Theoretical Mathematics & Applications, vol.1, no.1, 2011, 1-15
ISSN: 1792-9687 (print), 1792-9709 (online)
International Scientific Press, 2011

On contra Λ_r–continuous functions

M.J. Jeyanthi[1], S. Pious Missier[2] and P. Thangavelu[3]

Abstract

In this paper, we introduce a new class of function called contra Λr-continuous function. Some characterizations and several properties concerning contra Λr-continuity are obtained.

Mathematics Subject Classification : 54C08, 54C10

Keywords: Λ_r-open, Λ_r-continuous, contra Λ_r-continuous.

1 Introduction

Λ_r-open sets is recently introduced by the authors [6] and studied Λ_r-T_0, Λ_r-T_1 and Λ_r-T_2 spaces, Λ_r-regular spaces, Λ_r-normal spaces and variants of continuity

[1] Research Scholar, Research Department of Mathematics, Aditanar College of Arts and Science, Tiruchendur-628 216(T.N.), India, e-mail: jeyanthimanickaraj@gmail.com
[2] Post Graduate and Research Department of Mathematics, V.O.Chidambaram College, Thoothukudi-628 008(T.N.), India, e-mail: spmissier@gmail.com
[3] Department of Mathematics, Aditanar College, Tiruchendur-628216, India, e-mail:ptvelu12@gmail.com

Article Info: *Revised* : June 19, 2011. *Published online* : October 31, 2011

related to this concept in [6, 8, 7]. The purpose of the present paper is to introduce and investigate some of the fundamental properties of contra Λ_r-continuous functions and we obtain characterizations of contra Λ_r-continuous functions.

2 Preliminary Notes

Throughout the paper, (X, τ) (or simply X) will always denote a topological space. For a subset S of a topological space X, S is called regular-open [10] if S = *Int cl* S. Then the complement S^c (= X \ S) of a regular-open set S is called the regular-closed set. The family of all regular-open sets (resp. regular-closed sets) in (X, τ) will be denoted by $RO(X, \tau)$ (resp. $RC(X, \tau)$). A subset S of a topological space (X, τ) is called Λ_r-set [6] if S = $\Lambda_r(S)$, where

$$\Lambda_r(S) = \cap \{G \,/\, G \in RO(X, \tau) \text{ and } S \subseteq G\}.$$

The collection of all Λ_r-sets in (X, τ) is denoted by $\Lambda_r(X, \tau)$.

Throughout this paper, we adopt the notations and terminology of [6]. Let A be a subset of a space (X, τ). Then A is called a Λ_r-closed set if $A = S \cap C$ where S is a Λ_r-set and C is a closed set. The complement of a Λ_r-closed set is called Λ_r-open. The collection of all Λ_r-open (resp. Λ_r-closed) sets in (X, τ) is denoted by $\Lambda_rO(X, \tau)$ (resp. $\Lambda_rC(X, \tau)$). Also note that every open set is Λ_r-open; arbitrary union of Λ_r-open sets is Λ_r-open and arbitrary intersection of Λ_r-closed sets is Λ_r-closed; and intersection of two open sets is Λ_r-open.

A point $x \in X$ is called a Λ_r-cluster point of A if for every Λ_r-open set U containing x, $A \cap U \neq \emptyset$. The set of all Λ_r-cluster points of A is called the Λ_r-closure of A and it is denoted by Λ_r-*cl*(A). Then Λ_r-*cl*(A) is the intersection of Λ_r-closed sets containing A and it is the smallest Λ_r-closed set containing A. Also A is Λ_r-closed if and only if A = Λ_r-*cl*(A). The union of Λ_r-open sets contained in A is called Λ_r-interior of A and it is denoted by Λ_r-*int*(A). Before we enter into our work, we recall the following definitions.

Definition 2.1 A function f : X → Y is called

(i) contra-continuous [3], if $f^{-1}(V)$ is closed in X for each open set V of Y

(ii) Λ_r-continuous [7], if $f^{-1}(V)$ is a Λ_r-open set in X for each open set V in Y

(iii) Λ_r-irresolute [7], if $f^{-1}(V)$ is a Λ_r-open set in X for each Λ_r-open set V in Y

(iv) Λ_r^*-open [7], if the image of each Λ_r-open set in X is a Λ_r-open set in Y

(v) Λ_r^*-closed [7], if the image of each Λ_r-closed set in X is a Λ_r-closed set in Y

Definition 2.2 A topological space X is said to be

(i) Urysohn space [11], if for each pair of distinct points x and y in X, there exists two open sets U and V in X such that $x \in U$, $y \in V$ and $cl(U) \cap cl(V) = Ø$.

(ii) ultra normal [9], if each pair of nonempty disjoint closed sets can be separated by disjoint closed sets.

3 Contra Λ_r-continuous function

In this section, we introduce contra Λ_r-continuous functions, contra Λ_r-irresolute functions and perfectly contra Λ_r-irresolute functions and study their properties.

Definition 3.1 A function $f : (X, \tau) \to (Y, \sigma)$ is called contra Λ_r-continuous, if $f^{-1}(V)$ is Λ_r-closed in X for each open set V in Y.

Theorem 3.2 *For a function $f : (X, \tau) \to (Y, \sigma)$, the following are equivalent:*

(a) *f is contra Λ_r-continuous*

(b) *For every closed subset F of Y, $f^{-1}(F)$ is Λ_r-open in X*

(c) *For each $x \in X$ and each closed subset F of Y with $f(x) \in F$, there exists a Λ_r-open set U of X with $x \in U$, $f(U) \subseteq F$*

Proof. (a) ↔ (b) Obvious.

(b) → (c) Let F be any closed subset of Y and let $f(x) \in F$ where $x \in X$. Then by (b), $f^{-1}(F)$ is Λ_r-open in X. Also $x \in f^{-1}(F)$. Take $U = f^{-1}(F)$. Then U is a Λ_r-open set containing x and $f(U) \subseteq F$.

(c) → (b) Let F be any closed subset of Y. If $x \in f^{-1}(F)$, then $f(x) \in F$. Hence by (c), there exists a Λ_r-open set U_x of X with $x \in U_x$ such that $f(U_x) \subseteq F$. Then

$$f^{-1}(F) = \cup \{U_x : x \in f^{-1}(F)\},$$

and hence $f^{-1}(F)$ is Λ_r-open in X. □

Lemma 3.3 [1] *The following properties hold for subsets A, B of a space X:*

(a) *$x \in ker(A)$ if and only if $A \cap F \neq Ø$ for any $F \in C(X,x)$*

(b) *$A \subseteq ker(A)$ and $A = ker(A)$ if A is open in X*

(c) *If $A \subseteq B$, then $ker(A) \subseteq ker(B)$.*

Theorem 3.4 *Let $f : X \to Y$ be a bijective function. Then the following are equivalent:*

(a) *f is contra Λ_r-continuous*

(b) *$f(\Lambda_r\text{-}cl(A)) \subseteq ker(f(A))$ for every subset A of X*

(c) *$\Lambda_r\text{-}cl(f^{-1}(B)) \subseteq f^{-1}(ker(B))$ for every subset B of Y*

Proof. (a) → (b) Let A be any subset of X. Suppose $y \notin ker(f(A))$. By Lemma 3.3(a), there exists $F \in C(Y, f(x))$ such that $f(A) \cap F = Ø$. Then $A \cap f^{-1}(F) = Ø$. Since $f^{-1}(F)$ is Λ_r-open by (a), $\Lambda_r\text{-}cl(A) \cap f^{-1}(F) = Ø$. That implies $f(\Lambda_r\text{-}cl(A)) \cap F = Ø$ and so $y \notin f(\Lambda_r\text{-}cl(A))$. This shows that

$$f(\Lambda_r\text{-}cl(A)) \subseteq ker(f(A)).$$

(b) → (c) Let B be any subset of Y. Then by (b),

$$f(\Lambda_r\text{-}cl(f^{-1}(B))) \subseteq ker\, f(f^{-1}(B)) = ker\, B.$$

Therefore, $\Lambda_r\text{-}cl(f^{-1}(B)) \subseteq f^{-1}(ker\, B)$.

(c) → (a) Let V be open in Y. Then $\Lambda_r\text{-}cl(f^{-1}(V)) \subseteq f^{-1}(ker\, V) = f^{-1}(V)$ by (c) and Lemma 3.3(b). But $f^{-1}(V) \subseteq \Lambda_r\text{-}cl(f^{-1}(V))$. So $f^{-1}(V) = \Lambda_r\text{-}cl(f^{-1}(V))$. This means that $f^{-1}(V)$ is Λ_r-closed in X so that f is contra Λ_r-continuous. □

Remark 3.5 The Examples 3.6 and 3.7 show that the concepts of Λ_r-continuity and contra Λ_r-continuity are independent of each other.

Example 3.6 Let $X = \{a,b,c\}$, $Y = \{a,b,c,d\}$, $\tau = \{X, \emptyset, \{c\}, \{a,c\}, \{b,c\}\}$ and $\sigma = \{Y, \emptyset, \{a\}, \{b,c\}, \{a,b,c\}\}$. Then $\Lambda_r O(X,\tau) = \tau$ and

$$\Lambda_r C(X,\tau) = \{X, \emptyset, \{a\}, \{b\}, \{a,b\}\}.$$

Define a function

$$f: (X,\tau) \rightarrow (Y,\sigma) \quad \text{by} \quad f(a) = d, \quad f(b) = b \quad \text{and} \quad f(c) = c.$$

Then f is Λ_r-continuous. But f is not contra Λ_r-continuous since $\{b,c\}$ is open in (Y,σ) but $f^{-1}(\{b,c\}) = \{b,c\}$ is not Λ_r-closed in (X,τ).

Example 3.7 Let $X = Y = \{a,b,c,d\}$, $\tau = \{X, \emptyset, \{b,d\}, \{b,c,d\}, \{a,b,d\}\}$ and $\sigma = \{Y, \emptyset, \{a\}, \{b\}, \{a,b\}\}$. Then $\Lambda_r O(X,\tau) = \tau$ and

$$\Lambda_r C(X,\tau) = \{X, \emptyset, \{a\}, \{c\}, \{a,c\}\}.$$

Define a function

$$f: (X,\tau) \rightarrow (Y,\sigma) \quad \text{by } f(a) = a, \quad f(b) = c, \quad f(c) = b \quad \text{and} \quad f(d) = d.$$

Then f is contra Λ_r-continuous. But f is not Λ_r-continuous since $\{a\}$ is open in (Y,σ) but $f^{-1}(\{a\}) = \{a\}$ is not Λ_r-open in (X,τ).

Theorem 3.8 *If a function $f: (X,\tau) \rightarrow (Y,\sigma)$ is contra Λ_r-continuous and Y is regular, then f is Λ_r-continuous.*

Proof. Let $x \in X$ and V be an open set in Y with $f(x) \in V$. Since Y is regular, there exists an open set W in Y such that $f(x) \in W$ and $cl(W) \subseteq V$. Since f is contra Λ_r-continuous and cl(W) is a closed subset of Y with $f(x) \in cl(W)$, by Theorem 3.2 there exists a Λ_r-open set U of X with $x \in U$ such that $f(U) \subseteq cl(W)$. That is, $f(U) \subseteq V$. By Theorem 3.4 of [7], f is Λ_r-continuous. □

Recall that a topological space (X,τ) is said to be Λ_r-normal [8] if for every pair of disjoint closed sets A and B of X, there exists Λ_r-open sets U and V in X such that $A \subseteq U$, $B \subseteq V$ and $U \cap V = \emptyset$.

Theorem 3.9 *If $f : (X, \tau) \to (Y, \sigma)$ is closed, injective and contra Λ_r-continuous and Y is ultra normal, then X is Λ_r-normal.*

Proof. Let A and B be disjoint closed subsets of X. Since f is closed and injective, f(A) and f(B) are disjoint closed subsets of Y. Since Y is ultra normal, there exists two clopen sets U and V in Y such that $f(A) \subseteq U$, $f(B) \subseteq V$ and $U \cap V = \emptyset$. Since f is contra Λ_r-continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are Λ_r-open sets in (X, τ).
Also $A \subseteq f^{-1}(U)$, $B \subseteq f^{-1}(V)$ and $f^{-1}(U) \cap f^{-1}(V) = \emptyset$. This shows that X is Λ_r-normal. □

Recall that a space (X, τ) is Λ_r-T_2 [6] if for each pair of distinct points x and y in X, there exists a Λ_r-open set U and a Λ_r-open set V in X such that $x \in U$, $y \in V$ and $U \cap V = \emptyset$.

Theorem 3.10 *If a function $f : (X, \tau) \to (Y, \sigma)$ is injective, contra Λ_r-continuous and Y is a Urysohn space, then X is Λ_r-T_2.*

Proof. Let $x, y \in X$ with $x \neq y$. Since f is injective, $f(x) \neq f(y)$. Since Y is a Urysohn space, there exists open sets U and V in Y such that $f(x) \in U$, $f(y) \in V$ and $cl(U) \cap cl(V) = \emptyset$. Since f is contra Λ_r-continuous, by Theorem 3.2 there exists Λ_r-open sets A and B in X such that $x \in A$, $y \in B$ and $f(A) \subseteq cl(U)$, $f(B) \subseteq cl(V)$. Then $f(A) \cap f(B) = \emptyset$ and so $f(A \cap B) = \emptyset$. This implies that $A \cap B = \emptyset$ and hence X is Λ_r-T_2. □

Remark 3.11 Every contra-continuous function is contra Λ_r-continuous since every closed set is Λ_r-closed. But the converse need not be true.
For example, let X = Y = {a,b,c,d}, τ = {X, Ø, {a}, {b}, {a,b}} and
σ = {Y, Ø, {a}, {a,b}, {a,c}, {a,b,c}}. Then the closed sets of (X, τ) are X, Ø, {b,c,d}, {a,c,d}, {c,d} and Λ_r-closed sets of (X, τ) are X, Ø, {b,c,d}, {a,c,d}, {c,d}, {a},{b}. Define a function

$f : (X, \tau) \rightarrow (Y, \sigma)$ by f(a) = d, f(b) = a, f(c) = c and f(d) = c.

Then f is contra Λ_r-continuous but not contra-continuous since {a} is open in (Y, σ) but $f^{-1}(\{a\}) = \{b\}$ is not closed in (X, τ).

Definition 3.12 A topological space X is said to be Λ_r-connected, if X cannot be written as a disjoint union of two nonempty Λ_r-open sets.

A subset B of a topological space X is Λ_r-connected, if B is Λ_r-connected as a subspace of X.

Theorem 3.13 *For a topological space X, the following are equivalent:*

(i) *X is Λ_r-connected*

(ii) *The only subsets of X which are both Λ_r-open and Λ_r-closed are the sets X and Ø*

(iii) *Each Λ_r-continuous function of X into a discrete space Y with atleast two points is a constant function*

Proof. (i) → (ii) Let U be a both Λ_r-open and Λ_r-closed subset of X. Then X \ U is both Λ_r-open and Λ_r-closed. Since X is Λ_r-connected and X is the disjoint union of Λ_r-open sets U and X \ U, one of these must be empty.

Hence either U = Ø or U = X.

(ii) → (i) Suppose that X is not Λ_r-connected. Then $X = A \cup B$ where A and B are nonempty Λ_r-open sets such that $A \cap B$ = Ø. Since B = X \ A is Λ_r-open, A is both Λ_r-open and Λ_r-closed. By (ii), A = Ø or X. That is, either A = Ø or B = Ø , which is a contradiction. Therefore X is Λ_r-connected.

(ii) → (iii) Let $f : X \rightarrow Y$ be a Λ_r-continuous function from a topological space X into a discrete topological space Y. Then for each $y \in Y$, {y} is both open and closed in Y. Since f is Λ_r-continuous, $f^{-1}(y)$ is both Λ_r-open and Λ_r-closed in X. Hence X is covered by Λ_r-open and Λ_r-closed covering $\{f^{-1}(y) : y \in Y\}$.

By (ii), $f^{-1}(y)$ = Ø or X for each $y \in Y$. If $f^{-1}(y)$ = Ø for each $y \in Y$, then f fails to be a map. Hence there exists only one point $y \in Y$ such that $f^{-1}(y) = X$, which shows that f is a constant function.

(iii) → (ii) Let U be both Λ_r-open and Λ_r-closed in X. Suppose U $\neq$ Ø. Let f : X → Y be a Λ_r-continuous function from a topological space X into a discrete topological space Y defined by f (U) = {y} and f (X \ U) = {w}, where y,w$\in$Y and y $\neq$ w. By (iii), f is constant so that U = X. □

Theorem 3.14 *Let (X, τ) be a Λ_r-connected space and (Y, σ) be any topological space. If f : X → Y is surjective and contra Λ_r-continuous, then Y is not a discrete space.*

Proof. If possible, let Y be a discrete space. Let A be any proper nonempty subset of Y. Then A is both open and closed in (Y, σ). Since f is contra Λ_r-continuous, f^{-1}(A) is Λ_r-closed and Λ_r-open in (X, τ). Since X is Λ_r-connected, by Theorem 3.13, the only subsets of X which are both Λ_r-open and Λ_r-closed are the sets X and Ø . Hence f^{-1}(A) is either X or Ø. If f^{-1}(A) = Ø , then it contradicts to the fact that A$\neq$Ø and f is surjective. If f^{-1}(A) = X, then f fails to be a map. Hence Y is not a discrete space. □

Theorem 3.15 *If f : (X, τ) → (Y, σ) is surjective, contra Λ_r-continuous and X is Λ_r-connected, then Y is connected.*

Proof. Assume that Y is not connected. Then Y = A$\cup$B where A and B are nonempty open sets in Y such that A$\cap$B = Ø . Set U = Y \ A and V = Y \ B. Then U and V are nonempty closed sets in Y. Since f is surjective and contra Λ_r-continuous, f^{-1}(U) and f^{-1}(V) are nonempty Λ_r-open sets in (X, τ).

Now, f^{-1}(U) $\cap$ f^{-1}(V) = Ø and f^{-1}(U) $\cup$ f^{-1}(V) = X. This contradicts to the fact that X is Λ_r-connected and so Y is connected. □

Theorem 3.16 *A space X is Λ_r-connected if every contra Λ_r-continuous function from a space X into any T_0-space Y is constant.*

Proof. Suppose that X is not Λ_r-connected and every contra Λ_r-continuous function from X into a T_0-space Y is constant. Since X is not Λ_r-connected, by Theorem 3.13, there exists a proper nonempty subset A of X such that A is both

Λ_r-open and Λ_r-closed. Let Y = {a,b} and σ = {Y,Ø,{a},{b}} be a topology for Y. Let f : X → Y be a function such that f(A) = {a} and f(X \ A) = {b}. Then f is non constant and contra Λ_r-continuous such that Y is T_0, which is a contradiction. This shows that X must be Λ_r-connected. □

Theorem 3.17 *If f : (X, τ) → (Y, σ) is contra Λ_r-continuous and g : (Y, σ) → (Z, γ) is continuous, then $g\circ f$: (X, τ) → (Z, γ) is contra Λ_r-continuous.*

Proof. It directly follows from the definitions. □

Theorem 3.18 *Let f : (X, τ) → (Y, σ) be surjective, Λ_r-irresolute and Λ_r^*-open and g : (Y, σ) → (Z, γ) be any function. Then $g\circ f$ is contra Λ_r-continuous if and only if g is contra Λ_r-continuous.*

Proof. Suppose $g\circ f$ is contra Λ_r-continuous. Let F be any closed set in (Z, γ). Then $(g\circ f)^{-1}(F) = f^{-1}(g^{-1}(F))$ is Λ_r-open in (X, τ). Since f is Λ_r^*-open and surjective, $f(f^{-1}(g^{-1}(F))) = g^{-1}(F)$ is Λ_r-open in (Y, σ) and we obtain that g is contra Λ_r-continuous.

For the converse, suppose g is contra Λ_r-continuous. Let V be closed in (Z, γ). Then $g^{-1}(V)$ is Λ_r-open in (Y, σ). Since f is Λ_r-irresolute, $f^{-1}(g^{-1}(V)) = (g\circ f)^{-1}(V)$ is Λ_r-open in (X, τ) and so $g\circ f$ is contra Λ_r-continuous. □

Theorem 3.19 *Let f : X → Y be a function and g : X → X× Y the graph function of f, defined by g(x) = (x, f(x)) for every $x\in X$. If g is contra Λ_r-continuous, then f is contra Λ_r-continuous.*

Proof. Let U be an open set in Y. Then X×U is open in X×Y. Since g is contra Λ_r-continuous, $g^{-1}(X\times U) = f^{-1}(U)$ is Λ_r-closed in X. This shows that f is contra Λ_r-continuous. □

Theorem 3.20 *If f : X → Y is contra-continuous, g : X → Y is contra-continuous and Y is Urysohn, then E = {$x\in X$: f(x) = g(x)} is Λ_r-closed in X.*

Proof. Let $x \in X \setminus E$. Then $f(x) \neq g(x)$. Since Y is Urysohn, there exists open sets V and W in Y such that $f(x) \in V$, $g(x) \in W$ and $cl(V) \cap cl(W) = \emptyset$. Since f is contra-continuous, $f^{-1}(cl(V))$ is open in X. Since g is contra-continuous, $g^{-1}(cl(W))$ is open in X. Let $G = f^{-1}(cl(V))$ and $H = g^{-1}(cl(W))$ and set $A = G \cap H$. Then A is a Λ_r-open set containing x in X. Now,
$f(A) \cap g(A) \subseteq f(G) \cap g(H) \subseteq cl(V) \cap cl(W) = \emptyset$. This implies that $A \cap E = \emptyset$ where A is Λ_r-open. So x is not a Λ_r-cluster point of E. Hence $x \notin \Lambda_r\text{-}cl(E)$ and this completes the proof. □

Definition 3.21 A subset A of a topological space X is said to be Λ_r-dense in X if $\Lambda_r\text{-}cl(A) = X$.

Theorem 3.22 *Let $f : X \to Y$ be a contra-continuous function and $g : X \to Y$ be a contra-continuous function. If Y is Urysohn and $f = g$ on a Λ_r-dense set $A \subseteq X$, then $f = g$ on X.*

Proof. Let $E = \{x \in X : f(x) = g(x)\}$. Since f is contra-continuous, g is contra-continuous and Y is Urysohn, by Theorem 3.20, E is Λ_r-closed in X. By assumption, we have f = g on A where A is Λ_r-dense in X. Since $A \subseteq E$, A is Λ_r-dense and E is Λ_r-closed, we have

$$X = \Lambda_r\text{-}cl(A) \subseteq \Lambda_r\text{-}cl(E) = E.$$

Hence f = g on X. □

Definition 3.23 A space (X, τ) is said to be

(i) Λ_r-space, if every Λ_r-open set is open in X

(ii) locally Λ_r-indiscrete, if every Λ_r-open set is closed in X.

Theorem 3.24 *Let $f : X \to Y$ be a contra Λ_r-continuous function. Then*

(i) *f is contra-continuous, if X is a Λ_r-space*

(ii) *f is continuous, if X is locally Λ_r-indiscrete*

Proof. (i) and (ii) are directly follows from the definitions. □

Theorem 3.25 *Let $f : X \to Y$ be surjective, closed and contra Λ_r-continuous. If X is Λ_r-space, then Y is locally indiscrete.*

Proof. Let V be open in Y. Since f is contra Λ_r-continuous, $f^{-1}(V)$ is Λ_r-closed in X and hence closed in X since X is Λ_r-space. Since f is closed and surjective, $f(f^{-1}(V)) = V$ is closed in Y and so Y is locally indiscrete. □

Recall that a function $f : X \to Y$ is said to be contra λ-continuous [2] (resp., contra α-continuous [5], contra-precontinuous [4]), if $f^{-1}(V)$ is λ-closed (resp., α-closed, pre-closed) in X for each open set of Y.

Remark 3.26 Since every Λ_r-closed set is λ-closed, every contra Λ_r-continuous function is contra λ-continuous. But the converse need not be true which is shown by the following example.

Let X = Y = {a,b,c}, τ = {X, Ø, {a}, {b}, {a,b}} and σ = {X, Ø, {a}}. Then the function $f : (X, \tau) \to (Y, \sigma)$ defined by f(a) = a, f(b) = a and f(c) = c is contra λ-continuous but not contra Λ_r-continuous.

The following examples show that contra Λ_r-continuous and contra-precontinuous functions (resp., contra-α-continuous) are independent notions.

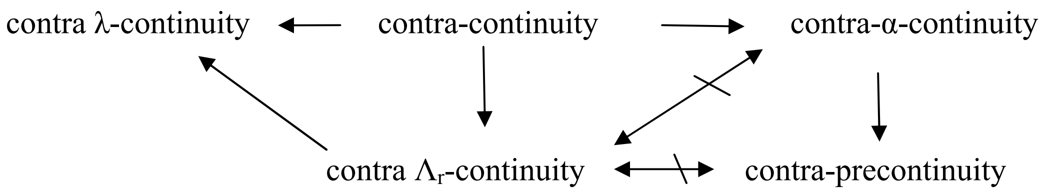
The function which is defined in Remark 3.11 is contra Λ_r-continuous but not contra-precontinuous and not contra-α-continuous.

Let X = Y = {a,b,c}, τ = {X,Ø,{a}} and σ = {Y, Ø, {a}, {b}, {a,b}}. Then $\Lambda_r O(X, \tau) = \tau$, $PO(X, \tau)$ = {X, Ø, {a}, {a,b}, {a,c}} and $\alpha(X, \tau)$ = {X, Ø, {a}, {a,b}, {a,c}}.

Define a function

$$f : (X, \tau) \to (Y, \sigma) \quad \text{by} \quad f(a) = c, \quad f(b) = b \quad \text{and} \quad f(c) = a.$$

Then f is contra-pre continuous and contra-α-continuous but not contra Λ_r-continuous.



In this diagram,

" A ⟶ B " means A implies B but not conversely

" A ↮ B " means A and B are independent of each other.

Definition 3.27 A function $f : (X, \tau) \to (Y, \sigma)$ is called contra Λ_r-irresolute, if $f^{-1}(V)$ is Λ_r-closed in (X, τ) for each Λ_r-open set V in (Y, σ).

Remark 3.28 The following examples show that the concepts of Λ_r-irresolute and contra Λ_r-irresolute are independent of each other.

Example 3.29 Let X = {a,b,c,d}, Y = {a,b,c,d,e}, τ = {X, Ø, {a}, {a,c}, {a,b,d}} and σ = {Y, Ø, {a}, {a,b}, {a,b,e}, {a,c,d}, {a,b,c,d}}. Then $\Lambda_r O(X, \tau) = \tau$, $\Lambda_r C(X, \tau)$ = {X, Ø, {c}, {b,d}, {b,c,d}} and $\Lambda_r O(Y, \sigma) = \sigma$.

Define a function

$$f : (X, \tau) \to (Y, \sigma) \text{ by } f(a) = a,\ f(b) = e,\ f(c) = c \text{ and } f(d) = e.$$

Then f is Λ_r-irresolute but not contra Λ_r-irresolute since {a} is open in (Y, σ), but $f^{-1}(\{a\}) = \{a\}$ is not Λ_r-closed in (X, τ).

Example 3.30 Let X = Y = {a,b,c,d}, τ = {X, Ø, {a}, {b,c}, {a,b,c}} and

σ = {Y, Ø, {a}, {a,b}, {a,c}, {a,d}, {a,b,c}, {a,b,d}, {a,c,d}}.

Then $\Lambda_r O(X, \tau)$ = {X, Ø, {a}, {b,c}, {a,b,c}, {b,c,d}, {a,d}},

$\Lambda_r C(X, \tau)$ = {X, Ø, {a}, {d}, {a,d}, {b,c}, {b,c,d}} and $\Lambda_r O(Y, \sigma) = \sigma$.

Define a function

$$f : (X, \tau) \to (Y, \sigma) \text{ by } f(a) = f(b) = f(c) = d \text{ and } f(d) = a.$$

Then f is contra Λ_r-irresolute but not Λ_r-irresolute since {a} is open in (Y,σ), but $f^{-1}(\{a\}) = \{d\}$ is not Λ_r-open in (X,τ).

Remark 3.31 Every contra Λ_r-irresolute function is contra Λ_r-continuous. But the converse need not be true as shown by the following example.
In Example 3.7, f is contra Λ_r-continuous but not contra Λ_r-irresolute.

Theorem 3.32 *A function $f : (X,\tau) \to (Y,\sigma)$ is contra Λ_r-irresolute if and only if $f^{-1}(V)$ is Λ_r-open in X for each Λ_r-closed set V in Y.*

Proof. Obvious. □

Theorem 3.33 *Let $f : (X,\tau) \to (Y,\sigma)$ and $g : (Y,\sigma) \to (Z,\gamma)$ be two functions. Then*

(a) *if g is Λ_r-irresolute and f is contra Λ_r-irresolute, then $g \circ f$ is contra Λ_r-irresolute*

(b) *if g is contra Λ_r-irresolute and f is Λ_r-irresolute, then $g \circ f$ is contra Λ_r-irresolute*

Proof. (a) Let V be Λ_r-open in Z. Since g is Λ_r-irresolute, $g^{-1}(V)$ is Λ_r-open in Y. Since f is contra Λ_r-irresolute, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is Λ_r-closed in X. This means that $g \circ f$ is contra Λ_r-irresolute. (b) is similar to (a). □

Theorem 3.34 *If $f : (X,\tau) \to (Y,\sigma)$ is contra Λ_r-irresolute and $g : (Y,\sigma) \to (Z,\gamma)$ is Λ_r-continuous, then $g \circ f$ is contra Λ_r-continuous.*

Proof. It directly follows from the definitions. □

Recall that a subset A of a topological space (X,τ) is called Λ_r-clopen [8] if A is both Λ_r-open and Λ_r-closed in X. The collection of all Λ_r-clopen sets in (X,τ) is denoted by $\Lambda_r CO(X,\tau)$.

Definition 3.35 A function f : $(X, \tau) \to (Y, \sigma)$ is called perfectly contra Λ_r-irresolute if $f^{-1}(V)$ is Λ_r-clopen in X for each Λ_r-open set V in Y.

Remark 3.36 Every perfectly contra Λ_r-irresolute function is contra Λ_r-irresolute and Λ_r-irresolute. The following two examples show that a contra Λ_r-irresolute function may not be perfectly contra Λ_r-irresolute, and a Λ_r-irresolute function may not be perfectly contra Λ_r-irresolute.

In Example 3.30, f is contra Λ_r-irresolute but not perfectly contra Λ_r-irresolute.

In Example 3.29, f is Λ_r-irresolute but not perfectly contra Λ_r-irresolute.

Theorem 3.37 *A function f : $(X, \tau) \to (Y, \sigma)$ is perfectly contra Λ_r-irresolute if and only if f is contra Λ_r-irresolute and Λ_r-irresolute.*

Proof. It directly follows from the definitions. □

We have the following relation for the functions defined above:

perfectly contra Λ_r-irresolute

↙ ↘

Λ_r-irresolute $\not\leftrightarrow$ contra Λ_r-irresolute

↓ ↓

Λ_r-continuous $\not\leftrightarrow$ contra Λ_r-continuous

In this diagram,

“ A $\longrightarrow$ B ” means A implies B but not conversely

“ A $\not\leftrightarrow$ B ” means A and B are independent of each other

References

[1] Ahmad Al-Omari and Mohd Salmi Md Noorani, Contra-ω-continuous and Almost contra-ω-continuous, *International Journal of Mathematics and Mathematical Sciences*, **2007**, Article ID 40469.

[2] F.G. Arenas, J. Dontchev and M. Ganster, On λ-sets and dual of generalized continuity, *Questions Answers Gen. Topology*, 15, (1997), 3-13.

[3] J. Dontchev, Contra-continuous functions and strongly S-closed spaces, *Internat. J. Math. Sci*., **19**, (1996), 303-310.

[4] Erdal Ekici, Almost contra-pre continuous functions, *Bull. Malaysian Math. Sc. Soc.* (second series), **27**, (2004), 53-65.

[5] S. Jafari and T. Noiri, Contra-α-continuous functions between topological spaces, *Iranian International Journal of Science*, **2**(2), (2001), 153-167.

[6] M.J. Jeyanthi, S. Pious Missier and P. Thangavelu, Λ_r-sets and separation axioms, *Malaysian Journal of Mathematical Sciences*, **5**(1), (2011), 45-60.

[7] M.J. Jeyanthi, S. Pious Missier and P. Thangavelu, On Λ_r-open sets and functions. (Submitted).

[8] S. Pious Missier, P. Thangavelu and M.J. Jeyanthi, On Λ_r-regular and Λ_r-normal spaces, *International J. of Math. Sci. and Engg. Appls.*, **4**(V), (Dec.2010), 225-235.

[9] R. Staum, The algebra of bounded continuous functions into a non-archimedean field, *Pacific J. Math.*, **50**, (1974), 169-185.

[10] M.H. Stone, Applications of the theory of boolean rings to the general topology, *Trans. A.M.S.*, **41**, 375-481.

[11] S. Willard, *General Topology*, Addison-Wesley, 1970.