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Improved Least Cost Method to Obtain a Better IBFS to the Transportation Problem

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Abstract

Transportation modeling is a technique that is used to way out the shipping of supplies from a number of sources to a number of destinations as well as to minimize the total shipment cost. This kind of problem is known as transportation problem (TP). Solution procedure of TP plays a vital role in operation research for its wide application in real world. In the solution procedure of a TP, finding an initial basic feasible solution (IBFS) is necessary to obtain the optimal solution. Least Cost Method (LCM) is one such procedure which is based on cost cells. This solution procedure starts with allocating as much shipments as possible to the cell with the smallest unit cost cell. In this paper we propose an effective improvement of LCM in the solution procedure to obtain a better IBFS for the TPs. To verify the performance of the proposed method, a comparative study is also carried out. Simulation results show that Improved Least Cost Method (iLCM)

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yields better IBFS in 80% cases than LCM.

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1 Introduction

It will be of interest to know that the linear programming had its origin during the Second World War (1939-1945). To fight the war man and material (resources) had to be managed in an effective way so that minimum losses occur to the land and its properties. The government in England studied the problems during war, particularly problems of armed forces, civil defenses and naval strategy etc. The study for the solutions of the above problems resulted in the linear programming solution procedures.

Linear programming is the most popular mathematical technique that deals with the use of limited resources in an optimal manner. The term, programming means planning to maximize profit or minimize cost or loss or minimum use of resources or minimizing the time etc. Such problems are called optimization problem.

There is a type of linear programming problem which may be solved using a simplified version of the simplex technique called transportation method. Because of its major application in solving problems involving several product sources and several destinations of products, this type of problem is very popular as the TP. In TP, main objective is to minimize the cost of distributing a product from a number of sources (e.g. factories) to a number of destinations (e.g. ware houses). This type of problem is also known as cost minimizing transportation problem.

The TP was introduced in 1941 when F.L. Hitchcock [1] presented a study entitled ‘The Distribution of a Product from Several Sources to Numerous

Localities'. The presentation is regarded as the first contribution to the TPs. In 1947, T.C. Koopmans [2] presented a study called 'Optimum Utilization of the Transportation System'. These two contributions are the fundamentals for the progress of TPs. But systematic procedure for finding solution for TPs were developed, primarily by Dantzig [3] in 1951, and then by Charnes, Cooper and Henderson [4] in 1953.The solution of TPs is basically a two steps process:

Step 1: To find an Initial Basic Feasible Solution;

Step 2: To find the Optimal solution.

Cost minimizing transportation problem, has been deliberated since long and is popularized by Abdur Rashid et al. [5], Adwell Mhlanga et al. [6], Aminur Rahman Khan et al. [7-10], Deshmukh N.M. [11], Edward J. Russell [12], Hamdy A. T. [13], Kasana & Kumar [14], Kirca and Satir [15], M. Sharif Uddin et al. [16], Mathirajan and Meenakshi [17], Md. Amirul Islam et al. [18,19], Md. Ashraful Babu et al. [20-22], Mohammad Kamrul Hasan [23], Mollah Mesbahuddin Ahmed et al. [24-27], Pandian & Natarajan [28], Ray and Hossain [29], Reinfeld & Vogel [30], Ulrich A. Wagener [31] and Utpal Kanti Das et al. [32-33].

The well-known existing methods for finding an IBFS for the TPs are North West Corner Method (NWCM) [4,8,13], Least Cost Method (LCM) [4,8,13] and Vogel's Approximation Method (VAM) [4,8,13,30]. These three techniques are mentioned here in the ascending order of their solution accuracy. Among these methods, the cost of the IBFS through VAM will be the least and very near to the optimal solution. For which VAM is considered as the best method for finding an IBFS for the TPs. NWCM started with allocating at the upper left corner cell or North West corner cell. It is based on position but not on transportation cost. Hence, this method usually yields a higher cost, which is much more than optimal cost.

But LCM is based on cost cells and, the solution procedure is started with allocating as much shipments as possible to the cell with the smallest unit cost cell. LCM usually gives better result than NWCM. Again, VAM is also based on cost and yields better solution than NWCM and LCM. In most of the cases, IBFS through VAM will be the least and very near to the optimal solution.

Quality of an IBFS of the TPs is measured by the computational efforts. There is no unique method which can be claimed as the best method for finding an IBFS. For this reason, main attention of this work is focused on the procedure for finding an IBFS rather than finding the optimal solution. Therefore an improved LCM method called iLCM is proposed in this paper to obtain a better IBFS.

Section 2 introduces transportation model, network representation and mathematical formulation to the TP. In section 3 existing methods for finding IBFS is described. In Section 4 the proposed method is presented. In section 5 we give numerical illustration and solution representation based on sampled data. In section 6 we compare the efficiency of the proposed method with other methods and Section 7 presents a conclusion.

2 Transportation Model and Mathematical Formulation

2.1 Transportation Model

The general and accepted form of the TP is presented by the following scheme:

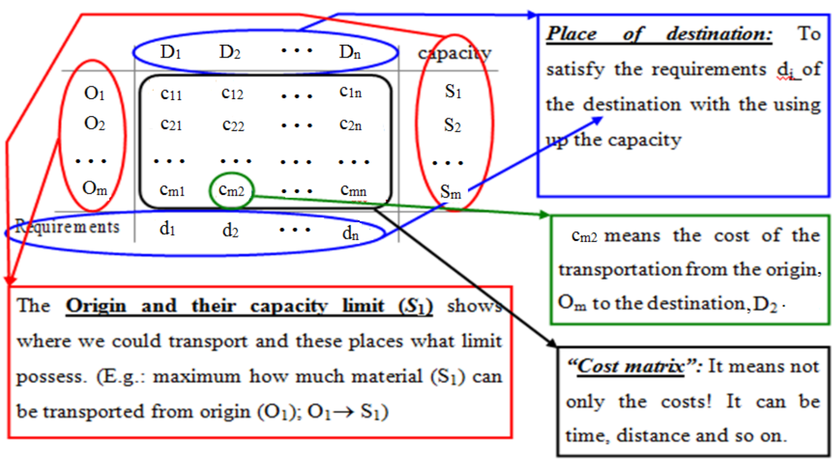


Figure 2.1: Transportation Problem Scheme

2.2 Network Diagram and Mathematical Formulation

To describe the model following notations are to be used:

- m Total number of sources/origins
- n Total number of destinations
- S_i Amount of supply at source i
- d_j Amount of demand at destination j
- c_{ij} Unit transportation cost from source i to destination j
- x_{ij} Amount to be shipped from source i to destination j

Using the above notations network representation of the TPs is shown in the Figure 2.2

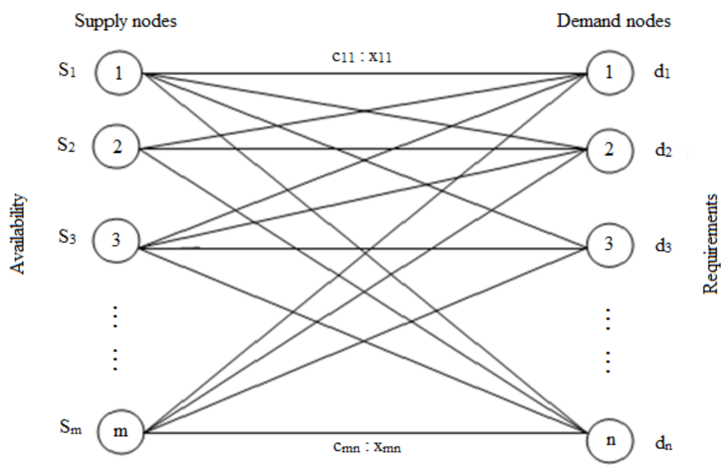


Figure 2.2: Network Diagram for Transportation Problem

The objective of the model is to determine the unknowns' x_{ij} that will minimize the total transportation cost while satisfying the supply and demand restrictions. Considering on this objective TP can be formulated as follows:

minimize: $$z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

subject to: $$\sum_{j=1}^{n} x_{ij} \le S_i \quad ; \quad i = 1, 2, \dots\dots, m$$

$$\sum_{i=1}^{m} x_{ij} \geq d_j \quad ; \quad j = 1, 2, \ldots \ldots, n$$

$$x_{ij} \geq 0 \text{ for all } i \text{ and } j.$$

3 Existing Methods for Finding an IBFS

A set of non-negative allocations, which satisfies the row and column restrictions, is known as IBFS. This is an initial solution of the problem, and is also known as a starting solution of TP. The IBFS may or may not be optimal. By improving upon the IBFS we obtain an optimal solution. Solution procedure of the existing methods mentioned in this article for finding an IBFS is discussed below:

3.1 Least Cost Method (LCM)

In Least Cost Method, basic variables are selected according to the every next least cost cell and the process of allocation is continued until all the demand and supply are allocated. Allocation procedure of this method is summarized below.

- Step 1: Balance the transportation problem.
- Step 2: Find the smallest cost cell c_{ij} in the transportation table. Allocate x_{ij}= min(s_i, d_j) at the cell (i, j).
- Step 3: If the allocation $x_{ij}=s_i$, and $x_{ij} \neq d_i$, cross out i-th row, reduce d_j to (d_j-s_i), and then go to Step-4. If $x_{ij}=d_j$, and $x_{ij} \neq s_i$, cross out j-th column, reduce s_j to (s_j-d_i), and then go to Step-4. If $x_{ij}=s_i=d_j$, cross out either the i-th row or j-th column but not the both and then reduce both s_j and d_j to zero.
- Step 4: Continue this process until all units are allocated. Whenever the smallest costs are not unique, make an arbitrary choice among the smallest costs.

- Step-5: Finally calculate the total transportation cost which is the sum of the product of cost and corresponding allocated value.

3.2 Vogel's Approximation Method (VAM)

This method is not quite simple as the computational procedure like LCM. But this method usually produces better IBFS. Solution procedure of this method is described step by step in below.

- Step 1: Balance the transportation problem.
- Step 2: Find the difference between the smallest and second smallest elements along every row and column. This difference is known as penalty. Enter the column penalties below the corresponding columns and row penalties to the right of the corresponding rows.
- Step-3: Select the highest penalty cost and observe the row or column along which this appears. If a tie occurs, choose any one of them randomly.
- Step-4: Identify the cost cell c_{ij} for allocation which has the least cost in the selected row/column. Make allocation x_{ij}= min(s_i, d_j) to the cell (i,j).
- Step-5: No further consideration is required for the row or column which is satisfied. If both the row and column are satisfied at a time, delete only one of the two, and the remaining row or column is assigned by a zero supply (or demand).
- Step-6: Calculate fresh penalty costs for the remaining sub-matrix as in Step-2 and allocate following the procedure of Steps 3, 4 and 5. Continue the process until all rows and columns are satisfied.
- Step-7: Finally calculate the total transportation cost which is the sum of the product of cost and corresponding allocated value.

4 Proposed Method

It is already mentioned that LCM is an uncomplicated calculation procedure. But result varies when there is a tie which needs to be arbitrarily broken in Least Cost Method. To overcome this problem of ties, an effective improvement is carried out and improved iLCM is proposed in this research. The illustration of the proposed iLCM is as follows:

- Step 1: Balance the transportation problem.
- Step 2: Find the smallest cost cell c_{ij} in the transportation table. Allocate x_{ij}= min (s_i , d_j) at the cell (i, j). In case of ties, select the cell where maximum allocation can be allocated. Again in case of same cost cells and same allocation values select the cell for which sum of demand and supply is maximum in the original transportation table. Finally if all these are same, choose the furthest cell from selected smallest cost cell.
- Step 3: Regulate the supply and demand requirements in the respective rows and columns. Then three cases arises:

  Case 1: If the allocation x_{ij}=s_i, and x_{ij}≠d_i, cross out i-th row, reduce d_j to (d_j-s_i). Now find the next smallest cost cell c_{kj} along the j-th column and allocates x_{kj}= min [s_k , (d_j - s_i)] at the cell (k, j). In case of ties, select the cell where maximum allocation can be made. Again in case of same cost cells and same allocations select the cell for which sum of demand and supply in the original transportation table is minimum. Now adjust the supply and demand requirements and delete either k-th row or j-th column, which is satisfied.

  Case 2: If the allocation x_{ij}=d_j, and x_{ij}≠s_i, cross out j-th column, reduce s_i to (s_i - d_j). Now find the next smallest cost cell c_{ip} along the i-th row and allocates x_{ip}= min [s_k , (s_i - d_j)] at the cell (i, p). In case of ties, follow the procedure of Case 1. Now adjust the supply and demand requirements and delete either i-th row or p-th column, which is satisfied.

Case 3: If the allocation x_{ij}=s_i, and x_{ij}=d_j, find out the smallest cost cell from the rest of the cost cells along the row and column, assigned zero supply (or demand) in that cell and cross out both the row and column.

- Step 4: Repeat Step-2 and Step-3, in the reduced transportation table until all the demand and supply are exhausted.
- Step 5: Finally calculate the total transportation cost which is the sum of the product of cost and corresponding allocated value.

5 Mathematical Illustration

5.1 Solution representation

Twenty sample cost minimizing transportation problem of different sizes, selected at random from some reputed journals published by several admirable authors mentioned in the Table 1 to illustrate the proposed method. We also use these examples to compare the solution obtained by our proposed method (iLCM) with the well known LCM and VAM.

Table 1: Sampled data with corresponding IBFS and Total Cost (using iLCM)

No.	Source	Data	IBFS	Total Cost
Ex-1	**[11]**	$[c_{ij}]_{3x3}$=[6 4 1; 3 8 7; 4 4 2] $[s_i]_{3x1}$=[50, 40, 60] $[d_j]_{1x3}$=[20, 95, 35]	x_{12}=15, x_{13}=35, x_{21}=20, x_{22}=20, x_{32}=60	**555**
Ex-2	**[27]**	$[c_{ij}]_{3x3}$=[4 3 5; 6 5 4; 8 10 7] $[s_i]_{3x1}$=[90, 80, 100] $[d_j]_{1x3}$=[70, 120, 80]	x_{12}=90, x_{22}=30, x_{23}=50, x_{31}=70, x_{33}=30	**1390**
Ex-3	**[27]**	$[c_{ij}]_{3x3}$=[15 7 25; 8 12 14; 17 19 21] $[s_i]_{3x1}$=[12, 17, 7] $[d_j]_{1x3}$=[12, 10, 14]	x_{11}=2, x_{12}=10, x_{21}=10, x_{23}=7, x_{33}=7	**425**
Ex-4	**[29]**	$[c_{ij}]_{3x4}$=[6 3 5 4; 5 9 2 7; 5 7 8 6] $[s_i]_{3x1}$=[22, 15, 8]	x_{12}=12, x_{13}=2, x_{14}=8, x_{23}=15, x_{31}=7, x_{34}=1	**149**

		$[d_j]_{1x4}$=[7, 12, 17, 9]		
Ex-5	**[6]**	$[c_{ij}]_{3x4}$=[1 2 1 4; 4 2 5 9; 20 40 30 10] $[s_i]_{3x1}$=[30, 50, 20] $[d_j]_{1x4}$=[20, 40, 30, 10]	x_{11}=0, x_{13}=30, x_{21}=10, x_{22}=40, x_{31}=10, x_{34}=10	**450**
Ex-6	**[8]**	$[c_{ij}]_{3x4}$=[6 1 9 3; 11 5 2 8; 10 12 4 7] $[s_i]_{3x1}$=[70, 55, 90] $[d_j]_{1x4}$=[85, 35, 50, 45]	x_{12}=35, x_{14}=35, x_{23}=50, x_{24}=5, x_{31}=85, x_{34}=5	**1165**
Ex-7	**[27]**	$[c_{ij}]_{3x4}$=[3 1 7 4; 2 6 5 9; 8 3 3 2] $[s_i]_{3x1}$=[300, 400, 500] $[d_j]_{1x4}$=[250, 350, 400, 200]	x_{12}=300, x_{21}=250, x_{23}=150, x_{32}=50, x_{33}=250, x_{34}=200	**2850**
Ex-8	**[20]**	$[c_{ij}]_{3x4}$=[19 30 50 12; 70 30 40 60; 40 10 60 20] $[s_i]_{3x1}$=[7, 10, 18] $[d_j]_{1x4}$=[5, 8, 7, 15]	x_{11}=2, x_{14}=5, x_{21}=3, x_{23}=7, x_{32}=8, x_{34}=10	**868**
Ex-9	**[11]**	$[c_{ij}]_{3x5}$=[4 1 2 4 4; 2 3 2 2 3; 3 5 2 4 4] $[s_i]_{3x1}$=[60, 35, 40] $[d_j]_{1x5}$=[22, 45, 20, 18, 30]	x_{12}=45, x_{13}=15, x_{21}=22, x_{24}=13, x_{33}=5, x_{34}=5, x_{35}=30	**295**
Ex-10	**[29]**	$[c_{ij}]_{3x5}$=[5 7 10 5 3; 8 6 9 12 14; 10 9 8 10 15] $[s_i]_{3x1}$=[5, 10, 10] $[d_j]_{1x5}$=[3, 3, 10, 5, 4]	x_{14}=1, x_{15}=4, x_{21}=3, x_{22}=3, x_{23}=4, x_{33}=6, x_{34}=4	**183**
Ex-11	**[24]**	$[c_{ij}]_{4x3}$=[2 7 4; 3 3 1 ; 5 4 7; 1 6 2] $[s_i]_{4x1}$=[5, 8, 7, 14] $[d_j]_{1x3}$=[7, 9, 18]	x_{11}=3, x_{12}=2, x_{23}=8, x_{32}=7, x_{41}=4, x_{44}=10	**80**
Ex-12	**[20]**	$[c_{ij}]_{4x4}$=[7 5 9 11; 4 3 8 6; 3 8 10 5; 2 6 7 3] $[s_i]_{4x1}$=[30, 25, 20, 15] $[d_j]_{1x4}$=[30, 30, 20, 10]	x_{12}=5, x_{13}=20, x_{14}=5, x_{22}=25, x_{31}=15, x_{34}=5, x_{41}=15	**435**
Ex-13	**[21]**	$[c_{ij}]_{4x4}$=[5 3 6 10; 6 8 10 7; 3 1 6 7; 8 2 10 12] $[s_i]_{4x1}$=[30, 10, 20, 10] $[d_j]_{1x4}$=[20, 25, 15, 10]	x_{11}=20, x_{13}=10, x_{23}=0, x_{24}=10, x_{32}=20, x_{42}=5, x_{43}=5	**310**
Ex-14	**[7]**	$[c_{ij}]_{4x6}$=[7 10 7 4 7 8; 5 1 5 5 3 3; 4 3 7 9 1 9; 4 6 9 0 0 8] $[s_i]_{4x1}$=[5, 6, 2, 9]	x_{13}=5, x_{22}=4, x_{23}=0, x_{26}=2, x_{31}=1, x_{33}=1, x_{41}=3, x_{44}=2, x_{45}=4	**68**

Ex-15	**[11]**	$[c_{ij}]_{4x6}$=[9 12 9 6 9 10; 7 3 7 7 5 5; 6 5 9 11 3 11; 6 8 11 2 2 10] $[s_i]_{4x1}$=[5, 6, 2, 9] $[d_j]_{1x6}$=[4, 4, 6, 2, 4, 2]	x_{13}=5, x_{22}=4, x_{23}=0, x_{26}=2, x_{31}=1, x_{33}=1, x_{41}=3, x_{44}=2, x_{45}=4	**112**
Ex-16	**[33]**	$[c_{ij}]_{5x5}$=[8 8 2 10 2; 11 4 10 9 4; 5 2 2 11 10; 10 6 6 5 2; 8 11 8 6 4] $[s_i]_{5x1}$=[40, 70, 35, 90, 85] $[d_j]_{1x5}$=[80, 55, 60, 80, 45]	x_{13}=40, x_{21}=30, x_{22}=40, x_{32}=15, x_{33}=20, x_{44}=45, x_{45}=45, x_{51}=50, x_{54}=35	**1565**
Ex-17	**[12]**	$[c_{ij}]_{5x5}$=[73 40 9 79 20; 62 93 96 8 13; 96 65 80 50 65; 57 58 29 12 87; 56 23 87 18 12] $[s_i]_{5x1}$=[8, 7, 9, 3, 5] $[d_j]_{1x5}$=[6, 8, 10, 4, 4]	x_{13}=8, x_{24}=4, x_{25}=3, x_{31}=5, x_{32}=4, x_{41}=1, x_{43}=2, x_{52}=4, x_{55}=1	**1102**
Ex-18	**[31]**	$[c_{ij}]_{5x6}$=[5 3 7 3 8 5; 5 6 12 5 7 11; 2 8 3 4 8 2; 9 6 10 5 10 9; 5 3 7 3 8 5] $[s_i]_{5x1}$=[3, 4, 2, 8, 3] $[d_j]_{1x6}$=[3, 4, 6, 2, 1, 4]	x_{12}=1, x_{14}=0, x_{16}=2, x_{21}=3, x_{24}=1, x_{36}=2, x_{43}=6, x_{44}=1, x_{45}=1, x_{52}=3	**121**
Ex-19	**[32]**	$[c_{ij}]_{5x7}$=[12 7 3 8 10 6 6; 6 9 7 12 8 12 4; 10 12 8 4 9 9 3; 8 5 11 6 7 9 3; 7 6 8 11 9 5 6] $[s_i]_{5x1}$=[60, 80, 70, 100, 90] $[d_j]_{1x6}$=[20, 30, 40, 70, 60, 80, 100]	x_{13}=40, x_{16}=20, x_{21}=20, x_{22}=0, x_{25}=60, x_{34}=70, x_{37}=0, x_{47}=100, x_{52}=30, x_{54}=0, x_{56}=60	**1900**
Ex-20	**[9]**	$[c_{ij}]_{6x6}$=[12 4 13 18 9 2; 9 16 10 7 15 11; 4 9 10 8 9 7; 9 3 12 6 4 5; 7 11 5 18 2 7; 16 8 4 5 1 10] $[s_i]_{6x1}$=[120, 80, 50, 90, 100, 60] $[d_j]_{1x6}$=[75, 85, 140, 40, 95, 65]	x_{12}=55, x_{16}=65, x_{23}=80, x_{31}=50, x_{41}=20, x_{42}=30, x_{44}=40, x_{51}=5, x_{53}=60, x_{55}=35, x_{65}=60	**2325**

5.2 Example illustration

Illustrative solution makes the algorithm understandable to the readers. Considering this, step by step allocations in various cost cells are explained below only for Ex-9 from Table 1.

Table 2: The given problem

From/To	D_1	D_2	D_3	D_4	D_5	Supply
S_1	4	1	2	4	4	60
S_2	2	3	2	2	2	35
S_3	3	5	2	4	4	40
Demand	22	45	20	18	30	

Initial basic feasible solution using proposed method is given below (Table 3):

Table 3: Initial Basic Feasible Solution obtained by iLCM

From/To	D_1	D_2	D_3	D_4	D_5	Supply
S_1	4	45 1	15 2	4	4	60
S_2	22 2	3	2	13 2	2	35
S_3	3	5	5 2	5 4	30 4	40
Demand	22	45	20	18	30	

- According to Step-1: It is found that the given problem is balanced.
- According to Step 2, minimum cost cell is (S_1, D_2) where the allocation is 45=min(45,60). For the allocation, column D_2 is crossed out and supply along S_1 row is reduced to 15=(60-45).
- As per Case-2 of Step-3, (S_1, D_3) is the smallest cost cell along S_1 row. Now allocate remaining 15=(60-45) at the cell (S_1, D_3) and cross out the S_1 row.
- Now in the reduced transportation table, 2 is the minimum cost that appears in the cells (S_2, D_1), (S_2, D_3), (S_2, D_4), (S_2, D_5) and (S_3, D_3). In these cells as per the rule of Step-2, maximum allocation 22 made at the cell (S_2, D_1). For this allocation D_1 column is crossed out. And remaining 13=(35-22) can be allocated at smallest cost cell (S_2, D_4) or (S_2, D_5) along the row S_2. Now sum of the supply and demand is 53=18+35 and 55=20+35 in the

cells (S_2, D_4) and (S_2, D_5) respectively. So, according to the rule of ties in case-1 of step-3, this allocation is made at cell (S_2, D_4) where the sum of demand and supply is minimum. Now cross out the S_2 row.

- Finally complete the allocation by allocating 5, 30 and 5 at the cells (S_3, D_3), (S_3, D_5) and (S_3, D_4) respectively.
- As per Step 5, transportation cost is,
  45x1+15x2+22x2+13x2+5x2+5x2+30x4=295.

6 Result and Analysis

The comparisons of the results are studied in this research to measure the effectiveness of the proposed method. This comparison is shown in following Table 4 and Chart-1 to Chart-4. It is to be mentioned that TORA software is used to obtain the solution of existing methods and also for the optimal solution. Again IBFS by iLCM is illustrated manually.

Table 4: The IBFS and Percentage of Deviation obtained by various methods with optimal solution

No.	Type of problem	Initial Basic Feasible Solution (S_R)			Optimal Result (F_R)	Percentage of Deviation from optimal result (D)			Status*		
		LCM	VAM	iLCM		LCM	VAM	iLCM	LCM	VAM	iLCM
Ex-1	3x3	555	555	555	555	0.00	0.00	0.00	**E**F_R	**E**F_R	**E**F_R
Ex-2	3x3	1450	1500	1390	1390	4.32	7.91	0.00	**U**F_R	**U**F_R	**E**F_R
Ex-3	3x3	433	425	425	425	1.88	0.00	0.00	**U**F_R	**E**F_R	**E**F_R
Ex-4	3x4	153	149	149	149	2.68	0.00	0.00	**U**F_R	**E**F_R	**E**F_R
Ex-5	3x4	560	450	450	450	24.44	0.00	0.00	**U**F_R	**E**F_R	**E**F_R
Ex-6	3x4	1165	1220	1165	1160	0.43	5.17	0.43	**U**F_R	**U**F_R	**U**F_R
Ex-7	3x4	2900	2850	2850	2850	1.75	0.00	0.00	**U**F_R	**E**F_R	**E**F_R
Ex-8	3x4	894	859	868	799	11.89	7.51	8.64	**U**F_R	**U**F_R	**U**F_R
Ex-9	3x5	305	290	295	290	5.17	0.00	1.72	**U**F_R	**E**F_R	**U**F_R
Ex-10	3x5	191	187	183	183	4.37	2.19	0.00	**U**F_R	**U**F_R	**E**F_R
Ex-11	4x3	83	80	80	76	9.21	5.26	5.26	**U**F_R	**U**F_R	**U**F_R

Ex-12	4x4	435	470	435	410	6.10	14.63	6.10	**U**F_R	**U**F_R	**U**F_R
Ex-13	4x4	310	285	310	285	8.77	0.00	8.77	**U**F_R	**E**F_R	**U**F_R
Ex-14	4x6	70	68	68	68	2.94	0.00	0.00	**U**F_R	**E**F_R	**E**F_R
Ex-15	4x6	114	112	112	112	1.79	0.00	0.00	**U**F_R	**E**F_R	**E**F_R
Ex-16	5x5	1685	1505	1565	1475	14.24	2.03	6.10	**U**F_R	**U**F_R	**U**F_R
Ex-17	5x5	1123	1104	1102	1102	1.91	0.18	0.00	**U**F_R	**U**F_R	**E**F_R
EX-18	5x6	134	116	121	116	15.52	0.00	4.31	**U**F_R	**E**F_R	**U**F_R
Ex-19	5x7	2080	1930	1900	1900	9.47	1.58	0.00	**U**F_R	**U**F_R	**E**F_R
Ex-20	6x6	2455	2310	2325	2170	13.13	6.45	7.14	**U**F_R	**U**F_R	**U**F_R

*Numerically S_R *is* equal to F_R or not. For equal (**E**F_R) or unequal (**U**F_R), notation is used.

The analysis of the new methodology has been carried out by solving twenty randomly chosen examples which are shown in the Table 1. Here, the formula $D = \frac{F_R - S_R}{F_R} \times 100$ is used to obtain the percentage of deviation from optimal result. This calculation is carried out to evaluate that how much nearer the S_R is to F_R.

Chart-1 shows a comparison between iLCM and LCM based on the result obtained from randomly chosen examples (shown in Table 1). In chart 1 we notice that iLCM yields better IBFS over LCM.

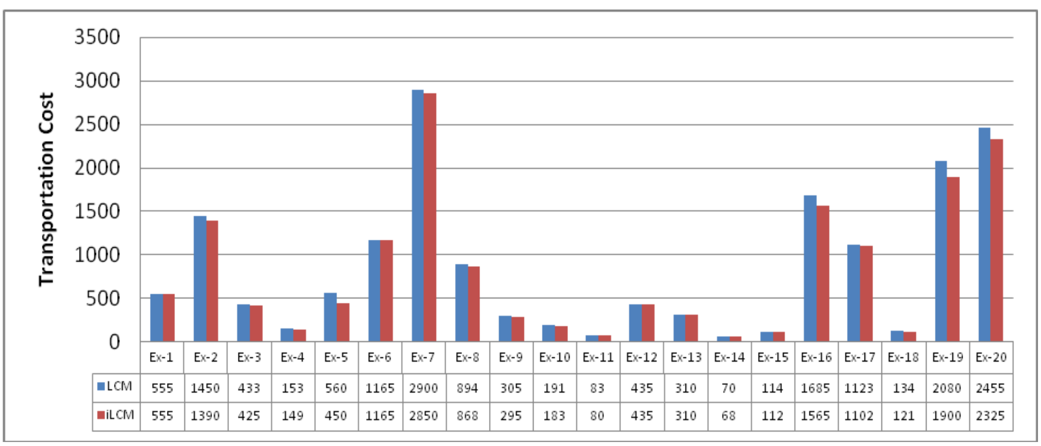
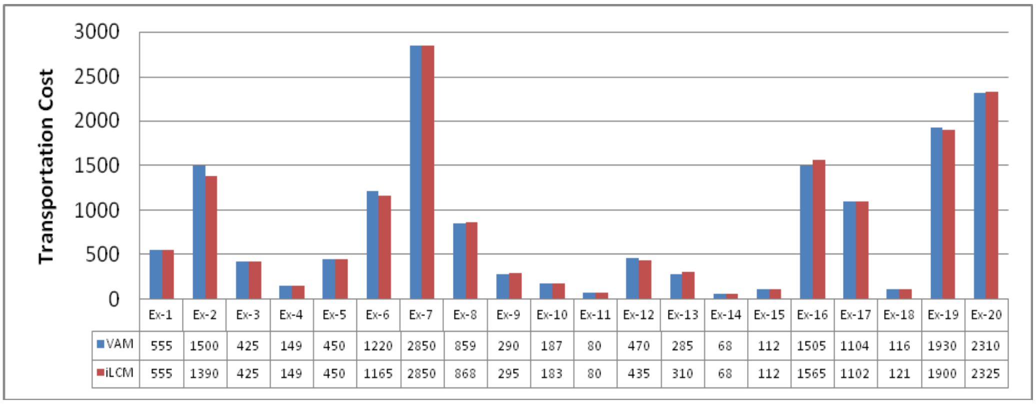


Chart-1: Comparison of the result between LCM and iLCM

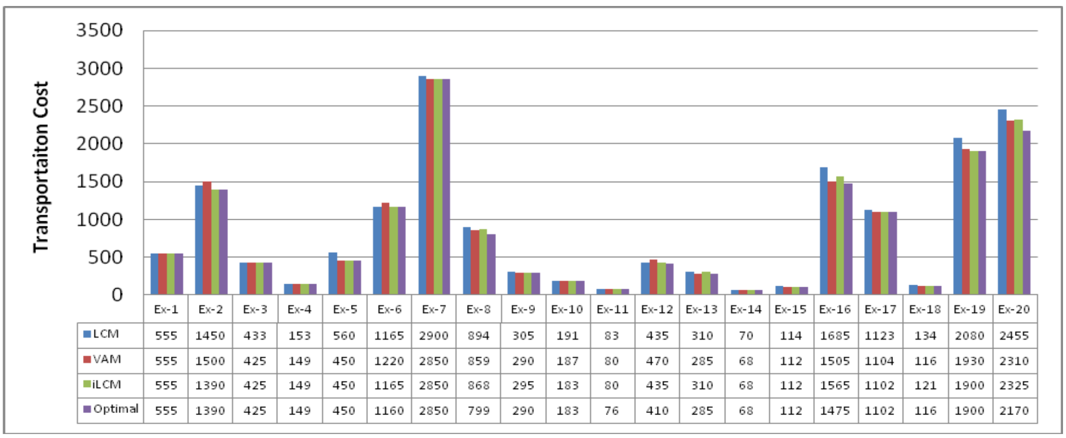
In chart-2 a comparative study is also made between VAM and iLCM based on the result obtained from randomly chosen examples (shown in Table 1).



	Ex-1	Ex-2	Ex-3	Ex-4	Ex-5	Ex-6	Ex-7	Ex-8	Ex-9	Ex-10	Ex-11	Ex-12	Ex-13	Ex-14	Ex-15	Ex-16	Ex-17	Ex-18	Ex-19	Ex-20
VAM	555	1500	425	149	450	1220	2850	859	290	187	80	470	285	68	112	1505	1104	116	1930	2310
iLCM	555	1390	425	149	450	1165	2850	868	295	183	80	435	310	68	112	1565	1102	121	1900	2325

Chart-2: Comparison of the result between VAM and iLCM

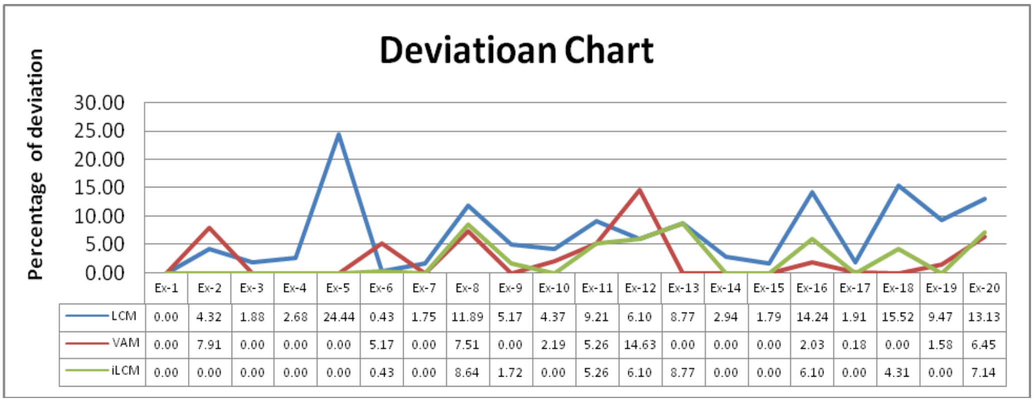
In chart-3 a comparative study of LCM, VAM, iLCM and optimal results are shown based on the result obtained from randomly chosen examples (shown in Table 1).



	Ex-1	Ex-2	Ex-3	Ex-4	Ex-5	Ex-6	Ex-7	Ex-8	Ex-9	Ex-10	Ex-11	Ex-12	Ex-13	Ex-14	Ex-15	Ex-16	Ex-17	Ex-18	Ex-19	Ex-20
LCM	555	1450	433	153	560	1165	2900	894	305	191	83	435	310	70	114	1685	1123	134	2080	2455
VAM	555	1500	425	149	450	1220	2850	859	290	187	80	470	285	68	112	1505	1104	116	1930	2310
iLCM	555	1390	425	149	450	1165	2850	868	295	183	80	435	310	68	112	1565	1102	121	1900	2325
Optimal	555	1390	425	149	450	1160	2850	799	290	183	76	410	285	68	112	1475	1102	116	1900	2170

Chart-3: Comparative Study of the Result obtained by LCM, VAM, iLCM with Optimal Result

According to the simulation results chart-4 shows that iLCM yields better IBFS than LCM in 80% cases and for rest of 20% its performance is similar to LCM. Again, once the iLCM is compared with VAM, it is observed that in 40% cases these two methods performs similar to each other, and in 30% cases performance of iLCM is superior than VAM and for rest of 30% cases VAM performs better than iLCM. It is also found that iLCM and VAM directly yields the same percentage of optimal solution. Simulation results in this study also show that iLCM directly yields 55% of optimal solutions whereas VAM directly yields 50% of optimal solutions. On the other hand, LCM provides very few direct optimal solutions.



	Ex-1	Ex-2	Ex-3	Ex-4	Ex-5	Ex-6	Ex-7	Ex-8	Ex-9	Ex-10	Ex-11	Ex-12	Ex-13	Ex-14	Ex-15	Ex-16	Ex-17	Ex-18	Ex-19	Ex-20
LCM	0.00	4.32	1.88	2.68	24.44	0.43	1.75	11.89	5.17	4.37	9.21	6.10	8.77	2.94	1.79	14.24	1.91	15.52	9.47	13.13
VAM	0.00	7.91	0.00	0.00	0.00	5.17	0.00	7.51	0.00	2.19	5.26	14.63	0.00	0.00	0.00	2.03	0.18	0.00	1.58	6.45
iLCM	0.00	0.00	0.00	0.00	0.00	0.43	0.00	8.64	1.72	0.00	5.26	6.10	8.77	0.00	0.00	6.10	0.00	4.31	0.00	7.14

Chart-4: Percentage of Deviation of the Results obtained by LCM, VAM and iLCM

7 Conclusion

In general TP is concerned with determining an optimal strategy for distributing commodities from a group of supply centers such as factories called sources to various receiving centers such as warehouses called destinations. In such a way it minimizes total distribution costs. In this work iLCM is proposed to obtain a better IBFS to the TP. The solution procedure of TP involves finding the IBFS first and then going through the necessary iterations to find the optimum solution. In our study, in most of the cases iLCM appeared to be the most efficient method in finding IBFS in terms of proximity to optimal solution. It may be fair to assume that the number of subsequent iterations may also be fewer if the IBFS is closest to optimal solution. Therefore this finding is important in saving time and resources for minimization of transportation costs and optimizing transportation processes which could help significantly to improve the organization's position in the market.

In this article the usefulness of iLCM has been carried out to justify its efficiency by solving twenty randomly chosen numerical examples where it is found that the method is suitable for solving TPs. According to the simulation results the proposed method iLCM provides a significant IBFS by ensuring minimum transportation cost.

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