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Extended $\bar{\partial}-$ Cohomology and Integral Transforms in Derived Geometry to QFT-equations Solutions using Langlands Correspondences

Francisco Bulnes[1]

Abstract

Through consider several geometrical Langlands correspondences are determined equivalences necessary to the obtaining in the quantized context from differential operators algebra (actions of the algebra on modules) and the holomorphic bundles in the lines bundle stacks required to the model the elements of the different physical stacks, the extension of their field ramifications to the meromorphic case. In this point, is obtained a result that establish a commutative diagram of rings and their spectrum involving the non-commutative Hodge theory, and using integral transforms to establish the decedent isomorphisms in the context of the geometrical stacks to a good **Opers,** level. The co-cycles obtained through integral transforms are elements of the corresponding deformed category to mentioned different physical stacks. This determines solution classes to the QFT-equations in field theory through the Spectrum of their differential operators.

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1 Introduction

In the geometrical Langlands programme, ramifications correspond to extensions of induced moduli stacks where meromorphic connections are induced to holomorphic connections. Thus in the context of the Penrose transforms is available to obtain the different cohomological solution classes of the field equations including the singularities of the space-time in an algebraic frame with a geometrical image on twisted lines bundles, when can be determined a sheaf $\mathcal{D}^{\lambda}_{k,I_y}$ of $\tilde{\mathcal{L}}_{\lambda}-$ twisted differential operators on the moduli space Bun_{G,I_y}, well-defined, which is our deformed sheaf necessary to establish the geometrical correspondences between objects of moduli stacks and differential operators that require meromorphic connections to determine the holomorphic connections of the corresponding derived category and their geometry. A justification on the nature of the our twisted derived category and their elements as ramifications of a field (to the field equations) is the followed through the Yoneda algebra [1], [2] where is searched extends the action of the endomorphism $\mathrm{End}(\mathbb{V}_{critical})$,through the Lie algebra action $\hat{\mathfrak{g}}$, that is the cohomological space $H^0(\hat{\mathfrak{g}}[[Z]], \mathbb{V}_{critical})$. The Yoneda extension algebra must be understood as a projective Harish-Chandra module to the pair $(\hat{\mathfrak{g}}, G[[Z]])$, corresponding to the hyper-cohomology space, at least of dimension $q = 1$, of the spectrum of the integral transform defined to a functor of type Zuckerman more an geometrical image of the ramification ∇_s. Then $H^0(\hat{\mathfrak{g}}[[Z]], \mathbb{V}_{critical}) = \mathbb{C}[\mathrm{Op}_{^LG}(\mathcal{D}^{\times})]$. The Zuckerman functor can be the extended through an adequate character.

2 Field Ramifications

From the theorem 2. 1, [3] is clear that the ramification to the part of connection ∇_s,must be inside the context of the moduli space $\mathcal{M}_{Higgs}(^LG, C)$.The induced lines bundle must be one from $T^{\vee}_{V}\mathrm{Bun}_{C}(\Sigma)$, with the condition of that it must be a divisor of holomorphic vector bundle.

One immediate consequence of this theorem 2. 1, and the application of an adequate meromorphic extension given in [4], but in the context of a divisor factor of the moduli space $\mathcal{M}_H({}^LG, C)$, [3] is the explicit induction, which could be the required using the moduli correspondences of this space.

Then is reliable analyze through cycle cohomology these moduli identities from the Hitchin mappings extended to deformations of the stacks $T^{\vee}\mathrm{Bun}_G$, and $T^{\vee}\mathrm{Bun}_{L_G}$, in similar manner. Likewise, these cohomological versions can give a factorization result of the solution classes to field equations to a corresponding dimension of the cohomology spaces considering as proper ramification the used in the stack moduli $T^{\vee}\mathrm{Bun}_{L_G}$, using the images of Cousin complexes (the corresponding to the Cousin cohomology) due to the Penrose transforms framework.

3 Cohomology of Cycles and Moduli problems

One important result from a point of view of schemes is the following:

Theorem 3.1. (F. Bulnes) [5]. *If we consider the category $M_{\mathcal{K}_F}(\hat{\mathfrak{g}},\ Y)$, then a scheme of their spectrum $V^{Def}_{critical}$, where Y, is a Calabi-Yau manifold comes given as:*

$$Hom_{\hat{\mathfrak{g}}}(X, V^{Def}_{critical}) \cong Hom_{Loc_{L_G}}(V_{critical}, M_{\mathcal{K}_F}(\hat{\mathfrak{g}}, Y)), \qquad (1)$$

Proof. [5].

Then we can to establish the following results considering the moduli problems between objects of an algebra, which has been realized using commutative rings extended by the Yoneda algebra, that is to say:

Theorem 3.2. *The Yoneda algebra* $Ext_{\mathcal{D}^s(Bun_G)}(\mathcal{D}^s, \mathcal{D}^s)$, *is abstractly* A_∞-*isomorphic to* $Ext^{\bullet}_{Loc_{L_G}}(\mathcal{O}_{Op_{L_G}}, \mathcal{O}_{Op_{L_G}})$.

Proof.[6],[7].

This result bring in particular that formal deformations of the sheaf $\mathcal{D}^s$, can be extended by deformation theory to the QFT case using path integrals.

Lemma (F. Bulnes) 3. 1. *Twisted derived categories corresponding to the algebra of functions* $\mathbb{C}[Op_{L_G}(D^\times)]$, *are the images obtained by the composition* $\mathcal{P}(\tau)$, *on* $\tilde{\mathcal{L}}_\lambda, \forall\lambda\epsilon\mathfrak{h}^*$, *and such that their Penrose transform is:*

$$\mathcal{P} : H^0({}^LG, \Gamma(Bun_G, \mathcal{D}^\times)) \cong Ker(U, \tilde{\mathcal{D}}_{\lambda,y}), \tag{2}$$

The lemma plays an important role to exhibit the influence of twistor transform to the obtaining the twisted nature of the derived categories $D^\times$, starting from the line bundle $\mathcal{L}_\lambda$. *Proof.* It is other form to write the twistor transform treatment followed in [8]. The image that stays is naturally a Penrose transform image.

Theorem (F. Bulnes) 3. 3. **[9]** *The following resolution of cohomological spaces is a geometrical resolution to the lines bundles given by* $\mathcal{L}^{\otimes 2} \cong \tilde{\mathcal{L}}_{[\bar{C}_{hV}(\theta)]} \otimes \zeta^{\otimes -(n-1)}$, *of [9] and that gives moduli stacks in (4):*

$$H^0(T^\vee Bun_G, \mathcal{O}))(\cong \mathbb{C}[Op_{L_G}(D)]) \to H^1(T^\vee Bun_G, \Omega^1) \to$$
$$H^2(T^\vee Bun_G(\Sigma), \Omega^2) \to \cdots \to H^\bullet(?, \Omega^\bullet) \to \dots, \tag{3}$$

Proof. [9].

4 Results

In the end of the demonstration of the theorem 3.2, [9] was exhibited the cohomological space $H^\bullet(?, \Omega^\bullet)$, as the space $H^\bullet(\mathbf{H}^\vee, \Omega^\bullet)$, where $\mathbf{H}^\vee =$

$T^{\vee}[Op_{L_G}(D)]$, which is included in the quasi-coherent category $M_{\mathcal{K}_F}(\hat{\mathfrak{g}}, Y)$, to the ramification problem. Then considering what we know on field theory in the frame of the derived categories, we can to enunciate the following diagram of co-cycles to the quasi-coherent category $M_{\mathcal{K}_F}(\hat{\mathfrak{g}}, Y)$, where the ramification of field that is the connection of the space $\Omega^{\bullet}(Op_{L_G}(D))$, is also a connection obtained under the following diagram of commutative rings, considering this space as the cotangent bundle space [9]:

Theorem 4.1. *One meromorphic extension of one flat connection given through a Hitchin construction we can give the following commutative co-cycles diagram to the category* $M_{\mathcal{K}_F}(\hat{\mathfrak{g}}, Y)$,

$$\begin{array}{ccccc} \mathbf{h}\epsilon H^0(T^{\vee}Bun_G, \mathcal{D}^s) & \xrightarrow{d} & H^1(T^{\vee}Bun_G, \mathcal{O}) & \xrightarrow{\cong} & \Omega^1[\mathbf{H}] \\ \cong\downarrow & & \cong_{\Phi\mu}\downarrow & & \downarrow\pi \\ a\epsilon\mathbb{C}[Op_{L_G}] & \xrightarrow{d} & \Omega^1[\mathcal{O}_{Op_{L_G}}] & \xrightarrow{d} & C\times B \end{array} \quad (4)$$

Proof. We consider the theorem 3. 3 of [10] and the study realized in [11] to demonstrate that in the derived categories class $\mathcal{D}^{\times}$ [2] the fibrations $R^1\chi_*(\mathcal{D}^s)$, analogues to $R^1\chi_*(\mathcal{O}^s)$, to this theorem, are equals to $R^1\chi_*(\mathbf{h})\epsilon\Omega^1[\mathbf{H}]$, [3] $\forall\chi : T^{\vee}$ Bun$_G \rightarrow \mathbf{H}$, a Hitchin mapping. Using first, the isomorphism in the

[2]The first differential in a spectral sequence $H^*(\mathcal{D})[[s]]$, implies $H^*(\mathcal{D}^s)$, for the deformation $\mathcal{D}\rightarrow\mathcal{D}^s$.

[3]For other side, using the diagram:

$$\begin{array}{ccccc} & & R^1\chi_* & & \\ & & \longrightarrow & & \\ | & & & & \downarrow \\ H^0(\Sigma,\Omega^1) & \xrightarrow{\Gamma} & H^1(\Sigma,\Omega^2) & \xrightarrow{\cong} & \Omega^1[\mathbf{H}] \\ \cong\downarrow & & \cong\downarrow & & \downarrow\pi_{\mathbf{H}} \\ \Omega^1(\Sigma^0,\mathfrak{g}) & \xrightarrow{d} & \Omega^2(\Sigma,\mathfrak{g}) & \xrightarrow{a} & C\times B \end{array}$$

Langlands correspondence $\mathfrak{c}$, such that $\Phi^i(\mathfrak{c}(\mathcal{O}_v)) = \mathcal{O}(\mathcal{O}_v) \boxtimes \wedge^i \mathbb{V}$, [4] arriving to $\Omega^i[Op_{L_G}]$, we have the equivalence of complexes (field solutions to similar differential equations) [5]

$$\{d\mathbf{h} = 0\} \cong^{L\Phi^\mu} \{da = 0\}, \tag{5}$$

which required the correspondence

$$\tilde{\mathfrak{c}} : D_{Coh}(T^\vee Bun_G, \mathcal{O}) \cong D_{Coh}({}^L T^\vee Bun_G, \mathcal{O}), \tag{6}$$

where $\tilde{\mathfrak{c}} = \mathfrak{c}(T^\vee \mathcal{O}_{Op_{L_G}}) = \mathfrak{c}(\mathcal{O}^{DG}_{SPu})$, [6]such that $\Phi_{\mathcal{O}^{DG}_{Spu}}(\mathbb{C}_u) \cong \Gamma q_{2!}(\mathcal{O}^L_{\tilde{\mathcal{N}}} \otimes_{\mathcal{N}} \mathbb{C}_u)$. But this functor can be categorized to a quantum level having the cohomology groups H^0, and H^1, in their quantized versions which derive in natural way. Indeed, due to that the Penrose transform determined by $\Phi_{\mathcal{O}^{DG}_{Spu}}(\mathbb{C}_u)$, transforms sheaves whose modules of critical level as the corresponding modules of the sheaf $\mathcal{D}^\times(\mathrm{Bun}_G\Sigma)$, (remember that this is the space of the twisted sheaves corresponding to the lines bundle $T^\vee_V \mathrm{Bun}_C(\Sigma)$,which must be a divisor of holomorphic bundle [8],[12]) only can be obtained and characterized by a parameterization of the space of connections given in the category $\mathrm{D}^b(\mathcal{Q}Coh(MOp^{nilp}_{L_G}))$, [7]which is identified with the category $D^b(M_{\mathcal{K}_\mathcal{C}}(\tilde{\mathfrak{g}}, Y))^{I_0}$.
By the Frenkel conjecture [13],[14] [8]we have that each quasi-coherent sheaf

[4]Of fact is had the integral transforms composition $\mathfrak{c} \circ \Phi^\mu =^L \Phi^\mu$.

[5]But the equation $d\mathbf{h} = 0$, haves their equivalent in a flag manifold as $H^q(\Sigma, ad_F(\mathfrak{u})) = 0$, with $q = 0$, or 1, uniquely, in the radical algebra component $\mathfrak{u}$. Then the unique survivors of the hyper-cohomology $H^q_{G[[Z]]}$, are the cohomology groups $H^{q-u}_{G[[Z]]}(\mathrm{Sym}T\mathrm{Bun}_G)$, to $\mathrm{Bun}_G \to BG[[z]]$.

[6]Here Sp_u is the Springer fiber defined by the space

$$Sp_u = \{\mathfrak{b}\prime \epsilon {}^L G \diagup {}^L B \mid u \epsilon \mathfrak{b}\prime\}.$$

[7]$MOp^{nilp}_{L_G}$, is the Miura space whose objects are resulted of the Miura transformation which consist in the map

$$\mathcal{D}^\times(\Omega^{-\rho}) \to Op_{L_G}(\mathcal{D}^\times),$$

where $\mathcal{D}^\times(\Omega^{-\rho})$ are the connections on the ${}^L H-$ bundle $\Omega^{-\rho}$, over $\mathcal{D}^\times$. This is the push-forward of the $\mathbb{C}^\times-$ bundle corresponding to the canonical line bundle Ω, with respect to the homomorphism $\rho : \mathbb{C} \to^L H$.

[8]Conjecture. There is an equivalence of categories

$$D^b(\mathfrak{g}_{\mathcal{KC}} - mod_{nilp})^{I^0} \cong D^b(\mathcal{Q}Coh(MOp^{nilp}_{LG})),$$

on MOp^{nilp}_{LG}, should correspond to an object of the derived category $D^b(M_{\mathcal{K}_C}(\tilde{\mathfrak{g}}, Y))^{I_0}$. The functor $\Phi_{\mathcal{O}^{DG}_{Spu}}$, is a Hecke functor [9] which is an integral transform that can associates eigen-sheaves of Hecke on $T^\vee Bun_{G,y}$,having that $T^\vee Bun_{G,y} = Bun_{G,y,\infty}$, where $Bun_{G,y,\infty} \cong G(\mathcal{K})\diagup B$, (which in the Hitchin map language $h : Bun_{Higgs} \to B$ (to a quantized)) is equivalent to the category $D_{coh}({}^L Bun, \mathcal{D})$.
But this category come from the fact of that the corresponding cohomologies $H^\bullet(Bun, SymT)$, and $H^\bullet(Bun, \mathcal{D}^s)$have been calculated on some finite union of Atiyah-Bott strata of Bun_G, for any twisted form $\mathcal{D}^s$, and $\mathcal{D}$, combining the corresponding Hitchin mapping.

Then the geometric Langlands conjecture in terms of Higgs bundles, consider a functor between the categories $D_{coh}({}^L Loc, \mathcal{O})$, with the action of the Hecke functors on $D_{coh}({}^L Bun, \mathcal{D})$.

But $MOp^{nilp}_{LG} \cong Op^{nilp}_{LG} \times \tilde{\tilde{\mathcal{N}}}\diagup {}^L G$, which can be deduced since exists a subjacent Steinberg variety structure to the Langlands correspondence $\mathfrak{c}$, such that $\tilde{\mathfrak{c}} = \mathfrak{c}(\mathcal{O}_{Sp_{uC}}(\mathbb{V}) \times \mathbb{C}^\times)$, where $\mathcal{O}_{Sp_{uC}} = \mathcal{O}_{\tilde{\mathcal{N}}} \otimes_{\mathcal{O}\mathcal{N}} \mathbb{C}$, and $\mathcal{N} \subset^{\mathcal{L}} \mathfrak{g}$, is the nilpotent cone whose springer resolution is $\tilde{\mathcal{N}}$.
Then by K-theory the subjacent Steinberg variety takes the form $St = \tilde{\mathcal{N}} \times_{\mathcal{N}} \tilde{\mathcal{N}}$, which have elements $C \times B$, satisfying that $Isom(d\mathbf{h}) = d(da), \forall a \in \mathbb{C}[Op_{L_G}(D)]$, that is to say, the element in $C \times B$ must be $adIsom(\pi_{V^*} L')$, to a Langlands correspondence $\tilde{\mathfrak{c}} = \Phi^i(\mathfrak{c}(\mathcal{O}_{nah_C}(\mathbb{V}))$. Then is established the equivalence between complexes

$$\{d(da)\} \underset{\longleftrightarrow}{{}^L\Phi^\mu} \{Isomd\mathbf{h}\}, \tag{7}$$

which is compatible with the actions of the algebra $\mathbf{FunOp}^{nilp}_{LG}$, of both categories.

[9]The Hecke functor ${}^L\Phi^\mu$, is defined as the integral transform

$${}^L\Phi^\mu : D^b({}^L Bun_G; \mathcal{D}) \to D^b({}^L Bun_G; \mathcal{D}^\times),$$

with the correspondence rule given as:

$$\mathcal{M} \mapsto q^\mu!(\mathcal{M}^L \otimes \mathbb{C}_{[dim^L\mathcal{H}^\mu]}).$$

In other words to the kernels of $\Omega^i, i = 1, 2, \ldots,$ are the that are in sheaf $\mathcal{O}_{Op_{L_G}}$, [10] that is to say, there is an extended Penrose transform such that the kernels set are the fields $\mathbf{h}$, with $Isom(d\mathbf{h}) = 0$,in the hyper-cohomology $\mathbb{H}^*(\Omega^1 \xrightarrow{d} \Omega^2 \xrightarrow{d} \ldots)$, [9] that is to say, the hyper-cohomology deduced through the Hitchin mapping χ_*, and the down line will have fields a, with $d(da) = 0$, in the hyper-cohomology $\mathbb{H}(\Omega^1 \xrightarrow{d} \Omega^2 \xrightarrow{d} \ldots)$.

Remarks: An equivalent commutative diagram that is direct consequence of the theorem 3. 2, in the context of differential operators algebra (differential graded algebra) is

$$\begin{array}{ccccc} H^\bullet(\mathfrak{g}[[z]], \mathbb{V}_{critical}) & \rightarrow & H^\bullet(\mathfrak{g}[[z]], \mathfrak{g}, \mathbb{V}_{critical}) & \xrightarrow{\cong} & \Omega^\bullet[\mathbf{H}] \\ \cong\downarrow & & \cong_\Phi\downarrow & & \downarrow\pi \\ \mathbb{C}[Op_{L_G}] \xrightarrow{d} & & \Omega^\bullet[Op_{L_G}] \xrightarrow{d} & & \mathbf{H}^\vee \end{array} \qquad (8)$$

where $\mathbf{H}^\vee = Spec_H SymT[Op_{L_G}(D)]$, we have that the Penrose transform appears in a natural way in the isomorphism given by $H^\bullet(\mathfrak{g}[[z]], \mathbb{V}_{critical}) \cong \mathbb{C}[Op_{L_G}]$,whose field solutions are to the equations $da = 0$, and an extended version to this integral transform to the deformed version of modules $H^\bullet(\mathfrak{g}[[z]], \mathfrak{g}, \mathbb{V}_{critical})$,considering the deformed Fourier-Mukai transform Φ, [8] comes given by the modules of critical level [11] (which come from the scheme given in the theorem 3. 1) giving solutions to the equations. Then the Yoneda algebra given by $\mathrm{Ext}_{\mathcal{D}^s(Bun_G)}(\mathcal{D}^s, \mathcal{D}^s)$, can establish an endomorphism of modules of critical level given by $\mathrm{End}_{\hat{\mathfrak{g}}}(\mathbb{V}_{critical}, \mathbb{V}_{critical})$, which implements an $A_\infty-$ isomorphic as said the Yoneda algebra theorem 3. 2, that is to say, to the complete sequence of critical Verma modules because it wants to extend the action of the $Ext-$ algebra of Verma modules $\mathbb{V}_{critical}$, that is to say, the extension of the module $\mathbb{V}_{critical}$, as a projective Harish-Chandra module to whole sequence. The a global functor to the diagram (4) until $\Omega^\bullet[Op_{L_G}]$, is one of these ${}^L\Phi^\mu(\mathcal{M}) = \mathcal{M} \boxtimes \rho^\mu(\mathbb{V})$, [12] with ${}^L\Phi^\mu$, a Hecke functor (re-

[10] $H^\bullet(T^\vee Bun_G, \mathcal{O}) \cong \mathcal{O}_{Op_{L_G}} = Ker(U, \nabla)$, where ∇, induces an holomorphic connection on lines bundles.

[11] The modules $\mathbb{V}_{critical} = U_{critical}\hat{\mathfrak{g}} \otimes_{\mathfrak{g}[[z]]} \mathbb{C}$.

[12] Thisi is a Hecke functor. Here $\mathcal{M}$, is a $D-$ module on ${}^L Bun_G$, which is a Hecke eigenmodule with eigenvalue $\mathbb{V} \in \mathrm{Loc}_{Sys}$, if for every μ, of G, is had that ${}^L\Phi^\mu(\mathcal{M}) = \mathcal{M} \boxtimes \rho^\mu(\mathbb{V})$

member that is being considered local Langlands parameters) to a certain anti-dominant character μ, which depends analytically of certain parameter λ, such that $\mathrm{End}_{\mathfrak{g}_{kc}}\mathbb{V}_\lambda \cong FunOp^{\lambda}_{^LG}$, having the categories $SymT[Op_{^LG}(D)]$. To their spectrum (in the Hamiltonian variety) we have that use the quantum version of the cohomology space $H^q(Bun_G\mathcal{D}^s) = \mathbb{H}^q_{G[[z]]}(\mathbf{G}, (\wedge^\bullet[\Sigma^0] \otimes \mathbb{V}_{critical}; \partial))$, where $\mathbf{G} := G((z))\diagup G[[\Sigma^0]]$, (the thick flag variety) where is clear that $\forall\phi \in G((z)), \phi \bullet G[[\Sigma^0]] \in \mathbf{G}$, then the elements of are the elements of $\mathfrak{g}[[z]]\diagup G$, (where we are using directly the theorem 3. 1) which are in terms of graded vector space $SpecSymT$, the elements of $\mathrm{D}_{coh}(^LBun, \mathcal{D}^\times)$, which are included in the quasi-coherent category $M_{\mathcal{K}_F}(\hat{\mathfrak{g}}, Y)$. Finally $Spec_G^{\mathfrak{g}[[z]]\diagup G}(\Omega^1(\mathbf{H})) = Y$.

Example 4. 1. As application to TFT, we consider the commutative diagram where a spectrum given by the theorem 3. 1, is the derived category $\mathcal{W}(H)$:

$$\begin{array}{ccccc} O_c(\varphi) \in H(mod\ f\ (C_{\text{-}*}(\Omega Z))) & \xrightarrow{R^{-1}} & H(\mathcal{M}) \longrightarrow & & C \\ \downarrow & & \downarrow embb & & \downarrow \\ C_{\text{-}*}(\Omega_\chi) & \xrightarrow{Diff} & \mathcal{W}(H) \ni \varphi & \xrightarrow{\mathcal{G}} & \mathcal{M} \end{array} \quad (9)$$

Specialized Notation

$\mathcal{P}-$ Penrose transform.

$\mathcal{D}^{\times}-$ Twisted sheaf of differential operators to our **Oper**, given by $\mathbf{Loc}_{^LG}(\mathcal{D}^{\times})$.

$K^{1/2}-$ Root square of the canonical line bundle on $\mathbf{Bun}_G$ corresponding to the critical level. This is a divisor vector bundle.

$\mathbf{Bun}_G(X)-$ Category of principal $G-$ bundles over $C \times X$ Also is the moduli stack of principal $G-$ bundles over C.

$\mathbf{Loc}_{^LG}(\mathcal{D}^{\times})-$ Set of equivalence classes of $^LG-$ bundles with a connection on $\mathcal{D}^{\times}$. This space shape a bijection with the set of gauge equivalence classes of the ramified operators, as defined in [15], [16].

$\mathcal{D}_{BRST}-$ The derived category on $D-$ modules of $Q_{BRST}-$ operators applied to the geometrical Langlands correspondence to obtain the "quantum" geometrical Langlands correspondence.

$\mathcal{H}_G- \cong (B \setminus G \diagup B)$, of bi-equivariant $D-$ modules on a complex reductive group G.

$\mathcal{D}^{\times}(Bun_G(\Sigma))-$ Its the category of the twisted Hecke categories $\mathcal{H}_{G,\lambda}$.

$\mathrm{Ch}_{G,[\lambda]}-$ Character sheaves used as Drinfeld centers in derived algebraic geometry. Their use connects different cohomologies in the Hecke categories context.

$\mathcal{M}_{Higgs}(^LG, C)-$Moduli space of the dualizing of the Higgs fields, that is to say, quasi-coherent $D-$ modules. Usually said quasi-coherent $D-$ modules are coherent $D-$ modules as $D-$branes.

$\mathcal{M}_{Higgs}(G, C)-$ Moduli space of the $Higgs$. Their fields are the $\theta \in T_V^{\vee} Bun_C(\Sigma))$.

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