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Numerical Investigations of the Particle Motion Equation in Superheated Stream Medium

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Abstract

This paper is to discuss some mathematical models to describe the particle acceleration in jet milling as we take the overheated stream and the air as the working medium, respectively. Furthermore, the corresponding numerical simulations are also investigated. Under the assumption of considering the heat transfer and without heat transfer, respectively, we discretize the particle motion equation and solve it by Newton iteration. These results show the trend of the velocity of a single particle in different density and different particle diameter sizes, respectively. Our results are benefit to the optimization of parameters design and system configuration in jet milling equipment.

Mathematics Subject Classification: 93A30
Keywords: Numerical investigation, Particle motion, Acceleration, Jet milling

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1 Introduction

It is well known that solid particles can break-up upon a collision. This breakage can play a significant role in many processes that utilize a large number of solid particles such as fluidized beds, pneumatic transport or granular flow. The principle of jet milling is that solids particles are accelerated by high-speed gas flow and fragmented due to multiple particle-particle collisions in interacting gas-particle jet. There are three kinds of commonly used working medium in jet milling: air, superheated steam and inert gas. Nowadays, withe the help of a lot of experiments and numerical simulations, how the different working medium has an impact on the effect of the particle acceleration is studied[3-5,7-13]. For instance, Chen *et al.*[1] thought that the energy consumption of superheated steam jet mill is far less than that of air flow jet mill. Ji *et al.*[6] analyzed the impact of the kind and the status of working medium in jet mill on the energy consumption and efficiency, and they also pointed out that, compared with air flow, the superheated stream has many advantages such as high pressure, high critical speed, high energy utilization ratio and high crushing strength. Yuan *et al.*[14] studied the flow field changes in Fluidized-bed jet mill under different working medium and derived simulation results of the velocity of air gas under different inlet pressure and different back pressure, respectively. These results show that if we take superheated steam as working medium, it has less loss of energy conversion and higher efficiency.

Although in recent years there has a great progress in the fundamental theory of jet milling, it is not enough in many ways which still need to be improved, especially on the technology of jet milling in superheated steam working medium. According to the principle of jet milling, the milling energy of particles comes from high-speed gas. Therefore, the analysis about the particle acceleration progress is a key point in the design of jet milling device. Whether the particles in the gas flow can be effectively accelerated and collided at its maximal velocity is the important condition to improve the efficiency of jet milling. Chen *et al.*[2] for the first time analyzed the particle acceleration process in the air working medium, and got the optimal acceleration distance. But on the whole, as far as the superheated steam medium is concerned, the research of particle motion process is still in the exploratory stage.

To investigate the particle motion in different working medium better, this paper discussed some mathematical models to describe particle acceleration and the numerical simulations show the law of the particle acceleration motion under overheated stream working medium, which is benefit to the optimization of parameters design and system configuration in jet milling equipment.

2 Particle motion equation

2.1 Mathematical model

We take usual assumptions[4] for one-dimensional monodispersed gas-particle: (1) the gas is in ideal; (2) a particle moves under the action of only the viscous gas-particle drag force; (3) gas-particle heat transfer is carried out by convection only; (4) a particle has a uniform temperature throughout its volume.

The motion equation for a single particle can be written as follows,

$$\frac{du_s}{dx} = 0.75C_d\frac{p}{\rho_s d_s u_s RT}(u-u_s)^2, \tag{1}$$

$$\frac{dT_s}{dx} = \frac{6\alpha(T-T_s)}{\rho_s c_s u_s d_s}, \tag{2}$$

where d_s is the particle size; p is the pressure; T and T_s are the gas and particle temperatures, respectively; u and u_s are the gas and particle velocities, respectively; α is the coefficient of particle-gas heat transfer; ρ_s is the particle density; $R = 8.314Mpa \cdot L/mol \cdot K$ is the gas constant; C_d is the particle drag force coefficient and its general expression can be written as,

(1) Stokes formula ($Re_s < 1$)

$$C_d = \frac{24}{Re_s}, \tag{3}$$

(2) Allen formula ($1 < Re_s < 1000$)

$$C_d = \frac{30}{Re_s^{0.625}}, \tag{4}$$

(3) Newton formula ($Re_s > 1000$)

$$C_d = 0.44, \tag{5}$$

where $Re_s = \dfrac{\rho_s d_s |u - u_s|}{\mu}$ is the Reynolds number and $\mu = 1.34 \times 10^{-5} N/m^2 \cdot s$ is the coefficient of gas dynamic viscosity.

Depending on the injecting high-speed gas flow, most particles can be accelerated in axis velocity area, but the existing problem is that some particles are accelerated at the edges of area (see Figure 1). Thus, except for the axis velocity of gas, we should also consider the mean velocity of quality of gas. Only in this way, can we scientifically evaluate the acceleration effect.

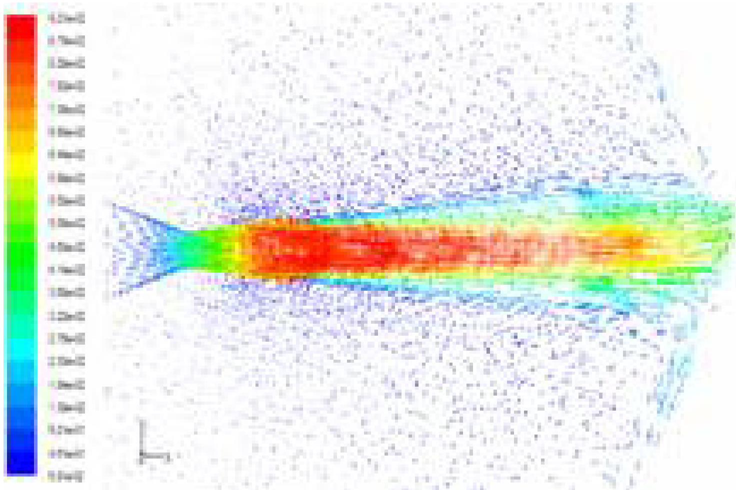


Figure 1: The velocity vectors of gas in fluidized-bed jet milling.

Assume that the axis velocity of gas changes in accordance with the jet. In initial part, that is, $\bar{x} \leq 4.4$, define the axis velocity of gas as

$$u(\bar{x}) = u, \tag{6}$$

and the mean velocity of quality of gas as

$$u(\bar{x}) = \frac{u}{1 + 1.52a\bar{x} + 5.28a^2\bar{x}^2}, \tag{7}$$

respectively, where $\bar{x}$ is the dimensionless distance, which equals to the ratio of the distance of jet stream section to the nozzle outlet and the diameter of the nozzle outlet, $u(\bar{x})$ is the velocity of gas in the dimensionless distance and $\alpha = 0.076$ is the coefficient of turbulence.

In main part, define the axis velocity of gas as

$$u(\bar{x}) = \frac{6.3u}{\bar{x} + 1.93}, \tag{8}$$

and the mean velocity of quality of gas as

$$u(\bar{x}) = \frac{3u}{\bar{x} + 1.93}, \tag{9}$$

respectively.

2.2 Simplified model and its simulations

Since there has no heat transfer under normal pressure and normal temperature, we take $\alpha = 0$ in equation (2). Then the equation (1) for the motion of a particle can be simplified as,

$$u_s \frac{du_s}{d\bar{x}} = \frac{0.75 C_d \rho d_e}{\rho_s d_s}(u(\bar{x}) - u_s)^2, \tag{10}$$

where d_e is the diameter size of the nozzle outlet.

2.2.1 Influence of the axis velocity on particle acceleration

Using air as the working medium, and by FLUENT software, we get the testing data of gas flow field in the 30 times dimensionless distance via solving on the standard $K - \varepsilon$ equation, that is, array $[\bar{x}, y]$, where $\bar{x}$ is the dimensionless distance, y is the parameter group including gas velocity, gas density and viscosity.

Next, let $d_e = 16mm$. We take $\rho_s = 2650kg/m^3$ and $\rho_s = 7950kg/m^3$, respectively. We also take $d_s = 50\mu m$,$150\mu m$, $250\mu m$, $500\mu m$ and $750\mu m$, respectively, and employ equation (6) and (8) to compute the velocity of gas. After discretization of equation (10) and by Newton iteration, we get the relationship between the particle acceleration and the dimensionless distance (see Figure 2).

It is clearly observed from Figure 2 that, when the curve of particle acceleration crosses the curve of the velocity of gas, particle reaches its maximal velocity, and the corresponding dimensionless distance is the optimal distance for nozzle jet.

In general, the smaller the particle diameter size is, the easier it is to be accelerated. When the particle density is low, the smaller the particle diameter size is, the more obvious the acceleration of velocity is; but if the density is

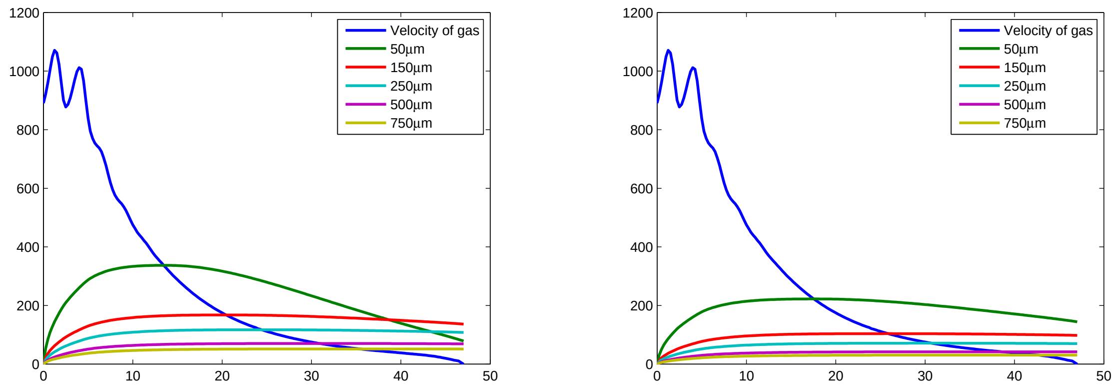


Figure 2: Left: the velocity of particle and the axis velocity of gas for $\rho_s = 2650kg/m^3$ without heat transfer; right: the velocity of particle and the axis velocity of gas for $\rho_s = 7950kg/m^3$ without heat transfer.

high, the speed change will not be obvious no matter the size of the particle is big or small.

For $\rho_s = 2650kg/m^3$, when particle diameter is $d_s = 50\mu m$, the $0-10$ times dimensionless distance is the particle's rapidly accelerating phase, the $10-20$ times dimensionless distance is its slow rise and slow reduction phase, and the $20-30$ times dimensionless distance is particle velocity's sharp decline phase. At 14 times dimensionless distance, particle reaches its maximal velocity of $378m/s$.

2.2.2 Influence of the mean velocity of quality on particle acceleration

Parameters ρ_s, d_s, d_e and C_d are taken as in section 2.2.1. Here we employ equation (7) and (9) to compute the velocity of gas. By the same computation method, we get the relationship between the particle acceleration and the dimensionless distance (see Figure 3).

From the above figures, we can see that the quality mean velocity of gas flow decays faster than the axis velocity of gas. In $0-25$ times dimensionless distance, the difference between the quality mean velocity and axis velocity increases along with the increase of dimensionless distance, because the gas flow is more and more divergent along with the increase of the dimensionless

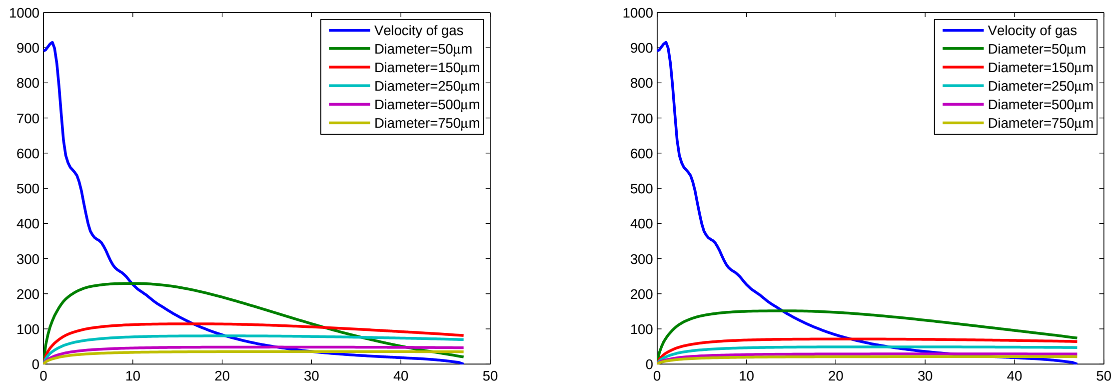


Figure 3: Left: the velocity of particle and the mean velocity of quality of gas for $\rho_s = 2650kg/m^3$ without heat transfer; right: the velocity of particle and the mean velocity of quality of gas for $\rho_s = 7950kg/m^3$ without heat transfer.

distance.

In $25 - 30$ times, the situation conversed. The main reason is that when gas collision happens in the milling center, the anti-shock wave will be formed, and then the axis velocity of gas will rapidly decay.

The trend of the particle velocity gets something similar to the above mentioned result, but the concrete numerical result is slightly different. For $\rho_s = 2650kg/m^3$ when particle diameter is $d_s = 50\mu m$, the $0 - 8$ times dimensionless distance is the particle's rapidly accelerating phase, the $8 - 17$ times dimensionless distance is its slow rise and slow reduction phase, and the $17 - 30$ times dimensionless distance is particle velocity's sharp decline phase. At 10 times dimensionless distance, particle reaches its maximal velocity of $223m/s$.

2.3 Numerical simulation of the axis velocity on particle acceleration

Using overheated stream as the working medium, we consider the numerical simulations of particle motion in equation (1) and (2). Let $\alpha = 0.23W/m^2 \cdot K$ and $c = 0.92J/kg \cdot K$.

(1) Take $p = 0.4Mpa, T = 453K$ and other parameters are taken as in

2.2.1.

We get the relationship between the particle acceleration and the dimensionless distance (see the left part of Figure 4 and Figure 5) and the relationship between the temperature of particle and the dimensionless distance, respectively (see the right part of Figure 4 and Figure 5).

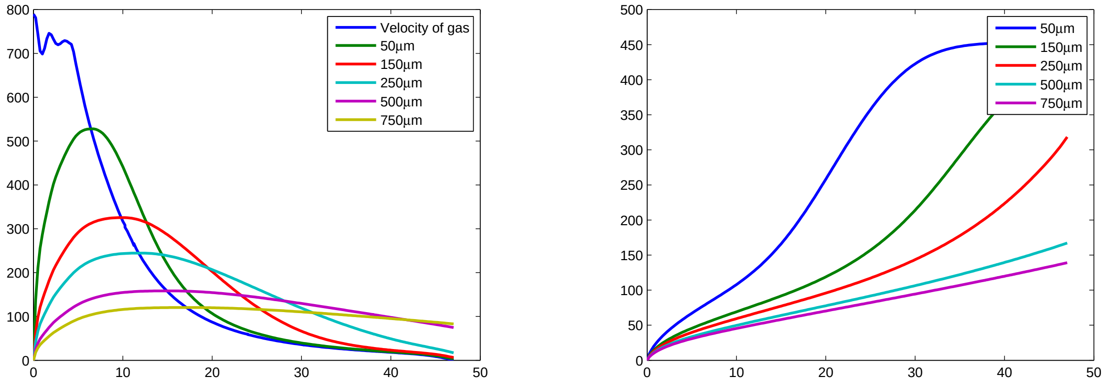


Figure 4: Left: the velocity of particle and the axis velocity of gas for $\rho_s = 2650kg/m^3, p = 0.4Mpa$ and $T = 453K$ with heat transfer; right: the temperature of particle for $\rho_s = 2650kg/m^3, p = 0.4Mpa$ and $T = 453K$ with heat transfer.

It can be seen that,

(A_1) Similarly to the result in section 2.2.1, the smaller the particle diameter size is, the easier it is to be accelerated;

(A_2) The smaller the particle diameter size is, the more obvious the temperature change is and it presents nonlinear increasing trend. But for large particle, the trend of temperature change is linear approximately;

(A_3) Compared to the results in air flow, the effect of the particle acceleration in overheated stream is far better. For $\rho_s = 2650kg/m^3$ when particle diameter is $d_s = 50\mu m$, the $0-6$ times dimensionless distance is the particle's rapidly accelerating phase, and the $6-20$ times dimensionless distance is particle velocity's sharp decline phase. At 6 times dimensionless distance, particle reaches its maximal velocity of $522m/s$ which is the 1.4 times to the maximal one in air flow.

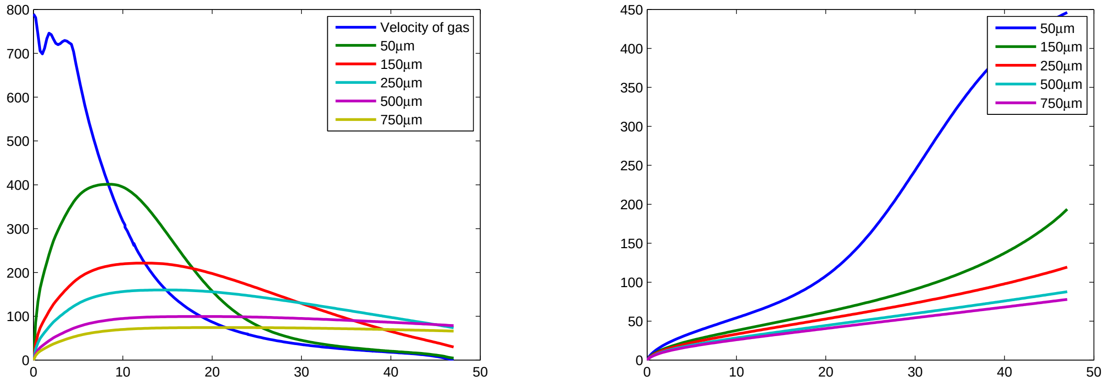


Figure 5: Left: the velocity of particle and the axis velocity of gas for $\rho_s = 7950kg/m^3, p = 0.4Mpa$ and $T = 453K$ with heat transfer; right: the temperature of particle for $\rho_s = 7950kg/m^3, p = 0.4Mpa$ and $T = 453K$ with heat transfer.

(2) Take $p = 0.5Mpa, T = 473K$.

To compare the influence of different parameters on particle acceleration, here we choose another set of pressure and temperature, and get the particle velocity and the temperature change curve(see Figure 6 and Figure 7).

Obviously, particle velocity and temperature changes are consistent with the above results, but the maximal speed of particles increased. For example, for $\rho_s = 2650kg/m^3$ when particle diameter is $d_s = 50\mu m$, its maximal velocity of particle is from $522m/s$ to $575m/s$.

2.4 Numerical simulation of the mean velocity of quality on particle acceleration

Similarly to the case in section 2.2.2, we can obtain some new numerical results.

(1) $p = 0.4Mpa, T = 453K$. The numerical results are as follows(see Figure 8 and Figure 9)

Compared to the result in section 2.3, the quality mean velocity of gas flow decays faster than the axis velocity of gas and the reason can be seen in section 2.2.2. Particle velocity and temperature changes are also similar to the results in section 2.3, but the concrete numerical values change. For $\rho_s = 2650kg/m^3$

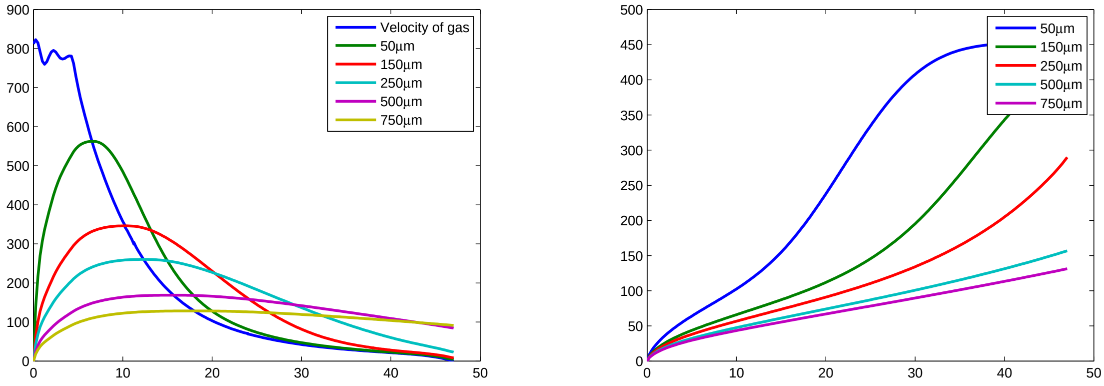


Figure 6: Left: the velocity of particle and the axis velocity of gas for $\rho_s = 2650kg/m^3, p = 0.5Mpa$ and $T = 473K$ with heat transfer; right: the temperature of particle for $\rho_s = 2650kg/m^3, p = 0.5Mpa$ and $T = 473K$ with heat transfer.

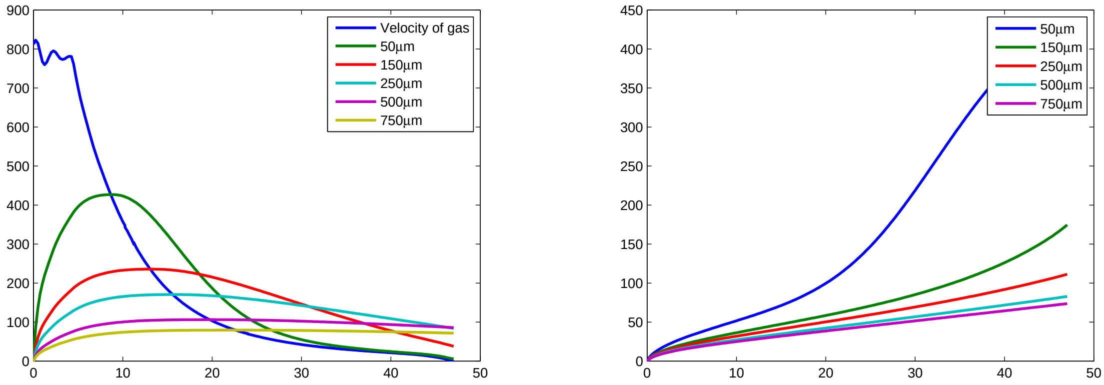


Figure 7: Left: the velocity of particle and the axis velocity of gas for $\rho_s = 7950kg/m^3, p = 0.5Mpa$ and $T = 473K$ with heat transfer; right: the temperature of particle for $\rho_s = 7950kg/m^3, p = 0.5Mpa$ and $T = 473K$ with heat transfer.

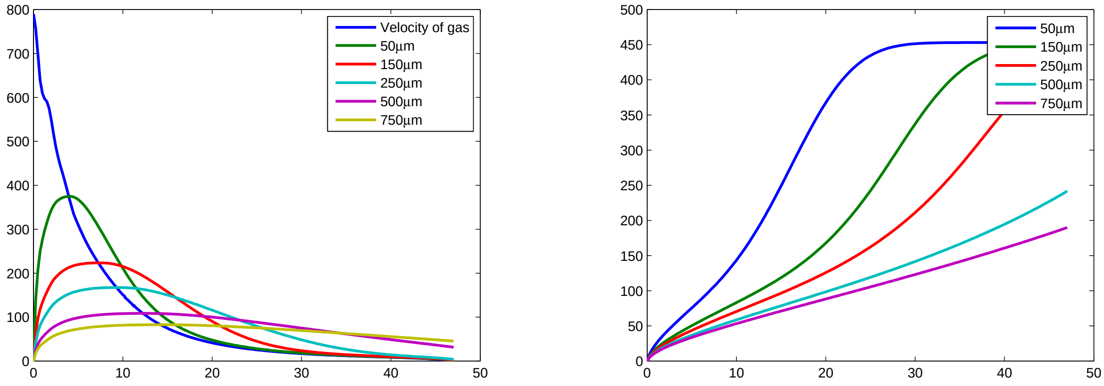


Figure 8: Left: the velocity of particle and the mean velocity of quality of gas for $\rho_s = 2650kg/m^3, p = 0.4Mpa$ and $T = 453K$ with heat transfer; right: the temperature of particle for $\rho_s = 2650kg/m^3, p = 0.4Mpa$ and $T = 453K$ with heat transfer.

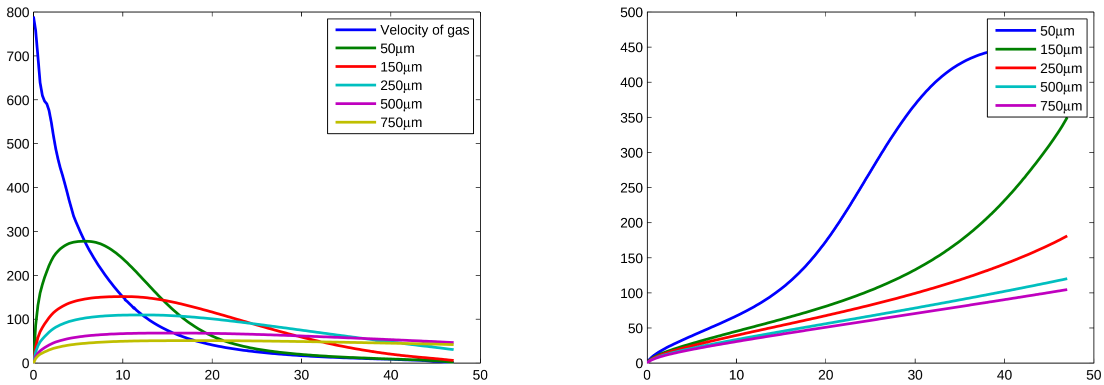


Figure 9: Left: the velocity of particle and the mean velocity of quality of gas for $\rho_s = 7950kg/m^3, p = 0.4Mpa$ and $T = 453K$ with heat transfer; right: the temperature of particle for $\rho_s = 7950kg/m^3, p = 0.4Mpa$ and $T = 453K$ with heat transfer.

when particle diameter is $d_s = 50\mu m$,, the $0-5$ times dimensionless distance is the particle's rapidly accelerating phase, and the $5-25$ times dimensionless distance is particle velocity's sharp decline phase. At 5 times dimensionless distance, particle reaches its maximal velocity of $382m/s$.

(1) $p = 0.5Mpa, T = 473K$. The numerical results are as follows(see Figure 10 and Figure 11).

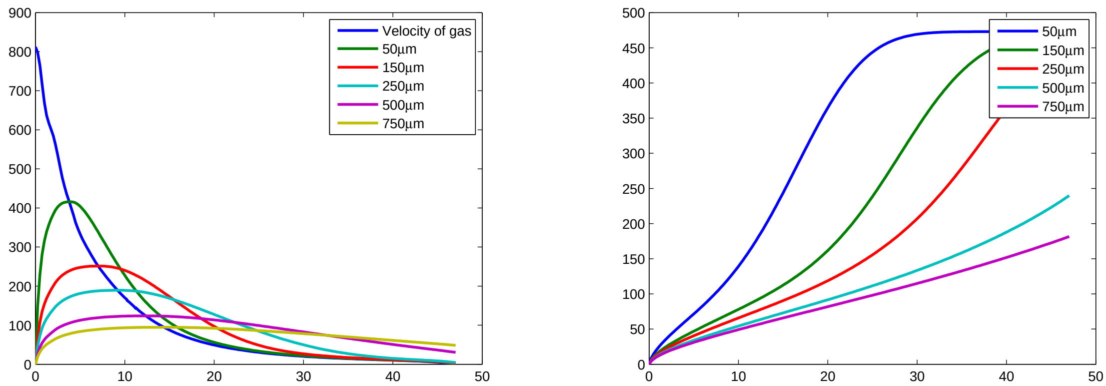


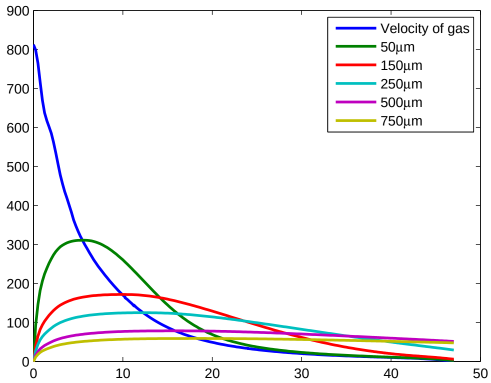
Figure 10: Left: the velocity of particle and the mean velocity of quality of gas for $\rho_s = 2650kg/m^3, p = 0.5Mpa$ and $T = 473K$ with heat transfer; right: the temperature of particle for $\rho_s = 2650kg/m^3, p = 0.5Mpa$ and $T = 473K$ with heat transfer.

The results and the reasons are similar to the above statement, and we do not explain it here repeatedly.

3 Results and Conclusions

By the numerical results in section 2, we derive the maximal velocity of particle (MVP) and the corresponding optimal dimensionless distance (ODD) in the particle acceleration progress under different conditions (see Table 1 and Table 2).

From the different simulation results of these models, both considering the axial velocity and the mean velocity of quality, we can see that the particle



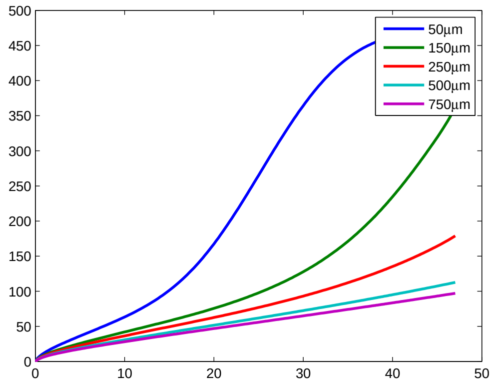


Figure 11: Left: the velocity of particle and the mean velocity of quality of gas for $\rho_s = 7950kg/m^3, p = 0.5Mpa$ and $T = 473K$ with heat transfer; right: the temperature of particle for $\rho_s = 7950kg/m^3, p = 0.5Mpa$ and $T = 473K$ with heat transfer.

Table 1: The MVP and the ODD without considering heat transfer

Velocity of gas	P(Mpa) and T(K)	Density (kg/m^3)	MVP(m/s)	ODD
Axis velocity	Normal	2650	370	14
		7950	202	17
Mean velocity of quality	Normal	2650	223	10
		7950	124	14

Table 2: The MVP and the ODD with considering heat transfer

Velocity of gas	P(Mpa) and T(K)	Density (kg/m^3)	MVP(m/s)	ODD
Axis velocity	$p = 0.4$,	2650	522	6
	$T = 453$	7950	403	8
	$p = 0.5$,	2650	575	6
	$T = 473$	7950	450	8
Mean velocity of quality	$p = 0.4$,	2650	381	4
	$T = 453$	7950	278	5
	$p = 0.5$,	2650	417	4
	$T = 473$	7950	308	5

acceleration in overheated medium is far better than that in air flow, especially for little particle. Meanwhile, we also get the trend of the velocity of the particle in different velocity of gas, different density and particle diameter sizes, respectively. The analysis of these results has great guidance to improve the technology of jet mill.

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