# Nonlinear Interactions and Volatility Spillovers between Stock and Foreign Exchange Markets: The STVEC-STGARCH-DCC Approach 

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#### Abstract

This study aims to investigate the interactions, volatility spillovers and smooth transition effects between stock and foreign exchange markets in emerging versus developed countries by the Smooth Transition Vector Error Correction-Smooth Transition GARCH with Dynamic Conditional Correlation model (STVE-STGARCH-DCC). The empirical results yield several findings. Firstly, boom stock markets in emerging countries will trigger their domestic currency appreciation, while prosperous stock markets in developed countries result in currency depreciation. Secondly, the conditional variances for stock markets mainly result from unexpected shocks, past volatility, and short-term impact effects, thus leading to a persistence of volatility in both emerging and developed markets. The conditional variances for foreign exchange markets display similar patterns but show weaker short-term impact effects and slower transition speeds. Thirdly, unexpected shocks in a stock market broadly affect its own stock volatility, while those only affect India's volatility in the rupee market. In contrast, unexpected shocks in foreign exchange markets mainly affect foreign exchange volatility, except for India; however, those influence their stock volatility only for emerging countries, such as India and South Africa. Lastly, developed markets are more efficient than emerging markets are.


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## 1. Introduction

The relationships between stock prices and exchange rates has attracted much attention from market participants, academicians as well as public officials, because stock and foreign exchange markets simultaneously have played central roles in the economic development of a country for a long time. International capital flows move easier and quicker than ever before since the integration and deregulation of international financial markets in the 1980s. The popularity of carry trades or arbitrage trading strategies further boosts the amount of hot money cross borders. So far, exchange rates have played a key role in the national flows of hot money in and out of stock markets. Moreover, stock volatility and changes in foreign exchange rates could lead to an apparent impact on the international investment portfolios. Thus, international investors often predict future market trends by assessing the interrelationships and volatility spillovers between these two markets. International investors often take a wait-and-see attitude toward imperfect financial markets, in which information asymmetry, noise traders, and heterogeneous arbitrageurs prevalently exist. As a result, these international investors with varied degrees of risk aversion often make diverse interpretations on the same financial events, leading them to make different decisions for their portfolio management. In this case, time series data could not fully reflect the contents of relevant public information. Thus, stock price and exchange rate movements may be adjusted gradually in a smooth manner, i.e., smooth transition occurs. As to the nonlinear interactions between stock and foreign exchange markets, most previous studies usually estimate the conditional mean equation based on a linear viewpoint which may not accurately reflect the dynamic adjustment processes under different market statuses. Taking dynamic adjustment processes and smooth transitional parameters into consideration, the smooth transition autoregressive (STAR) models can be applied to time series data as an extension of autoregressive models and as a generalization of threshold autoregressive (TAR) models. The STAR model allows time-series of economic variables to smoothly switch from one regime to the other, rather than a sudden jump. In addition, it is more flexible to permit a higher order of model parameters through the smooth transition function. Integrated with the conditional mean equation of the STAR model, this study estimates the conditional variance equation of the smooth transition generalized autoregressive conditional heteroskedastic (STGARCH) model, which owns asymmetric and nonlinear smooth transition mechanisms of volatility. Compared to the Glosten-Jagannathan-Runkle generalized autoregressive conditional heteroscedastic (Glosten, Jagannathan, and Runkle, 1993, hereafter GJR-GARCH) and exponential generalized autoregressive conditional heteroscedastic (Nelson, 1991, hereafter EGARCH) models, the STGARCH model captures the leverage effect of volatility by a smooth transition structure. To sum up, our conditional mean and variance equations involve the smooth transition vector error correction (STVEC) model, STGARCH model, as well as dynamic conditional correlation (DCC). By using the STVEC-STGARCHDCC model, this research seeks to examine nonlinear interactions and volatility
spillovers between stock and foreign exchange markets. Specifically, we first analyze how cross-market assets affect stock returns and changes in foreign exchange rates, respectively, under different market statuses. Next, we discuss the effects of volatility by testing whether unexpected shocks, past volatility, or the U.S. stock volatility affect self- and cross-market volatility. Finally, the impacts of critical financial crises on the dynamic conditional correlations between stock returns and changes in foreign exchange rates are investigated. Our sample includes emerging and mature markets, since it is expected that the transaction costs and the degree of information response are different in emerging versus mature markets. The remainders of this study are organized as follows. Section 2 surveys the related literature. Section 3 describes the data and methodology. Section 4 presents the empirical results, interpretations and implications. Section 5 concludes this paper.

## 2. Literature Review

Previous studies on the relationships between stock and foreign exchange markets often use non-stationary variables to analyze their correlations or test the integrated effects between these two markets. Nieh and Lee (2001) established the vector error correction model (VECM); however, they document that stock and foreign exchange markets do not display a long-term co-movement relationship. Instead, there is a short-term dynamic relationship and these two markets only have the oneday predicting power to each other. On the other hand, the existing literature often estimates the GARCH models, proposed by Bollerslev (1986), to measure the timevarying relationship between stock and foreign exchange markets. For example, Ajayi, Friedman and Mehdian (1998) presented that stock prices unidirectionally affect foreign exchange rates in mature markets but not in emerging ones. Yang and Doong (2004) adopted a multivariable EGARCH model to explore the volatility spillovers between these two markets. Their results support the existence of volatility spillovers from stock markets of G7 industrialized countries to foreign exchange markets, but not the other way around. The relationships between real exchange rate return and stock return can be different under different states of stock markets and economic development (Pan, Fok, and Liu, 2007). The empirical evidences between stock prices and foreign exchange rates are mixed although theories suggested causal relationships between these variables (Lin, 2012; Tsai, 2012). Ülkü and Demirci (2012) investigated the relationships between stock and foreign exchange markets of emerging and developed economies in eastern and western European countries. Their analysis variables include exchange rates, stock indices, and global market indices, showing that a significant co-movement between stock and foreign exchange markets is driven by the stock returns in developed economies. The co-movements between stock prices and exchange rates also depend on both stock market depth and transition status. Moreover, the time-series of these economic variables seem smoothly switch from one regime to the other rather than a sudden jump.
Regarding to smooth transition effect, Teräsvirta and Anderson (1992) used
industrial production indices to detect the nonlinearity of business cycles. As a transition variable, the business cycle can be divided into two states, namely expansion and contraction. Their nonlinear STAR models detect the response of industrial production indices to large negative shocks (e.g., oil price shocks). Röthig and Chiarella (2007) followed up to adopt the futures returns as a two-regime transition variable to estimate STAR models, They find a similar structure of nonlinearities with regard to the different regimes, transition variables, and the value at which the transition occurs. Moreover, Liu and Chen (2016) detected the interactions among house prices, interest rates, and stock prices by applying STVEC-GARCH model in which the positive (negative) transition variable means the expansion (contraction) regime. They reconfirm that a nonlinear and cointegrated relation among the three variables.
It is intuitive to combine a nonlinear conditional mean equation with a conditional variance equation. For example, Lee, Liu, and Chiu (2008) combined the conditional mean equation of the STAR model with the conditional variance equation of the GRACH $(1,1)$ model to explore the dynamic adjustment process of daily settlement prices of spots and futures in the Brent crude oil markets. Yaya and Shittu (2014) specified an asymmetric STAR model with linear and nonlinear GARCH approaches by Monte Carlo approach. Their work has a more applicable dynamic adjustment process than do the traditional STAR models, exhibiting a higher level of goodness-of-fit. Generally speaking, financial asset prices often exhibit volatility clustering as well as asymmetric effects between the positive and negative shocks which are so-called a leverage effect. Precisely, the asymmetric effect means that the market volatility caused by negative news (i.e. unanticipated price decrease) is greater than that by positive news (i.e. unanticipated price increase). In addition to volatility asymmetry, there might exist smooth transition attributes of assets volatility under different economic statuses in the real world (Gonzalez-Rivera, 1996; Lundbergh and Teräsvirta, 1998; Nam, 2002; Sollis, 2009; Yaya and Shittu, 2014; Liu and Chen, 2016; Escribano and Torrado, 2018). Therefore, it is greatly appropriate to connect the nonlinear conditional mean equation to the conditional variance equation with smooth transition mechanism to explain the nonlinear price movements and volatility spillovers of financial assets (e.g., an extension of GJR-GARCH models).

## 3. Data and Methodology

### 3.1 Data Sources and Measurement

This study examines interactions and volatility spillovers between stock and foreign exchange markets in BRICS countries such as India and South Africa with available data. To compare with well-developed markets, we also investigate well-developed countries such as Germany and Japan.
Stock indices and foreign exchange rates among our sample countries are from the DataStream database. Our daily-data sample ranges from January 1, 2002 to December 31, 2018. The currency quotation is essential for anyone wanting to trade
currencies in foreign exchange markets. We are looking at the currency pair in terms of indirect quotation for each sample country, which the U.S. dollar would be the quote currency (i.e., in USD), and the currency of each sample country would be the base currency.
This study denotes the original prices of stock prices and foreign exchange rates as $P_{i, t}$. Subscript $i=1$ is for stock price and $i=2$ is for exchange rate; subscript $t$ represents trading day. We measure the daily stock return or the change rate of foreign exchange rate by $R_{i, t}=\left(\ln P_{i, t}-\ln P_{i, t-1}\right)$, where $P_{i, t}$ is the closing stock price or exchange rate, and $R_{i, t}$ is the stock return or change rate of foreign exchange rate. Specifically, stock return is defined as the natural logarithm of the closing index to the previous close index expressed as a percentage. Similarly, the change rate of foreign exchange rates is measured by the natural logarithm of the closing rate to the previous closing rate expressed as a percentage. Based on our currency quotation, the increase (decrease) in the change rates of foreign exchange rates indicate the currency appreciation (depreciation).

### 3.2 Methodology

### 3.2.1 Smooth Transition Autoregressive Model

(1) Model Specifications for the STAR Model

Teräsvirta and Anderson (1992) estimated the univariate STAR model, which allows the economic indicator to smoothly switch between two distinct regimes rather than a sudden jump from one to the other. The $k$-day lagged univariate STAR model to express the time series $y_{t}$ as follows:

$$
\begin{equation*}
y_{t}=\left(u_{1}+\sum_{j=1}^{k} \beta_{1, j} y_{t-j}\right)\left(1-F\left(s_{t-d}\right)\right)+\left(u_{2}+\sum_{j=1}^{k} \beta_{2, j} y_{t-j}\right) F\left(s_{t-d}\right)+\varepsilon_{t}, \tag{1}
\end{equation*}
$$

where $u_{1}$ and $u_{2}$ are respectively intercepts corresponding to different market conditions. The error terms follow the normal distribution, i.e., $\varepsilon_{t} \sim N\left(0, \sigma^{2}\right)$. $F\left(s_{t-d}\right)$ indicates the continuous transition function, lying between zero and one; $d$ denotes the lagged number of the transitional variables. This STAR model smoothly switches between $F\left(s_{t-d}\right)=0$ and $F\left(s_{t-d}\right)=1$. The time series $y_{t}$ relies on $\left(u_{1}+\sum_{j=1}^{k} \beta_{1, j} y_{t-j}\right)$ much more than on $\left(u_{2}+\sum_{j=1}^{k} \beta_{2, j} y_{t-j}\right)$, as $F\left(s_{t-d}\right)$ approaches zero. On the contrary, $y_{t}$ principally depends on $\left(u_{2}+\sum_{j=1}^{k} \beta_{2, j} y_{t-j}\right)$, when $F\left(s_{t-d}\right)$ is close to one. Thus, the STAR model, by economic implications,
demonstrates the weighted average of two linear regression models, corresponding to two different weights of $\left(1-F\left(s_{t-d}\right)\right)$ and $F\left(s_{t-d}\right)$, respectively. Following Teräsvirta and Anderson's (1992) approach, we consider both logistic and exponential transition functions as follows.

## A. Logistic Transition Function

The logistic transition function can be defined as follows:
$F\left(y_{t-d} ; \gamma, c\right)=\frac{1}{1+\exp \left[-\gamma\left(y_{t-d}-c\right)\right]}, \gamma>0$
where $\gamma$ indicates the transition speed from one regime to the other one, and $c$ denotes the threshold value. The transition function $F\left(y_{t-d} ; \gamma, c\right)$ equal to 0.5 when the transitional variables $y_{t-d}$ equals the threshold value of $c$. In an upper regime, the transition function $F\left(y_{t-d} ; \gamma, c\right)$ equal to one as the transitional variables $y_{t-d}$ or $\left(y_{t-d}-c\right)$ is positively infinite, such as boom expansion periods or bull markets. On the contrary, in a lower regime, the transition function $F\left(y_{t-d} ; \gamma, c\right)$ is zero as the transitional variables $y_{t-d}$ or $\left(y_{t-d}-c\right)$ is negatively infinite, such as economic contraction or bear markets.
Economic expansion and contraction can be applied to two regimes in the STAR model. The bull and bear markets are generally used to describe whether stock prices are increasing and decreasing, respectively. A bull market is a financial market of a group of assets in which prices are increasing or are expected to rise, while a bear market is characterized by decreasing prices and typically shows a lack of investment confidence. The logistic STAR (LSTAR) model can be applied to a bull market when the transitional variables $y_{t-d}$ positively deviates from the threshold value of $c$ and thus lead to $F\left(y_{t-d} ; \gamma, c\right)$ is close to one. In a similar way, the LSTAR model is available for a bear market when the transitional variables $y_{t-d}$ negatively diverges from the threshold value of $c$, causing $F\left(y_{t-d} ; \gamma, c\right)$ quite near zero. In this LSTAR model, the transitional function displays an asymmetric dynamic adjusted process with respect to the threshold value $c$, in which $\gamma$ stands for the speed of smooth regime switching. The larger the $\gamma$ is, the higher the speed of the regime switching will be.

## B. Exponential Transition Function

Suppose that the transition function is an exponential form as follows:
$F\left(y_{t-d} ; \gamma, c\right)=1-\exp \left[-\gamma\left(y_{t-d}-c\right)^{2}\right], \gamma>0$

In an exponential STAR (ESTAR) model, financial assets are in an equilibrium or stable status (i.e., non-bubble or non-crisis periods) when the transitional variables $y_{t-d}$ equal the threshold value of $c$. At this time, the value of the transition function, $F\left(y_{t-d} ; \gamma, c\right)$, equals zero. We refer to the non-bubble or non-crisis periods as a middle regime. When the transitional variables $y_{t-d}$ or $\left(y_{t-d}-c\right)$ is positively (negatively) infinite, the financial market is classified as an expansion (contraction) period. In the meantime, the value of the transition function, $F\left(y_{t-d} ; \gamma, c\right)$, equals one, being regarded as an outer regime (i.e., economic expansion or contraction). The exponential transition function in the ESTAR model is symmetrical with respect to the threshold value, in which the transitional variables $y_{t-d}$ would be the highest (lowest) responding to expansion (contraction) under an outer regime. Thus, the ESTAR dynamic transition process is similar between expansion and contraction statuses in an outer regime, in which $\gamma$ stands for the same speed of smooth regime switching. However, the transition process in a middle regime differs from that in an outer regime. As a consequence, the ESATR model is more suitable to capture the assets price movements for either the expansion/contraction statuses in an outer regime or the stable status in a middle regime, while it is less applicable to detect the bull and bear markets. The main reasons are that economic expansion and contraction display the analogous transition speed, whereas bull versus bear markets experience different dynamic transition process. In short, this research applies the LSTAR-type model to capture the movements of time series $y_{t}$ under the bull and bear markets, while we put the ESTAR-type model in use for the expansion or contraction statuses in an outer regime and for the stable status in a middle regime.

## (2) Estimation Method for the Smooth Transition Autoregressive Model

## A. Linear Regression Model

This study adopts a bivariate analysis to determine the empirical relationships between stock returns and changes in foreign exchange rates. Unlike the uni-variate analysis, the bivariate analysis can more precisely infer the linkages between the two variables. We first construct a vector autoregressive (VAR) model or a vector error correction model (VECM) to capture the assets price movements in a linear manner.

## B. Transitional Variables and Linearity Tests

Teräsvirta (1994) suggested a third order Taylor polynomial to replace the transition function with $\gamma=0$ and estimates the auxiliary regression model as follows:
$y_{t}=\beta_{0}+\beta_{1}^{\prime} w_{t}+\sum_{j=1}^{p} \beta_{2 j} y_{t-j} y_{t-d}+\sum_{j=1}^{p} \beta_{3 j} y_{t-j} y_{t-d}^{2}+\sum_{j=1}^{p} \beta_{4 j} y_{t-j} y_{t-d}^{3}+v_{t}$,
where $y_{t}$ indicates stock returns or changes in foreign exchange rates; $d$ denotes the lagged number of the $y_{t}$ variables and $w_{t}=\left(y_{t-1}, \ldots \ldots, y_{t-p}\right)^{\text {}}$. To test the linearity in this auxiliary regression, we perform the Lagrange multiplier (LM) for testing for the null hypothesis: $H_{01}: \beta_{2 j}=\beta_{3 j}=\beta_{4 j}=0 \quad j=1, \ldots, p$. We repeat the LM-test using various scenarios of delay parameters for this null hypothesis that the linear specification for the auxiliary regression is appropriate. The linear specification will be valid, if we do not reject the null hypothesis; by the contrary, the linearity hypothesis does not hold when we reject the null hypothesis. We obtain the optimal lagged numbers (i.e., $d$-value), based on the minimum $p$-value among the rejected null hypotheses. In practice, LM-statistic is calculated by estimating both restricted and unrestricted regression models as follows:
$L M_{0}=\frac{\left(S S R_{0}-S S R_{1}\right) / 3 m}{S S R_{1} / T-4 m-1}$.

We state here that $S S R_{0}$ is the sum of squared residuals from the restricted regression model with $3 m$ restrictions imposed; $S S R_{1}$ is the sum of squared residuals from the auxiliary regression model without restrictions. $T$ represents the number of observations. The LM-statistic follows an $F$-distribution with two degrees of freedom ( $3 m, T-4 m-1$ ). We conclude there exist a linear relationship between the dependent and independent variables when the LM-statistic does not reject the hypothesis of the linearity.

## C. Transitional Variables and Non-linearity Tests

We further turn to the next step to look at which transition function is better when the LM-statistics reject the linearity hypothesis of $H_{01}$ in the above auxiliary regression model. The corresponding null hypotheses are as follows:

$$
\begin{aligned}
& H_{02}: \beta_{4 j}=0 \quad j=1, \ldots, p \\
& H_{03}: \beta_{3 j}=0 \mid \beta_{4 j}=0 \quad j=1, \ldots, p \\
& H_{04}: \beta_{2 j}=0 \mid \beta_{3 j}=\beta_{4 j}=0 \quad j=1, \ldots, p
\end{aligned}
$$

To rule out the possible wrong judgements, Teräsvirta (1994) suggested an optimal model which the $p$-value is the minimum among $p$-values corresponding to its overall $L M$-statistics of rejecting hypotheses of $H_{02}, H_{03}$, and $H_{04}$, respectively. In brief, the exponential transition function can be regarded as the optimal one if the $p$-value corresponding to its $L M$-statistic is the minimum among the rejected hypotheses for $H_{03}$. Otherwise, the logistic transition function is taken as the optimal transition function.

### 3.2.2 STGARCH Model

We proceed to introduce the STGARCH model that characterizes the nonlinear conditional variance equation. The specification of the $\operatorname{STGARCH}(1,1)$ model is as follows:
$h_{i, t}=\alpha_{0, i}+\alpha_{1, i} \varepsilon_{t-1}^{2}+\left(\alpha_{2, i} \varepsilon_{t-1}^{2}\right) F\left(\varepsilon_{t-1}, \gamma\right)+\beta_{i} h_{t-1}$,
$F\left(\varepsilon_{t-1}, \gamma\right)=\frac{1}{1+\exp \left(\gamma \times \varepsilon_{t-1}\right)}-\frac{1}{2} \quad \gamma>0$,
where $h_{i, t}$ indicates the conditional variance; $\varepsilon_{t-1}^{2}$ denotes the unexpected shocks $; \varepsilon_{t-1}$ is the transitional variables in transition function; and $\gamma$ is the adjustment speed of the smooth transition function. The transition function, $F\left(\varepsilon_{t-1}, \gamma\right)$, lies between -0.5 and 0.5 . To make sure of the positive conditional variance equation, necessary conditions include $\alpha_{0, i}>0, \alpha_{1 i} \geq 0, \alpha_{1 i} \geq \frac{1}{2}\left|\alpha_{2 i}\right|$, and $\beta_{i} \geq 0$. Note that $\alpha_{2 i}$ can be positive or negative. Since the volatility for bad news is generally larger than that for good news, both parameters $\gamma$ and $\alpha_{2 i}$ should be simultaneously positive or negative.
Based on the continuous transition function $F\left(\varepsilon_{t-1}, \gamma\right)$, the volatility of the STGARCH $(1,1)$ model can be smoothly transferred between different regimes. The conditional variance equation is asymmetric between positive and negative shocks, meaning that a leverage effect occurred. The transition function would be positive that its value is larger than or equals to zero but less than $1 / 2$, i.e.,
$0 \leq F\left(\varepsilon_{t-d}, r\right)<\frac{1}{2}$, under a piece of bad news $\varepsilon_{t-1} \leq 0$. The transition function would be $1 / 2$ (i.e., $F\left(\varepsilon_{t-d}, r\right)=\frac{1}{2}$ ) for $\varepsilon_{t-d}=-\infty$. At this time, the conditional variance is the maximum and $h_{\mathrm{i}, \mathrm{t}}=\alpha_{0, i}+\left(\alpha_{1, i}+\frac{1}{2} \alpha_{2, i}\right) \varepsilon_{t-1}^{2}+\beta_{i} h_{t-1}$. By the contrary, the transition function would be negative that its value is larger than or equal to $1 / 2$ but less than zero, i.e., $-\frac{1}{2} \leq F\left(\varepsilon_{t-d}, r\right)<0$, under a piece of good news $\varepsilon_{t-1} \geq 0$.
The transition function would be $-1 / 2$ (i.e., $F\left(\varepsilon_{t-d}, r\right)=-\frac{1}{2}$ ) for $\varepsilon_{t-d}=\infty$. In the meanwhile, the minimum conditional variance $h_{i, t}=\alpha_{0, i}+\left(\alpha_{1, i}-\frac{1}{2} \alpha_{2, i}\right) \varepsilon_{t-1}^{2}+\beta_{i} h_{t-1}$. If $\alpha_{2}$ is significantly positive $\left(\alpha_{2}>0\right)$, then there exists a leverage effect. The leverage effect documents that conditional volatility increases with bad news but decreases with good news (Black, 1976). If the transition variable, $\varepsilon_{\mathrm{t}-1}$, equals zero, then the value of the transition function $F\left(\varepsilon_{t-1}, \gamma\right)$ equals zero, and the STGARCH $(1,1)$ model will be the $\operatorname{GARCH}(1,1)$ specification.
Up to the present, the autoregressive conditional heteroscedasticity (ARCH) effects may exist in residuals from the STAR model. Some existing literature adopts the STAR-GARCH model to solve this problem. Taken as a whole, this study uses conditional mean and variance models in a smooth transition manner to examine the nonlinearly dynamic interactions and volatility spillovers between stock and foreign exchange markets. Specifically, we adopt the STVEC-STGARCH-DCC model, taking both the smooth transition and leverage effect of volatility into considerations to obtain the empirical results.

## 4. Empirical Results and Implications

### 4.1 Descriptive Statistics

Table 1 reports the descriptive statistics of stock returns and changes in foreign exchange rates. Panels A and B correspond to stock returns and changes in nominal foreign exchange rates, respectively. These variables are expressed as a percentage. We first perform the Jarque-Bera test to analyze whether our sample data have the skewness and kurtosis matching the normal distribution. The null hypothesis for the Jarque-Bera test is a joint hypothesis of both skewness and excess kurtosis being zero (i.e., normal distribution). The results present that all stock returns of Germany, Japan, India, South Africa, and U.S. reject the null hypothesis of a normal distribution, displaying these stock return series do not follow a normal distribution. Similarly, overall changes in nominal foreign exchange rates among our sample
countries do not follow the normal distribution.
Next, we carry out the Ljung-Box $Q$ test to examine whether the autocorrelation of time series exists at multiple lags jointly. The null hypothesis for this Ljung-Box $Q$ test is that the first $m$ autocorrelations are jointly zero (i.e., no autocorrelation). Our results show that all stock returns and changes in nominal foreign exchange rates reject the null hypothesis of no autocorrelation, except for euro. The evidence indicates that these time series exist significantly first order sequence correlation. We also perform the Ljung-Box $Q$ test for the squared series, symbolized as $Q^{2}$, showing that autocorrelation prevails in these squared time series (i.e., ARCH effects). Based on the preliminary results, we should take the autoregressive conditional heteroskedasticity model into consideration.

Table 1: Descriptive statistics of stock returns and changes in nominal foreign exchange rates

|  | Germany | Japan | India | South <br> Africa | U.S. |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Panel A. Stock Returns |  |  |  |  |  |
| Mean | 0.0145 | 3.35E-06 | 8.64E-05 | 4.83E-04 | 9.12E-04 |
| Std. Dev. | 0.7251 | 0.0162 | 0.0182 | 0.0142 | 1.4521 |
| Maximum | 0.0987 | 0.1420 | 162.0000 | 0.0693 | 0.0915 |
| Minimum | -0.0743 | -0.1321 | -0.2315 | -0.0622 | -0.0825 |
| Skewness | 0.1126 | -0.4215 | -0.3125 | -0.1425 | 0.8120 |
| Kurtosis | 7.2315 | 9.1257 | 9.3671 | 7.0351 | 15.3261 |
| J-B | 2,831.21*** | 6,326.15*** | 7,012.35*** | 1,234.38*** | 9,012.34*** |
| $Q$ | 23.21** | $28.31^{* * *}$ | $57.11^{* * *}$ | $34.22^{* * *}$ | 60.26 *** |
| $Q^{2}$ | 1,802.15*** | 3,262.25*** | $562.38^{* * *}$ | 2,125.36*** | 2,712.25*** |
| Number of Obs. | 2,390 | 2,390 | 2,390 | 2,390 | 2,390 |
| Panel B. Changes in Nominal Foreign Exchange Rates |  |  |  |  |  |
| Mean | $2.01 \mathrm{E}-05$ | $1.99 \mathrm{E}-04$ | $2.51 \mathrm{E}-04$ | $2.38 \mathrm{E}-04$ |  |
| Std. Dev. | 0.0071 | 0.0052 | 0.0061 | 0.1225 |  |
| Maximum | 0.0512 | 0.0415 | 0.0411 | 0.0825 |  |
| Minimum | -0.0012 | -0.0362 | -0.0321 | -0.0514 |  |
| Skewness | -0.2315 | 0.3652 | -0.0281 | -0.3625 |  |
| Kurtosis | 7.0125 | 6.1254 | 12.3614 | 6.7825 |  |
| J-B | 1,215.26*** | 930.68*** | 6,825.64*** | 1,214.26*** |  |
| $Q$ | 17.21 | 3,214.20*** | $34.31^{* * *}$ | $32.62^{* * *}$ |  |
| $Q^{2}$ | 501.26*** | $451.26^{* * *}$ | $580.25^{* * *}$ | 991.57*** |  |
| Number of Obs. | 2,390 | 2,390 | 2,390 | 2,390 |  |

Note: **, **, and * indicate significance at the $1 \%, 5 \%$, and $10 \%$ levels, respectively. $Q$ and $Q^{2}$ denotes the Ljung-Box $Q$ statistics for the raw series and squared series, respectively.

### 4.2 Unit-Root Tests

A unit root test is a popular approach to test whether a time series is stationary. The corresponding null hypothesis is generally defined as the presence of a unit root (i.e., non-stationary time series), while the alternative hypothesis is either stationarity without a trend, stationarity with a trend, or explosive root depending on the method used. This current study uses the Phillips-Perron (PP) test to assess the null hypothesis of a unit root in a univariate time series (Phillips and Perron, 1988). Table 2 demonstrates the Phillips-Perron unit root tests for stock indices and nominal foreign exchange rates. Panels A and B correspond to the PP tests for natural logarithm series and return series, respectively. Panel A of Table 2 shows all natural logarithm series fail to reject the null hypothesis, indicating that these time series have their corresponding unit roots, and thus are non-stationary. Panel B of Table 2, in contrast, reveals all return series reject the unit-root null hypothesis, displaying that all stock returns and changes in nominal foreign exchange rates are stationary time series. Afterwards, this study relies upon these stationary series, rather than the non-stationary raw data.

Table 2: Phillips-Perron unit-root tests for stock indices and nominal foreign exchange rates

|  | With Intercept | With Intercept <br> and Time Trend | Without <br> Intercept or <br> Time Trend |
| :--- | :---: | :---: | :---: |
| Panel A. Natural Logarithm Series for Stock Indices and Nominal |  |  |  |
| Foreign Exchange Rates |  |  |  |


\left.| Panel B. Return Series for Stock Indices and Nominal Foreign Exchange |  |  |  |
| :---: | :---: | :---: | :---: |
| Rates |  |  |  |$\right]$.

Note: ${ }^{* * *}$, **, and *indicate significance at the $1 \%, 5 \%$, and $10 \%$ levels, respectively. The number in [ ] indicates the periods of lags.

### 4.3 Johansen Co-integration Tests

We proceed to conduct the Johansen co-integration procedure for testing cointegration of several I (1) time series, which relies on the trace test and the maximum eigenvalue test but the inferences might be a little bit different to each other. The Johansen co-integration allows more than one co-integrating relationship, so it is more generally applicable than the Engle-Granger test ${ }^{4}$. The null hypothesis for the trace test is that the number of co-integration vectors is $r=r^{*}<k$ versus the alternative hypothesis that $r=k$. Comparatively, the null hypothesis for the maximum eigenvalue test is like the trace test but the alternative is $r=r^{*}+1$. Table 3 demonstrates the results for the Johansen co-integration tests. We find stock indices and nominal foreign exchange rates have the long-run co-integrating relations. Thus, we should add the error-correction terms into our STAR model. The error-correction terms (ECTs) bear a relation to the fact that the observed values in time $t-1$ deviate from the long run equilibrium relationship and such deviation influences its short-run dynamics. Subsequently, the ECTs force the variable back to the long-run equilibrium.

Table 3: Johansen co-integration tests

| $\mathbf{H}_{\mathbf{0}}$ | Trace | Max-Eigen | Trace | Max-Eigen |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Germany |  | Japan |  |  |
| $r=0$ | $21.5665^{* *}$ | $19.3621^{* *}$ | $22.4157^{* *}$ | $16.3142^{* *}$ |  |
| $r \leq 1$ | 2.515 | 2.3125 | 2.2621 | 2.1521 |  |
|  | India |  |  | South Africa |  |
| $r=0$ | $189.7121^{*}$ | $16.1124^{* *}$ | $22.1189^{* *}$ | $16.1125^{* *}$ |  |
| $r \leq 1$ | 3.1458 | 3.0261 | 5.9257 | 5.8925 |  |

Note: ${ }^{* * *},{ }^{* *}$, and ${ }^{*}$ indicate significance at the $1 \%, 5 \%$, and $10 \%$ levels, respectively. $\mathrm{H}_{0}$ states the null hypothesis that the number of co-integration vectors equals the given number.

### 4.4 Linearity Tests and Choosing the Transition Function Forms

The linearity test aims to verify whether there exists a liner relationship between the dependent and independent variables. In practice, it is an important topic to examine how foreign exchange markets affect stock markets, and vice versa, to some extent under different regimes. This study empirically investigates whether stock returns and changes in foreign exchange rates interact with each other in varying degrees, conditional on different market statuses. Suppose that all linearity hypotheses are rejected. Using changes in foreign exchange rates as the transitional variables, we examine the non-linear dynamic adjustment process of stock returns under currency

[^1]appreciating versus depreciating regimes. By the contrary, using stock returns as the transitional variables, we discuss the non-linear dynamic adjustment process of changes in foreign exchange rates under bull versus bear markets.
Table 4 displays the results for the linearity tests. Stock returns and changes in foreign exchange rates are reported in Panels A and B, respectively. Germany's stock returns and changes in foreign exchange rates reject the linearity null hypotheses at $d=4$ and 1, respectively. Japan's stock returns and changes in foreign exchange rates correspond to $d=8$ and 6, respectively. Similarly, India's $d=9$ and 7 and South Africa's $d=9$ and 1 correspond to stock returns and changes in foreign exchange rates, respectively. After $d$ is determined for each country, we can judge the logistic or exponential transition function would be the reasonable function for stock returns and changes in foreign exchange rates across our sample countries.

Table 4: Linearity tests

| Delay <br> Parameter | Germany | Japan | India | South Africa |
| :---: | :---: | :---: | :---: | :---: |
| Panel A. $\boldsymbol{p}$-value for Linearity Tests of Stock Returns |  |  |  |  |
| $d=1$ | 0.1832 | 0.2236 | 0.1526 | 0.1925 |
| $d=2$ | 0.0812 | $1.521 \mathrm{E}-05$ | 0.0041 | 0.0020 |
| $d=3$ | 0.0421 | 0.2125 | $4.6214 \mathrm{E}-04$ | 0.2925 |
| $d=4$ | $\mathbf{2 . 2 0 2 1 E - 0 5 \#}$ | 0.0134 | 0.6021 | 0.0086 |
| $d=5$ | $0.1628 \mathrm{E}-04$ | 0.4316 | 0.07125 | $1.6215 \mathrm{E}-07$ |
| $d=6$ | 0.0925 | 0.0912 | 0.1566 | $6.0257 \mathrm{E}-05$ |
| $d=7$ | 0.0064 | 0.0099 | 0.0834 | $1.8925 \mathrm{E}-05$ |
| $d=8$ | 0.9892 | $\mathbf{0 . 2 0 5 E}-\mathbf{0 8} \#$ | 0.3625 | $5.6721 \mathrm{E}-05$ |
| $d=9$ | 0.2832 | 0.06321 | $\mathbf{4 . 6 9 8 2} \#$ | $\mathbf{1 . 3 5 2 1 E}-\mathbf{0 7 \#}$ |
| Panel B. $\boldsymbol{p}$-value for Linearity Tests of Changes in Foreign Exchange Rates |  |  |  |  |
| $d=1$ | $\mathbf{0 . 0 0 4 8}$ | 0.2128 | 0.1025 | $\mathbf{5 . 3 6 1 4 \mathrm { E } - 0 6 \#}$ |
| $d=2$ | 0.2298 | 0.5826 | 0.0033 | $3.5672 \mathrm{E}-04$ |
| $d=3$ | 0.1582 | 0.1526 | 0.0062 | 0.0351 |
| $d=4$ | 0.0202 | 0.0326 | 0.1525 | $4.921 \mathrm{E}-05$ |
| $d=5$ | 0.0153 | $2.4125 \mathrm{E}-08$ | 0.0020 | 0.0492 |
| $d=6$ | 0.0153 | $\mathbf{2 . 4 1 2 5 E}-\mathbf{0 8} \#$ | 0.0020 | 0.0492 |
| $d=7$ | 0.2236 | 0.08125 | $\mathbf{2 . 4 6 1 2 E - 0 6 \#}$ | 0.0031 |
| $d=8$ | 0.1625 | 0.0014 | 0.2025 | $7.8214 \mathrm{E}-05$ |
| $d=9$ | 0.1925 | 0.0182 | 0.1725 | 0.0052 |

Note: We consider the delay parameters, $d$, over the ranges $1<=d<=9$. \# indicates the $p$-value is the minimum among the cases of rejection for the linearity hypotheses.

We now turn to explore which transition function is better among LM-statistics in the above auxiliary regression models. Table 5 reports that stock returns of Japan and changes in foreign exchange rates of Germany and India are more applicable to the exponential transition function, and other cases are classified as the logistic transition functions.

Table 5: Tests for transition functions

|  | Germany | Japan | India | South <br> Africa |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Panel A. Stock Returns |  |  |  |  |  |  |  |  |  |
|  | $d=4$ | $d=8$ | $d=9$ | $d=9$ |  |  |  |  |  |
| $H_{02}: \beta_{4 j}=0$ | $\mathbf{2 . 6 1 5 2 E - 0 4 \#}$ | 0.0214 | 4.5421E-03\# | 9.8215E-04\# |  |  |  |  |  |
| $H_{03}: \beta_{3 j}=0 \mid \beta_{4 j}=0$ | $1.6251 \mathrm{E}-03$ | $\mathbf{2 . 7 1 2 5 E - 0 7 \#}$ | 0.0151 | $3.9214 \mathrm{E}-03$ |  |  |  |  |  |
| $H_{04}: \beta_{2 j}=0 \mid \beta_{3 j}=\beta_{4 j}=0$ | 0.2785 | 0.0170 | 0.2936 | 0.3839 |  |  |  |  |  |
| Panel B. Changes in foreign exchange rates |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  | $d=1$ | $d=6$ | $d=7$ | $d=1$ |
| $H_{02}: \beta_{4 j}=0$ | 0.3234 | $\mathbf{1 . 0 2 5 1 E - 0 7 \#}$ | $5.8932 \mathrm{E}-04$ | $\mathbf{3 . 8 1 4 6 E - 0 3 \#}$ |  |  |  |  |  |
| $H_{03}: \beta_{3 j}=0 \mid \beta_{4 j}=0$ | $\mathbf{5 . 2 4 6 4 E - 0 4 \#}$ | 0.0725 | $\mathbf{1 . 3 4 6 2 E - 0 4 \#}$ | 0.2826 |  |  |  |  |  |
| $H_{04}: \beta_{2 j}=0 \mid \beta_{3 j}=\beta_{4 j}=0$ | 0.5825 | 0.2025 | 0.1426 | 0.0071 |  |  |  |  |  |

### 4.5 STVEC-STGARCH Model Specifications

This section starts by giving formal definitions of the conditional mean and variance equations, respectively. The conditional means and variances should be modelled jointly since they are interacted through the parameters of the joint distribution. Throughout the following discussion, subscripts $i$ and $j$ refer to stock returns and changes in nominal foreign exchange rates, respectively. Subscript $t$ denotes days.

### 4.5.1 Conditional Mean Equations

To empirically investigate the interactions between stock and foreign exchange markets, this study estimates the bivariate STVEC models as the conditional mean equations as follows:

$$
\begin{align*}
r_{i, t}= & \left(\varphi_{i, 0}+\varphi_{i, z} Z_{t-1}+\sum_{k=1}^{P} \varphi_{i, k} r_{i, t-k}+\sum_{k=1}^{q} \varphi_{i j, k} r_{j, t-k}+\varphi_{i, U} r_{U S A, t-1}\right) \times\left[1-F\left(r_{j, t-d}\right)\right] \\
& +\left(\delta_{i, 0}+\delta_{i, z} Z_{t-1}+\sum_{k=1}^{P} \delta_{i, k} r_{i, t-k}+\sum_{k=1}^{q} \delta_{i j, k} r_{j, t-k}+\delta_{i, U} r_{U S A, t-1}\right) \times\left[F\left(r_{j, t-d}\right)\right]+\varepsilon_{i, t} \tag{7}
\end{align*}
$$

$$
\begin{align*}
r_{j, t}= & \left(\varphi_{j, 0}+\varphi_{j, z} Z_{t-1}+\sum_{k=1}^{P} \varphi_{j, k} r_{j, t-k}+\sum_{k=1}^{q} \varphi_{j i, k} r_{i, t-k}+\varphi_{j, U} r_{U S A, t-1}\right) \times\left[1-F\left(r_{i, t-d}\right)\right]  \tag{8}\\
& +\left(\delta_{j, 0}+\delta_{j, z} Z_{t-1}+\sum_{k=1}^{P} \delta_{j, k} r_{j, t-k}+\sum_{k=1}^{q} \delta_{j i, k} r_{i, t-k}+\delta_{j, U} r_{U S A, t-1}\right) \times\left[F\left(r_{i, t-d}\right)\right]+\varepsilon_{j, t}
\end{align*}
$$

where $r_{i, t}$ and $r_{j, t}$ are stock returns and changes in foreign exchange rates, respectively. $r_{\mathrm{USA}, t-1}$ denotes the one-day lagged U.S. stock returns representing the effects from world information center. In the first mean equation for stock returns, the long-run trend lies between $\varphi_{i, 0}$ and $\delta_{i, 0}$ depending on the transition functions; $Z_{t-1}$ is error-correction term at time $t-1 ; \varphi_{i, z}$ and $\delta_{i, z}$ indicate the equilibrium adjustment speed in different regimes, respectively. $\varphi_{i, k}$ and $\delta_{i, k}$ denote the autocorrelations for stock returns and changes in foreign exchanges, respectively. $\varphi_{i j, k}$ represents the sensitivity of stock returns to changes in foreign exchange rates; by contrast, $\delta_{i j, k}$ is the sensitivity of changes in foreign exchange rates to stock returns. $\varphi_{i, U}$ and $\delta_{j, U}$ stand for the impacts of the U.S. stock returns on stock returns of other countries in different regimes. Similarly, in the second equation for changes in foreign exchange, the long-run trends of changes in foreign exchange rates are $\varphi_{j, 0}$ and $\delta_{j, 0}$ for different regimes. In the first regime for changes in foreign exchanges, $\varphi_{j, z}, \varphi_{j, k}$, and $\varphi_{j i, k}$ correspond to the equilibrium adjustment speed, the level of autocorrelation; and the sensitivity of changes in foreign exchange rates to stock returns, respectively. Likewise, in the second regime, $\delta_{j, z}, \delta_{j, k}$, and $\delta_{j i, k}$ are separately in accordance with the equilibrium adjustment speed, the level of autocorrelation, and the sensitivity of changes in foreign exchange rates to stock returns. $\varphi_{j, U}$ and $\delta_{j, U}$ indicates the impacts of the U.S. stock returns on changes in foreign exchange rates of other countries in the first and second regimes, respectively.
$F\left(r_{j, t-d}\right)$ and $F\left(r_{i, t-d}\right)$ indicate the types of transition functions which cause nonlinear dynamics in the mean equations. Generally speaking, the logistic and exponential functions can be defined by Equations (9) and (10), respectively, as follows:

$$
\begin{align*}
& F\left(S_{t-d} ; \gamma, c\right)=\frac{1}{1+\exp \left[-\gamma\left(S_{t-d}-c\right)\right]}, \quad \gamma>0  \tag{9}\\
& F\left(S_{t-d} ; \gamma, c\right)=1-\exp \left[-\gamma\left(S_{t-d}-c\right)^{2}\right], \quad \gamma>0 \tag{10}
\end{align*}
$$

Note that $F\left(y_{t-d} ; \gamma, c\right)$ is the transition function; $\gamma$ indicates the transition speed from one regime to the other; and $c$ denotes the threshold value. In our bivariate STVEC model, $\gamma_{i}$ and $\gamma_{j}$ denote the transition speed between different regimes
for stock returns and changes in foreign exchange rates, respectively. $c_{i}$ and $c_{j}$ indicate the long-run equilibrium value or the threshold value of transitional variables for stock returns and changes in foreign exchange rates, respectively. Specifically, using the logistic transition function, when the foreign exchange market is under the currency depreciation regime as the value of the transition function equals zero, our conditional mean equation demonstrates yields several implications for stock returns. The long-run trend is $\varphi_{i, 0}$; the equilibrium adjustment speed is $\varphi_{i, z}$; the level of stock return autocorrelation is $\varphi_{i, k}$; the sensitivity of stock returns to changes in foreign exchange rates is $\varphi_{i j, k}$; and the impact of the U.S. stock returns from S\&P 500 indices on stock returns of other countries is $\varphi_{i, U}$. When the foreign exchange market is under the currency appreciating regime as the value of the logistic transition function equals one, stock returns display the following effects. The long-run trend is $\delta_{i, 0}$; the equilibrium adjustment speed is $\delta_{i, z}$; the level of stock return autocorrelation is $\delta_{i, k}$; the sensitivity of stock returns to changes in foreign exchange rates is $\delta_{i j, k}$; and the impact of the U.S. stock returns on stock returns of other countries is $\delta_{i, U}$. Similarly, when the stock market is under the bear market as the values of the logistic transition function equal zero, the economic effects for changes in foreign exchange rates are as follows. The long-run trend of changes in foreign exchange rates is $\varphi_{j, 0}$; the equilibrium adjustment speed is $\varphi_{j, z}$; the level of autocorrelation is $\varphi_{j, k}$; the sensitivity of changes in foreign exchange rates to stock returns is $\varphi_{j i, k}$; and the impact of the U.S. stock returns on changes in foreign exchange rates of other countries is $\varphi_{j, U}$. By contrast, the economic effects of changes in foreign rates under the bull market as the value of the transition logistic function equals one are as follows. The long-run trend is $\delta_{j, 0}$; the equilibrium adjustment speed is $\delta_{j, z}$; the level of autocorrelation is $\delta_{j, k}$; the sensitivity of changes in foreign exchange rates to stock returns is $\delta_{j i, k}$; and the impact of the U.S. stock returns on changes in foreign exchange rates of other countries is $\delta_{j, U}$.
We now turn to the exponential function, when the foreign exchange market is under the stable status as the value of the transition function is zero, the fist conditional mean equation for stock returns displays the following economic effects. The long-run trend of stock returns is $\varphi_{i, 0}$; the equilibrium adjustment speed is $\varphi_{i, z}$; the level of stock return autocorrelation is $\varphi_{i, k}$; the sensitivity of stock returns to changes in foreign exchange rates is $\varphi_{i j, k}$; and the impact of the U.S. stock returns on stock returns of other countries is $\varphi_{i, U}$. Relatively, when the foreign exchange market is the situation of the outer regime as the value of the exponential transition function equals one. Several implications emerge from the conditional mean equations. The long-run trend of stock returns is $\delta_{i, 0}$; the equilibrium adjustment speed is $\delta_{i, z}$, the level of stock return autocorrelation is $\delta_{i, k}$, the sensitivity of stock returns to changes in foreign exchange rates is $\delta_{i j, k}$; and the impact of the U.S. stock returns on stock returns of other countries is $\delta_{i, U}$. As to the second mean
equation for changes in foreign exchanges, in a stable stock market based on the value of the exponential transition function being zero, the foreign exchange market show that the long-run trend is $\varphi_{j, 0} \cdot \varphi_{j, z}, \varphi_{j, k}, \varphi_{i j, k}$, and $\varphi_{j, U}$ respectively present the equilibrium adjustment speed, autocorrelation level; the sensitivity of changes in foreign exchange rates to stock returns, and the impact of the U.S. stock returns on changes in foreign exchange rates of other countries. Likewise, when the stock market is under the extreme statuses with the value of the exponential transition function of one, $\delta_{j, 0}, \delta_{j, z}, \delta_{j, k}$, and $\delta_{j i, k}$ correspond to the long-run trend of changes in foreign exchange rates, the equilibrium adjustment speed, the level of autocorrelation, and the sensitivity of changes in foreign exchange rates to stock returns, respectively. In this case, the impact of the U.S. stock returns on changes in foreign exchange rates of other countries is $\delta_{j, U}$.

### 4.5.2 Conditional Variance Equations

To explore whether volatility of stock and foreign exchange markets influences each other, this study estimates the bivariate STGARCH-DCC models as the conditional variance equations as follows:

$$
\begin{align*}
& \begin{array}{l}
\boldsymbol{h}_{i, t}=\omega_{i, \mathrm{O}}+\alpha_{i i} \varepsilon_{i, t-1}^{2}+\left(\lambda_{i i} \varepsilon_{i, t-1}^{2}\right) F\left(\varepsilon_{i, t-1}, \gamma_{i i}\right)+\boldsymbol{\beta}_{i i} \boldsymbol{h}_{i, t-1} \\
\quad+\alpha_{i j} \varepsilon_{j, t-1}^{2}+\left(\lambda_{i j} \varepsilon_{j, t-1}^{2}\right) F\left(\varepsilon_{j, t-1}, \gamma_{i j}\right)+\theta_{i} r_{U S A, t-1}^{2} \\
F\left(\varepsilon_{i, t-1}, \gamma_{i i}\right)=\frac{1}{1+\exp \left(\gamma_{i i} \varepsilon_{i, t-1}\right)}-\frac{1}{2}, \quad \gamma_{i i}>0, \\
\begin{array}{r}
F\left(\varepsilon_{j, t-1}, \gamma_{i j}\right)=\frac{1}{1+\exp \left(\gamma_{i j} \varepsilon_{j, t-1}\right)}-\frac{1}{2}, \quad \gamma_{i j}>0, \\
\\
\quad+\alpha_{j, t}=\omega_{j i, 0}+\alpha_{i, t-1}^{2}+\left(\lambda_{j i} \varepsilon_{i, t-1}^{2}\right) F\left(\varepsilon_{i, t-1}^{2}, \gamma_{j i}\right)+\theta_{j} r_{U S A, t-1}^{2} \\
F\left(\varepsilon_{j, t-1}, \gamma_{j j}\right)=\frac{1}{1+\exp \left(\gamma_{j j} \varepsilon_{j, t-1}\right)}-\frac{1}{2}, \quad \gamma_{j j}^{2}>0
\end{array} \\
F\left(\varepsilon_{i, t-1}, \gamma_{j i}\right)=\frac{1}{1+\exp \left(\gamma_{j i} \varepsilon_{i, t-1}\right)}-\frac{1}{2}, \quad \gamma_{j i}>0
\end{array} \tag{11}
\end{align*}
$$

where $h_{i, t}$ and $h_{j, t}$ are the conditional variances of stock returns and changes in foreign exchange rates, respectively. In our bivariate STGARCH-DCC models,
$\omega_{i, 0}$ and $\omega_{j, 0}$ indicate the long-run trends of conditional variances for stock returns and changes in foreign exchange rates, respectively. $\varepsilon_{i, t-1}^{2}$ and $\varepsilon_{j, t-1}^{2}$ denote the unexpected shocks from stock returns and changes in foreign exchange rates, respectively. Their corresponding coefficients on $\alpha_{i i}$ and $\alpha_{j j}$ separately measure the ARCH effects for stock returns and changes in foreign exchange rates. $F\left(\varepsilon_{i, t-1}^{2}, \gamma_{i i}\right), \quad F\left(\varepsilon_{i, t-1}^{2}, \gamma_{i j}\right), \quad F\left(\varepsilon_{j, t-1}^{2}, \gamma_{j j}\right)$, and $F\left(\varepsilon_{j, t-1}^{2}, \gamma_{j i}\right)$ indicate the transition functions which cause nonlinear dynamics in the variance equations. $\gamma_{i i}$ and $\gamma_{j j}$ measure the speed of the smooth but asymmetric volatility transition between different regimes for stock returns and changes in foreign exchange rates, respectively. $\gamma_{i j}$ measures the impact of the volatility transition of foreign exchange market on stock market, while $\gamma_{j i}$ assesses the impact of the volatility transition of stock market on foreign exchange market. $\lambda_{i i}$ and $\lambda_{j j}$ stand for the asymmetric ARCH effects for stock returns and changes in foreign exchange rates, respectively. $\alpha_{i j}$ denotes the impact of volatility of changes in foreign exchange rates on the conditional variance of stock returns, while $\alpha_{j i}$ indicates the impact of volatility of stock returns on the conditional variance of changes in foreign exchange rates. $\lambda_{i j}$ expresses the spillover effects of volatility of changes in foreign exchange rates on the conditional variance of stock returns, while $\lambda_{j i}$ represents those of volatility of stock returns on the conditional variance of changes in foreign exchange rates. Lastly, $r_{U S A, t-1}^{2}$ indicates the squared one-day lagged U.S. stock returns representing the return volatility effects from world information center. $\theta_{i}$ and $\theta_{j}$ separately measure the impacts of the volatility of the U.S. stock returns on the conditional variances of stock return and changes in foreign exchange rates of other markets, respectively.

### 4.5.3 Conditional Covariance Equations

$$
\begin{align*}
h_{i j, t} & =\frac{q_{i j, t}}{\sqrt{q_{i i, t} q_{j j, t}}} \times \sqrt{h_{i i, t} h_{i j, t}} \\
& =\frac{\left[\bar{\rho}_{i j}+\alpha\left(z_{i, t-1} z_{j, t-1}-\bar{\rho}_{i j}\right)+\beta\left(q_{i j, t-1}-\bar{\rho}_{i j}\right)\right] \times \sqrt{h_{i i, t} h_{i j, t}}}{\sqrt{(1-\alpha-\beta) \bar{\rho}_{i i}+\alpha\left(z_{i, t-1}^{2}\right)+\beta\left(q_{i i, t-1}\right)} \sqrt{(1-\alpha-\beta) \bar{\rho}_{j j}+\alpha\left(z_{j, t-1}^{2}\right)+\beta\left(q_{i j, t-1}\right)}},  \tag{17}\\
q_{i j, t} & =\bar{\rho}_{i j}+\alpha\left(z_{i, t-1} z_{j, t-1}-\bar{\rho}_{i j}\right)+\beta\left(q_{i j, t-1}-\bar{\rho}_{i j}\right)  \tag{18}\\
q_{i i, t} & =(1-\alpha-\beta) \bar{\rho}_{i i}+\alpha\left(z_{i, t-1}^{2}\right)+\beta\left(q_{i i, t-1}\right) \tag{19}
\end{align*}
$$

$q_{j j, t}=(1-\alpha-\beta) \bar{\rho}_{j j}+\alpha\left(z_{j, t-1}^{2}\right)+\beta\left(q_{j j, t-1}\right)$
$\mu_{t}=\left[\begin{array}{l}\mu_{i, t} \\ \mu_{j, t}\end{array}\right]$
$\varepsilon_{t} \sim \mathrm{~N}\left(0, H_{t}\right)$
$\left[\begin{array}{ll}h_{i, t}^{2} & h_{i j, t} \\ h_{j i, t} & h_{j, t}^{2}\end{array}\right]$
$\mu_{i j, t}$ represents the standardized residuals $\mu_{i j, t}=\frac{\varepsilon_{i j, t}}{\sqrt{h_{i j, t}}}$
Suppose that $\bar{\rho}_{i j}$ indicates the unconditional correlation between stock returns and changes in foreign exchange rates, showing a long-run relationship. $u_{i, t-1}$ and $u_{j, t-1}$ denote the one-period lagged standardized residuals for stock returns and changes in foreign exchange rates, respectively. We state here that $h_{i j, t}$ is the conditional covariance between stock returns and changes in foreign exchange rates. $h_{i i, t}$ and $h_{j j, t}$ are the conditional variances of stock returns and changes in foreign exchange rates, respectively. $q_{i j, t}$ time-varying covariance matrix of $\mu_{t}$ between stock and exchange rate market at period $\mathrm{t}, q_{i i, t}$ time-varying covariance matrix of $\mu_{t}$ of stock market at period t and $q_{j j, t}$ time-varying covariance matrix of $\mu_{t}$ of exchange rate market at period $t$. Both $\alpha$ and $\beta$ are non-negative scalar parameters, meeting the condition of $\alpha+\beta<1$.

### 4.5.4 Dynamic Conditional Correlation Models

This study uses the dynamic conditional correlation (DCC), proposed by Engle (2002), for estimating time varying conditional correlations. The DCC model parameterizes the conditional correlation directly and have some computational advantages over multivariate GARCH models. ${ }^{5}$
$\rho_{i j, t}=\frac{h_{i j, t}}{\sqrt{h_{i i, t} h_{j j, t}}}$,
where $\rho_{i j, t}$ is the conditional correlation between stock returns and changes in foreign exchange rates.

[^2]
### 4.5.5 Empirical results and Interpretations for Conditional Mean Equations

Table 6 presents estimated parameters for conditional mean equations. Panels A and B correspond to the values of the transition functions with zero and one, respectively. The results reveal that increase in stock returns of well-developed markets usually brings about their currency depreciation, while increase in stock returns of emerging markets generally leads to their currency appreciation. For example, in the Germany stable stock market, $1 \%$ increase in the current-day stock returns will cause a $0.1026 \%$ decrease in the next-day changes in foreign exchange rates. By the contrary, under the stable stock market in India, $1 \%$ increase in the current-day stock returns will lead to the increase in the next-day changes in foreign exchange rates by $0.0242 \%$. In a similar fashion, under the bull stock market in South Africa, $1 \%$ increase in the current-day stock returns will lead to the increase in the next-day changes in foreign exchange rates by $0.1011 \%$. Putting together, stock price increase in emerging markets may more attract hot money inflows and then trigger their domestic currency appreciation. It seems that investors favor boom stock markets in emerging countries and plunge a lot of capital flows into these markets, thus leading their currency appreciation. However, the situation is very different for well-developed markets. Regardless of bull or stable markets, well-developed markets may be filled with abundant capital funds and may even worry about inflation potential problems so that prosperous stock markets result in their currency depreciation, instead of appreciation.
As to the role of the U.S. stock market reported in Table 6, the U.S. S\&P500 index returns mainly significantly and positively affect various stock markets, no matter of well-developed or emerging markets, showing that the U.S. stock market has a far-reaching impact on other stock markets. For example, the S\&P500 index returns significantly and positively affect most stock markets, except for Germany stock market under currency appreciation status and South Africa stock market under the currency depreciation status. On the other hand, in the stable stock markets for Germany and India, $1 \%$ increase in the S\&P500 index returns will lead to $0.1021 \%$ and $0.0034 \%$ currency appreciation for euros and rupees, respectively. In the bear stock markets for South Africa, $1 \%$ increase in the S\&P500 index returns will cause $0.1024 \%$ currency appreciation for South African rand. Inversely, under the Japanese bear stock market, $1 \%$ increase in the S\&P500 index returns will negatively affect Japanese yen by $0.1216 \%$. Moreover, the S\&P500 index returns negatively affect Japanese yen under a bull stock market and India rupee under the India's volatile stock markets, while they positively influence South Africa rand under a bull stock market. In short, the U.S. stock market is positively related to most stock markets around the world; however, its impacts on the foreign exchange markets are quite diversified.
The smooth transition speeds of well-developed markets lie between 2.0619 and 7.0151, while those of emerging markets range from 0.3108 to 1.5126 . The larger the speed of smooth transition is, the quicker the market will adjust to the equilibrium. The results present that well-developed markets have quicker
adjustment speed and thus higher level of market efficiency, when the asset prices deviate from the equilibrium value. As to the threshold value, the higher the value is, the higher the degree of tolerance of its economic system will be. Our results show that well-developed markets do not have significantly larger threshold value than do emerging markets - that is, emerging markets do not necessarily easily change the structure or switch the regimes than do well-developed markets, when facing with external shocks.

Table 6: Estimated results for conditional mean equations

|  | Germany |  | Japan |  | India |  | South Africa |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Stock Market | Foreign Exchange Market | Stock <br> Market | Foreign Exchange Market | Stock <br> Market | Foreign Exchange Market | Stock <br> Market | Foreign Exchange Market |
| Panel A. The Value of the Transition Function Equals Zero |  |  |  |  |  |  |  |  |
|  | Currency Depreciation Status | Stable Stock Market | Currency Stable Status | Bear Stock Market | $\begin{gathered} \text { Currency } \\ \text { Depreciation } \end{gathered}$ Status | Stable Stock Market | Currency Depreciati on Status | Bear Stock Market |
| $\varphi_{\text {self }, 0}$ | $\begin{gathered} 0.0005 \\ (1.4623) \\ \hline \end{gathered}$ | $\begin{gathered} 0.0005 \\ (1.6121) \\ \hline \end{gathered}$ | $\begin{aligned} & \hline-9.2 \mathrm{E}-04 \\ & (-0.9010) \\ & \hline \end{aligned}$ | $\begin{aligned} & 3.1 \mathrm{E}-04 \\ & (1.0891) \\ & \hline \end{aligned}$ | $\begin{gathered} 9.1 \mathrm{E}-04^{* * *} \\ (2.8226) \\ \hline \end{gathered}$ | $\begin{gathered} 9.8 \mathrm{E}-05^{* * *} \\ (2.4301) \\ \hline \end{gathered}$ | $\begin{aligned} & 0.0215^{* *} \\ & (2.4301) \\ & \hline \end{aligned}$ | $\begin{gathered} 0.0010 \\ (0.4306) \\ \hline \end{gathered}$ |
| $\varphi_{z}$ | $\begin{gathered} 0.0052 \\ (0.2412) \end{gathered}$ | $\begin{array}{\|c} \hline-0.0612^{* * * *} \\ (-2.7002) \\ \hline \end{array}$ | $\begin{aligned} & \hline-0.0024^{* *} \\ & (-2.7121) \\ & \hline \end{aligned}$ | $\begin{gathered} 0.2502 \\ (0.7012) \end{gathered}$ | $\begin{gathered} -0.324^{* *} \\ (-2.4115) \end{gathered}$ | $\begin{gathered} \hline 0.0124 \\ (0.4512) \end{gathered}$ | $\begin{aligned} & \hline-0.0005^{*} \\ & (-1.9940) \\ & \hline \end{aligned}$ | $\begin{array}{\|l\|} \hline-0.00005 \\ (-0.1025) \\ \hline \end{array}$ |
| $\varphi_{\text {self }, 1}$ | $\begin{aligned} & -0.2126^{* *} \\ & (-4.3721) \\ & \hline \end{aligned}$ | $\begin{gathered} -0.0216 \\ (-0.3534) \\ \hline \end{gathered}$ | $\begin{aligned} & -2.8 \mathrm{e}-04 \\ & (-0.2834) \\ & \hline \end{aligned}$ | $\begin{gathered} -0.0502 \\ (-1.0112) \\ \hline \end{gathered}$ | $\begin{aligned} & 0.1125^{* * *} \\ & (3.9251) \\ & \hline \end{aligned}$ | $\begin{gathered} -0.0562^{* * *} \\ (-3.1212) \\ \hline \end{gathered}$ | $\begin{array}{\|l\|} \hline 0.0401^{* *} \\ (1.9420) \\ \hline \end{array}$ | $\begin{array}{\|c\|} \hline-0.0524 \\ (-1.4101) \\ \hline \end{array}$ |
| $\varphi_{\text {cross }}$ | $\begin{gathered} -0.0059 \\ (-0.1021) \\ \hline \end{gathered}$ | $\begin{gathered} -0.1026^{*} \\ (-1.9236) \\ \hline \end{gathered}$ | $\begin{gathered} -0.0208 \\ (-0.3401) \\ \hline \end{gathered}$ | $\begin{gathered} 0.0321 \\ (1.5120) \\ \hline \end{gathered}$ | $\begin{gathered} -0.1054 \\ (-0.5021) \\ \hline \end{gathered}$ | $\begin{aligned} & 0.0242^{* * *} \\ & (4.0115) \\ & \hline \end{aligned}$ | $\begin{gathered} \hline-0.8211 \\ (-1.7782) \\ \hline \end{gathered}$ | $\begin{array}{c\|} \hline 0.0524 \\ (0.7115) \\ \hline \end{array}$ |
| $\varphi_{U S A}$ | $\begin{aligned} & \hline 0.3652^{* * *} \\ & (4.7210) \\ & \hline \end{aligned}$ | $\begin{gathered} 0.1021^{*} \\ (1.9246) \\ \hline \end{gathered}$ | $\begin{aligned} & 0.6021^{* * *} \\ & (10.1102) \\ & \hline \end{aligned}$ | $\begin{gathered} -0.1216^{* * *} \\ (-4.2112) \\ \hline \end{gathered}$ | $\begin{aligned} & 0.1673^{* * *} \\ & (5.4212) \\ & \hline \end{aligned}$ | $\begin{gathered} 0.0034^{*} \\ (1.9215) \\ \hline \end{gathered}$ | $\begin{gathered} 0.8425 \\ (1.0245) \\ \hline \end{gathered}$ | $\begin{array}{\|l\|} \hline 0.1024^{* * *} \\ (3.2104) \\ \hline \end{array}$ |
| Panel B. The Value of the Transition Function Equals One |  |  |  |  |  |  |  |  |
|  | Currency Appreciation Status | Volatile Stock <br> Market | Volatile Foreign Exchange Status | Bull Stock Market | Currency Appreciation Status | Volatile Stock Market | Currency Appreciati on Status | Bull Stock Market |
| $\delta_{\text {self }, 0}$ | $\begin{gathered} -0.0176 \\ (-0.3411) \\ \hline \end{gathered}$ | $\begin{gathered} 0.0004 \\ (1.6925) \\ \hline \end{gathered}$ | $\begin{array}{r} \hline 2.9 \mathrm{E}-03 \\ (1.4121) \\ \hline \end{array}$ | (1.3202) | $\begin{aligned} & \hline 5.1 \mathrm{E}-04 \\ & (0.5025) \\ & \hline \end{aligned}$ | $\begin{gathered} 1.7 \mathrm{E}-03^{* * *} \\ (3.6451) \\ \hline \end{gathered}$ | $\begin{aligned} & \hline 0.0061^{* *} \\ & (2.2525) \\ & \hline \end{aligned}$ | $\begin{array}{\|c\|} \hline-0.0252^{* * *} \\ (-3.2115) \\ \hline \end{array}$ |
| $\delta_{z}$ | $\begin{gathered} -0.5059 \\ (-0.5426) \end{gathered}$ | $\begin{aligned} & \hline-0.0616^{* *} \\ & (-2.3015) \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline-0.0034^{* *} \\ & (-2.6210) \end{aligned}$ | $\begin{gathered} -0.2015 \\ (-0.6815) \end{gathered}$ | $\begin{aligned} & -0.0072^{* *} \\ & (-2.4995) \\ & \hline \end{aligned}$ | $\begin{gathered} -0.0916 \\ (-0.5821) \end{gathered}$ | $\begin{array}{\|l\|} \hline-0.0116^{* *} \\ (-2.4112) \\ \hline \end{array}$ | $\begin{gathered} -0.0101^{* * *} \\ (-3.3215) \\ \hline \end{gathered}$ |
| $\delta_{\text {self, }, 1}$ | $\begin{gathered} 0.5067 \\ (0.2925) \\ \hline \end{gathered}$ | $\begin{gathered} -0.0317 \\ (-0.5441) \end{gathered}$ | $\begin{gathered} -0.3025 \\ (-1.6225) \\ \hline \end{gathered}$ | $\begin{gathered} -0.0467^{*} \\ (-1.9025) \\ \hline \end{gathered}$ | $\begin{gathered} -0.2615 \\ (-1.6627) \\ \hline \end{gathered}$ | $\begin{gathered} -0.9014^{* * *} \\ (-5.4112) \\ \hline \end{gathered}$ | $\begin{gathered} -0.2827 \\ (-0.7115) \\ \hline \end{gathered}$ | $\begin{gathered} 0.0308 \\ (0.9201) \\ \hline \end{gathered}$ |
| $\delta_{\text {cross }}$ | $\begin{aligned} & -0.70145 \\ & (-0.3011) \end{aligned}$ | $\begin{gathered} -0.0102 \\ (-0.7101) \end{gathered}$ | $\begin{gathered} -0.2815 \\ (-0.9364) \\ \hline \end{gathered}$ | $\begin{gathered} -0.0282^{* * *} \\ (-2.9201) \\ \hline \end{gathered}$ | $\begin{gathered} 0.3919 \\ (1.6205) \end{gathered}$ | $\begin{gathered} -0051 \\ (-0.4521) \end{gathered}$ | $\begin{gathered} 0.0165 \\ (0.1025) \end{gathered}$ | $\begin{aligned} & 0.1011^{* * *} \\ & (4.6225) \end{aligned}$ |
| $\delta_{U S A}$ | $\begin{gathered} -1.2514 \\ (-0.2346) \\ \hline \end{gathered}$ | $\begin{gathered} 0.0062 \\ (0.4125) \\ \hline \end{gathered}$ | $\begin{aligned} & \hline 0.6025^{* * *} \\ & (8.2004) \\ & \hline \end{aligned}$ | $\begin{gathered} -0.1509^{* * *} \\ (-11.301) \\ \hline \end{gathered}$ | $\begin{aligned} & 0.2611^{* * *} \\ & (3.4218) \\ & \hline \end{aligned}$ | $\begin{gathered} -0.2248^{* * *} \\ (-5.1011) \\ \hline \end{gathered}$ | $\begin{aligned} & 0.2628^{* *} \\ & (2.5125) \\ & \hline \end{aligned}$ | $\begin{aligned} & 0.3157^{* * *} \\ & (6.3821) \\ & \hline \end{aligned}$ |
| $\gamma_{\text {self }}$ | $\begin{aligned} & 3.1125^{* * *} \\ & (3.4051) \end{aligned}$ | $\begin{aligned} & 2.6251^{* *} \\ & (2.8745) \end{aligned}$ | $\begin{gathered} 7.0151^{*} \\ (1.9026) \end{gathered}$ | $\begin{aligned} & 2.0619^{* *} \\ & (2.8110) \end{aligned}$ | $\begin{gathered} \hline 0.3154 \\ (1.6810) \end{gathered}$ | $\begin{aligned} & \hline 0.9410^{* *} \\ & (2.9115) \end{aligned}$ | $\begin{aligned} & 0.3108^{* *} \\ & (2.2321) \end{aligned}$ | $\begin{array}{c\|} \hline 1.5126 \\ (1.7205) \end{array}$ |
| $c_{\text {self }}$ | $\begin{gathered} 0.0418 \\ (0.5112) \\ \hline \end{gathered}$ | $\begin{gathered} \hline 0.0044 \\ (0.5026) \\ \hline \end{gathered}$ | $\begin{gathered} -0.0275 \\ (-1.3205) \\ \hline \end{gathered}$ | $\begin{array}{\|l\|} \hline 0.0310^{* * *} \\ (12.7899) \\ \hline \end{array}$ | $\begin{aligned} & 1.56 \mathrm{e}-03 \\ & (1.5351) \\ & \hline \end{aligned}$ | $\begin{gathered} \hline 0.0021^{*} \\ (1.8505) \\ \hline \end{gathered}$ | $\begin{array}{\|c\|} \hline-0.2014^{*} \\ (-1.8201) \\ \hline \end{array}$ | $\begin{array}{\|l\|l} \hline-0.01225 \\ (-0.9210) \\ \hline \end{array}$ |

Note: ${ }^{* * *}$, **, and *indicate significance at the $1 \%, 5 \%$, and $10 \%$ levels, respectively.

### 4.5.6 Empirical results and Interpretations for Conditional Variance Equations

Table 7 reports estimated parameters for conditional variance equations. Panels A and B correspond to the STGARCH-DCC models for stock returns and changes in foreign exchange rates, respectively. Panel A of Table 7 reveals the estimated
coefficients of $\alpha_{i i}$ are significantly positive across countries, indicating ARCH effects in each stock market. In other words, unexpected shocks of stock markets significantly affect their own volatility, regardless of well-developed or emerging markets.
The estimated coefficients of $\beta_{i i}$ are significantly positive across countries that is, the autocorrelation of volatility for each stock market is significantly positive, exhibiting a prevailing phenomenon of volatility clustering. The results show the estimated coefficients of $\lambda_{i i}$ are significantly positive across countries, displaying asymmetric effects of volatility for each stock market. These shocks of positive versus negative news in each stock market significantly affect its own stock volatility and display leverage effects, while those in foreign exchange markets only significantly affect its corresponding stock volatility for India and South Africa but show much weaker effects. Moreover, the estimated coefficients of $\lambda_{i j}$ are significantly positive only for emerging markets. In short, the results present that there exists short-term persistence in unexpected shocks of stock markets regardless of well-developed or emerging markets, but unexpected shocks of foreign exchange markets only affect emerging markets' conditional variances. Moreover, the estimated coefficients of $\gamma_{i i}$ are significantly positive across countries, whereas those of $\gamma_{i j}$ are significantly positive only for emerging markets. The transition speeds between different regimes are quicker in stock self-markets than in crossmarkets. For example, the impacts of the smooth but asymmetric volatility transition of India and South Africa foreign exchange markets on their corresponding stock market are significantly positive, but not applicable for Germany and Japan. Lastly, the U.S. stock volatility significantly affects the conditional variances of other stock markets.
Panel B of Table 7 presents that unexpected shocks of foreign exchange markets significantly affect their own volatility in both well-developed and emerging markets. There still exists a prevailing phenomenon of volatility clustering in foreign exchange markets. The asymmetric effects of unexpected shocks in foreign exchange markets happens, except for India. It is worth mentioning that the asymmetric effects do not hold in India's currency market, while the unexpected shocks in the stock market do influence the currency volatility. It seems that crossmarket effects exhibit stronger power in India. Lastly, the U.S. stock volatility significantly affects the conditional variances of foreign exchange rates across countries, but the impacts are much weaker than those in stock markets.

Table 7: Estimated results for conditional variance equations

|  | Germany |  | Japan |  | India |  | South Africa |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Coeff. | $t$-value. | Coeff. | $t$-value. | Coeff. | $t$-value. | Coeff. | $t$-value. |
| Panel A. Stock Returns |  |  |  |  |  |  |  |  |
| $\omega_{i, 0}$ | 4.2E-06*** | 4.1020 | 4.1E-06*** | 4.2670 | $8.1 \mathrm{E}-06^{* * *}$ | 5.7010 | 6.0E-06*** | 7.0120 |
| $\alpha_{i i}$ | $0.1025^{* * *}$ | 7.9320 | $0.0802^{* * *}$ | 4.8210 | $0.2025^{* * *}$ | 9.3310 | $0.0712 * *$ | 8.1125 |
| $\alpha_{i j}$ | 0.0195 | 0.2020 | 0.0201 | 1.2469 | $0.1421^{*}$ | 1.8210 | 0.0134* | 5.1120 |
| $\beta_{i i}$ | $0.9467^{* * *}$ | 46.2125 | $0.9025^{* * *}$ | 62.3041 | $0.8268^{* * *}$ | 444.1165 | $0.9025^{* * *}$ | 14.2200 |
| $\lambda_{i i}$ | $0.1625^{* *}$ | 14.3281 | $0.0521^{* * *}$ | 7.0113 | 0.1250 *** | 7.0126 | $0.0511^{* * *}$ | 7.2014 |
| $\lambda_{i j}$ | 0.1625 | 0.7315 | -0.0045 | -0.2191 | $0.2025^{* * *}$ | 5.0110 | $0.0182^{* * *}$ | 4.1120 |
| $\gamma_{i i}$ | $0.0502^{* * *}$ | 14.6236 | $0.0966^{* * *}$ | 7.0110 | $0.0558^{* * *}$ | 7.0129 | $0.0825^{* * *}$ | 4.1125 |
| $\gamma_{i j}$ | 0.0108 | 0.9125 | 0.0202 | 0.0910 | $0.0125^{* * *}$ | 5.1426 | $0.0812^{* * *}$ | 7.0120 |
| $\theta_{i}$ | $0.0220^{*}$ | 1.9217 | $0.0365^{* * *}$ | 4.8245 | 0.0192** | 2.3107 | $0.0310^{* * *}$ | 4.0160 |
| Panel B. Changes in Foreign Exchange Rates |  |  |  |  |  |  |  |  |
| $\omega_{i, 0}$ | 1.0E-06** | 2.4202 | 2.7E-06** | 2.0012 | 4.0E-08*** | 3.3988 | 4.6E-06*** | 5.8331 |
| $\alpha_{i j}$ | $0.0344^{* * *}$ | 5.8835 | $0.0408^{*}$ | 3.4997 | $0.1435^{* * *}$ | 9.0972 | 0.0634*** | 7.1375 |
| $\alpha_{i i}$ | 0.0004 | 1.4658 | 0.0018 | 1.1044 | $7.9 \mathrm{E}-06$ | 0.1465 | 0.0010 | 0.2907 |
| $\beta_{i j}$ | $0.9489^{* * *}$ | 48.4582 | $0.8630^{* * *}$ | 15.1133 | $0.8256^{* * *}$ | 41.9086 | $0.8871^{* * *}$ | 25.2434 |
| $\lambda_{i j}$ | $0.0316^{* *}$ | 2.3144 | $0.0177^{* * *}$ | 4.8556 | 0.0046 | 0.4736 | $0.0919^{* * *}$ | 4.2619 |
| $\lambda_{i i}$ | $0.0176^{* *}$ | 2.3489 | 0.0470 | 0.6522 | 0.0741** | 1.9704 | 0.0066 | 1.2726 |
| $\gamma_{i j}$ | $0.0001^{* * *}$ | 3.1201 | $0.0935^{* * *}$ | 4.9962 | 0.0064 | 0.4737 | $0.0162^{* * *}$ | 4.2640 |
| $\gamma_{i i}$ | $0.0134^{* *}$ | 2.8377 | -0.0018 | -0.6566 | $0.0145^{* *}$ | 2.0457 | 0.0874 | 1.2723 |
| $\theta_{j}$ | 0.0013 * | 1.9304 | $0.0093 * *$ | 2.0042 | -4.2E-05*** | -2.7161 | 0.0078** | 2.1983 |

Note: ${ }^{* * *}$, **, and *indicate significance at the $1 \%, 5 \%$, and $10 \%$ levels, respectively.

### 4.5.7 Goodness-of-fit Test and Diagnostic Checking

Table 8 presents diagnostics for standardized residuals. Using Ljung-Box Q tests at lags 10 and 20, we test whether standardized residuals exhibit serial correlations. The results show that the $Q$-tests do not reject the null hypothesis of no autocorrelation. We conclude that the standardized residuals from our STVEC models are not auto-correlated. The squared residuals also do not have serial correlation, displaying no ARCH effects in the model. We also refer to Engle and Ng (1993) and proceed to conduct non-parametric sign tests to test whether there exists asymmetry in the volatility of the residuals based on the sign bias test (SBT), the negative sign bias test (NSBT), the positive sign bias test (PSBT), and the joint test (JT). The null hypothesis of a sign test states that the signs of positive and negative are of equal size. Our results (Table 9) indicate that there do not exist
asymmetric bias effects in the volatility of the residuals from our estimated models, reconfirming a better goodness of fit in our model. Therefore, these diagnostic checks verify that the STVEC-STGARCH-DCC model we estimated is an appropriate specification for these two markets.

Table 8: Diagnostics for standardized residuals

|  |  | $\boldsymbol{u}_{\boldsymbol{i t}}$ <br> $=\boldsymbol{\varepsilon}_{\boldsymbol{i t}} / \sqrt{\boldsymbol{h}_{\boldsymbol{i t}}}$ | $\mathrm{u}_{\mathrm{jt}}=\varepsilon_{\mathrm{jt}} /$ <br> $\sqrt{\mathrm{h}_{\mathrm{it}}}$ | $\boldsymbol{u}_{\boldsymbol{i t}}^{2}$ | $\boldsymbol{u}_{\boldsymbol{j t}}^{2}$ | $\boldsymbol{u}_{\boldsymbol{i t}} \boldsymbol{u}_{\boldsymbol{j t}}$ |
| :---: | :--- | :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{Q}(10)$ | 7.825 | 5.112 | 13.211 | 11.382 | 13.262 |
|  | $\mathrm{Q}(20)$ | 16.361 | 12.114 | 18.365 | 16.231 | 19.371 |
| Japan | $\mathrm{Q}(10)$ | 15.231 | 13.108 | 8.002 | 6.824 | 12.165 |
|  | $\mathrm{Q}(20)$ | 26.215 | 18.271 | 14.361 | 8.971 | 22.387 |
| India | $\mathrm{Q}(10)$ | 17.215 | 15.215 | 10.397 | 8.321 | 6.2100 |
|  | $\mathrm{Q}(20)$ | 23.211 | 21.832 | 21.118 | 13.211 | 17.215 |
| South <br> Africa | $\mathrm{Q}(10)$ | 12.146 | 5.1170 | 5.3210 | 5.412 | 9.3820 |
|  | $\mathrm{Q}(20)$ | 13.215 | 12.116 | 14.268 | 18.081 | 22.415 |

Note: ${ }^{* * *}$, ${ }^{* *}$, and ${ }^{*}$ indicate significance at the $1 \%, 5 \%$, and $10 \%$ levels, respectively.
Table 9: Sign tests

| Countries | Methods | Stock Returns | Foreign Exchange Rates |
| :--- | :---: | :---: | :---: |
| Germany | SBT | 1.3281 | 1.6135 |
|  | NSBT | 0.0031 | $4.15 \mathrm{E}-06$ |
|  | PSBT | 3.3215 | 0.0032 |
|  | JT | 4.6105 | 3.3125 |
| India | SBT | 0.9825 | 0.3805 |
|  | NSBT | 0.0315 | 0.3510 |
|  | PSBT | 1.4965 | 0.4315 |
|  | JT | 4.9152 | 0.5715 |
|  | SBT | 2.6102 | 1.9025 |
|  | NSBT | 1.9025 | 0.0058 |
|  | PSBT | 1.4075 | 0.1836 |
|  | JT | 3.2160 | 2.5102 |
|  | SBT | 0.0415 | 3.0152 |
|  | NSBT | 0.0410 | 0.9625 |
|  | PSBT | 2.3021 | 3.8351 |
|  | JT | 5.8969 | 4.0151 |

Note: ***, **, and *indicate significance at the $1 \%, 5 \%$, and $10 \%$ levels, respectively.

### 4.5.8 Estimated Coefficients of Conditional Correlations via Conditional Covariances and Variance

This section focuses on the dynamic conditional correlations between stock returns and changes in foreign exchange rates via conditional covariances (Equations (17) $\sim(24)$ ). Figure 1 demonstrates, based on the STVEC-STGARCH-DCC models, the conditional correlations between the two markets. Panels A to D correspond to Germany, Japan, India, and South Africa, respectively. Conditional correlations are very important for investors to properly construct diversified portfolios. The results show that the conditional correlations in Germany and South Africa change the signs from negative to positive correlations. The Japanese conditional correlations are almost negative, while the India's are positive.


Figure 1: Conditional correlations between stock returns and changes in foreign exchange rates

To check whether the conditional correlations vary due to the external shocks of important financial crisis events. We use Inclan and Tiao's (1994) iterated cumulative sums of squares (ICSS) algorithm to investigate the points of shocks in the variance. Based on the estimation, there are three structural break dates which can be detected, including the subprime mortgage crisis (July 1, 2007 and September 14, 2008), financial tsunami crisis (September 15, 2008 to April 22, 2010)
and European debt crisis (April 23, 2010 and February 28, 2011). Now, we further check whether the conditional correlations vary due to the external shocks of important financial crisis events. Table 10 presents dynamic conditional correlations between stock returns and changes in foreign exchange rates sorted by important financial crisis events. The empirical results reveal that the impacts of these financial crisis events promote dynamic conditional correlations between stock returns and changes in foreign exchange rates. For example, both 2008 financial tsunami and European debt crises obviously affect the dynamic conditional correlations in Germany, India, and South Africa. As to the effects of the subprime mortgage crisis, Japan, India, and South Africa are comparatively more influenced by the subprime mortgage crisis than are other countries. In Japan, the dynamic conditional correlations between stock returns and changes in foreign exchange rates are negative, regardless of non-crisis or crisis periods. In short, international mega crises, such as the 2008 financial crisis and European debt crisis, have far-reaching impacts on the dynamic conditional correlations between stock returns and changes in foreign exchange rates, while regional crisis, e.g., subprime mortgage crisis, only affect the dynamic conditional correlations in some countries.

Table 10: Dynamic conditional correlations between stock returns and changes in foreign exchange rates sorted by important financial crisis events

| Countries | Non-crisis | Subprime <br> Mortgage <br> Crisis | Financial <br> Tsunami <br> Crisis | European <br> Debt <br> Crisis |
| :---: | :---: | :---: | :---: | :---: |
| Germany <br> (stocks vs. euro) | -0.0907 | 0.0502 | 0.1996 | 0.2379 |
| Japan <br> (stocks vs. Japanese yen) | -0.1831 | -0.4617 | -0.4619 | -0.4185 |
| India <br> (stocks vs. rupee) | 0.2014 | 0.3182 | 0.5004 | 0.4635 |
| South Africa <br> (stocks vs. rand) | 0.2014 | 0.3182 | 0.5004 | 0.4635 |

## 5. Concluding Remarks

In the contemporary financial markets, a well-maintained portfolio is very essential to any investor's success. To determine an optimal asset allocation, market statuses, interactions and spillover effects between stock and foreign exchange markets are very important for an investment portfolio technique. This study investigates the nonlinear interactions and volatility spillovers between stock and foreign exchange markets in both emerging and developed countries. By estimating nonlinear STVEC-STGARCH-DCC models, the smooth transition coefficients or parameters can measure the market efficiency in our sample countries.
On average, our conditional mean equations present that stock price increase in emerging markets tend to trigger their domestic currency appreciation, whereas prosperous stock markets in developed countries lead to currency depreciation. The
empirical results from conditional variance equations reveal that the conditional variances for stock markets mainly result from unexpected shocks, past correlated volatility, and short-term impact effects, thus leading to a long-term persistence of volatility, regardless of emerging or developed markets. Foreign exchange markets display similar patterns of conditional variances but show weaker short-term impact effects and slower transition speeds. Moreover, unexpected shocks in a stock market highly affect its own volatility, while those only affect India's currency volatility. In contrast, unexpected shocks in foreign exchange markets primarily affect its own volatility, except for India. However, those influence their corresponding stock volatility only for emerging countries, such as India and South Africa. Lastly, the transition speeds are higher for developed countries than for emerging ones, implicating that markets are more efficient in developed markets than in emerging markets.
Under different market statuses, such as the bull/bear stock markets, stable/outer stock statuses, currency appreciation/depreciation, and currency stable/volatile statuses, the spillover effects between stock returns and changes in foreign exchange rates are more or less varied. Stock investors still need to care about the external shocks from the foreign exchange markets. Likewise, foreign exchange traders should pay close attention to both stock and foreign exchange markets. In addition, developed countries are more efficient than emerging markets, investors should attach importance to relevant information and information transparency when investing in emerging countries.

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[^1]:    ${ }^{4}$ The Engle-Granger test is based on the Dickey-Fuller or the augmented test for a unit root in the residuals from an estimated co-integrating vector. It assesses the null hypothesis of no cointegration for two time series. One of the advantages of the Johansen co-integration procedure is easier than other methods, and thus relatively costless to be compared with other tests.

[^2]:    ${ }^{5}$ For example, the number of parameters in GARCH-DCC models in the correlation process is independent of the number of series to be correlated. Thus, large correlation matrices can be estimated.

