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The limit in the category of Q-P quantale modules

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Abstract

In this paper, firstly, the definition of Q-P quantale modules and some relative concepts were introduced. we prove that the category of Q-P quantale modules is a pointed and connected category. Based on which, we give the structure of the limit of this category, so it is complete. Secondly, we talk about some properties of the inverse systems of the category of Q-P quantale modules, we construct the inverse limit of the category of Q-P quantale modules. Introducing the definition of a mapping between two inverse systems, we get the limit mapping in the category of Q-P quantale modules. At last, The definition of bimorphism of Q-P quantale modules is given.

Keywords: Q-P quantale modules; Morphisms; Category; Limit; Inverse systems;

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1 Introduction

Quantale was introduced by C.J.Mulvey in 1986 in order to provide a lattice theoretic setting for studying non-commutative C*-algebras[1]. On the other hand, the concept was expected to relate to the semantics of non-commutative logics, for example that of quantum mechanics. A quantale-besed (non-commutative logic theoretic) approach to quantum mechanics was developed by Piazza. It is known that quantales are one of the semantics of linear logic. The systematic introduction of quantale theory came from the book [2], which written by K.I.Rosenthal in 1990. In particular, each frame (and therefore each complete Boolean algebra) is a quantale. Other examples include the power-set of a semigroup as well as the set of all relations on a set. Quantale theory provides a powerful tool in studying noncommutative structures, it has a wide applications, especially in studying noncommutative C*-algebra theory [3], the ideal theory of commutative ring[4], linear logic [5] and so on. Following C.J.Mulvey, the quantale theory have been studied by many researches [6-16].The inverse limit in the category of topological molecular lattices and the limit in the category of topological molecular lattices was studied deeply in [18-20].we give the structure of the limit of this category, so it is complete. Secondly, we talk about some properties of the inverse systems of the category of Q-P quantale modules, we construct the inverse limit of the category of Q-P quantale modules.

2 Preliminaries

Definition 2.1[2] A quantale is a complete lattice Q with an associative binary operation & satisfying: a&($\sup_\alpha$b$_\alpha$)=$\sup_\alpha$(a&b$_\alpha$)and ($\sup_\alpha$b$_\alpha$)&a=$\sup_\alpha$(b$_\alpha$&a)for all a$\in$ Q and b$_\alpha$ $\subseteq$Q.

Definition 2.2:[6] Let Q be a quantale, a left module over Q(briefly, a left Q-module)is a sup-lattice M, together with a module action $\cdot$:Q$\times$M$\longrightarrow$M satisfying

(1) $(\bigvee\limits_{i\in I} a_i)\cdot m=\bigvee\limits_{i\in I}(a_i\cdot m)$;

(2) $a\cdot(\bigvee\limits_{j\in J} m_j)=\bigvee\limits_{j\in J}(a\cdot m_j)$;

(3) $(a\&b)\cdot m=a\cdot(b\cdot m)$. for all a,b,a$_i$ $\in$Q, m,m_j $\in$M.

The right modules are defined analogously.

If Q is untial and e·m=m for every m∈M, we say that M is unital.

Definition 2.3[6] Let M and N are Q-quantales. A mapping f : M⟶N is said to be module homomorphism if $f(\bigvee_{i\in I} m_i) = \bigvee_{i\in I} f(m_i)$, and $f(a\cdot m) = a\cdot f(m)$for all a ∈Q, m, m_i ∈M.

Definition 2.4 Let Q and P are quantales , a double quantale module over Q and P is a sup-lattice M, together with a module atction $T : Q\times M\times P \longrightarrow M$ satisfying

(1) $T(\bigvee_{i\in I} a_i, m, \bigvee_{j\in J} b_j) = \bigvee_{i\in I}\bigvee_{j\in J} T(a_i, m, b_j)$;

(2) $T(a, (\bigvee_{k\in K} m_k), b) = \bigvee_{k\in K} T(a, m_k, b)$;

(3) $T(a\&b, m, c\&d) = T(a, T(b, m, c), d)$. for all$a, b, a_i$ ∈Q,c, d, b_j ∈Q, $m, m_k \in$ M.

we shall denote by (M, T) the Q-P quantale module M over Q and P.

Definition 2.5 Let (M_1, T_1) and (M_2, T_2) are Q-P quantale modules. A mapping f : $M_1 \longrightarrow M_2$is saied to be Q-P quantale module homomorphism if satisfying

(1) $f(\bigvee_{i\in I} m_i) = \bigvee_{i\in I} f(m_i)$;

(2) $f(T_1(a, m, b)) = T_2(a, f(m), b)$for all a∈ Q, b∈ P, $m_i \in M$.

Definition 2.6 Let (M, T_M) be Q-P quantale module, N is the subset of M, N is said to be submodule of M if N is closed under arbitrary join and T_M(a, n, b)∈N for all a∈ Q, b∈ P, n∈N.

If $Q = P$ is unital quantale with unit e, we define $T(e, m, e) = m$ for all $m \in M$.

Definition 2.7. Let Q, P be a quantale, (M_1, T_1) and (M_2, T_2) are Q-P quantale modules. A mapping f : $M_1 \longrightarrow M_2$ is said to be a $Q-P$ *quantale module homomorphism* if f satisfies the following conditions:

(1) $f(\bigvee_{i\in I} m_i) = \bigvee_{i\in I} f(m_i)$;

(2) $f(T_1(a, m, b)) = T_2(a, f(m), b)$ for all $a \in Q$,$b \in P$, $m_i, m \in M$.

Definition 2.8. Let (M, T_M) be a $Q-P$ quantale module over Q, P, N be a subset of M, N is said to be a *submodule* of M if N is closed under arbitrary join and $T_M(a, n, b) \in N$ for all $a \in Q$,$b \in P$, $n \in N$.

3 The limit in the category of Q-P quantale module

In this section, we will talk about the structure of limit in the category of Q-P quantale module.

Definition 3.1. Let Q, P be a quantale, $_{\mathbf{Q}}\mathbf{Mod}_{\mathbf{P}}$ be the category whose objects are the Q-P quantale modules of Q, P, and morphisms are the Q-P quantale module homomorphisms, i.e.,

$\mathcal{O}b(_{\mathbf{Q}}\mathbf{Mod}_{\mathbf{P}})$={M : M is Q-P quantale modules},

$\mathcal{M}or(_{\mathbf{Q}}\mathbf{Mod}_{\mathbf{P}})$={$f : M \longrightarrow N$ is the Q-P quantale modules homorphism}.
Hence, the category $_{\mathbf{Q}}\mathbf{Mod}_{\mathbf{P}}$ is a concrete category.

Theorem 3.1 Let I is a small category , $D : I \longrightarrow$ **LFQuant** is a functor, then the limit of functor D is that $\overline{S} = \overline{\pi} \mid^{S}: S \longrightarrow L$, $\overline{\pi} : \prod\limits_{i\in I} D(i) \longrightarrow L$ for all $u : i \longrightarrow j$ is a I morphis such that $f(j) = D(u)(f(i))$, and satisfy $p_i(f) = f(i)$ for all $i \in I, f \in (\prod\limits_{i\in I} D(i), T)$.

Proof. Define a order on L is that $f \leq g$ if and only if $f(i) \leq g(i)$ in $D(i)$ for all $i \in I$.

(1) Let $u : i \longrightarrow j, D(u) \in \mathcal{M}or(_{\mathbf{Q}}\mathbf{Mod}_{\mathbf{P}})$, then $D(u)(0(i)) = 0(j)$, so $0 \in L$.

Suppose $\{f_k \mid k \in K\} \subseteq L$, define $(\bigvee\limits_{k\in K} f_k)(i) = \bigvee\limits_{k\in K} f_k(i)(\forall i \in I)$for all $i \in I$, then for all $u : i \longrightarrow j$, we have $D(u)((\bigvee\limits_{k\in K} f_k)(i)) = D(u)(\bigvee\limits_{k\in K} f_k(i)) = \bigvee\limits_{k\in K} D(u)(f_k(i)) = \bigvee\limits_{k\in K} f_k(j) = (\bigvee\limits_{k\in K} f_k)(j)$, thus $\bigvee\limits_{k\in K} f_k \in L$;

(2) For all $a \in Q, b \in P, f \in L$, define$(T(a, f, b))(i) = T_{D_{(i)}}(a, f(i), b)$(for all $i \in I$), then for all $u : i \longrightarrow j$is a Q-P quantale module morphism, $D(u)(T(a, f, b)(i)) = D(u)(T_{D_{(i)}}(a, f(i), b)) = T_{D_{(j)}}(a, D(u)(f(i)), b) = T_{D_{(j)}}(a, f(j), b) = (T(a, f, b))(j)$, so $T(a, f, b) \in L$.

Therefore L is a submodule of $\prod\limits_{i\in I} D(i)$,i.e.,$L \in \mathcal{O}b(_{\mathbf{Q}}\mathbf{Mod}_{\mathbf{P}})$. By the definition of L, we can know that is a natural source with functor D.

Let $(\widehat{L}, T_{\widehat{L}})$is a Q-P quantale module, $(\widehat{L}, (\widehat{p_i})_{i\in I})$is a natural resource about functor, then$D(u)(\widehat{p_i}(m)) = \widehat{p_j}(m)$for all $u : i \longrightarrow j, m \in \widehat{L}$. Define $h : \widehat{L} \longrightarrow L$such that for all$m \in \widehat{L}, h(m) = f_m$, and $f_m(i) = \widehat{p_i}(m)$ for all $i \in I$. Since for all$m \in \widehat{L}$, $u : i \longrightarrow j$ is a Q-P quantale module morphism, we have $D(u)(f_m(i)) = D(u)(\widehat{p_i}(m)) = \widehat{p_j}(m) = f_m(j)$, so $f_m \in L$, therefore h is a well defined.

We will prove h is a Q-P quantale module morphism,

(1) For all$\{m_k \mid k \in K\} \subseteq \widehat{L}$, $i \in I$,

$h(\bigvee_{k\in K} m_k)(i) = f_{\bigvee_{k\in K} m_k}(i) = \widehat{p_i}(\bigvee_{k\in K} m_k) = \bigvee_{k\in K} \widehat{p_i}(m_k) = \bigvee_{k\in K} f_{m_k}(i) = \bigvee_{k\in K} h(m_k)(i) = (\bigvee_{k\in K} h(m_k))(i), i.e., h(\bigvee_{k\in K} m_k) = \bigvee_{k\in K} h(m_k);$

(2) For all $a \in Q, b \in P, m \in \widehat{L}$, $i \in I$,

$h(T_{\widehat{L}}(a,m,b))(i) = (f_{T_{\widehat{L}})(a,m,b)}(i) = \widehat{p_i}(T_{\widehat{L}}(a,m,b)) = T_{D_{(i)}}(a,\widehat{p_i}(m),b) = T_{D_{(i)}}(a,f_m(i),b)$

$= T_{D_{(i)}}(a,(h(m)(i)),b) = (T(a,h(m),b))(i), i.e., h(a,m,b) = (a,h(m),b).$

So h is a Q-P quantale module morphism.

It's clear $\widehat{p_i} = p_i \circ h$ for all $i \in I$.In fact, $(p_i \circ h)(m) = p_i(h(m)) = h(m)(i) = f_m(i) = \widehat{p_i}(m)$ for all $m \in L$.

Let $h' : \widehat{L} \longrightarrow L$ is a Q-P quantale module morphism such that $\widehat{p_i} = p_i \circ h'$ for all $i \in I$, then $h(m)(i) = f_m(i) = \widehat{p_i}(m) = (p_i \circ h')(m) = h'(m)$ for all $m \in \widehat{L}$, $i \in I$, so $h = h'$.

Therefore is the limit of functor D .

Theorem 3.3 The category of $_{\mathbf{Q}}\mathbf{Mod}_{\mathbf{P}}$ is completed.

4 Inverse system and Inverse limit in the category of Q-P quantale module

Theorem 4.1 Let I is a downward-directed set, $F : I \longrightarrow {}_{\mathbf{Q}}\mathbf{Mod}_{\mathbf{P}}$ is a functor, then F is said to be inverse system in the category of $_{\mathbf{Q}}\mathbf{Mod}_{\mathbf{P}}$.

Remark 4.2 We can give another definition of inverse system is as follow. Let I is a downward-directed set, there is a Q-P quantale module (A_i, T_i) for all $i \in I$, If $i, j \in I$ and $i \leq j$, then exist a Q-P quantale module morphism $F_{ij} : A_i \longrightarrow A_j$ for all $i, j, k \in I$. If $i \leq j \leq k$, then $F_{ik} = F_{jk} \circ F_{ij}$ and $F_{ii} = id_{A_i}$. We can said $S = \{A_i, F_{ij}, I\}$ is a inverse system of a Q-P quantale module, $F_{ij}$is said to be a skeletal mapping.

Definition 4.3 Let $S = \{A_i, F_{ij}, I\}$ is a inverse system of a Q-P quantale module, $\{x_i\}_{i\in I} \in (\prod_{i\in I} A_i, T)$, $i, j \in I$,If$i \leq j$, then$F_{ij}(x_i) = x_j$, $\{x_i\}_{i\in I}$is said to be Silk thread of$S$.

Theorem 4.4 Let $S = \{A_i, F_{ij}, I\}$is a inverse system of a Q-P quantale module, then W is a submodule of $(\prod\limits_{i\in I} A_i, T)$.

Proof. (1) Let $\{m_k\}_{k\in K} \subseteq W$, and for all $k \in K$, $m_k = \{x_{ki}\}_{i\in I}$. Since$F_{ij}$is a Q-P quantale module morphism, then $F_{ij}(\bigvee\limits_{k\in K} x_{ki}) = \bigvee\limits_{k\in K} F_{ij}(x_{ki}) = \bigvee\limits_{k\in K} x_{kj}$, i.e., $\bigvee\limits_{k\in K} m_k \in W$.

(2)For $\{x_i\}_{i\in I} \in W, a \in Q, b \in P$,$i, j \in I$, If$i \leq j$, then $F_{ij}(T_i(a, x_i, b)) = T_j(a, F_{ij}(x_i), b)$
$= T_j(a, x_j, b)$, so$\{T_i(a, x_i, b)\}_{i\in I} \in W$.

Theorem 4.5 Let I is a downward-directed set, $F : I \longrightarrow {}_{\mathbf{Q}}\mathbf{Mod}_{\mathbf{P}}$ is a inverse system of Q-P quantale module, then $(W, (p_i)_{i\in I})$is the limit of F, and $p_i(\{x_i\}_{i\in I}) = x_i$ for all $\{x_i\}_{i\in I} \in W$.

Let $F : I \longrightarrow {}_{\mathbf{Q}}\mathbf{Mod}_{\mathbf{P}}$ and $K : I' \longrightarrow {}_{\mathbf{Q}}\mathbf{Mod}_{\mathbf{P}}$ are the inverse systems of F and K respectively, $(W, (p_i)_{i\in I})$ and $(W', (p'_{i'})_{i'\in I'})$ are the inverse limit of F and K respectively. Specifically says, let I and I'are the downward-directed sets, F and K are the downward-directed sets, satisfy$F(i) = A_i, K(i') = A_{i'}$, for all $i \in I$, $i' \in I$, (A_i, T_i) and $(A_{i'}, T_{i'})$are Q-P quantale modules, by the theorem 5.3, we can see that (W, T) and $(W', T')$is the submodule of $(\prod\limits_{i\in I} A_i, T)$ and $(\prod\limits_{i\in I} A_{i'}, T')$respectively. For all $i, j \in I, i', j' \in I'$, since$i \leq j, i' \leq j'$, then$F(i \longrightarrow j) = F_{ij} : F(i) \longrightarrow F(j), K(i' \longrightarrow j') = K_{i'j'} : K(i') \longrightarrow K(j')$ are the Q-P quantale module morphisms. For all$i, j, k \in I, i', j', k' \in I'$, if $i \leq j \leq k, i' \leq j' \leq k'$, then$F_{ik} = F_{jk} \circ F_{ij}, K_{i'k'} = K_{j'k'} \circ K_{i'j'}$ and $F_{ii} = id_{A_i}, K_{i'i'} = id_{A_{i'}}$, F_{ij} and $K_{i'j'}$ is said to be Skeleton mapping of F and K respectively.

Definition 4.6 Let I is a downward-directed set, $I' \subseteq I$. For all $i \in I$, exist $i' \in I'$ such that $i' \leq i$, $I'$is said to be cofinal set of$I$.

Definition 4.7 Let $F : I \longrightarrow {}_{\mathbf{Q}}\mathbf{Mod}_{\mathbf{P}}$ and $K : I' \longrightarrow {}_{\mathbf{Q}}\mathbf{Mod}_{\mathbf{P}}$ are inverse systems in the ${}_{\mathbf{Q}}\mathbf{Mod}_{\mathbf{P}}$, $(\varphi, \{f_{i'}\}_{i'\in I'})$ is said to be mapping from F to K, if satisfy

(1) $\varphi : I' \longrightarrow I$ is a monotone and $\varphi(I')$ is a cofinal set of I

(2) $f_{i'} : F(\varphi(i')) \longrightarrow K(i')$ is a Q-P quantale module morphism for all $i' \in I'$, and satisfy that $K_{i'j'} \circ f_{i'} = f_{j'} \circ F_{\varphi(i')\varphi(j')}$for all $i', j' \in I', i' \leq j'$, then the diagram

$$\begin{array}{ccc} F(\varphi(i')) & \xrightarrow{f_{i'}} & K(i') \\ {\scriptstyle F_{\varphi(i')\varphi(j')}}\downarrow & & \downarrow{\scriptstyle K_{i'j'}} \\ F(\varphi(j')) & \xrightarrow[f_{j'}]{} & K(j') \end{array}$$

commutes.

Theorem 4.8 Let $F : I \longrightarrow {}_{\mathbf{Q}}\mathbf{Mod}_{\mathbf{P}}$ and $K : I' \longrightarrow {}_{\mathbf{Q}}\mathbf{Mod}_{\mathbf{P}}$are inverse systems in the ${}_{\mathbf{Q}}\mathbf{Mod}_{\mathbf{P}}$,$(W, (p_i)_{i\in I})$ and $(W', (p'_{i'})_{i'\in I'})$ is the inverse limit of F and K respectively, then the mapping $(\varphi, \{f_{i'}\}_{i'\in I'})$between F and K can induce a Q-P quantale module morphism$f : W \longrightarrow W'$, such that $f(\{x_i\}_{i\in I}) = \{x'_{i'}\}_{i'\in I'}$for all $\{x_i\}_{i\in I} \in T$, $x'_{i'} = (f_{i'} \circ p_{\varphi(i')})(\{x_i\}_{i\in I})$ for all $i' \in I'$.

Proof. At first, we will prove the mapping f is well-defined.

For all$i', j' \in I', i^{'} \leq j'$, since$\phi$is a monotone, then $\varphi(i') \leq \varphi(j')$. By the definition 5.7(2), we know that $K_{i'j'} \circ f_{i'} = f_{j'} \circ F_{\varphi(i')\varphi(j')}$. $F_{\varphi(i')\varphi(j')}(x_{\varphi(i')}) = x_{\varphi(j')} = p_{\varphi(j')}(\{x_i\}_{i\in I})$ for all $\{x_i\}_{i\in I} \in W$, then $K_{i'j'}(x^{'}_{i'}) = K_{i'j'} \circ f_{i'} \circ p_{\varphi(i')}(\{x_i\}_{i\in I}) = K_{i'j'} \circ f_{i'}(x_{\varphi(i')}) = f_{j'} \circ F_{\varphi(i')\varphi(j')}(x_{\varphi(i')}) = f_{j^{'}} \circ p_{\varphi(j^{'})}(\{x_i\}_{i\in I}) = x^{'}_{j^{'}}$, i.e.,$\{x'_{i'}\}_{i'\in I'} \in W'$.

(1) For all $\{g_s\}_{s\in S} \subseteq W, i' \in I'$, we have that

$f(\bigvee_{s\in S} g_s)(i') = (f_{i'} \circ p_{\varphi(i')})(\bigvee_{s\in S} g_s) = f_{i'} \circ (\bigvee_{s\in S} g_s)(\varphi(i')) = f_{i'}(\bigvee_{s\in S} g_s(\varphi(i')) = \bigvee_{s\in S} (f_{i'}(g_s(\varphi(i')))$

$= \bigvee_{s\in S} (f_{i'} \circ p_{\varphi(i')})(g_s) = (\bigvee_{s\in S} f(g_s)(i')$,i.e.,$f(\bigvee_{s\in S} g_s) = \bigvee_{s\in S} f(g_s)$, Since$f(0) = 0$, then$f$ preserve arbitrary sups.

(2) For all $a \in Q, b \in P, \forall\{x_i\}_{i\in I} \in W, i' \in I'$,

we have that $f(T(a, \{x_i\}_{i\in I}, b))(i') = (f(\{T_i(a, x_i, b)\}_{i\in I}))(i') = (f_{i'}\circ p_{\varphi(i')})(\{T_i(a, x_i, b)\}_{i\in I})$ $= f_{i'}(T_{\varphi(i')}(a, x_{\varphi(i')}, b)) = T'_{i'}(a, f_{i'}(x_{\varphi(i')}), b) = \{T'(a, f(\{x_i\}_{i\in I}), b)\}_{i\in I}(i')$,

then $f(T(a, \{x_i\}_{i\in I}, b)) = T'(a, f(\{x_i\}_{i\in I}), b)$.

Therefore $f$is a Q-P quantale module morphism.

Definition 4.9 Let $F : I \longrightarrow {}_{\mathbf{Q}}\mathbf{Mod}_{\mathbf{P}}$ and $K : I' \longrightarrow {}_{\mathbf{Q}}\mathbf{Mod}_{\mathbf{P}}$ are inverse systems in the ${}_{\mathbf{Q}}\mathbf{Mod}_{\mathbf{P}}$,$(W, (p_i)_{i\in I})$ and $(W', (p'_{i'})_{i'\in I'})$ is the inverse limit of F and K respectively,then mapping $f$is said to be the limit mapping in the ${}_{\mathbf{Q}}\mathbf{Mod}_{\mathbf{P}}$.

Theorem 4.10 Let $F : I \longrightarrow {}_{\mathbf{Q}}\mathbf{Mod}_{\mathbf{P}}$ and $K : I' \longrightarrow {}_{\mathbf{Q}}\mathbf{Mod}_{\mathbf{P}}$ are inverse systems in the ${}_{\mathbf{Q}}\mathbf{Mod}_{\mathbf{P}}$, $(W,(p_i)_{i\in I})$ and $(W',(p'_{i'})_{i'\in I'})$ is the inverse limit of F and K respectively, f is a limit mapping. Since $f_{i'}$is a Q-P quantale module morphism for all $i'\in I$.

Proof. For all $g_1,g_2\in T$ and $g_1\neq g_2$. Since$f(g_1)=f(g_2)$, then$f(g_1)(i')=(f_{i'}\circ p_{\varphi(i')})(g_1)=(f_{i'}\circ p_{\varphi(i')})(g_2)=f(g_2)(i')$ for all $i'\in I'$. Because $f_{i'}$is a monotone, then $p_{\varphi(i')}(g_1)=p_{\varphi(i')}(g_2)$, so$g_1(\varphi(i'))=g_2(\varphi(i'))$. Since$\varphi(I')I'$is the cofinal set of$I$, then for all$i\in I$, exist$\varphi(j')\in\varphi(I')$such that$\varphi(j')\leq i$, then$g_1(i)=f_{\varphi(j')i}(g_1(\varphi(j')))=f_{\varphi(j')i}(g_2(\varphi(j')))=g_2(i)$, Contradictory,thus$f(g_1)\neq f(g_2)$,therefore$f$is a monomorphism.

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