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A Sarima Fit To Monthly Nigerian Naira-British Pound Exchange Rates

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Abstract

The time plot of the series NPER exhibits an overall downward trend with a deep depression in late 2008. No regular seasonality is evident. A 12-month differencing yields a series SDNPER which has an overall slightly upward trend with no clear seasonality. A nonseasonal differencing of SDNPER yields a series DSDNPER with an overall horizontal trend. The visual inspection of its time plot hardly gives an impression of any regular seasonality. However its autocorrelation function shows a significant negative spike at lag 12, indicating a 12-month seasonality and a seasonal moving average component of order one. Moreover the partial autocorrelation plot has significant spikes at lags 12 and 24, suggesting the involvement of a seasonal autoregressive component of order two. Consequently, a $(0, 1, 0)x(2, 1, 1)_{12}$ SARIMA model is hereby proposed, fitted and shown to be adequate.

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1 Introduction and Literature Review

Modelling of Nigerian Naira foreign exchange rates with other currencies has engaged the attention of many researchers, a few of whom are Olowe[1], Etuk([2], [3], [4]), etc. Many economic and financial time series are known to exhibit some seasonality in their behavior. Foreign exchange rates are among such series, their observed volatility notwithstanding. For instance, Etuk[2] has shown that monthly Nigerian Naira-US Dollar exchange rates are seasonal with period 12 months. He fitted an $(0, 1, 1)x(1, 1, 1)_{12}$ seasonal autoregressive integrated moving average (SARIMA) model to it and on its basis forecasted the 2012 values. He has modeled daily Naira-Dollar exchange rates by a $(2, 1, 0)x(0, 1, 1)_7$ SARIMA model after having observed a 7-day seasonality (Etuk[3]). He has also fitted another $(0, 1, 1)x(1, 1, 1)_{12}$ SARIMA model to the monthly Naira-Euro exchange rates (Etuk[4]). In this write-up, interest is in the fitting of a SARIMA model to monthly Nigerian Naira-British Pound exchange rates. Perhaps there are no earlier attempts to model the series by SARIMA methods.

Box and Jenkins[5] introduced a SARIMA model as an adaptation of an autoregressive integrated moving average (ARIMA) model, which they earlier proposed, to specifically explain the variation of seasonal time series. SARIMA modeling has been quite successful. A few other authors who have written extensively on the theoretical properties as well as on the practical applications of SARIMA models, highlighting their relative benefits are Priestley[6], Madsen[7], Gerolimetto[8], Martinez and Soares da Silva[9], Prista et al[10], Saz[11], Surhatono[12], Oduro-Gyimah et al [13], Sami et al[14] and Bigovic[15].

2 Materials and Methods

The data for this research work is the monthly Naira-Pound exchange rates from 2004 to 2011 published under the Data and Statistics heading of the Central Bank of Nigeria website www.cenbank.org.

2.1 Sarima Models

A time series $\{X_t\}$ is said to follow an autoregressive moving average (ARMA) model of order p and q denoted by ARMA(p, q) if

$$X_t - \alpha_1 X_{t-1} - \alpha_2 X_{t-2} - \ldots - \alpha_p X_{t-p} = \varepsilon_t + \beta_1 \varepsilon_{t-1} + \beta_2 \varepsilon_{t-2} + \ldots + \beta_q \varepsilon_{t-q} \qquad (1)$$

where the α's and the β's are constants such that (1) is stationary and invertible and the sequence of random variables $\{\varepsilon_t\}$ is a white noise process.
Let (1) be put as

$$A(L)X_t = B(L)\varepsilon_t \qquad (2)$$

where $A(L) = 1 - \alpha_1 L - \alpha_2 L^2 - \ldots - \alpha_p L^p$ and $B(L) = 1 + \beta_1 L + \beta_2 L^2 + \ldots + \beta_q L^q$ and L is the backward shift operator defined by $L^k X_t = X_{t-k}$. It is well known that for (1) to be stationary and invertible the zeros of A(L) and B(L) must be outside the unit circle respectively.

Many real life time series are nonstationary. For such a time series Box and Jenkins[5] propose that differencing up to an order d could render it stationary. Suppose the stationary d^{th} order difference of X_t is denoted by $\nabla^d X_t$. Clearly $\nabla = 1-L$. Putting $\nabla^d X_t$ in lieu of X_t in (1) yields an autoregressive integrated moving average (ARIMA) model of order p, d and q, denoted by ARIMA(p, d, q) in $\{X_t\}$.
Suppose a time series $\{X_t\}$ is seasonal of period s. For such a series a SARIMA model of order $(p, d, q)x(P, D, Q)_s$ is defined by

$$A(L)\Phi(L^s)\nabla^d \nabla_s^D X_t = B(L)\Theta(L^s)\varepsilon_t \qquad (3)$$

where $\Phi(L)$ and $\Theta(L)$ are respectively polynomials of order P and Q with coefficients such that the model is stationary and invertible respectively. $\Phi(L)$ and $\Theta(L)$ are respectively the seasonal autoregressive and moving average operators of the model.

2.2 Model Estimation

The software Eviews was used for model fitting. Time series analysis invariably begins with the time plot. At this stage a lot about the nature of the series could be evident. For instance any seasonal tendency or otherwise could show up. Generally no regular seasonal pattern is obvious. The autocorrelation function (ACF) better reveals a seasonal nature or otherwise. A significant spike at the seasonal lag is an indication of seasonality; a negative spike indicates a seasonal moving average component and a positive one an autoregressive component. To avoid unnecessary model complexity it has been advised that d + D be at most equal to 2. An autoregressive model of order p has a partial autocorrelation function (PACF) that cuts off at lag p. On the other hand a moving average model of order q has an ACF that cuts off at lag q.

After determination of the orders p, d, q, P, D, Q and s, the rest of the parameter estimation process could be done. Eviews is based on the least error sum of squares criterion. This involves an iterative process after an initial estimate of the solution is made, the process converging to an optimal solution.

After model estimation, the model is subjected to goodness-of-fit tests to ascertain its adequacy. Analysis of its residuals is done. Assuming the model is adequate its residuals should be uncorrelated and should follow a normal distribution.

3 Results

The time plot of the series NPER in figure 1 reveals an overall slightly negative trend with a deep depression in late 2008. Ne regular seasonality is observable. Seasonal (i.e. 12-point) differencing of NPER yields a series SDNPER with a slightly positive secular trend and no regular seasonality still (See Figure 2). A nonseasonal differencing of SDNPER yields a series DSDNPER with an overall horizontal trend with no observable regular seasonality (See Figure 3). The ACF of DSDNPER of Figure 4 however shows a significant spike at lag 12, indicating seasonality of period 12 and a seasonal moving average component of order one. The PACF has significant spikes at lags 12 and 24 suggesting a seasonal autoregressive component of order two. Therefore a $(0, 1, 0)x(2, 1, 1)_{12}$ SARIMA model

$$DNDNPER_t - \alpha_{12} DSDNPER_{t-12} - \alpha_{24} DSDNPER_{t-24} = \varepsilon_t + \beta_{12}\varepsilon_{t-12} \qquad (4)$$

is proposed. The estimation of (4) as summarized in Table 1 yields

$$DSDNPER_t + 1.1697 DSDNPER_{t-12} + 0.7014 DSDNPER_{t-24} = \varepsilon_t - 0.8611\varepsilon_{t-12} \qquad (5)$$

We note that all three coefficients are statistically significant, each being more than twice its standard error. The regression is very highly significant with a p-value of 0.000000. As high as 61% of the variation in DSDNPER is accounted for by the fitted model (5). Figure 5 shows a very close agreement between the fitted model and the data. Figure 6 shows that the residuals are uncorrelated. Therefore the fitted model is adequate.

4 Conclusion

Fitted to the monthly exchange rate series NPER is the $(0, 1, 0)x(2, 1, 1)_{12}$ SARIMA model (5). By various arguments it has been shown to be adequate.

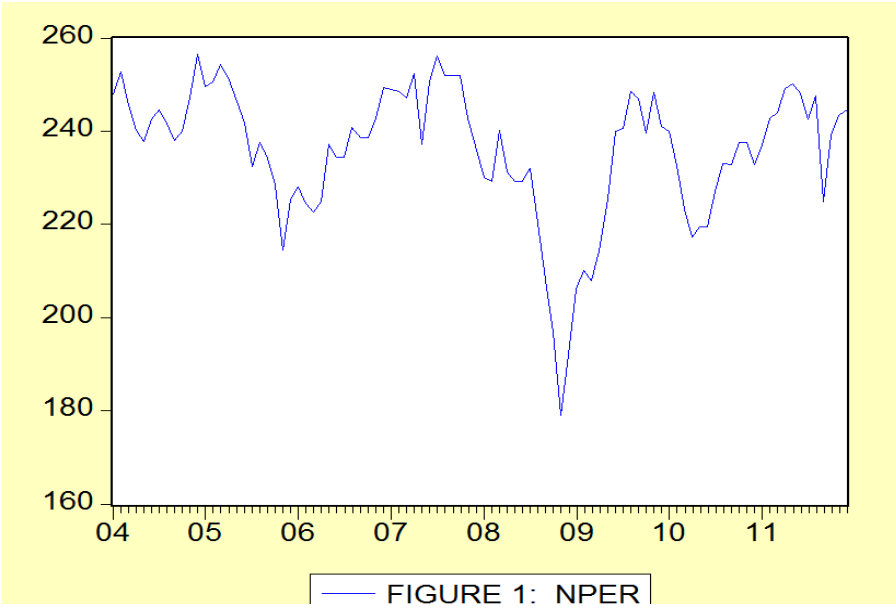
260
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FIGURE 1: NPER

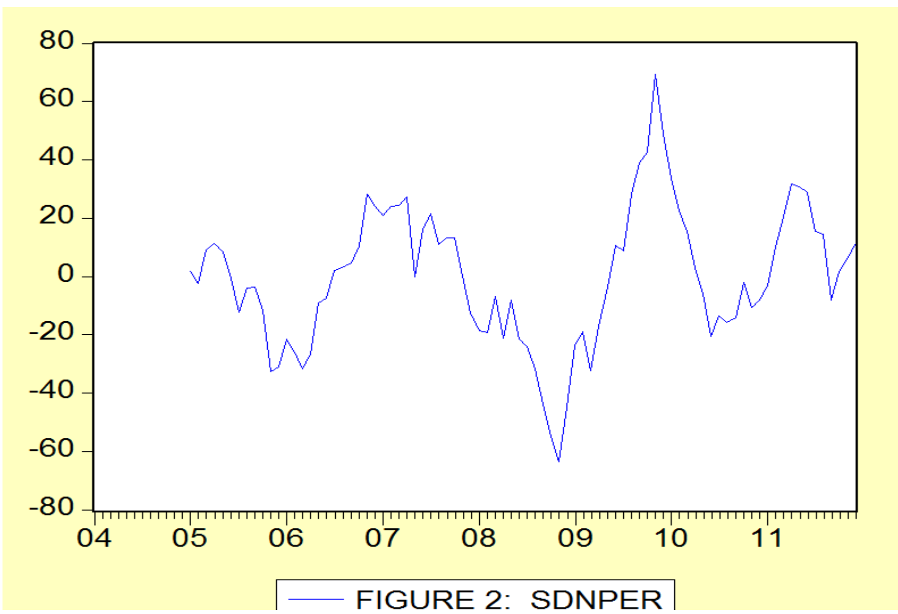
80
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FIGURE 2: SDNPER

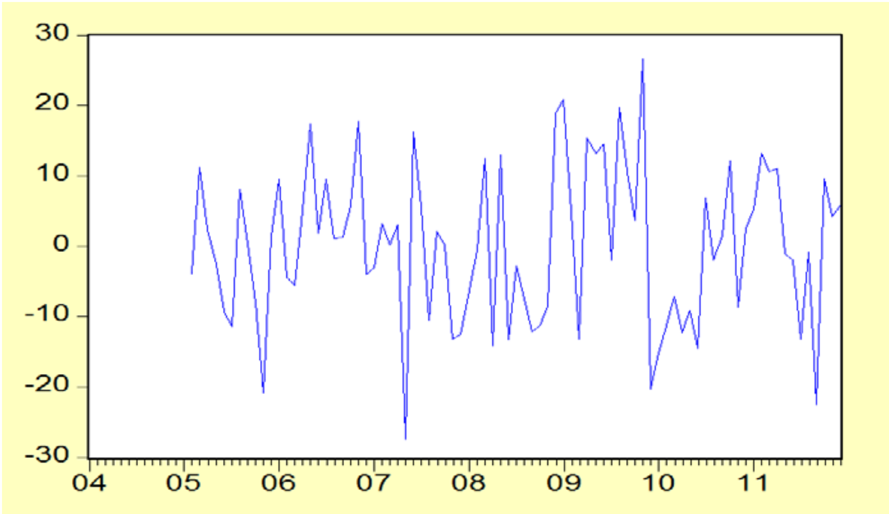
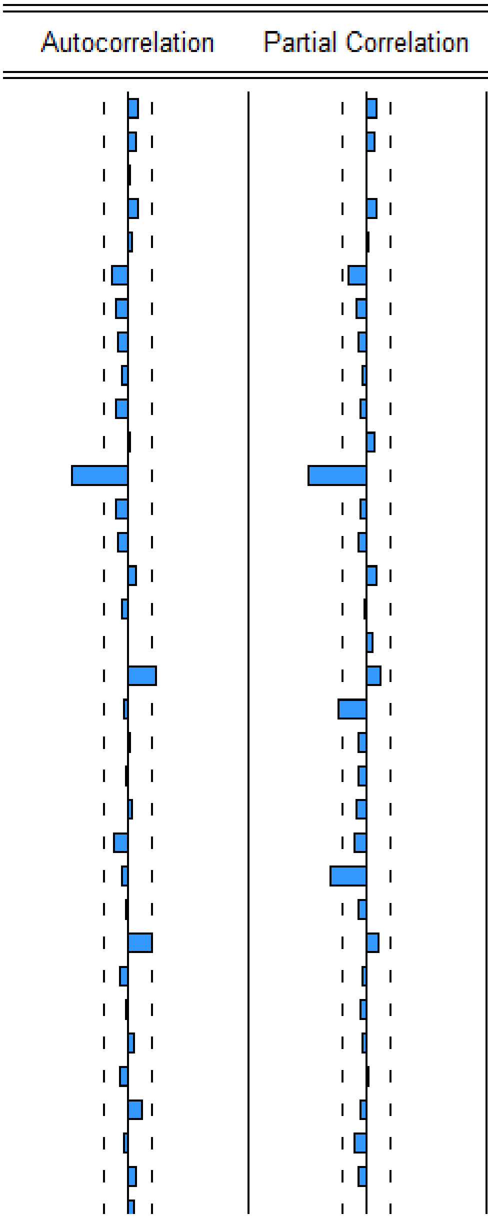


FIGURE 3: DSDNPER



	AC	PAC	Q-Stat	Prob
1	0.096	0.096	0.7899	0.374
2	0.068	0.059	1.1903	0.551
3	0.019	0.007	1.2224	0.748
4	0.095	0.090	2.0302	0.730
5	0.027	0.009	2.0942	0.836
6	-0.136	-0.153	3.7812	0.706
7	-0.100	-0.082	4.7174	0.694
8	-0.091	-0.069	5.4935	0.704
9	-0.056	-0.036	5.7971	0.760
10	-0.099	-0.057	6.7464	0.749
11	0.011	0.058	6.7592	0.818
12	-0.478	-0.503	29.503	0.003
13	-0.104	-0.053	30.602	0.004
14	-0.098	-0.081	31.579	0.005
15	0.081	0.098	32.262	0.006
16	-0.046	-0.016	32.483	0.009
17	0.000	0.043	32.483	0.013
18	0.239	0.123	38.665	0.003
19	-0.031	-0.253	38.769	0.005
20	0.024	-0.082	38.836	0.007
21	-0.022	-0.074	38.892	0.010
22	0.027	-0.087	38.974	0.014
23	-0.126	-0.104	40.827	0.012
24	-0.055	-0.318	41.190	0.016
25	-0.009	-0.081	41.199	0.022
26	0.201	0.107	46.191	0.009
27	-0.065	-0.036	46.726	0.011
28	-0.020	-0.056	46.776	0.014
29	0.052	-0.037	47.125	0.018
30	-0.061	0.012	47.613	0.022
31	0.119	-0.050	49.548	0.019
32	-0.039	-0.114	49.760	0.024
33	0.068	-0.063	50.404	0.027
34	0.043	-0.001	50.671	0.033

Figure 4: Correlogram of DSDNPER

Table 1: Model Estimation

Dependent Variable: DSDNPER
Method: Least Squares
Date: 02/06/13 Time: 13:59
Sample(adjusted): 2007:02 2011:12
Included observations: 59 after adjusting endpoints
Convergence achieved after 10 iterations
Backcast: 2006:02 2007:01

Variable	Coefficient	Std. Error	t-Statistic	Prob.
AR(12)	-1.169705	0.095910	-12.19581	0.0000
AR(24)	-0.701397	0.102847	-6.819840	0.0000
MA(12)	0.865311	0.029901	28.93895	0.0000

R-squared	0.614571	Mean dependent var	-0.156102
Adjusted R-squared	0.600806	S.D. dependent var	11.97690
S.E. of regression	7.567225	Akaike info criterion	6.935039
Sum squared resid	3206.722	Schwarz criterion	7.040677
Log likelihood	-201.5837	F-statistic	44.64629
Durbin-Watson stat	2.043437	Prob(F-statistic)	0.000000

Inverted AR Roots	.97+.19i	.97 -.19i	.93+.32i	.93 -.32i
	.74+.65i	.74 -.65i	.65 -.74i	.65+.74i
	.32 -.93i	.32+.93i	.19 -.97i	.19+.97i
	-.19+.97i	-.19 -.97i	-.32+.93i	-.32 -.93i
	-.65+.74i	-.65 -.74i	-.74 -.65i	-.74+.65i
	-.93 -.32i	-.93+.32i	-.97 -.19i	-.97+.19i
Inverted MA Roots	.95+.26i	.95 -.26i	.70+.70i	.70 -.70i
	.26+.95i	.26 -.95i	-.26 -.95i	-.26+.95i
	-.70 -.70i	-.70 -.70i	-.95 -.26i	-.95+.26i

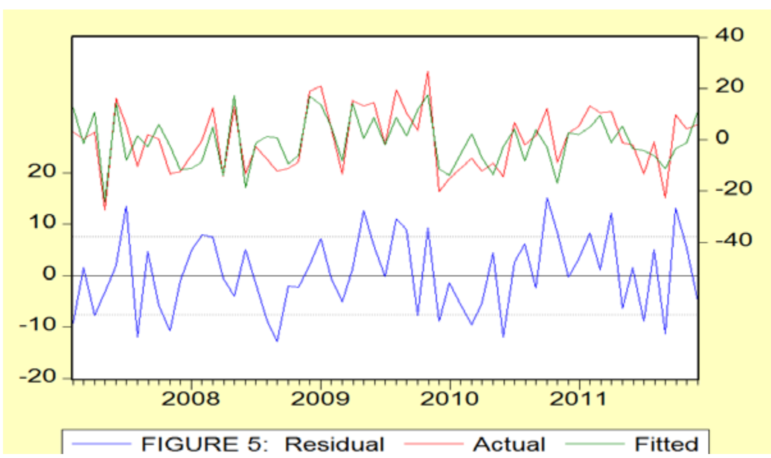


FIGURE 5:

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.043	-0.043	0.1133	
		2	0.122	0.121	1.0594	
		3	0.067	0.078	1.3487	
		4	-0.044	-0.054	1.4764	0.224
		5	-0.203	-0.231	4.2198	0.121
		6	0.130	0.126	5.3691	0.147
		7	-0.222	-0.159	8.7753	0.067
		8	-0.110	-0.138	9.6298	0.086
		9	-0.074	-0.084	10.026	0.124
		10	-0.221	-0.225	13.620	0.058
		11	0.001	0.053	13.620	0.092
		12	-0.034	-0.102	13.710	0.133
		13	-0.085	-0.106	14.281	0.161
		14	-0.069	-0.163	14.662	0.198
		15	0.231	0.155	19.030	0.088
		16	0.001	0.070	19.030	0.122
		17	0.251	0.105	24.444	0.040
		18	0.137	0.067	26.090	0.037
		19	0.147	0.102	28.047	0.031
		20	-0.050	-0.032	28.274	0.042
		21	0.045	-0.061	28.462	0.055
		22	-0.047	0.049	28.673	0.071
		23	0.055	0.095	28.979	0.088
		24	-0.176	-0.121	32.152	0.057

Figure 6: Correlogram of the Residuals

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