(1), [ragheb\_br\_lr@yahoo.com](mailto:ragheb_br_lr@yahoo.com)(2)

**Abstract**

In this paper, we find new inequalities of the real parts of the zeros of polynomials, and a new inequalities of the zeros and critical points of polynomials.

Keywords: Real Parts, Zeros of Polynomials, critical points.

1.Introduction

The problem of finding regions that contain some or all eigenvalues of matrices or zeros of polynomials has a long history. For example Cauchy gave an easily-calculated circular bound for complex coefficient polynomial zeros. Very recently research on this problem was based on Cauchy’s work. For matrix eigenvalues, Gerschgorin theorem is very powerful tool for improving existing bounds or computing new ones. It can be used to find disks whose union contains all eigenvalues of a complex matrix. Such results may be applied to analyze the stability or the relative stability of discrete-time systems. They may also be applied to determine the eigenvalue distribution in certain disks for continuous-time systems, by shifting the origin of the s-plane. In recent years, numerous papers and comprehensive books have been published, for finding circular bounds of polynomial zeros which have real or complex coefficients

**Definition 1[2].**

*Let* f *be a polynomial of degree , with complex coefficients, and let  be the zeros of* f*.*

*Let ,I, and J be the identity matrix of order (n-1) and the (n-1) × (n-1) matrix with all entries equal to 1, respectively. Then the (n-1) × (n-1) derivative companion matrix of* f *is given by*

**

*which is called a D-companion matrix of f.*

**Theorem 1[3].**

*If , then*

**

**Lemma 1[4].**

*If z any zero of , then*

*.*

**Lemma2[5].**

*For , we have*

**

**Lemma 3[5].**

*If the zeros of  are arranged in such a way that is , then*

**

*for  and*

.

**Lemma 4[7].**

*Let  be a conjugate normal matrices. Then*

*.*

**Lemma 5[2].**

*Let  if , then*

**

**Lemma 6[6].**

*Let . Then for ,*

**

**Lemma 7[1].**

*Let  be the zeros of polynomials f of degree and  be the critical points of f. Then for  we have*

****

***2.Main Results***

**Theorem 2.1**

*If z is any zero of  for any scaler r. Then*

******

***Proof.***

The companion matrix of p(z) is



By Lemma 1 we have  , and hence



**Example 2.1**

*If ,then*

**

**Theorem 2.2**

*If z is any zero of . Then*

**

***Proof.***

.

Thus,

,

where  is partitioned matrix



with , and  is the n× n tridiagonal matrix

.

The eigenvalue of  are

,

,

and

 for .

It is well known that the eigenvalue of  are

 for .

and hence by Lemma 2 we have,

**

for . 

**Example 2.2**

*If ,then*

**

**Theorem 2.3**

*If . Then .*

***Proof .*** By Lemma 5, we have



Put , for i, j=1,2,…,n and p=2, we have .

**Theorem 2.4**

*Let  have two eigenvalues. Then*

**

**Proof.** By Lemma 6, we have



By taking p=2, we have the result.

**Theorem 2.5**

*Let if  Then*

*.*

***Proof.***  By Lemma 5, we have

.

and by Lemma 6, we have



Hence, the result is hold

.

**Corollary 2.1**

*Let  Then .*

***Proof .*** By Theorem 2.5, we have

**

By taking p=2 in Theorem 5, we get the result.

**Theorem 2.6**

*If the zeros of for is any complex number are arranged in such a way that is , then*

**

*for k=1,2,..,n-1 and*

**

***Proof.***

By Lemma 3 we have



By taking  for j=1,2,…,n , where 

and .

Hence, we get the result.

**Theorem 2.7**

*If  be conjugate normal matrix, then*

**

***Proof .***

By Lemma 4, we have



By taking B=A , we have.





But .

Hence, we get the result.

**Theorem 2.8**

*Let  be the zeros of polynomials f of degree and  be the critical points of f. Then we have*

***Proof.***

By Lemma 7 for , we have



By Definition 1 and Theorem 1 and taking p=2, we have

****

By square both sides we get the results of (4).

For (5), we have





By square both sides we get the result of (5).

Now for (6) we have,

where

.

Note that rank(E)≤ 1





Hence,  , where 0 is of multiplicity n-2. So  for j=2,3,..,n-2

Now,





By square both sides we get the result of (6).

**References**

[1] M. Adm, F. Kittaneh,*Bounds and Majorization Relations for the Critical Points of Polynomials*, Linear Algebra and its Applications 436 (2012) 2494–2503.

[2] W. Cheung, T. Ng, *A companion matrix approach to the study of zeros and critical points of a polynomial*, J. Math. Anal. Appl.319 (2006) 690–707.

[3] R. Bhatia, *Matrix Analysis*, Springer, New York, 1997. MR1477662 (98i:15003)

[4] M. Fujii and F. Kubo, *Buzano’s inequality and bounds for roots of algebraic equations*, Proc.Amer. Math. Soc. 117(1993), 359–361. MR1088441 (93d:47014)

[5] F. Kittaneh, *Bounds and a majorization for the real parts of the zeros of polynomials*, Proc. Amer.Math. Soc. 135 (2007) 664–695.

[6] H. Linden*, Bounds for the zeros of polynomials from eigenvalues and singular values of some companionmatrices*, Linear Algebra Appl. 271 (1998) 41–82.

[7] H. D. Sterck and Minghualin, *Some Majorization Inequalities for Coneigevalues*, a publication of the International Linear Algebra Society

Volume 23, pp. 669-677, August 2012.