

Effect of Grashof Parameter on Velocity for Unsteady Fluid Flow Past an Infinite Vertical Porous Flat Plate

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Abstract

Natural convection of an incompressible fluid flow past an infinite vertical flat plate with constant suction velocity is investigated. In this work, we investigate the effect of Grashof parameter on velocity profiles. Unsteady flow is considered and restricted to laminar domain. The equations governing the flow are discretized and solved iteratively using java for various values of Grashof parameter ranging from 0 to 1. Small values of constant suction velocity and constant magnetic field were chosen ($v = 2.5$ and $m^2 = 5.0$) respectively. The result obtained are then analysed using tables and graphs. The effect of increase of Grashof parameters was addressed. An increase in Grashof parameter leads to an increase in velocity profile near the plate but as the distance from the plate increase the velocity profile decreases.

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1 Introduction

Over the past 20 years a revolution in Magneto hydrodynamic (MHD) studies has taken place. Magneto hydrodynamics is the science of motion of electrically conducting fluid under magnetic fields. It analyses the dynamics of the interaction of electrically conducting fluids and electromagnetic fields. The flow of an electrically conducting fluid under a magnetic field in general gives rise to induced electric currents. The magnetic field exerts mechanical forces on the induced electric currents. The induced electric current flows in the direction perpendicular to both the magnetic field and the direction of motion of the fluid. However, the induced currents also generate their own magnetic field, which in turn affects the original magnetic field.

The production of sheeting material, which includes both metal and polymer sheets, arises in a number of industrial manufacturing processes. The fluid dynamics due to a stretching surface have important application in many engineering processes. Thus the rate of cooling and mass transfer have to be controlled effectively in order to attain the expected quality of the final product. In recent years, considerable interest has been given to the theory of rotating fluids due to its application in cosmic and geophysical sciences. Studies on boundary layer flows of viscous fluids due to uniform stretching sheet have been done by Hayat *et. al* [4]. Many researchers have done experiments and study on electrically conducting fluids with magnetic fields. Many researches have been done in acceleration, pattern of flows, forces causing motion, heat transfer, mass transfer etc. The induced magnetic field at different time levels for a given Hartmann number and magnetic Prandtl number was analysed by Rao *et. al* [8].

The flow of fluids through porous media in rotating system is of interest to a petroleum engineer, who is concerned with the movement of oil and gas through the reservoir and also hydrologist who is interested in the study of migration of underground flow. Study of flows through porous media in rotating systems also find applications in geothermal energy system Herrero *et.al* [5]. MHD free convection heat and mass transfer of a heated generating fluid past an impulsively started infinite vertical porous plate with Hall current and radiation absorption was carried out by Kinyanjui *et.al* [7] Laminar convection in a uniformly heated vertical porous channel was carried out by Chandrasekhar *et.al* [2]. Chaturvedi [3] studied MHD flow past an infinite porous media with variable suction.

Unsteady MHD convective heat permeable moving plate with heat absorption study was carried out by Chamka A.J [1], whereas network simulation method applied to radiation and viscous dissipation effects on MHD unsteady free convective over vertical porous plate was carried out by Jordan J.Z [6]. Tania S.K and Samad M.A [9] studied the effect of radiation, heat generation and viscous dissipation on MHD free convection flow along a stretching sheet. In spite of all these contributions the area of unsteady fluid flow past an infinite vertical porous plate has receive little attention from researchers and that is the motivation for caring out this research work.

Nomenclature

F_i	Body force, N
g	Acceleration due to gravity m^2s^{-2}
\vec{B}	Specific heat at constant pressure $g^{-1}k^{-1}$
Gr	Grashof number
Pr	Prandtl number
H	Magnetic field intensity, wbm^{-2}

H_0	Fixed magnetic field intensity, wbm^{-2}
i, j, k	Unit vectors in the x^*, y^*, z^* directions respectively
M	Magnetic parameter
k	Thermal conductivity
$q(u, V)$	Velocity of the fluid, ms^{-1}
t	Dimensional time, s
T^*	Dimensional temperature of the fluid, k
T_∞^*	Dimensional temperature of the fluid at the plate k
T_y^*	Dimensionless temperature of the fluid
u^*, v^*	Velocity components, ms^{-1}
x^*, y^*	Cartesian co-ordinates in dimensional form
H	Specific enthalpy $kJkg^{-1}$
\vec{E}	Magnetic induction vector
\hat{E}	Electric field
C	Dimensionless concentration of the species
P	Pressure of the fluid Nm^{-2}

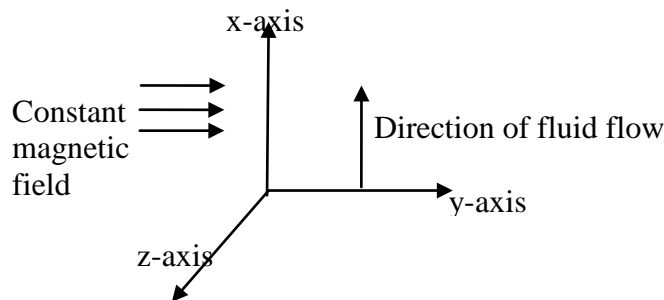
GREEK SYMBOLS

Symbol	Quantity
β	Coefficient of volumetric expansion, K^{-1}
σ	Electrical conductivity, $\Omega^{-1}m^{-1}$
μ_e	Magnetic permeability, Hm^{-1}
ρ	Fluid density Kgm^{-3}
ν	Kinematics viscosity of the fluid m^2s^{-1}
θ	Dimensionless temperature of the fluid
μ	Coefficient of viscosity

∇	Gradient operator, $\left(i \frac{\partial}{\partial x} + j \frac{\partial}{\partial y} + k \frac{\partial}{\partial z} \right)$
∇^2	Laplacian operator, $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right)$
ρ_∞	Free stream fluid density Kgm^{-3}
∇t	Time interval
$\nabla x, \nabla y$	Distance interval
<i>MHD</i>	Magnetohydrodynamic
Hot	Higher order terms
ϕ	Viscous dissipation

2 Mathematical analysis

A two dimensional fluid flow where the x-axis is taken along the plate in a vertically upward direction which is the direction of the flow and y-axis is taken normal to plate. Since the plate is infinite in length and for a two dimensional free convective fluid flow the physical variables are functions of x, y and t as shown below in flow configuration.



The non-dimensional equations governing the laminar two-dimensional incompressible flow of the fluid are described below.

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + V \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + Gr\theta + M^2 u \quad (1)$$

and

$$\frac{\partial V}{\partial t} + u \frac{\partial V}{\partial x} + V \frac{\partial V}{\partial y} = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + Gr\theta + M^2 V \quad (2)$$

The non-dimensional variables that appear in the equations are as a result of the following scaling variables.

$$t = \frac{t^* U^2}{\nu}, \quad x = \frac{x^* U}{\nu}, \quad y = \frac{y^* U}{\nu} \quad u = \frac{u^*}{U} \quad V = \frac{V^*}{U}, \quad \theta = \frac{T^* - T_\infty^*}{T_y - T_\infty^*}, \quad H = \frac{H^*}{H_0}$$

With superscript (*) star being dimensional variables and the non-dimensional

parameters being Grashof number $Gr = \frac{\nu g \beta (T_y - T_\infty)}{U^3}$, and

$$M^2 = \frac{\sigma \mu^2 H_0^2 \nu}{U^2 \rho} \text{-Hartmann number.}$$

The initial conditions are;

$$\text{at } y = 0, \quad u_{(0,i)}^0 = 1, \quad V_{(0,i)}^0 = 0$$

$$\text{at } y > 0, \quad u_{(j,i)}^0 = 0, \quad V_{(j,i)}^0 = 0$$

for $j > 0$ and all i

The boundary conditions;

$$\text{at } y = 0, \quad u_{(0,i)}^n = 1, \quad V_{(0,i)}^n = 0$$

$$\text{at } x = 0, \quad u_{(j,0)}^n = 1, \quad V_{(j,0)}^n = 0$$

3 Method of solution

The equations governing fluid flow are solved numerically using finite difference method with a uniform mesh, then the difference equations are solved iteratively using java computer program.

For this forward difference approximation is used for time derivative and central difference approximations are used for all spatial derivatives. The constant velocity and magnetic field were set at 2.5 and 5.0 respectively.

4 Result and discussions

The computations were carried out for a range of Grashof parameter varying from 0.5, 0.7 and 1.0 respectively. The results are depicted in form of tables and graphs for both primary velocity and secondary velocity as shown below.

Table 1: Table for primary velocity

Y	U1	U4	U8
0.0	1.000000000	1.000000000	1.000000000
0.1	0.852135665	0.817776475	0.737017285
0.2	0.735475307	0.671836221	0.551999851
0.3	0.643495945	0.557906303	0.423621530
0.4	0.571013344	0.470800631	0.335531553
0.5	0.513935243	0.405357986	0.275622962
0.6	0.469046822	0.356922478	0.235159823
0.7	0.433824263	0.321537459	0.207963187
0.8	0.406279716	0.295975932	0.189732588
0.9	0.384838246	0.277686505	0.177515328
1.0	0.368243634	0.264702162	0.169307803
1.1	0.355487874	0.255538692	0.163764381
1.2	0.345758898	0.249097184	0.159989323
1.3	0.338401656	0.244577600	0.157390670
1.4	0.332888629	0.241406109	0.155579216
1.5	0.328796897	0.239176378	0.154299647

1.6	0.325789722	0.237603666	0.153384242
1.7	0.323601325	0.236489959	0.152722170
1.8	0.322024016	0.235698156	0.152239380
1.9	0.320897178	0.235133403	0.151885603
2.0	0.320097781	0.234729883	0.151626042
2.1	0.319532231	0.234441625	0.151436100
2.2	0.319129356	0.234236184	0.151298051
2.3	0.318834350	0.234090304	0.151198914
2.4	0.318603446	0.233986852	0.151129056
2.5	0.318399065	0.233912510	0.151081198
2.6	0.318185116	0.233855762	0.151049594
2.7	0.317922081	0.233804769	0.151029164
2.8	0.317561422	0.233744662	0.151014358
2.9	0.317038744	0.233653697	0.150997443
3.0	0.316264985	0.233497482	0.150965722
3.1	0.315114694	0.233220203	0.150896963
3.2	0.313410153	0.232731341	0.150751917
3.3	0.310899750	0.231885833	0.150462223
3.4	0.307228622	0.230455065	0.149911253
3.5	0.301899454	0.228085625	0.148904606
3.6	0.294221994	0.224243174	0.147126357
3.7	0.283252738	0.218141358	0.144077996
3.8	0.267734499	0.208663772	0.139001842
3.9	0.246065025	0.194306671	0.130805522

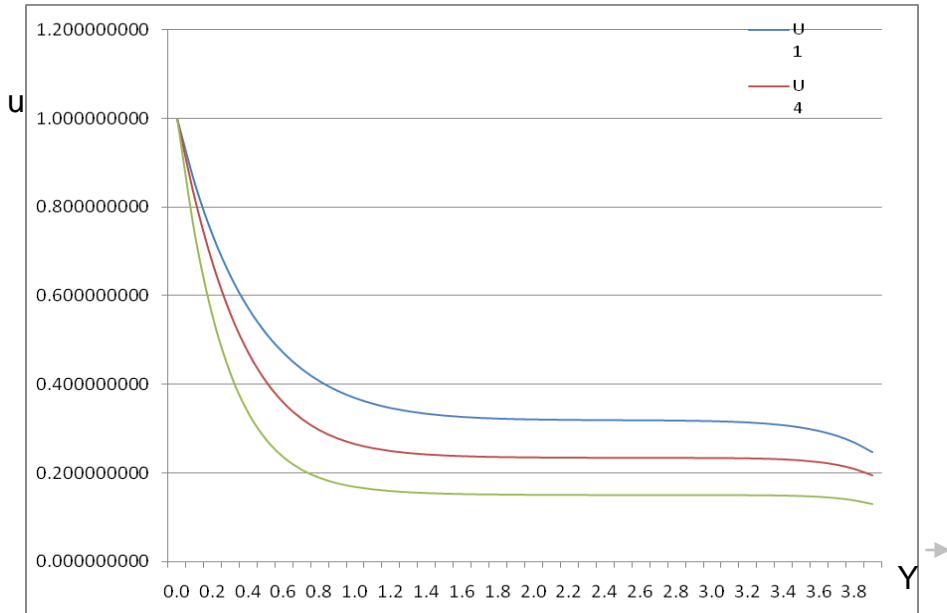


Figure 1: Primary velocity profile

Table 2: Table for secondary velocity

Y	V1	V4	V8
0.0	0.000000000	0.000000000	0.000000000
0.1	-0.046354116	-0.110238964	-0.141147326
0.2	-0.073573448	-0.161057063	-0.220827550
0.3	-0.088103303	-0.178850410	-0.260612328
0.4	-0.094493754	-0.179513206	-0.275624750
0.5	-0.095882247	-0.172253695	-0.276115964
0.6	-0.094386378	-0.162162765	-0.268815441
0.7	-0.091402712	-0.151877031	-0.257987732
0.8	-0.087825083	-0.142620777	-0.246216080
0.9	-0.084200871	-0.134840100	-0.234964660
1.0	-0.080841986	-0.128578030	-0.224973191
1.1	-0.077903551	-0.123689469	-0.216528980

1.2	-0.075439565	-0.119959433	-0.209651087
1.3	-0.073442067	-0.117164376	-0.204212286
1.4	-0.071868298	-0.115100988	-0.200017481
1.5	-0.070659078	-0.113597102	-0.196852050
1.6	-0.069750738	-0.112513307	-0.194509779
1.7	-0.069082371	-0.111740151	-0.192807253
1.8	-0.068599741	-0.111193643	-0.191589600
1.9	-0.068256922	-0.110810439	-0.190730939
2.0	-0.068016422	-0.110543376	-0.190131844
2.1	-0.067848411	-0.110357617	-0.189715333
2.2	-0.067729445	-0.110227428	-0.189422287
2.3	-0.067640947	-0.110133521	-0.189206828
2.4	-0.067567612	-0.110060834	-0.189031850
2.5	-0.067495782	-0.109996590	-0.188864734
2.6	-0.067411799	-0.109928461	-0.188673098
2.7	-0.067300290	-0.109842650	-0.188420372
2.8	-0.067142312	-0.109721665	-0.188060918
2.9	-0.066913280	-0.109541522	-0.187534367
3.0	-0.066580613	-0.109268089	-0.186758864
3.1	-0.066101026	-0.108852168	-0.185622931
3.2	-0.065417470	-0.108222937	-0.183975718
3.3	-0.064455781	-0.107279299	-0.181615670
3.4	-0.063121215	-0.105878857	-0.178277979
3.5	-0.061295324	-0.103824594	-0.173621978
3.6	-0.058833997	-0.100850271	-0.167221037
3.7	-0.055568220	-0.096607535	-0.158559901
3.8	-0.051310270	-0.090661428	-0.147048425
3.9	-0.045869719	-0.082507175	-0.132066520

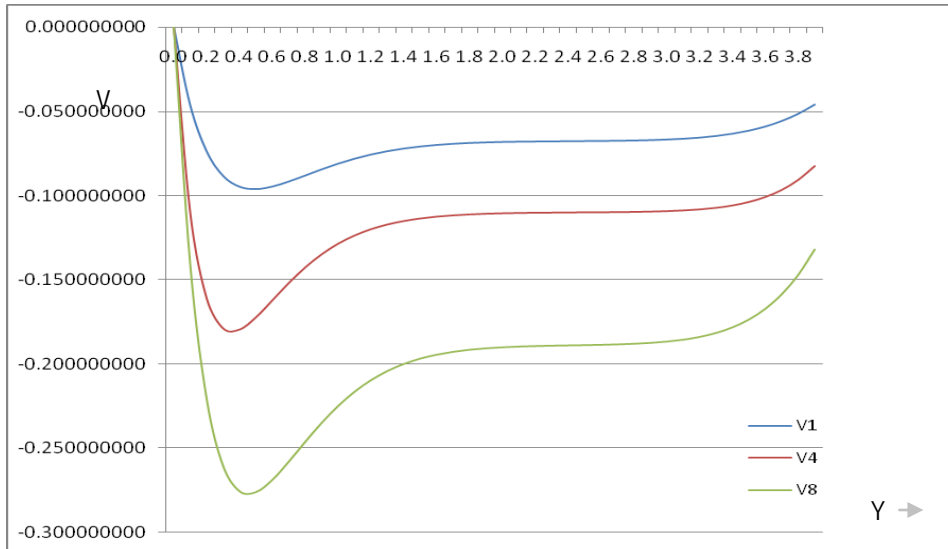


Figure 2: Secondary velocity profile

From Table 1 $Gr > 0$, we observe that, the primary velocity profile is greater near the plate and decreases away from the plate. We also observe from table 2 for $Gr > 0$ that the secondary velocity profile decreases nearer the plate but as distance from the plate increases, the velocity profile decreases.

An increase in the Grashof parameter increases the secondary velocity profile but as distance from the plate increases, the curve exhibits a decrease in velocity profile from the curves V_1 , V_4 , V_8 respectively. In case of primary velocity profile the velocity decrease with increase of Grashof parameter. As shown by Figure 1.

5 Conclusion

Analysis of the effect of various parameters on velocities on unsteady free convection incompressible fluid flow was investigated. In all the cases considered the velocity was resolved into two components restricted to the laminar boundary layer. An increase in Grashof parameter leads to an increase in velocity profiles near the plate and a decrease away from the plate.

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