Journal of Applied Finance & Banking, vol. 8, no. 1, 2018, 35-52
ISSN: 1792-6580 (print version), 1792-6599 (online)
Scienpress Ltd, 2018

An Algorithm Exploiting Episodes of Inefficient Asset Pricing to Derive a Macro-Foundation Scaled Metric for Systemic Risk: A Time-Series Martingale Representation

Richard W. Booser[1]

Abstract

This paper employs an event study, the Global Financial Crisis. Episodes of inefficient pricing, the externality, are exploited as a measure of systemic risk. The theoretical asset pricing model, the martingale representation, is shown to be a valid algorithm to identify episodes of efficient and inefficient pricing in time series. Systemic risk metrics are derived from episodes of inefficient pricing, utilizing a shadow volatility metric. The algorithm is forward looking, deriving macro-foundation metrics from actual agent market behavior. The algorithm provides precise risk metrics for magnitude and diffusion using US and Canadian treasury markets. Given the US dollar's role as the de-facto world reserve currency, scaled metrics derived from the US treasury market provide a globalized systemic benchmark. The risk metrics signal the crisis buildup and calibrate around the crisis epicenter date of September 2008. The risk metrics are heuristically consistent with the stylized facts of financial crises and support the extraordinary US policy response to the crisis. The algorithm output is validated by time-series analysis.

JEL classification numbers: C22, C61, G01, G12, G14
Keywords: Systemic Risk, Time Series, Optimization, Financial Crises, Asset Pricing

1 Introduction

[1] East Stroudsburg University, USA.

Article Info: *Received* : August 23, 2017. *Revised* : September 15, 2017
Published online : January 1, 2018

The Global Financial Crisis of 2008 is described similarly to the 100-year flood. The financial crisis was a systemic global event, leading to a global recession. The financial crisis provides a rich data environment for the assessment of systemic risk. The use of US and Canadian treasury yield curve data allows for a cross-county comparison of systemic risk profiles, along with support for the validation of the martingale representation as an empirical time-series algorithm. Since the US and Canada are similarly large, geographically contiguous trading partners, the two countries provide a natural comparison for the evaluation of crisis impact. The divergence in financial crisis impact was quite apparent as large US financial institutions required bailouts, while large Canadian financial institutions did not. The empirical results indicate cross-country systemic risk profiles that are not completely uniform. The systemic risk magnitudes differ, whereby the diffusion processes are more similar.

The theoretical martingale representation, the standard core analytical model used in modern asset pricing, is utilized as an empirical time-series algorithmic platform. The algorithm is forward looking, and exploits episodes of inefficient pricing, the externality, as a measure of systemic risk. A scaled risk metric is derived from actual agent market behavior found in the special status, US Treasury market. The scaled metric provides a macro-foundation, complete systems approach to identifying and measuring systemic risk. The scaled metric stands in contrast to the conventional micro-foundation, default correlation partial or incomplete systems network approach. The use of the algorithm does not require complex mathematical abstractions, complex network construction, simplifying assumptions of agent micro-behavior or convenient well-behaving functional forms to guarantee tractability. The algorithm is market behavior based, and is very robust in that it captures episodes of inefficient pricing under both normal and crisis conditions. The algorithm's risk metric output captures both distinct phases of systemic risk in signaling the elevation of risk buildup prior to the crisis and the emergent risk systemics calibrating around the crisis event date. [1]

Operationally, the algorithm is a dynamic one-period, re-setting empirical platform used to identify episodes of efficient and inefficient pricing in time series. Episodes of inefficient pricing, the externality, are then exploited as a measure of systemic risk using a shadow volatility metric. The shadow volatility metric, imputing attributes associated with a dual variable, counterfactually re-establishes efficient pricing in the inefficient pricing segment of the time series. More formally, satisfying the primal problem of optimal (efficient) pricing requires the use of the shadow volatility metric to re-establish positive state prices in the martingale representation through the restoration of efficient pricing.

The algorithm output provides valid and precise risk metrics that include signaling state prices, systemic risk magnitudes, and risk diffusion patterns. As market agent opinion shifts, volatility moves to extremes, as financial markets display cycles alternating between an appetite for more risk assets and a flight to quality assets, particularly for the treasury bonds of the industrialized world. [2] In effect, the algorithm captures actual market sentiment reversal from a risk-on to a risk-off paradigm. The state prices signal a pre-crisis elevation of risk. The scaled metric derived from the shadow volatility metric measures the crisis impact intensity resulting from a reversal in market energy, and is interpretively similar with the tremor event moment magnitude scaled metric measuring tectonic plate energy. The diffusion metrics provide insight into the risk dynamics along the treasury yield curve. The scaled metric allows for consistent comparison of systemic risk impact and contagion. For the US crisis impact epicenter, the scaled metric is 32.25. The corresponding scaled metric for Canada is 13.75.

As applied, the state prices associated with short-term US maturities display signaling properties relative to pre-crisis risk elevations. For both the US and Canada, the emergent risk metrics calibrate around the epicenter crisis event date of September 2008. The metrics are heuristically consistent with the extraordinary US monetary policy response to the crisis and the historical stylized facts of major financial crisis events. The validity of both the shadow volatility metric and the algorithm-derived scaled metric is confirmed by time-series analysis.

2 Brief Literature Review

Arbitrage-free asset pricing is the accepted norm in finance. Examples of anomalies do not suggest that an efficient market and exploitable arbitrage opportunities are compatible. [3] Large persistent violations must be considered an externality. A risk tolerance paradigm shift creating market inefficiencies is an expression of an externality.

Risk appetites may also display path dependency characteristics. Agent choices are impacted by the way the game is evolving for the player. Research from behavioral economics indicates that in certain situations, agents may be less risk averse and actively seek more risk. Players using house money that are ahead or players behind but anticipating a break-even outcome shift their risk profiles to less risk averse positions. [4] Fragility may also be associated with path dependency. Agents focus on success, say profitability, without first emphasizing risk control to ensure survival. To a rational agent, the logical sequence of events should emphasize survival strategies before success strategies. In other words, the order of events taken is of primary importance over the destination or outcome. [5] Overall, the history of the path may play a crucial role in agent risk assessment and agent allocations. Path dependency imputes the analysis of risk by using time series analysis.

Financial crises are systemic events resulting from sudden regime shifts, characterized most basically by market agents exiting from bank debt and creating insolvency within the banking system. [6] Systemic events reflect systemic risk, aggregate or macro behavior in a system. Systemic risk may involve breakdowns such as adverse network effects from an internal shock, insolvency of key institutional factors, and liquidity bottlenecks. [7]

Network structure plays a key role. The identical factors that contribute to network resilience may also contribute to network fragility, as a financial contagion displays a phase transition characteristic. Below a certain threshold, shocks enhance stability in densely connected networks. Above a certain shock threshold, densely connected networks propagate shocks leading to increased fragility. [8] Clustered networks, networks of financial institutions holding identical portfolios, tend to default together, whereby un-clustered networks display more default dispersion. The impact of long-term financing and network structure is neutral. In contrast, network structure matters relative to short-term financing. [9]

The cost of a public policy intervention to correct a negative spillover, an externality, might be correctly considered a measure of systemic risk. Public policy intervention is a societal cost related to the correction of a negative externality. The associated cost of the externality response is a measure of full systemic risk. [10] Rather than solely a public policy action, the externality could reflect a market response or some combination of public policy and market response. Exploiting episodes of inefficient pricing, an externality, is argued to be a viable measure for systemic risk.

There are two distinct phases to systemic risk. The run-up phase in the backdrop before the crisis and the materialization as the crisis event occurs. To measure systemic risk, one must be able to overcome the significant empirical challenge of boiling down large sources and amounts of data to a singular, meaningful risk statistic or metric. When considering systemic analysis, there has been a priority given to propagation and amplification in the financial sector, and particularly for interactions and types of financial institutions. Following this line of research, measuring systemic risk begins with quantifying firm risk. The natural sequence is to start at the micro-firm level, and develop risk allocation rules to accurately allocate total or marginal contributions to systemic risk across various types of financial institutions or other relevant market agents. Risk allocation rules abound, such as proportional allocation, Euler or gradient allocation, with-and-without allocation, and, from game theory, Shapley value. Systemic risk measures include systemic expected shortfall (SES), distressed insurance premium (DIP), CoVaR analysis, contingent claims analysis, and a copula approach. These risk allocation rules and systemic risk measures all build from the micro-level to the systemic level. A good measure of systemic risk must insure that the sum of all risk contributions equals the total, and the appropriate amount of marginal risk taken on by any agent or institution is guided by incentives. [1] The important question remains whether building from the micro-foundation level out provides a valid risk-based measure as one must identify all relevant micro-units and correctly allocate risk contributions system-wide. This micro-based conventional approach is a partial or incomplete systems approach in that it does not provide a complete system, macro-economy measure of systemic risk. The martingale representation algorithm, in using the special status of the US treasury market, abandons the daunting abstracting micro-detail, and provides a complete system, macro-economy systemic metric. The algorithm, through pre-crisis state price signaling and the episodes of inefficient pricing calibrating around the epicenter event date, satisfies the two distinct phases to systemic risk.

The seminal work on surveying systemic risk analytics was prepared for the US Department of the Treasury. The survey identified 31 quantitative measures of systemic risk from the economics and finance literature. Ten different definitions of systemic risk were identified from published research. The 31 analytical measures were organized into broad categories:

Macroeconomic Measures, Granular Foundations and Network Measures, Forward-Looking Risk Measures, Stress-Test Measures, Cross Sectional Measures, and Measures of Illiquidity and Insolvency. The analytical measures were sub-categorized in terms of ex ante early warning, ex ante counterfactual simulation and stress tests, contemporaneous fragility, contemporaneous crisis monitoring, ex post forensic analysis, and ex post orderly resolution. Most relevant to this paper, the identification of seven forward-looking risk measures – contingent claims analysis, Mahalanobis distance, the option iPoD, multivariate density estimators, simulating the housing sector, consumer credit, principle component analysis – does not include the theoretical martingale representation model. In addition, the ex post forensic approach does not include the theoretical representation model. [11] More specifically, none of the 31 analytical measures identified include the theoretical martingale representation model as applied to yield curve time-series or other data sources.

In finance, the presumption of efficient pricing imputes a theoretical Walrasian equilibrium, consistent with the notion of the "invisible hand." The notion of an idealized Walrasian system is violated by the reality of imperfectly competitive market conditions, and the presumption of a guiding invisible hand typically fails to materialize at a non-cooperative equilibrium. Under multi-equilibria conditions, the lack of cooperation can result in a non-cooperative equilibrium that is inefficient, and yet there are no incentives to agents to unilaterally move to a better equilibrium. The resulting inefficient equilibrium is due to lack of coordination. Invoking coordination allows a movement to the optimal position. [12] A crisis driven paradigm shift in market sentiment from risk-on to risk-off reflects a multi-equilibria condition, whereby effective policy coordination intervention is needed to provide signals to agents to move back to an efficient equilibrium position. These two equilibria extremes reflect systemic risk, the measurable magnitude of a market agent move from an optimal to a sub-optimal equilibrium. The martingale representation algorithm follows this line of reasoning to quantify systemic risk through a scaled metric.

3 The Stylized Facts of Financial Crises

The paper incorporates an event study involving a major financial crisis. The common characteristics of financial crises or panics involve acute liquidity shortages and contagion. Walter Bagehot, in 1873, indicted that central banks should act as a lender of last resort and lend freely. Beyond a liquidity event, you have contagion and possibly the impairment of the credit granting function. Contagion may be a rational response as bank failures increase counterparty risk, impacting a very large number of institutions. [13] Financial panics happen under both fiat currency and gold standard regimes. Under the International gold standard from 1879 to 1913, a major rule of the game was to address short-run liquidity crises resulting from a gold drain with a central bank lending freely to the domestic banking sector. [14] The contagion effect reflects systemic risk, and volatility is exacerbated by contagion as countries or assets are grouped into categories of risk that are perceived as being very correlated. [2] It is difficult to have a sense for the event or events causing a specific panic. In some cases, there may not be a logical reason. An event occurs leading to a failure in confidence in the financial system. The loss of confidents, possibly driven by instinct, results in runs on banks, squeezing reserve positions. Solvent banks become insolvent. Banks face liquidity squeezes and call loans which tighten credit, raising short-term interest rates, spawning credit disruptions and business failures. The ensuing disruption leads to an economic recession. [15]

4 The Martingale Representation Algorithm

The theoretical representation model is commonly known as a martingale. The representation model is theoretically correct in that it is forward looking. The model provides for efficient market pricing under the assumption of arbitrage-free valuations. Empirically, the combination of the martingale model and rational expectations is generally viewed as satisfying market efficiency criteria. [16] The martingale model implies risk-neutral agents.

Arbitrage-free is formally defined as not allowing for a zero time-t cash investment with the potential for receiving a non-zero investment return at time T. Alternatively, it does not allow for receiving time-t cash to make an investment with zero liabilities at time T. Such portfolios cannot be feasible at given current prices when arbitrage-free conditions apply. In the martingale representation model, given actual asset prices at time-t, arbitrage-free requires that all elements in the state price vector exist and be greater than zero. [17] The violation of positive state prices plays a key role in deriving the systemic risk scaled metric.

While the representation expressed by the theoretical model is not observable in the real world (only one state-of-the world will be observed), the theoretical representation model used as algorithm is a powerful empirical platform. The martingale representation is formally expressed in matrix notation. The actual martingale equations are hidden within the representation's matrix notation.

$$\begin{array}{ccc} \mathbf{t_o} & & \mathbf{t_1} \\ \begin{bmatrix} B_1 \\ B_n \end{bmatrix} & = & \begin{bmatrix} 1 & 1 \\ B_{n-1}+\sigma & B_{n-1}-\sigma \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} \end{array} \tag{1}$$

Variables Q_1 and Q_2 are the state prices. The elements of the vector at time-t_o record the respective bond prices. Elements of the payoff matrix record the two-state values for the bonds at time-t_1. Solving the matrix multiplication and adjusting for the forward measure yields two equations.

$$B_1 = B_1\,[(Q_1/\,B_1) + (Q_2/\,B_1)] \tag{2}$$
$$B_n = B_1\,[(B_{n-1}+\sigma)\,(Q_1/\,B_1) + (B_{n-1}-\sigma)\,(Q_2/\,B_1)] \tag{3}$$

The forward measure, $Q_i/\,B_1$, is the synthetic probability of each state occurring. An equation restated in the following generalized ratio form is a martingale.

$$B_t/B_1 = E_p\,[B_T] \quad (4)$$

The existence of positive state prices, Q_1 and Q_2, are required to provide efficient pricing in equations (2) and (3).

Operationally, the martingale representation is used as a one-period, re-setting algorithm to empirically determine episodes of efficient and inefficient pricing in time-series data. The one-period, re-setting format is justified both empirically and theoretically. In each new time-period of one month, it is reasonable to expect a new information set, I_t, based on new news arriving. Theoretically, given rational expectations, since all available information in the information set should be reflected in the bond price, and since new information arrives randomly, a new information set should result in new bond pricing. Empirically, the data clearly and uniformly displays new bond prices every month.
Solving the representation's matrix algebra requires the time-to bond prices, B_t, and the forward time-t_1 volatility, σ. Equation (1) is sufficient (equations 2 and 3 may remain hidden) to derive the values of state prices given market bond prices at time-to and the forward volatility at time-t_1. The up state-one movement equals $B_{n-1}+\sigma$ and the down state-two movement equals $B_{n-1}-\sigma$. The use of volatility (σ) multiplied by the square root of the time interval is the common practice in modern finance. [18] The time frame from time-to to time-t_1 is always one year, so the $\sqrt{\Delta}$ term is dropped. The assignment of future up-down values using B_{n-1} rather than B_n reflects the fact that the two-year bond will be a one-year bond at time-t_1, and so on. The volatility (σ) is for the B_{n-1} bond, and represents the sample standard deviation over the 12-month forward (t_1) period. Positive state prices impute efficient pricing. For each month, the algorithm's matrix algebra is solved for the state prices Q_1 and Q_2. Each forward volatility value in the volatility time series is associated with its own set of state prices, elements found in the state price vector. Volatility is an ex post measure. In terms of the representation matrix, the bond values are ex ante prices. The algorithm confirms efficient pricing, arbitrage-free outcomes when ex ante bond prices and the ex post volatility yield state prices that exist and are all greater than zero.

The algorithm is linear but it is obviously not a linear programming methodology. Still, some conceptual attributes of the primal and dual problems are reflective of playing a similar role in the martingale representation through state prices and the shadow volatility metric. The primal objective of optimal (efficient) pricing is satisfied through the provision of a positive state price vector. The dual variable is the shadow (price) volatility metric, counterfactually re-establishing efficient pricing and positive state prices in the time series. The algorithm is applied to time-series data displaying episodes of both efficient and inefficient pricing. The shadow volatility metric is similarly the imputed value of the input volatility resource that provides for optimal time-series pricing, just as the linear programming shadow prices are the imputed values of the scarce resource contributions that are owed to the resulting primal optimal (profit) value. [19]

The study uses monthly US and Canadian government bond interest rate data taken from the Federal Reserve Board and Bank of Canada historical time-series data bases. The bond prices represent the standard conventional zero-coupon bond values as derived from the interest rate data. Yield curve interest rates are available for the one-year, two-year, three-year, five-year and seven-year government issues. The four-year and six-year rates are interpolated. Because of data availability limitations, Canadian dollar LIBOR rates were substituted for one-year rates from September of 2005 to April of 2006. Pre- and post-crisis data (approximately 36 months before and 36 months after the crisis event) is purposely used, consistent with an event study. The yield curve data sets run from September 2005 to September 2011, plus a 12-month forward-looking data requirement running through September 2012. Model and data constraints limit the use of the algorithm to the one-year to six-year length of the yield curve.

5 Risk Metric Output from the Algorithm

The algorithm generates a time series of state prices. The state price values calibrate around the financial crisis event date of September 2008. State prices shift from positive to negative values, and signal pre-crisis elevated risk levels as US shorter-maturity Q_2 state prices (state prices may be viewed as elementary insurance contracts) gradually approach 0.95. Negative state prices represent sub-optimal, inefficient pricing of assets. For both the United States and Canada, truncated Q_1 and Q_2 state price time-series sequences are provided in Table 1 and Table 2, respectively. The date of the first shift to negative state prices is recorded in italics.

For US state prices, some state prices turn negative before September 2008 and other state prices react one month later in October 2008. One-year, three-year, and four-year maturity state prices begin to turn negative in August 2008. The two-year maturity state price begins to turn negative in July 2008. The five-year and six-year maturity state prices begin to turn negative in October 2008.

Table 1: US state prices

US

Dates	1Y Q1	Q2	2Y Q1	Q2	3Y Q1	Q2
2008-03	0.3884319	0.5964017	0.2887435	0.6960901	0.126215	0.8586185
2008-04	0.1095763	0.8733213	0.2252818	0.7576158	0.1231311	0.8597665
2008-05	0.0140361	0.9657797	0.1379016	0.8419142	0.150766	0.8290498
2008-06	0.0197917	0.9565801	0.0194763	0.9568955	0.1100725	0.8662993
2008-07	0.0354408	0.9422675	*-0.0480126*	*1.0257209*	0.0240093	0.953699
2008-08	*-0.000693*	*0.9793581*	-0.2024626	1.1811277	*-0.1422123*	*1.1208774*
2008-09	-0.0461556	1.0274136	-0.4754465	1.4567045	-0.5269104	1.5081684
2008-10	-0.5107997	1.4967985	-1.2927187	2.2787175	-1.2641839	2.2501827
2008-11	-0.9318846	1.9212979	-1.6953625	2.6847758	-1.1521137	2.141527
2008-12	-2.7418746	3.7369985	-1.9092983	2.9044222	-0.9740281	1.969152

Dates	4Y Q1	Q2	5Y Q1	Q2	6Y Q1	Q2
2008-03	0.0962382	0.8885954	0.1743814	0.8104522	0.1627704	0.8220632
2008-04	0.1112059	0.8716917	0.2070713	0.7758263	0.2024648	0.7804328
2008-05	0.1541844	0.8256314	0.217476	0.7623398	0.2150901	0.7647257
2008-06	0.1282859	0.8480859	0.2221941	0.7541777	0.2313158	0.745056
2008-07	0.0556983	0.9220099	0.1592307	0.8184776	0.1734629	0.8042453
2008-08	*-0.042894*	*1.0215591*	0.1124148	0.8662503	0.1401079	0.8385572
2008-09	-0.2559814	1.2372394	0.0315554	0.9497025	0.0773013	0.9039567
2008-10	-0.7467139	1.7327128	*-0.1596659*	*1.1456648*	*-0.0723588*	*1.0583576*
2008-11	-0.6743195	1.6637328	-0.2098768	1.19929	-0.1217794	1.1111926
2008-12	-0.4856719	1.4807958	-0.1488598	1.1439837	-0.0743936	1.0695175

For Canadian state prices, one-year to four-year maturity state prices turn negative two to three months later than comparable US state prices. Five-year and six-year maturity state prices respond identically to US state prices. The one-year maturity state price begins to turn negative in November 2008. The two-year maturity state price begins to turn negative in September 2008. The three-year, four-year, five-year and six-year maturity state prices begin to turn negative in October 2008.

Table 2: Canadian state prices

Canada

Dates	1Y Q1	Q2	2Y Q1	Q2	3Y Q1	Q2
2008-03	0.4382511	0.536883	0.413495	0.5616391	0.3525197	0.6226144
2008-04	0.4502395	0.5235652	0.3240099	0.6497947	0.3933753	0.5804294
2008-05	0.2962115	0.6757948	0.3451289	0.6268773	0.3593717	0.6126345
2008-06	0.4941671	0.4744497	0.3763343	0.5922825	0.4257824	0.5428345
2008-07	0.3729582	0.5983871	0.2491112	0.7222341	0.3277185	0.6436269
2008-08	0.3932877	0.5806118	0.2651585	0.708741	0.321471	0.6524285
2008-09	0.0889974	0.8852817	*-0.1038397*	*1.0781187*	0.1736073	0.8006717
2008-10	0.4304623	0.5494495	-0.6780309	1.6579427	*-0.3289343*	*1.3088461*
2008-11	*-0.2085498*	*1.1927049*	-1.11398	2.0981351	-0.5364787	1.5206338
2008-12	-1.0761796	2.0677513	-1.1502229	2.1417946	-0.6367172	1.6282889

	4Y Q1	Q2	5Y Q1	Q2	6Y Q1	Q2
2008-03	0.3312824	0.6438517	0.3370496	0.6380845	0.3291938	0.6459403
2008-04	0.389291	0.5845136	0.3552816	0.618523	0.3499458	0.6238588
2008-05	0.354108	0.6178982	0.3447175	0.6272887	0.3447977	0.6272085
2008-06	0.4218188	0.546798	0.3858944	0.5827225	0.3820655	0.5865513
2008-07	0.3225584	0.6487869	0.2814651	0.6898802	0.2817298	0.6896155
2008-08	0.3160214	0.657878	0.263947	0.7099525	0.2595774	0.7143221
2008-09	0.2353532	0.7389259	0.1464777	0.8278013	0.1420753	0.8322038
2008-10	*-0.176684*	*1.1565958*	*-0.0915929*	*1.0715047*	*-0.1234079*	*1.1033197*
2008-11	-0.312856	1.2970111	-0.0741372	1.0582923	-0.1000361	1.0841912
2008-12	-0.3610214	1.3525931	-0.2365464	1.228118	-0.2391585	1.2307301

The shift to negative state prices is uniformly characterized by compressions in ex post volatilities. Ex ante bond prices are incongruent with ex post volatilities. Market sentiment rushes to hold cash or risk-free, cash-equivalent short-term US Treasuries. The Panic of 1907, arguably the most similar financial panic to this crisis, shows similar behavior. The banking system collapsed almost overnight. Reserves were depleted as people rushed to hoard cash or its equivalent, gold. The circulation of available cash disappeared, and liquidity vanished. [20] When liquidity vanishes, there is no trading and volatilities compress. In the

subsequent volatility time-series analysis, the failure to reject the unit root is consistent with the argument that financial panics are ultimately liquidity crises.
To restore optimal or efficient pricing in time series displaying episodes of inefficient pricing, an imputed or shadow volatility metric is used. Each realized volatility value found in the inefficient pricing segment of the volatility time series is multiplied by a common multiple (X) to counterfactually restore all negative state prices found in the state price time series to positive state prices. The common multiple adjustment to the volatility time series re-establishes efficient pricing by removing the inefficient pricing externality and counterfactually restoring positive state price vectors. The common multiple magnitude is referred to as the X-factor. The X-factor adjusted portion of the realized volatility time series is the shadow volatility metric, the time-series volatility adjustment that re-establishes the optimization of the primal objective of efficient pricing. This follows from the fact that a shadow value or price is the imputed economic measure of value relative to the optimal objective value. [21]

The X-factor is a dimensionless, scaled metric measure of systemic risk. In terms of a generalized interpretation of the X-factor magnitude, although mathematically quite different in construct, it is interpretively analogous to the moment magnitude scaled metric used to measure tremor events. The X-factor value is the metric measuring the magnitude of systemic risk. Equivalently, it is also the measure for the magnitude of the economic externality. The X-factor is an extreme value measure in that it is the common volatility adjustment multiple required to counterfactually remove all the inefficient pricing found in the time series and counterfactually fully restore efficient pricing and positive state prices. Given the use of US treasury data and the US dollar's status as the de-facto world reserve currency, the X-factor provides a benchmark macro-foundation systemic metric for the 2008 "global" financial crisis.

For the US data, the X-factor values for the one-year to six-year maturities are 32.25, 14.75, 14.5, 9, 5.25 and 5, respectively. For the Canadian data, the X-factor values for the one-year to six-year maturities are 13.75, 6.75, 4.25, 4.5, 3.25, and 3.25, respectively. The corresponding X-factors are of larger magnitudes for the US, indicating larger externalities resulting from the financial crisis. Based on the X-factor values found along the yield curve, the risk diffusion process displays a tendency to diminish. Equivalently, the absolute size of the externality tends to diminish moving out along the yield curve. Since the crisis originated in the US, and the Canadian X-factors display lower values, the metrics suggest a cross-country diminishing contagion diffusion process. Table 3 records the X-factor values for each volatility maturity for both countries.

Table 3: Scaled X-factor metrics for US and Canada

	1Y	2Y	3Y	4Y	5Y	6Y
US	32.25X	14.75X	14.50X	9.00X	5.25X	5.00X
Can	13.75X	6.75X	4.25X	4.50X	3.25X	3.25X

The nature of the calculation for the X-factor metric suggests complete independence between individual X-factor values by maturity. The very small sample size (N=6) limits any meaningful inferential statistical analysis. The standard principle for diffusion in finance imputes a Brownian Motion diffusion process.

The X-factor sequences recording year-to-year percentage changes are provided for both countries. The one-year maturities, sometimes referred to as epicenters given their largest magnitudes, are sequenced as Xo.

US

$$\%\Delta X_1+\%\Delta X_2+\%\Delta X_3+\%\Delta X_4+\%\Delta X_5 = (-0.543)+(-0.017)+(-0.379)+(-0.417)+(-0.048) \quad (5)$$

Canada

$$\%\Delta X_1+\%\Delta X_2+\%\Delta X_3+\%\Delta X_4+\%\Delta X_5 = (-0.509)+(-0.370)+(0.059)+(-0.278)+(0.00) \quad (6)$$

Based on visual inspection of the two sequences, a normalized first-order diffusion process looks to follow a random process, where most magnitudes are associated with negative values.

Arguably, the epicenter magnitude (Xo) may conceivably be the actual driver of the diffusion process as agent market behavior reflects a (quasi) complete-markets approach through a pro rata allocation of additional risk moving out along the yield curve. The summation of the ratios of the epicenter X-factor value, Xo, to each of the five subsequent X-factor values, Xi, is provided for both countries:

$$US: \quad \Sigma\, Xi/Xo = .457+.450+.279+.163+.155 = 1.504 \quad (7)$$

$$Canada: \quad \Sigma\, Xi/Xo = .491+.309+.327+.236+.236 = 1.600 \quad (8)$$

The two epicenter-based pro rata allocation sequences comprising the summations look to follow a diffusion process that generally diminishes moving out along the yield curve. Interestingly, in total, the two pro rata risk allocations converge to similar values of 1.504 and 1.6. One interpretation is that market agents allocated, on a complete-markets pro rata basis, an additional 150 to 160 percent of systemic risk. Despite different epicenter magnitudes of 32.25 and 13.75, market agents allocated the same additional risk. The pro rata allocations differ between maturity values and between market agents of different countries, but in total, the pro rata risk allocations are seemingly epicenter uniform.

The X-factor metrics, for the most part, reveal diffusion processes displaying diminished allocations of systemic risk: (1) when moving away from the epicenter magnitudes, (2) for cross-country contagion from the originating country to the contagion county, and (3) from a complete-markets, pro rata risk allocation perspective.

6 Time-Series Analysis of the Algorithm Output

The representation algorithm empirics are organized around time-series data as opposed to a single data point. Evaluating for covariance-stationary time-series properties is therefore a natural extension of the representation algorithm empirics.

Two types of time-series volatility data sets are tested for unit root - the original, realized volatility time series and a constructed shadow volatility time series. The shadow volatility time series is built by splicing the positive state price volatilities with the shadow volatility metric (X-factor) adjusted volatilities that counterfactually restore realized negative state prices to positive state prices.

The model specification used is the standard re-parameterized model (non-zero unconditional mean) to test for unit root.

$$\Delta\sigma = \alpha + (\Phi\text{-}1)\,\sigma_{-1} + \epsilon \qquad (9)$$

To correct for autocorrelation, the equation is augmented by adding first differences lagged variables ($\Delta\sigma_{-1}$, $\Delta\sigma_{-2}$) until the autocorrelation is reduced to white noise. [22] Some equations do not need to be augmented, while other equations require one or two augmentations. The test for autocorrelation is the standard Durbin h statistic. [23] For all the volatility time series recorded in Table 4, the Durbin h statistic fails to reject the absence of autocorrelation at the five percent (5%) level of significance.

Table 4: US realized volatility time-series analysis

Year	Φ-1	Stnd Error	S-NS	ADF Statistic	Durbin h	Rho	ACA
1Y	-0.0241	0.0119	NS	-2.0204	-0.2466	-0.0293	2
2Y	-0.0343	0.0149	NS	-2.3024	-0.5149	-0.0610	2
3Y	-0.0644	0.0163	S	-3.9393	1.4626	0.1719	1
4Y	-0.0800	0.0190	S	-4.2127	1.0425	0.1221	1
5Y	-0.0977	0.0225	S	-4.3290	0.6890	0.0802	1
6Y	-0.1054	0.0245	S	-4.3015	0.5420	0.0629	1

US shadow volatility time-series analysis

Year	Φ-1	Stnd Error	S-NS	ADF Statistic	Durbin h	Rho	ACA
1Y	-0.4001	0.0953	S	-4.1985	0.2852	0.0197	0
2Y	-0.2450	0.0781	S	-3.1358	0.2803	0.0247	0
3Y	-0.0699	0.0428	NS	-1.6345	0.4231	0.0464	0
4Y	-0.0620	0.0396	NS	-1.5646	1.3785	0.1529	0
5Y	-0.0664	0.0330	NS	-2.0074	-0.2720	-0.0310	1
6Y	-0.0689	0.0339	NS	-2.0290	-0.2189	-0.0249	1

Canadian realized volatility time-series analysis

Year	Φ-1	Stnd Error	S-NS	ADF Statistic	Durbin h	Rho	ACA
1Y	-0.0526	0.0176	S	-2.9892	0.6073	0.0712	1
2Y	-0.0657	0.0205	S	-3.2054	0.1901	0.0222	1
3Y	-0.0786	0.0244	S	-3.2223	-0.0929	-0.0107	1
4Y	-0.0867	0.0257	S	-3.3721	0.1106	0.0128	1
5Y	-0.1008	0.0285	S	-3.5376	0.2208	0.0254	1
6Y	-0.1107	0.0293	S	-3.7711	0.2447	0.0281	1

Canadian shadow volatility time-series analysis

Year	Φ-1	Stnd Error	S-NS	ADF Statistic	Durbin h	Rho	ACA
1Y	-0.0708	0.0411	NS	-1.7220	0.4267	0.0474	1
2Y	-0.0775	0.0429	NS	-1.8073	-0.0927	-0.0101	0
3Y	-0.0637	0.0329	NS	-1.9373	-0.3921	-0.0447	1
4Y	-0.0662	0.0336	NS	-1.9703	-0.2695	-0.0306	1
5Y	-0.1600	0.0623	NS	-2.5661	0.1172	0.0117	0
6Y	-0.1541	0.0614	NS	-2.5063	0.1545	0.0155	0

For the US and Canadian data, Table 4 contains both the original realized and shadow volatility time-series analysis information, including the maturity dates, estimated parameters, standard errors, covariance-stationary property (S-NS), augmented Dickey-Fuller tau (ADF), number of autocorrelation adjustments (ACA), Durbin h statistic, and rho for the error term.

Given the Dickey-Fuller tau (ADF) statistics for the original realized volatilities (time series that include both efficient and inefficient pricing episodes), only the US one-year and two-year realized volatility time series are found to be non-stationary processes with ADF statistics of -2.0204 and -2.3024, respectively. All other US and all Canadian realized volatility time series are found to be stationary processes at the five percent (5%) level of significance. For the US data, ADF statistics range from a value of -3.9393 to a value of -4.3290. For the Canadian data, ADF statistics range from a value of -2.9892 to a value of -3.7711. In most of the original realized volatility time series tested, post crisis episodes of inefficient pricing are not associated with volatility compressions significant enough to indicate a non-stationary stochastic process.

For the X-factor constructed shadow volatility time series, the US one-year and two-year maturities are found to be stationary processes at the five percent (5%) level of significance with ADF statistics of -4.1985 and -3.1358, respectively. For all other US and all Canadian shadow volatility time series, there is a failure to reject the unit root, indicating non-stationary volatility time series processes. For the US data, the ADF statistics range from a value of -1.5646 to a value of -2.029. For the Canadian data, the ADF statistics range from a value of -1.7220 to a value of -2.5661.

For all the time series, original realized volatility time series switch from stationary (non-stationary) to non-stationary (stationary) when moving to their respective shadow volatility time series. The uniform time series covariance-stationary process reversals found when moving from original realized time series to shadow volatility time series validate the shadow volatility metric as having attributes of a dual variable. Inefficient pricing under a stationary process must revert to a non-stationary process to restore efficient pricing, and vice versa. Given ex ante bond prices, the shadow volatility metric consistently adjusts the time series back to optimal (efficient) pricing conditions. As found from the unit root test, the provision of efficient pricing in the representation algorithm can co-exist with either stationary or non-stationary time-series volatility processes.

For both the US and Canadian data, one anomaly does appear in the one-year volatility time series. Specifically, both US and Canadian one-year time series display episodes of negative state prices prior to the September 2008 crisis epicenter. Based on the representation algorithm, the US had a brief inefficient pricing episode from July 2006 to November 2006, where an X-factor adjustment of 3.5 is required to re-instate efficient pricing. Canada had a longer episode of inefficient pricing from December 2005 to December 2006, where an X-factor adjustment of 3.75 is required to re-instate efficient pricing. These pre-crisis, inefficient pricing episodes are not unusual as the representation algorithm is quite robust in that mispricing is also identified outside of a major financial event. When X-factor adjusting US realized (July to November 2006) and shadow volatility time series for these pre-crisis, inefficient pricing episodes, the time-series statistics remain generally consistent with the US one-year statistics recorded in Table 4. When X-factor adjusting the Canadian original realized time-series data (December 2005 to December 2006), the realized time series is a non-stationary process, resulting in a deviation from the statistical data recorded in Table 4. The X-factor adjustment to the Canadian shadow volatility time series is a non-stationary process, and remains consistent with the statistics recorded in Table 4. The full statistics are recorded in Table 5.

Table 5: Pre-crisis adjusted US realized and shadow volatility time series

Year	Φ-1	Stnd Error	S-NS	ADF Statistic	Durbin h	Rho	ACA
1Y	-0.0832	0.0490	NS	-1.6968	-0.0144	-0.0015	0
1Y	-0.4096	0.0961	S	-4.2618	0.3312	0.0225	0

Pre-crisis adjusted Canadian realized and shadow volatility time series

Year	Φ-1	Stnd Error	S-NS	ADF Statistic	Durbin h	Rho	ACA
1Y	-0.0612	0.0432	NS	-1.4163	1.6108	0.1765	0
1Y	-0.0841	0.0451	NS	-1.8625	0.3861	0.0423	1

Aside from the one violation described above, as recorded, the Dickey-Fuller statistics for unit root are consistent across the time-series tested. The covariance-stationary properties recorded are reliable in the context of the martingale representation algorithm.

From the time-series analysis, the US one-year and two-year original realized volatilities record large magnitude volatility compressions resulting in non-stationary time-series processes. All other original realized volatility time series are found to be stationary processes, not displaying large volatility compressions.

Based on the time-series analysis, in a financial crisis, systemic risk is not uniform. This finding is more than likely counter to the seemingly rational perception that large crises or panics have large uniform across-the-board impacts. In fact, there is more market resilience than probably acknowledged, and less market allocated systemic risk moving further out the yield curve.

7 Risk Metric Output Policy Implications

The martingale representation algorithm is a valid empirical platform for identifying episodes of efficient and inefficient pricing, and for measuring and evaluating systemic risk dynamics. Extrapolating from the systemic risk empirics derived from the two-country event study, several key findings have policy implications.

Systemic risk metrics should not be expected to be uniform for (1) all asset maturities and (2) on a cross-country basis, indicating elements of resilience. Contrary to differing cross-country systemic risk magnitudes, individual country diffusion patterns might be expected to more uniform, but not necessarily independent of their respective epi-center magnitudes.

Based on asset maturity, episodes of inefficient pricing should not be expected to be uniform. Longer dated maturities will most likely lag the epicenter event date. For the crisis originating location, shorter maturities will most likely lead the epicenter event date. For contagion locations, the shorter maturities will most likely lag the epicenter event date. Inefficient pricing dynamics found along the yield curve seem to be complicated.

The extreme market response by economic agents to risk-aversion, as measured by large US systemic risk metric magnitudes of 32.25 and 14.75, are shown to confirm the use of extraordinary monetary policy by US authorities during the aftermath of September 2008. The metrics demonstrate the extreme risk-off market response and severe liquidity dislocations in US money markets.

The paradigm shift from a market sentiment (confidence) of risk-on to risk-off can be sudden and violent, suggesting that market opinion has its own volatility that can quickly and significantly reverse episodes of optimal, efficient pricing and result in dramatic shifts in systemic risk profiles and contagion. Financial crises or panics reflect sudden reversals in market opinion or confidence, and the resulting risk-aversion impacts to liquidity conditions and contagion can be enormous to multiple segments of a market economy. Therefore, the ideology of unregulated financial markets guaranteeing complete information and complete markets that yield optimal risk-return pricing and dampening effects on market instability should be seriously re-evaluated. Macro-prudential financial regulation most likely is a very prudent endeavor.

8 Conclusion

The standard core asset valuation model in modern analytical finance, a martingale representation, is used as a one-period, re-setting algorithm to effectively differentiate episodes of efficient and inefficient pricing in time series. As applied, the algorithm is shown to be extremely simple to utilize, and powerful and robust in calibrating and measuring systemic risk and diffusion dynamics. The algorithm derives precise, consistent and plausible systemic risk metrics based on actual agent market behavior. The use of the shadow volatility metric provides a scaled metric quantifying systemic risk through the re-establishment of efficient pricing in the time series. The algorithm-derived metrics signal the impending pre-crisis risk buildup and calibrate around the critical financial crisis event date of September 2008. The time-series analysis validates the use of the shadow volatility metric as exhibiting attributes of a dual variable and confirms efficient pricing that is consistent with both stationary and non-stationary processes. The algorithm sacrifices abstraction and micro-foundation detail in favor of providing a complete system, economy-wide measure of systemic risk. The risk metrics are heuristically consistent with the historical stylized facts of financial crises related to confidence, liquidity conditions and contagion. The risk metrics confirm the extraordinary policy response by US officials. The algorithm is robust in that it effectively exploits inefficient pricing episodes under both normal and crisis conditions.

References

[1] M. Brunnermeier and M. Oehmke, Bubbles, Financial Crises and Systemic Risk, NBER Working Paper W18398, (2012).

[2] J. Ocampo, C. Rada, and L. Taylor, Growth and Policy in Developing Countries: A Structuralist Approach, Columbia University Press, New York, 2009.

[3] B. Malkiel, The Efficient Market Hypothesis and Its Critics, The Journal of Economic Perspectives, 17(1), (2003), 75-76.

[4] R. Thaler, Misbehaving: The Makings of Behavioral Economics, W. W. Norton and Company, New York, 2015.

[5] N. Taleb, Antifragile: Things that Gain from Disorder, Random House, New York, 2012.

[6] G. Gorton, Some Reflections on the Recent Financial Crisis, NBER Working Paper W18397, (2012).

[7] L. Hansen, Challenges in Identifying and Measuring Systemic Risk, University of Chicago and NBER Working Paper, (2012).

[8] D. Acemoglu, A. Ozdaglar, and A. Tahbez-Salehi, Systemic Risk and Financial Stability in Networks, American Economic Review, 105(2), (2015), 564-608.

[9] F. Allen, A. Babus, and E. Carletti, Financial Connections and Systemic Risk, NBER Working Paper W16177, (2010).

[10] M. Brunnermeier, and P. Cheridito, Measuring and Allocating Systemic Risk, Princeton University Working Paper, (2014).

[11] D. Bisias, M. Flood, A. Lo, and S. Valavanis, The Survey of Systemic Risk Analytics, Office of Financial Research, US Department of the Treasury, Working Paper Series W0001, (2012).
[12] J. Silvestre, The Market-Power Foundations of Macroeconomic Policy, Journal of Economic Literature, 31(1), (1993), 105-109.
[13] A. Blinder, After the Music Stopped: The Financial Crisis, the Response, and the Work Ahead, The Penguin Press, New York, 2013.
[14] R. McKinnon, International Money in Historical Perspective, Journal of Economic Literature, 31(1), (1993), 3-4.
[15] G. Akerlof and R. Shiller, Animal Spirits: How Human Psychology Drives the Economy, and Why It Matters for Global Capitalism, Princeton University Press, New Jersey, 2009.
[16] S. LeRoy, Efficient Capital Markets and Martingales, The Journal of Economic Literature, 27(4), (1989), 1594-1595.
[17] S. Neftci, Principles of Financial Engineering, Elsevier Academic Press, California, 2004.
[18] S. Neftci, An Introduction to the Mathematics of Financial Derivatives, Elsevier Academic Press, California, 2000.
[19] W. Baumol, Economic Theory and Operations Analysis, Prentice-Hall, New Jersey, 1977.
[20] R. Lowenstein, America’s Bank: The Epic Struggle to Create the Federal Reserve, The Penguin Press, New York, 2015.
[21] H. Taha, Operations Research: An Introduction, Macmillan Publishing Company, New York, 1987.
[22] G. Gonzalez-Rivera, Forecasting for Economics and Business, Pearson Education, New Jersey, 2013.
[23] G. Judge, R. Hill, W. Griffiths, H. Lutkepohl, and TC. Lee, Introduction to the Theory and Practice of Econometrics, John Wiley and Sons, New York, 1982.