Optimization of Model-Reference Variable-Structure Controller Parameters for Direct-Current Motor

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Abstract

Modified Model-Reference Variable-Structure (MRVS) control methods are presented in the paper. Elimination of control signal chattering in order to reduce energy consumption is further shown. Four different integral optimization criterions with the following integral performance indices are described: integral of absolute value of function, integral of squared function, integral of time multiplied by absolute value of function and integral of time multiplied by squared function. Error signal, amount of energy consumption and their combination are used as functions in these integral performance indices for optimization of MRVS controller parameters. The presented integral optimization criterions are tested by computer simulations on modified MRVS direct-current motor control. MRVS

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Article Info: *Received* : September 3, 2012. *Revised* : September 30, 2012 *Published online* : November 20, 2012

controller parameters optimization results in further reduction of energy consumption. Modified MRVS control methods are tested with respect to load change, complex sinusoidal external disturbance and measurement noise and the simulation results show satisfactory robustness of all modified MRVS control methods.

Mathematics Subject Classification: 93A30

Keywords: Direct-current motor, Model-reference variable-structure control, Optimization, Robustness

1 Introduction

DC servomotors are very often used in robotics. There is a big problem of achieving and maintaining accurate robot trajectory tracking during high speed robot motions with large dynamic forces. This can be obtained, together with proper robustness, by using combination of Model-Reference Adaptive Control (MRAC) and Variable-Structure Control (VSC), as it is shown in [1,3,6,10].

Much better results regarding minimal tracking error and amount of energy consumption can be achieved with controller parameters optimization, as can be seen in [5,8]. Therefore, two types of combined MRVS control methods and four different integral optimization criterions are further presented.

2 Two Modified MRVS Control Methods

The original scheme of MRVS control method is consisting of two control loops: outer and inner and is given on Figure 1.

The outer control loop consists of the second order reference model and VS

controller which parameters λ and γ are optimized.

The input signal to the second order system is adjusted by the outer loop, so the errors between the model states and the system states become zero.

Motor shaft angle $\theta(t)$ and velocity $\dot{\theta}(t)$ are system state variables $x_1(t)$ and $x_2(t)$, respectively.



Figure 1: The scheme of MRVS control with controller parameters optimization

The following common error can be defined for state variables on Figure 1:

$$\sigma(e, \dot{e}) = \lambda \cdot e(t) + \dot{e}(t),$$

where:

$$e(t) = x_{1M}(t) - x_1(t),$$

$$\dot{e}(t) = x_{2M}(t) - x_2(t).$$

The goal of the combined control is elimination of errors:

$$\sigma(e, \dot{e}) = 0,$$

which can be achieved by using the following control law with the sign function, as shown in [2,11]:

$$u_{A}(t) = \gamma \cdot \operatorname{sgn}(\sigma(t)) = \gamma \cdot \frac{\sigma(t)}{|\sigma(t)|}.$$
(1)

Because of sign function in (1), signal $u_A(t)$ has chattering and therefore there is

a great dissipation of energy. To reduce this chattering, this sign function in control signal (1) can be replaced by continuous control signal proposed in [7]:

$$u_{A}(t) = \gamma \cdot \frac{\sigma(t)}{|\sigma(t)| + \delta},$$
(2)

or by a continuous approximation with a high gain, i.e. saturation function, which is suggested in [13]:

$$u_{A}(t) = \gamma \cdot sat(\sigma(t)) = \begin{cases} -\gamma, \sigma < -\delta \\ \gamma \cdot \frac{\sigma}{\delta}, -\delta < \sigma < \delta \\ \gamma, \sigma > \delta \end{cases},$$
(3)

or by an exponential function [9]:

$$u_{A}(t) = \gamma \cdot \operatorname{sgn}(\sigma(t)) \cdot \left(1 - e^{-\frac{|\sigma|}{\delta}}\right), \tag{4}$$

where δ denotes the thickness of boundary layer.

The inner control loop (Figure 2) is composed of the process and the controller, which act together as the second order system. It consists of a PD-controller with proportional gain P and derivational gain D, amplifier with gain K_{AM} , DC motor and sensors for motor shaft angle and velocity measurement.



Figure 2: The scheme of the second order system of the inner DC motor control

The parameters of DC motor on Figure 2 are: resistance R_a and inductance L_a

of armature winding
$$\left(K_a = \frac{1}{R_a}, T_a = \frac{L_a}{R_a}\right)$$
, torque constant K, moment of inertia

 J_m , viscous motor friction coefficient b_{vm} .

The transfer function of open-loop system on Figure 2 is, according to [9]:

$$G_{os}(s) = \frac{K_{tot} \cdot \left(1 + \frac{D}{P} \cdot s\right)}{s \cdot \left(1 + T_{1PR} \cdot s\right) \cdot \left(1 + T_{2PR} \cdot s\right)}$$

where:

$$\begin{split} K_{tot} &= \frac{P \cdot K_{AM} \cdot K_a \cdot K}{b_{vm} + K_a \cdot K^2}, \\ T_{1PR} &= \frac{1}{2} \cdot \frac{T_a \cdot b_{vm} + J_m}{b_{vm} + K_a \cdot K^2} \cdot \left(1 - \sqrt{1 - \frac{4 \cdot T_a \cdot J_m \cdot \left(b_{vm} + K_a \cdot K^2\right)}{\left(T_a \cdot b_{vm} + J_m\right)^2}} \right), \\ T_{2PR} &= \frac{1}{2} \cdot \frac{T_a \cdot b_{vm} + J_m}{b_{vm} + K_a \cdot K^2} \cdot \left(1 + \sqrt{1 - \frac{4 \cdot T_a \cdot J_m \cdot \left(b_{vm} + K_a \cdot K^2\right)}{\left(T_a \cdot b_{vm} + J_m\right)^2}} \right), \end{split}$$

To get the second order closed-loop inner system shown on Figure 2, the derivational coefficient of the PD-controller is set to eliminate the smaller motor time constant:

$$D = P \cdot T_{1PR}.$$
 (5)

Then the closed-loop transfer function of the second order system on Figure 2 is [9]:

$$G_{cs}(s) = \frac{1}{1 + \frac{1}{K_{tot}} \cdot s + \frac{T_{2PR}}{K_{tot}} \cdot s^{2}}.$$
(6)

The second order system can be also described by the following transfer function:

$$G_{cs}(s) = \frac{K_s}{1 + 2 \cdot \xi_s \cdot T_s \cdot s + T_s^2 \cdot s^2}$$
(7)

The comparison of equations (6) and (7) gives the system coefficients:

$$K_{s} = 1$$
, $T_{s} = \sqrt{\frac{T_{2PR}}{K_{tot}}}$, $\xi_{s} = \frac{1}{2 \cdot \sqrt{K_{tot} \cdot T_{2PR}}}$,

which leads to the following controller parameter setting [9]:

$$P = \frac{b_{vm} + K_a \cdot K^2}{K_{AM} \cdot K_a \cdot K} \cdot \frac{1}{4 \cdot T_{2PR}} \cdot \frac{1}{\xi_s^2}$$
(8)

The proportional coefficient of PD-controller given in (8) can be set according to demand of having no overshooting $\xi_s = 1$.

In the reality, the ideal PD-controller has to be replaced by the real PD-controller with the following transfer function:

$$G_{PDreal}(s) = \frac{P + D \cdot s}{1 + T_{v} \cdot s}$$

where T_{v} is very small time constant.

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Analogous to (6), the second order reference model on Figure 1 can be described as:

$$G_M(s) = \frac{K_M}{1 + 2 \cdot \xi_M \cdot T_M \cdot s + T_M^2 \cdot s^2}$$
(9)

The reference model in (9) can be chosen to be faster then the second order inner system with DC motor.

The efficiency of the applied MRVS controller can be improved by optimization of its parameters (Figure 1) according to the optimization criterion with the desired performance index, as shown in [5,8].

To obtain minimal error and energy consumption, VS controller parameters λ and γ have to be optimized (Figure 1) by different optimization criterions which are further presented.

3 Four Integral Optimization Criterions

The selection of an appropriate performance index is very important. Commonly used performance indices are based on integral performance measures, such as:

- integral of absolute value of error:

$$J_{IAE} = \int_{0}^{\infty} \left| e(t) \right| dt,$$

- integral of squared error:

$$J_{ISE} = \int_{0}^{\infty} e^{2}(t) dt,$$

- integral of time multiplied by absolute value of error:

$$J_{ITAE} = \int_{0}^{\infty} t \cdot |e(t)| dt,$$

- integral of time multiplied by squared error:

$$J_{ITSE} = \int_{0}^{\infty} t \cdot e^{2}(t) dt$$

The integral performance indices J_{IAE} , J_{ISE} , J_{ITAE} , J_{ITSE} have to be minimized to acquire the DC motor MRVS controller parameters for the best system performance. After optimization, the resulting system is optimal with respect to the selected criterion.

The amount of energy consumption is calculated [4]:

$$E = \int_{0}^{T_{sim}} U_a \cdot I_a \cdot dt, \quad if \ U_a \cdot I_a > 0, \tag{10}$$

where T_{sim} denotes simulation time, U_a is armature voltage and I_a is armature current of a DC motor. All these optimization criterions are applied on three modified MRVS control methods of DC motor and the results of computer simulations are further explained.

4 Simulation Results

All four proposed optimization criterions applied on MRVS controller parameters optimization are tested by computer simulations in Matlab in the case of step input signal $u_{ref}(t) = 5 \cdot S(t)$ acting on DC motor with the following parameters: resistance $R_a = 16.35 [\Omega]$ and inductance $L_a = 0.3004 [H]$ of armature winding, torque constant K = 1.211 [Vs], moment of inertia $J_m = 0.0157 [kg \cdot m^2]$, viscous motor friction coefficient $b_{vm} = 0.015 [kg \cdot m^2 \cdot s^{-1}]$.

The simulation time $T_{sim} = 2[s]$ and amplifier coefficient $K_{AM} = 10$ are chosen. The maximal allowed model tracking error for DC motor is set to $e_{max} = 0.05 \cdot u_{ref}$. The second order system with DC motor is chosen to have critical damping $\xi_s = 1$, so the following coefficients of PD-controller are calculated according to (8) and (5): proportional coefficient $P = 0.268 [V \cdot rad^{-1}]$ and derivative coefficient $D = 0.00561 [V \cdot s \cdot rad^{-1}]$.

The reference model is chosen to be 3 times faster then the second order system with DC motor. The goal is to find MRVS controller parameters λ and γ which are optimal according to chosen optimization criterion.

First it is necessary to determine the initial values of parameters λ and γ . The simulaton results show that values of $\lambda = 10 [s^{-1}]$ and $\gamma = 40$ insure maximal allowed model tracking error of 5%, but chattering of control signal and amount of energy consumption are too big E = 64.5784 [J], so the original MRVS control method has to be modified by choosing proper value for thickness of boundary layer δ on previous explained ways. The simulation results show that minimal thickness of boundary layer at which the amount of energy consumption rapidly decreases is $\delta = 0.1$. The influence of thickness of boundary layer δ on model tracking error e(t) is shown on Figure 3 for continuous control function with $\lambda = 10 [s^{-1}]$ and $\gamma = 42$, on Figure 4 for control law with saturation function with $\lambda = 10 [s^{-1}]$ and $\gamma = 40$ and on Figure 5 for control law with exponential function with $\lambda = 10 [s^{-1}]$ and $\gamma = 40$.



Figure 3: The influence of parameter δ on e(t) for continuous control function (2)



Figure 4: The influence of parameter δ on e(t) for saturation control function (3)



Figure 5: The influence of parameter δ on e(t) for exponential control function (4)

As can be seen from Figures 3-5, the thickness of boundary layer δ has to be minimal because bigger values δ increase model tracking error e(t), so the best value for the thickness of boundary layer δ is minimum, i.e. $\delta = 0.1$. Parameter δ decreases the amount of energy consumption (10) to the following values: E = 44.3899 [J] for continuous control function, E = 45.5841 [J] for saturation control function and E = 44.5811 [J] for exponential control function. The maximal allowed model tracking error in all of these methods remains equal, i.e. $e_{\text{max}} = 0.05 \cdot u_{ref}$.

It is also interesting to analyze the influence of VS controller parameters λ and γ on model tracking error e(t), for continuous control function (Figures 6 and 9), for control law with saturation function (Figures 7 and 10) and exponential function in control law (Figures 8 and 11).



Figure 6: The influence of parameter λ on e(t) for continuous control function (2)



Figure 7: The influence of parameter λ on e(t) for saturation control function (3)



Figure 8: The influence of parameter λ on e(t) for exponential control function (4)



Figure 9: The influence of parameter γ on e(t) for continuous control function (2)



Figure 10: The influence of parameter γ on e(t) for saturation control function (3)



Figure 11: The influence of parameter γ on e(t) for exponential control function (4)

The results of simulations on Figures 6-8 show that the increasing of parameter λ is making system transient response faster without changing of model tracking error maximum, with satisfactory behaviour at $\lambda \ge 10 \lceil s^{-1} \rceil$.

From Figures 9-11 it can be seen that the increasing of another VSC parameter γ causes decreasing of maximal allowed tracking error e_{max} .

From Figures 6-11 it can also be seen that the simulation results are very similar for all three modified MRVS control methods.

The next goal is to reduce model tracking error e(t) and the amount of energy consumption even more by optimizing the MRVS controller parameters.

Therefore, optimizations of MRVS controller parameters λ and γ with finding minimum values of integral performance indices J_{IAE} , J_{ISE} , J_{ITAE} , J_{ITSE} are performed by using function *fminsearch* in Matlab. The results of these optimizations are given in Tables 1, 2, 3 for modified MRVS controllers with continuous, saturation and exponential function in their control laws.

criterion	IAE	ISE	ITAE	ITSE
δ	0.1	0.1	0.1	0.1
$\lambda_{ m opt}$	34.2108	35.6206	31.1744	35.1828
${\cal Y}_{ m opt}$	173.2484	182.3955	55.0077	168.6228
J _{opt}	0.0136	0.0020	8.6734E-4	1.1376E-4
e _{max} [%]	4.162	4.162	4.2042	4.162
Е [J]	107.7331	113.6582	88.0445	111.4068

 Table 1: Optimal parameters according to minimal error criterion for modified

 MRVS controller with continuous function

criterion	IAE	ISE	ITAE	ITSE
δ	0.1	0.1	0.1	0.1
$\lambda_{ m opt}$	31.2465	33.1328	31.1153	34.8232
${\mathcal Y}_{\mathrm{opt}}$	51.5258	52.7662	51.6632	94.5837
J _{opt}	0.0133	0.0020	8.4148E-4	1.1374E-4
e _{max} [%]	4.162	4.162	4.162	4.162
E [J]	92.2579	98.5690	91.9981	109.0425

 Table 2:
 Optimal parameters according to minimal error criterion for modified

MRVS controller with saturation function

 Table 3: Optimal parameters according to minimal error criterion for modified MRVS controller with exponential function

criterion	IAE	ISE	ITAE	ITSE
δ	0.1	0.1	0.1	0.1
$\lambda_{ m opt}$	34.2286	35.4564	30.9469	35.0978
${\cal Y}_{ m opt}$	165.2828	92.486	51.6937	89.2377
$\mathbf{J}_{\mathrm{opt}}$	0.0136	0.0020	8.4608E-4	1.1375E-4
e _{max} [%]	4.162	4.162	4.1665	4.162
$\mathrm{E}\left[\mathrm{I} ight]$	107.8351	110.5805	89.5097	108.9373

The results presented in Tables 1-3 show that optimizations reduce all integral performance indices and maximal model tracking errors, in comparison with the corresponding initial values, for all MRVS control methods. On the other hand, the amount of energy consumption is 2-2.56 times enlarged for all MRVS control methods, which means that another optimization criterion has to be found.

criterion	continuous	saturation
δ	0.1	0.1
λ_{opt}	3.1793	12.3011
γ_{opt}	-5.0254	-5
$J_{opt} = E_{\min} \left[J \right]$	7.8411E-5	3.6350E-16
e _{max} [%]	99.9048	99.8369

Table 4: Optimal parameters according to minimal energy criterion

Another optimization of MRVS controller parameters λ and γ can be done by finding minimum value of the amount of energy consumption (10). This optimization is also performed by using function *fminsearch* in Matlab and the results are given in Table 4 for both modified MRVS controllers with continuous function and saturation.

The simulation results given in Table 4 show that optimization of controller parameters λ and γ according to minimal energy criterion is not a good solution. Although the amount of energy consumption is significantly reduced, maximal model tracking error e_{max} is 20 times enlarged, so the optimization results are not as desired.

The best optimization results are expected to achieve by combining both error and energy optimization criterions, as follows:

$$\begin{split} J_{IAEN} &= \int_{0}^{T_{sim}} \left(\left| e(t) \right| + w \cdot U_a \cdot I_a \right) \cdot dt , \qquad & \text{if } U_a \cdot I_a > 0, \\ J_{ISEN} &= \int_{0}^{T_{sim}} \left(e^2(t) + w \cdot U_a \cdot I_a \right) \cdot dt , \qquad & \text{if } U_a \cdot I_a > 0, \end{split}$$

$$\begin{split} J_{ITAEN} &= \int_{0}^{T_{sim}} \left(t \cdot \left| e(t) \right| + w \cdot U_a \cdot I_a \right) \cdot dt , \qquad \text{if } U_a \cdot I_a > 0, \\ J_{ITSEN} &= \int_{0}^{T_{sim}} \left(t \cdot e^2(t) + w \cdot U_a \cdot I_a \right) \cdot dt , \qquad \text{if } U_a \cdot I_a > 0, \end{split}$$

where *w* denotes weighting factor.

These new integral performance indices J_{IAEN} , J_{ISEN} , J_{ITAEN} , J_{ITSEN} are minimized with selected factor w = 1 and the results are given in Tables 5-7 for modified MRVS controllers with continuous, saturation and exponential function in their control laws.

Table 5: Optimal parameters according to minimal error and energy criterion formodified MRVS controller with continuous function

criterion	IAE	ISE	ITAE	ITSE
δ	0.1	0.1	0.1	0.1
λ_{opt}	10.125	5.8772	9.6374	6.5761
γ_{opt}	0.29424	1.1937	0.20046	0.39561
J_{opt}	3.0879	4.2675	2.2104	2.6438
e_{\max} [%]	55.0067	50.6178	55.4946	54.4875
E[J]	1.5880	2.1634	1.5290	1.6427

From Tables 5-7 can be seen that a proper combination of error and energy optimization criterions, i.e. finding minimum values of the sum of integral performance indices J_{IAE} , J_{ISE} , J_{ITAE} , J_{ITSE} and the amount of energy consumption E is the best solution because it decreases e_{max} and increases E. In these optimizatins factor w has to be changed untill the desired values of e_{max} and E are achieved.

criterion	IAE	ISE	ITAE	ITSE
δ	0.1	0.1	0.1	0.1
λ_{opt}	11.444	5.6372	11.2129	6.148
γ_{opt}	0.27259	1.1473	0.1859	0.38199
$m{J}_{opt}$	3.0905	4.2792	2.2104	2.6439
e_{\max} [%]	55.1092	50.7889	55.5642	54.5422
E[J]	1.5792	2.1458	1.5230	1.6361

 Table 6: Optimal parameters according to minimal error and energy criterion for modified MRVS controller with saturation function

 Table 7: Optimal parameters according to minimal error and energy criterion for modified MRVS controller with exponential function

criterion	IAE	ISE	ITAE	ITSE
δ	0.1	0.1	0.1	0.1
λ_{opt}	10.879	5.6782	10.2291	6.438
γ_{opt}	0.27596	1.158	0.1905	0.39098
\boldsymbol{J}_{opt}	3.0891	4.2724	2.2096	2.6428
e_{\max} [%]	55.0918	50.7396	55.5401	54.4962
$E\left[J\right]$	1.5798	2.1475	1.5245	1.6411

From Tables 5-7 can also be noticed that very similar results for e_{max} and E are obtained for all different modified MRVS control methods by using the same optimization criterion.

Robustness of all presented modified MRVS control methods is further

tested. The analysis is performed for load change (Figures 12 and 13), influence of white measurement noise (Figures 14 and 15) and complex sinusoidal external disturbance (Figures 16 and 17).

The influence of load change on the robustness of all MRVSC methods is analysed by adding step functions to motor shaft as different loads from 0 to 0.1 [Nm]. The results related to e_{max} and E are shown in Figures 12 and 13. One can see that values of e_{max} are very close for all three modified control methods, while the energy consumption is increased for modified MRVS control method with saturation.



Figure 12: Maximal error for MRVSC methods in the case of load change



Figure 13: Energy consumption for MRVSC methods in the case of load change



Figure 14: Maximal error for MRVSC methods in the case of measurement noise



Figure 15: Energy consumption for MRVSC methods in the case of measurement noise



Figure 16: Maximal error for MRVSC methods in the case of complex external disturbance



Figure 17: Energy consumption for MRVSC methods in the case of complex external disturbance

The robustness of studied MRVSC methods is examined in the presence of white measurement noise added to motor shaft speed signal (varied in the range from 0% to 20% of measured speed). The results given in Figures 14 and 15 show that added measurement noise slightly increases energy consumption in all modified MRVSC methods.

The robustness of modified MRVSC methods is also tested to the influence of complex sinusoidal external disturbance added to motor shaft:

$$T_d = T_{Ad} \cdot \left(\sin\left(0.1 \cdot t\right) + \sin\left(t\right) + \sin\left(10 \cdot t\right) + \sin\left(100 \cdot t\right) \right)$$

where T_{Ad} changes from 0 to 0.1 [Nm]. From the results displayed in Figures 16 and 17, it can be seen that the influence of such disturbances is very similar for all modified MRVSC methods, although the MRVSC method with saturation function in the control law has the greatest value of consumed energy.

5 Conclusion

The efficiencies of all four optimization criterions applied to three modified MRVS control methods are proved in the case of DC motor control in reducing of energy consumption and maximal model tracking error. Robustness of these MRVSC methods is also tested in the case of load changes, white measurement noise, complex sinusoidal external disturbance and the results are satisfactory. Therefore, it will be very useful to analyse the effectiveness of these optimization criterions applied to more complex case of MRVS robot control.

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