Slightly Weak Separation axioms

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Abstract We introduce to notions of $\emptyset(w)-T_1$, $\emptyset(w)-T_2$, $\emptyset(w)-regular$ and $\emptyset(w)-normal$ on μ −weak structure ($\mu-WS$). And investigate a characterizations for such notions and relationships among $\emptyset(w)-T_1$, $\emptyset(w)-T_2$, $\emptyset(w)-T_3$ and $\emptyset(w)-T_4$. And some characterizations of w-regular and w-normal spaces in $\mu-WS$ have been given.

Keywords: μ −weak structure, $\emptyset(w)-T_1$, $\emptyset(w)-T_2$, $\emptyset(w)-T_3$ and $\emptyset(w)-T_4$

Notation and Preliminaries

Recently Á. Császár [6] defined the notion of *weakstructures* and showed that these structures can replace in many situations generalized topologies or minimal structures. Let us say that $w \subseteq P$ is a weak structure (briefly WS) on X if $\phi \in \omega$. Clearly each generalized topology and each minimal structure is a WS . Let w be a weak structure on X and $A \subseteq X$. Á. Császár defined (as in the general case) $i_\omega A$ as the union of all w -open subsets of A (e.g. ϕ) and $c_w A$ as the intersection of all w -closed sets containing A (e.g. X). Define the family $w_\mu = \{v \cap \mu : v \in w\}$ is the w_μ-structure induced over $\mu \subset X$ by w (by short $\mu-WS$). The elements of w_μ are called w_μ-open sets; a set v is a w_μ-closed set if $\mu - v \in w_\mu^c$. We note that w_μ^c is the family of all w_μ-closed sets. Let $v \in \mu-WS$ we define the w_μ-interior (by short i_w) of μ as the finest w_μ - open sets contained in v that is, $i_\mu w(v) = \cup_{\zeta \in w_\mu}\{\zeta : \zeta \subseteq v\}$. Let $v \in \mu-WS$ we define the w_μ-closure (by short c_w) of μ as the smallest w_μ-closed sets which contained in v that is, $c_\mu w(v) = \cap_{\zeta \in w_\mu}\{\mu-\zeta : v \subseteq \mu-\zeta\}$. . Let $M_{\emptyset(w)}$ denote the union of all elements of X. Let (X,τ) be a topological space and A is a subset of X. The closure of A and the interior of A are denoted by $Cl(A)$ and $Int(A)$, respectively. In this paper in section 1. We introduce the concepts of $\emptyset(w)-T_1$, $\emptyset(w)-T_2$, $\emptyset(w)-T_3$ and $\emptyset(w)-T_4$. And investigate a characterizations for such notions and relationships among them are introduces. In section 2. Some characterizations of w -regular and w -normal spaces in $\mu-WS$ have been given.

1- $\emptyset(w) - T_i, i = 1,2,3,4$

Definition 1.1 Let w be a w eak structure on X, $\mu \subset X$. Then X is called a $\emptyset(w) - T_1$ if for $x_1, x_2 \in M_{\emptyset(w)}$ $with$ $x_1 \neq x_2$. Then there exists V, U are $\emptyset(w) - open$ set , $x_1 \in U, x_2 \notin U$ and $x_2 \in V, x_1 \notin V$.

Theorem 1.1 Let w be a $w$eak structure on $X, \mu \subset X$. Then

For the converse, let $x, y \in M_{\emptyset(w)}$ $with$ $x_1 \neq x_2$. Then by hypothesis,

$(X - M_{\emptyset(w)}) \cup \{x_1\}$ and $(X - M_{\emptyset(w)}) \cup \{x_2\}$ are $\emptyset(w) - closed$. Set

$U = M_{\emptyset(w)} - \{x_2\}$ and $V = M_{\emptyset(w)} - \{x_1\}$. Then V, U are $\emptyset(w) - open$ set , $x_1 \in U, x_2 \notin U$ and $x_2 \in V, x_1 \notin V$. Hence X is a $\emptyset(w) - T_1$.

Definition 1.2 Let w be a $w$eak structure on $X, \mu \subset X$. Then X is called a relative $\emptyset(w) - T_2$ if for every $x_1, x_2 \in M_{\emptyset(w)}$ $with$ $x_1 \neq x_2$, there exists $\emptyset(w) - open$ sets $U, V \in w$ such that $x_1 \in U, x_2 \in V$ and $U \cap V = \emptyset$.

Remark 1.1 it is clear that every $\emptyset(w) - T_2$ and $\emptyset(w) - T_1$, but the converse need not be true in general .

Lemma 1.1 Let w be a $w$eak structure on $X, \mu \subset X$. Every $\emptyset(w) - closed$ set includes $X - M_{\emptyset(w)}$.

Proof. For every $w - open$ sets U in $X, \mu \subset X$, $U \subseteq M_{\emptyset(w)} \subseteq U^c$.

Theorem 1.2 Let w be a $w$eak structure on $X, \mu \subset X$. Then the following properties are equivalent:

(1) X is $\emptyset(w) - T_2$

(2) Let $x \in M_{\emptyset(w)}$. For each $z \in M_{\emptyset}(w)$ $with$ $x_1 \neq x_2$, there is a $\emptyset(w) - open$ set U containing x such that $z \notin C_w(U)$.

(3) For $x \in M_{\emptyset(w)}$, $\cap \{C_w(U): U \in w \ and \ x \in U\} \supseteq \cup (X - M_{\emptyset(w)})$

(4) The set $\Delta \cup M_{\emptyset(w)} \times M_{\emptyset(w)})^c$ is $\emptyset(w) - closed$ set in $X \times X$, where the diagonal $\Delta = \{(x, x): x \in X\}$.

Proof (1) $\Rightarrow$ (2)) For $x \in M_{\emptyset(w)}$, let $z \in M_{\emptyset(w)}$ $with$ $x_1 \neq x_2$. Then there exist disjoint $\emptyset(w) - open$ set U and V containing x and z, respectively. From lemma 1.1 $z \notin C_w(U)$.

(2) $\Rightarrow$ (3) For $x \in M_{\emptyset}(w)$, let $z \in M_{\emptyset(w)}$ $with$ $x_1 \neq x_2$. Then by (2) , there is a $\emptyset(w) - open$ set U containing x such that $z \notin C_w(U)$, so by above lemma

$z \notin \cap \{C_w(U): U \in w \ and \ x \in U\} \supseteq \cup (X - M_{\emptyset(w)})$. Thus we find that $\cap \{C_w(U): U \in w \ and \ x \in U\} = \{x\} \cup (X - M_{\emptyset(w)}$.

(3) $\Rightarrow$ (4) We show that $X \times X - (\Delta \cup M_{\emptyset(w)} \times M_{\emptyset(w)})^c)$ is $\emptyset(w) - open.$ For the proof , let (x, y) be any element in $X \times X - (\Delta \cup M_{\emptyset(w)} \times M_{\emptyset(w)})^c)$. Then

$x, z \in M_{\emptyset(w)}$ and $x \neq z$. Since

$z \notin \cap \{C_w(U): U \in w \ and \ x \in U\} = \{x\} \cup (X - M_{\emptyset(w)}$ }, there exists some $U \in w$ such that $x \in U$ and $z \notin C_w(U)$. Since $U \cap (X - C_w(U)) = \emptyset$ and $X - C_w(U)$ is $\emptyset(w) - open$ set containig z, $U \times (X - C_w(U))$ is a $\emptyset(w) - open$ set containig (x, z) such that $U \times (X - C_w(U) \cap (\Delta \cup M_{\emptyset(w)} \times M_{\emptyset(w)})^c)$. Hence this implies $X \times X - (\Delta \cup M_{\emptyset(w)} \times M_{\emptyset(w)})^c)$ is $\emptyset(w) - open$ in $X \times X$

(3) $\Rightarrow$ (4) let $x, z \in M_{\emptyset(w)}$ and $x \neq z$. Then $(x, z) \notin \Delta \cup M_{\emptyset(w)} \times M_{\emptyset(w)})^c$. Since $\Delta \cup (M_{\emptyset(w)} \times M_{\emptyset(w)})^c$ is $\emptyset(w) - open$, by lemma 1.1 there exists a $\emptyset(w) - open$ set $U \times V$ containig the point (x, z) such that

$U \times V \cap (\Delta \cup M_{\emptyset(w)} \times M_{\emptyset(w)})^c) = \emptyset$. Hence we can say that there exist $U, V \in w$ such that $x \in U$, $z \in V$ and $U \cap V = \emptyset$.

Definition 1.3 Let w, w' be a weak structures on X and Y respectively, $\mu \subset X$. Then a function $f: (X, w) \to (Y, w')$ is said to be

(1) $\emptyset(w, w') - continuous$ if $U' \in w'$ implies that $f^{-1}(U') \in w$

(2) $\emptyset(w, w') - open$ if $U \in w$ implies that $f(U) \in w'$

Lemma 1.2 Let w, w' be a weak structures on $X, \mu \subset X$. If $f: (X, w) \to (Y, w')$ is a $\emptyset(w, w') - open$, then $f(M_{\emptyset(w)}) \subseteq M_{\emptyset(w')}$.

Proof Since $M_{\emptyset(w)} \in w$, $f(M_{\emptyset(w)}) \in M_{\emptyset(w')}$ and so $f(M_{\emptyset(w)}) \subseteq M_{\emptyset(w')}$

Theorem 1.3 Let $f: (X, w) \to (Y, w')$ be an injective, $\emptyset(w, w') - open$ and $\emptyset(w, w')$-continuous function on a weak structures (X, w) and (Y, w') . If Y is $\emptyset(w') - T_2$, then X is $\emptyset(w) - T_2$.

Proof Let $x_1, x_2 \in M_{\emptyset(w)}$ with $x_1 \neq x_2$. Then $f(x_1) \neq f(x_2)$ and from lemma 1.2, we have $f(x_1), f(x_2) \in M_{\emptyset(w)}$. Since Y is $\emptyset(w') - T_2$, there exist $U', V' \in w'$ such that $f(x_1) \in U', f(x_2) \in V'$ and $U' \cap V' = \emptyset$. This implies $f^{-1}(U'), f^{-1}(V') \in \emptyset(w), x_1 \in f^{-1}(U'), x_2 \in f^{-1}(V')$ and $f^{-1}(U') \cap f^{-1}(V') = \emptyset$. Hence X is $\emptyset(w) - T_2$.

Theorem 1.4 Let $f:(X,w)\to(Y,w')$ be an injective, $\emptyset(w,w')$-open function on a weak structures (X,w) and (Y,w') . If X is $\emptyset(w)-T_2$, and

$f\left(M_{\emptyset(w)}\right)=M_{\emptyset(w')}$ then Y is $\emptyset(w)-T_{2.}$

Proof Let $x_1,x_2\in M_{\emptyset(w')}$ $with$ $y_1\neq y_2$. Then from $f\left(M_{\emptyset(w)}\right)=M_{\emptyset(w')}$

There exist $x_1,x_2\in M_{\emptyset(w)}$ such that $f(x_{1)}=y_1$, $f(x_2)=y_2$. Since X is $\emptyset(w)-T_2$, there exist $U,V\in\emptyset(w)$ such that $x_1\in U,x_2\in V$ and $U\cap V=\emptyset$. Thus

$f(U),f(V)\in\emptyset(w'),\ y_1\ \in f(U)\ \ and\ \ y_2\ \in f(V).$ And injective of f ,

$f(U\cap V)=f(U)\cap f(V)=\emptyset$ and so Y is $\emptyset(w)-T_{2.}$

Definition 1.4 Let w be a weak structure on $X,\mu\subset X$. Then X is said to be relative $\emptyset(w)-regular$ (simply, $\emptyset(w)-regular$) if for $x\in M_{\emptyset(w)}$ and $\emptyset(w)-closed$ set F with $x\notin F$, there exist $U,V\in\emptyset(w)$ such that $x\in U,\ F\cap M_{\emptyset(w)}\subseteq V$ and $U\cap V=\emptyset$. And if X is $\emptyset(w)-T_{1.}$ And $\emptyset(w)-regular$, then it is said to be $\emptyset(w)-T_{3.}$

Theorem 1.5 Let w be a weak structure on $X,\mu\subset X$. Then X is said to be $\emptyset(w)-regular$ if and only if for $x\in M_{\emptyset(w)}$ and a $\emptyset(w)-open$ set U containing x, there $\emptyset(w)-open$ set V containing x such that $x\in V\subseteq C_w(V)\cap M_{\emptyset(w)}\subseteq U$ is a

Proof Assume that X is said to be $\emptyset(w)-regular$. Then for $x\in M_{\emptyset(w)}$ and a $\emptyset(w)-open$ set U containing x and the $\emptyset(w)-closed$ set U^c have disjoint $\emptyset(w)-open$ sets U,W with $x\in V$, $U^c\cap M_{\emptyset(w)}\subseteq W$. Since $V\subseteq W^c$ and W^c is $\emptyset(w)-closed$, it follow $C_w(V)\subseteq W^c$. This implies $C_w(V)\cap(U^c\cap M_{\emptyset(w)})\subseteq C_w(V)\cap W=\emptyset$, thus $C_w(V)\cap M_{\emptyset(w)}\subseteq U.$

For the converse, let F be any $\emptyset(w)-closed$ set and $x\notin F$ for for $x\in M_{\emptyset(w)}$. Then since F^c is $\emptyset(w)-open$ set containing x, by hyothesis, there is a $\emptyset(w)-open$ set V containing x such that $x\in V\subseteq C_w(V)\cap M_{\emptyset(w)}\subseteq F^c$, thus $C_w(V)\cap M_{\emptyset(w)}\cap F=\emptyset$, so that $M_{\emptyset(w)}\cap F\subseteq C_w(V^c)$. Hence X is $\emptyset(w)-regular.$

Definition 1.5 Let w be a weak structure on $X,\mu\subset X$. Then X is said to be relative $\emptyset(w)-normal$ (simply, $\emptyset(w)-normal$) if for $\emptyset(w)-closed$ set F_1 and F_2 with $F_1\cap F_2=X-M_{\emptyset(w)}$ there exist $U,V\in\emptyset(w)$ such that $F_1\cap M_{\emptyset(w)}\subseteq U,\ F_2\cap M_{\emptyset(w)}\subseteq V$ and $U\cap V=\emptyset$. And if X is $\emptyset(w)-T_{1.}$ And $\emptyset(w)-regular$, then it is said to be $\emptyset(w)-T_4$.

Theorem 1.6 Let w be a weak structure on $X,\mu\subset X$. Then X is said to be $\emptyset(w)-normal$ if and only if for a $\emptyset(w)-open$ set F and a $\emptyset(w)-open$ set U with $F\cap M_{\emptyset(w)}\subseteq U$, There is a $\emptyset(w)-open$ set V containing x such that

$F\subseteq V\subseteq C_w(V)\cap M_{\emptyset(w)}\subseteq U.$

Proof It is similar to the proof of Theorem 1.5

2 characterizations w -regular spaces and w -normal spaces

In this section characterizations of w-regular and w-normal spaces in $\mu - WS$ have been given.

Definition 2.1 Let (X,τ) be a topological space and w be a $\mu - WS$ on X. Then (X,τ) is said to be w -regular if for each closed set F of X and each $x \notin F$, there exist disjoint w -open sets U and V such that $x \in U$ and $F \subseteq V$

Theorem 2.1 Let (X,τ) be a topological space and w be a $\mu - WS$ on X. Consider the following statements:

1. X is w -regular.

2. For each $x \in X$ and each $U \in \tau$ with $x \in U$, there exists $V \in \mathrm{w}$ such that $x \in V \subseteq c_{\mathrm{w}}(V) \subseteq U$.

Then the implication (1) $\Rightarrow$ (2) holds. If $i_{\mathrm{w}}(A) \in \mathrm{w}$ for every w -closed A of X, then the statements are equivalent.

Proof

1. $\Rightarrow$ (2). Let $x \notin (X-U)$, where $U \in \tau$. Then there exist disjoint G, $V \in w$ such that $(X-U) \subseteq G$ and $x \in V$. Thus $V \subseteq X-G$ and hence $x \in V \subseteq c_{\mathrm{w}}(V) \subseteq c_{\mathrm{w}}(X-G) = X-G \subseteq U$.

2. $\Rightarrow$ (1). Let F be a closed set and $x \notin F$. Then $x \in X-F \in \tau$ and hence there exists $V \in \mathrm{w}$ such that $x \in V \subseteq c_{\mathrm{w}}(V) \subseteq X-F$. Therefore, $F \subseteq X - c_{\mathrm{w}}(V) = i_{\mathrm{w}}(X-V) \in \mathrm{w}$.

Definition 2.2 Let w be a $\mu - WS$ on a topological space (X,τ). Then $A \subseteq X$ is called a generalized w-closed set (or simply $g\mathrm{w}$ -closed set) if $c_{\mathrm{w}}(A) \subseteq U$ whenever $A \subseteq U \in \tau$. The complement of a g_{w} -closed set is called a generalized w -open (or simply g_{w} -open) set.

Theorem 2.2 *Let (X,τ) be a topological space and w be a $\mu - WS$ on X, and consider the following statements:*

1. X is w -regular.

2. For each closed set F and $x \notin F$, there exists $U \in \mathrm{w}$ and $g\mathrm{w}$ -open set V such that $x \in U$, $F \subseteq V$ and $U \cap V = \phi$.

3. For each $A \subseteq X$ and each closed set F with $A \cap F = \phi$, there exist $U \in$ w and a gw -open set V such that $A \cap U \neq \phi$, FV and $U \cap V = \phi$. Then the implications (1) $\Rightarrow$ (2)$\Rightarrow$ (3) hold. If $i_{\text{w}}(A) \in$ w for every gw -open set A of X, then the statements are equivalent.

Proof

1. $\Rightarrow$ (2). Obvious.

2. $\Rightarrow$ (3). Let $A \subseteq X$ and F be a closed set with $A \cap F = \phi$. Then for $a \in A$, $a \notin F$, and hence by (2), there exist $U \in$ w and a gw -open set V such that $a \in U$, $F \subseteq V$ and $U \cap V = \phi$. Hence $A \cap U \neq \phi$, $F \subseteq V$ and $U \cap V = \phi$.

3. $\Rightarrow$ (1). Let $x \notin F$, where F is closed in X. Since $F \cap x = \phi$, by (3) there exist U and a gw -open set set W such that $x \in U$, $F \subseteq W$ and $U \cap W = \phi$. Then by Theorem 2.11 we have $F \subseteq i_{\text{w}}(W) = V \in$ w and hence $U \cap V = \phi$.

Definition 2.3 Let (X, τ) be a topological space and w be a $\mu - WS$ on X. Then (X, τ) is said to be w -normal if for any two disjoint closed sets A and B there exist two disjoint w -open sets U and V such that $A \subseteq U$ and $B \subseteq V$.

Remark 2.1

For a $\mu - WS$ w on on a topological space (X, τ), every w -closed set is a g_{w} -closed set. In fact, if A is a w -closed set with $A \subseteq U \in \tau$, then $A = c_{\text{w}}(A) \subseteq U$, so that A is g_{w} -closed.

Theorem 2.3 Let (X, τ) be a topological space and w be a $\mu - WS$ on X. Then A is gw -open if and only if $F \subseteq i_{\omega}(A)$ wherever $F \subseteq A$ and F is closed.

Proof

Let A be a gw -open set and $F \subseteq A$, where F is closed. Then $X - A$ is a gw - closed set contained in an open set $X - F$. Hence $c_{\omega}(X - A) \subseteq X - F$, i.e. $X - i_{\text{w}}(A) \subseteq X - F$. So $F \subseteq i_{\text{w}}(A)$. Conversely, suppose that $F \subseteq i_{\text{w}}(A)$ for any closed set F whenever $F \subseteq A$. Let $X - A \subseteq U$, where $U \in \tau$. Then $X - U \subseteq A$ and $X - U$ is closed. By assumption, $X - U \subseteq i_{\text{w}}(A)$ and hence $c_{\text{w}}(X - A) = X - i_{\text{w}}(A) \subseteq U$. Therefore $(X - A)$ is gw - closed and hence A is gw - open .

Theorem 2.4 Let w be a μ − WS on a topological space (X, τ), and consider the following statements:

(1) X is w -normal.

(2) For any pair of disjoint closed sets A and B of X, there exist disjoint $g\mathrm{w}$-open sets U and V of X such that $A \subseteq U$ and $B \subseteq V$.

(3) For each closed set A and each open set B containing A, there exists a $g\mathrm{w}$-open set U such that $A \subseteq U \subseteq c_{\mathrm{w}}(U) \subseteq B$.

Then the implications (1) $\Rightarrow$ (2) $\Rightarrow$ (3) hold. If $i_{\mathrm{w}}(A) \in \mathrm{w}$ and $c_{\mathrm{w}}(A)$ is w-closed for every $g\mathrm{w}$-open set A of X, then the statements are equivalent.

Proof

1. $\Rightarrow$ (2). Let A and B be a pair of disjoint closed sets of X. Then by (1) there exist disjoint w-open sets U and V of X such that $A \subseteq U$ and $B \subseteq V$. Then (2) follows from Remark 2.1

2. $\Rightarrow$ (3). Let A be a closed set and B be an open set containing A. Then A and $X-B$ are two disjoint closed sets. Hence by (2) there exist disjoint $g\mathrm{w}$-open sets U and V of X such that $A \subseteq U$ and $X-B \subseteq V$. Since V is $g\mathrm{w}$-open and $X-B$ is a closed set with $X-B \subseteq V$, by Theorem 2.11, $X-B \subseteq i_{\mathrm{w}}(V)$. Hence $c_{\mathrm{w}}(X-V) = X - i_{\mathrm{w}}(V) \subseteq B$. Thus $A \subseteq U \subseteq c_{\mathrm{w}}(U) \subseteq c_{\mathrm{w}}(X-V) \subseteq B$.

3. $\Rightarrow$ (1). Let A and B be two disjoint closed subsets of X. Then A is a closed set and $X-B$ is an open set containing A. Thus by (3) there exists a $g\mathrm{w}$-open set U such that $A \subseteq U \subseteq c_{\mathrm{w}}(U) \subseteq X-B$. Thus by Theorem 2.3

and $B \subseteq X - c_{\mathrm{w}}(U)$, where $i_{\mathrm{w}}(U)$ and $X - c_{\mathrm{w}}(U) = i_{\mathrm{w}}(X-U)$ $A \subseteq i_{\mathrm{w}}(U)$ are disjoint sets. Since U is $g\mathrm{w}$-open, $i_{\mathrm{w}}(U) \in \mathrm{w}$ and $i_{\mathrm{w}}(X-U) \in \mathrm{w}$. Hence X is w-normal.

Theorem 2.5 Let (X,τ) be a topological space and w be a $\mu-$WS on X, and consider the following statements:

1. For each g-closed set A and each open set B containing A, there exists an w-open set U such that $Cl(A) \subseteq U \subseteq c_{\mathrm{w}}(U) \subseteq B$.

2. For each closed set A and each g-open set B containing A, there exists an w-open set U such that $A \subseteq U \subseteq c_{\mathrm{w}}(U) \subseteq Int(B)$.

3. For each g-closed set A and each open set B containing A, there exists a $g\mathrm{w}$-open set U such that $Cl(A) \subseteq U \subseteq c_{\mathrm{w}}(U) \subseteq B$.

4. For each closed set A and each open set B containing A, there exists a $g\mathrm{w}$-open set U such that $A \subseteq U \subseteq c_{\mathrm{w}}(U) \subseteq B$.

5. For each closed set A and each g-open set B containing A, there exists a gw-open set U such that $A \subseteq U \subseteq c_{w}(U) \subseteq Int(B)$.

Then the implications (1) $\Rightarrow$ (2) $\Rightarrow$ (3) $\Rightarrow$ (4) $\Rightarrow$ (5) hold. If $i_{\omega}(A) \in$ w for every gw-open A of X, then the statements are equivalent.

Proof

1. $\Rightarrow$ (2). Let A be a closed set and B be a g-open set containing A. Then $A \subseteq Int(B)$. Since A is closed and $Int(B)$ is open, by (1) there exists a w-open set U such that $A \subseteq U \subseteq c_{w}(U) \subseteq Int(B)$.

2. $\Rightarrow$ (3) and (3) $\Rightarrow$ (4) are obvious.

3. $\Rightarrow$ (5). Let A be a closed set and B be a g-open set containing A. Since B is g-open and A is closed, since . Thus by (4), there exists a gw-open set U such that $A \subseteq U \subseteq c_{w}(U) \subseteq Int(B)$.

4. $\Rightarrow$ (1). Let A be a g-closed subset of X and B be an open set containing A. Then $Cl(A) \subseteq B$, where B is g-open. Thus by (5) there exists a gw-open set G such that $Cl(A) \subseteq G \subseteq c_{w}(G) \subseteq Int(B) \subseteq B$. Since G is gw-open and $Cl(A) \subseteq G$, by Theorem 2.3, $Cl(A) \subseteq i_{\omega}(G)$. Put $U = i_{w}(G)$. Then $U \in$ w and $Cl(A) \subseteq U \subseteq c_{w}(U) = c_{w}(i_{w}(G)) \subseteq c_{w}(G) \subseteq B$.

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