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A de Broglie Wave Solution to the Maxwell-Schrodinger-Einstein Equations

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Abstract

We examine de Broglie matter waves in the rest frame of a mass undergoing circular motion. The matter waves are based on the de Broglie orbital condition. The fields of the matter waves satisfy Maxwell's equations, the Schrodinger equation and Einstein's equations, providing an electromagnetic, quantum mechanical, gravitational coupling.

Mathematics Subject Classification: 34A05; 34A09

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1 Introduction

Unified field theory, in particular physics is an attempt to describe all fundamental forces and the relationship between elementary particles in terms of a

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single theoretical framework [1-2]. In physics, fields that mediate interactions between separate objects can describe forces. In the mid 19th century James Clerk Maxwell formulated the first field theory in his theory of electromagnetism [3]. Then, in the early part of the 20th century, Albert Einstein developed general relativity, a field theory of gravitation [4-5]. Later, Einstein and others attempted to construct a unified field theory in which electromagnetism and gravity would emerge as different aspects of a single fundamental field [6-7]. They failed, and to this day gravity remains beyond attempts at a unified field theory.

At subatomic distances, fields are described by quantum field theories, which apply the ideas of quantum mechanics to the fundamental field. In the 1940s quantum electrodynamics (QED), the quantum field theory of electromagnetism, became fully developed [8-9].

The electroweak interaction is the unified description of two of the four known forces: electromagnetism and the weak interaction [10-11]. Although these two forces appear very different at everyday low energies, the theory models them as two different aspects of the same force.

It is generally believed that a successful grand unified theory (GUT) will still not include gravity. The problem here is that theorists do not yet know how to formulate a workable quantum field theory of gravity based on the exchange of a hypothesized graviton [12-14]. The current quest for a unified field theory is largely focused on superstring theory and in particular, on an adaptation known as M-theory [15].

We have discovered that a slow de Broglie matter field is a coupled solution to electromagnetism, quantum mechanics and general relativity. The slow matter wave packet is considered in the rest frame of an orbiting mass and based on the de Broglie condition for integral wavelengths [16-17]. For a kilogram mass orbiting within a one meter radius, the wave packet has a velocity of approximately $10^{-34} m/s$. For these waves, mass transforms into length. The "charge" of this field is curvature, but Maxwell's equations are precisely satisfied.

In this paper, we present a detailed derivation of the field equations.

2 Theory

Given a mass M moving in a circular orbit of constant radius r at a constant velocity $-v$, there is the de Broglie relationship:

$$\lambda = \frac{h}{p}, \tag{1}$$

where λ is the wavelength of the de Broglie wave, h is Planck's constant [18] and p is the magnitude of the momentum of the mass. The de Broglie orbital condition [19] is:

$$n\lambda = 2\pi r. \tag{2}$$

In the rest frame of the mass, define the following de Broglie wave fields for the rotating wave packet:

$$\vec{B} \equiv \frac{v}{n}\hat{\theta}. \tag{3}$$

$$\vec{E} \equiv \theta\hat{\theta} \tag{4}$$

with

$$\theta = \frac{\omega}{n}t \tag{5}$$

for

$$\omega = \frac{v}{r}. \tag{6}$$

The divergence of $\vec{B}$ in cylindrical coordinates is

$$\vec{\nabla}\cdot\vec{B} = \frac{1}{r}\frac{\partial}{\partial r}\left(rB_r\right) + \frac{1}{r}\frac{\partial B_\theta}{\partial \theta} + \frac{\partial B_z}{\partial z} = 0. \tag{7}$$

The curl of $\vec{B}$ is

$$\vec{\nabla}x\vec{B} = \frac{\hat{r}}{r}\left(\frac{\partial B_\theta}{\partial \theta} - \frac{\partial\left(rB_\theta\right)}{\partial z}\right) - \hat{\theta}\left(\frac{\partial B_z}{\partial r} - \frac{\partial B_r}{\partial z}\right) + \frac{\hat{z}}{r}\left(\frac{\partial\left(rB_\theta\right)}{\partial r} - \frac{\partial B_r}{\partial \theta}\right) = 0. \tag{8}$$

The divergence of $\vec{E}$ is

$$\vec{\nabla}\cdot\vec{E}=\frac{1}{r}\frac{\partial}{\partial r}\left(rE_r\right)+\frac{1}{r}\frac{\partial E_\theta}{\partial\theta}+\frac{\partial E_z}{\partial z}=\frac{1}{r}. \tag{9}$$

The curl of $\vec{E}$ is

$$\vec{\nabla}x\vec{E}=\frac{\hat{r}}{r}\left(\frac{\partial E_\theta}{\partial\theta}-\frac{\partial\left(rE_\theta\right)}{\partial z}\right)-\hat{\theta}\left(\frac{\partial E_z}{\partial r}-\frac{\partial E_r}{\partial z}\right)+\frac{\hat{z}}{r}\left(\frac{\partial\left(rE_\theta\right)}{\partial r}-\frac{\partial E_r}{\partial\theta}\right)=0. \tag{10}$$

Define the density ρ with the equation:

$$\int_0^{2\pi}\rho r\,d\theta=1. \tag{11}$$

or

$$\rho=\frac{1}{2\pi r}. \tag{12}$$

Define the function $\vec{J}$ with the equation:

$$2\pi\vec{J}+\frac{\partial\vec{E}}{\partial t}=0. \tag{13}$$

Based on equation (4) and (5), (13) becomes:

$$2\pi\vec{J}+\frac{\omega}{n}\hat{\theta}=0.$$

or

$$\vec{J}=-\frac{\omega}{2\pi n}\hat{\theta}. \tag{14}$$

Equations (12) and (14) satisfy the equation of continuity [20]:

$$\vec{\nabla}\cdot\vec{J}+\frac{\partial\rho}{\partial t}=0. \tag{15}$$

If we let the permittivity $\in_0$ and the permeability μ_0 be such that:

$$\in_0=\frac{1}{2\pi}\quad and\quad \mu_0=2\pi \tag{16}$$

so the velocity of light is

$$c=\frac{1}{\sqrt{\in_0\,\mu_0}}=1. \tag{17}$$

Then the system of field equations become:

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0} \qquad \vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} x \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \vec{\nabla} x \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}. \tag{18}$$

These are Maxwell's equations [21] for an electromagnetic field. Some additional support for these equations is provided by Ampere's law [22]:

$$\int_0^{2\pi} \vec{B} \cdot d\vec{\ell} = \mu_0 I , \tag{19}$$

where I is the enclosed current.

$$\int_0^{2\pi} \vec{B} \cdot d\vec{\ell} = \int_0^{2\pi} \frac{vr}{n} d\theta = \frac{2\pi rv}{n} = \lambda v . \tag{20}$$

$$\int_0^{2\pi} \frac{vr}{n} d\theta = \frac{hv}{p}. \tag{21}$$

or

$$\int_0^{2\pi} pr\, d\theta = nh . \tag{22}$$

Equation (22) is the de Broglie condition. From (20) we see that the current is

$$I = \frac{rv}{n}. \tag{23}$$

The radiation flux of the electromagnetic field is given by the Poynting vector [23]:

$$\vec{S} = \frac{1}{\mu_0}\left(\vec{E} x \vec{B}\right) = 0 . \tag{24}$$

The energy density [24] is

$$u = \frac{1}{2}\left(\epsilon_0 \vec{E}^2 + \frac{1}{\mu_0}\vec{B}^2\right). \tag{25}$$

The conservation law for the field energy [25] is

$$\frac{\partial u}{\partial t}+\vec{\nabla}\cdot\vec{S}=-\vec{J}\cdot\vec{E}\,. \tag{26}$$

$$\vec{\nabla}\cdot\vec{S}=0\,. \tag{27}$$

Therefore,

$$\frac{\partial u}{\partial t}=-\vec{J}\cdot\vec{E}\,. \tag{28}$$

The energy circulates through the electric field. Therefore, the total energy U_B stored in the field is due to $|\vec{B}|$:

$$U_B=\frac{\vec{B}^2}{4\pi\rho}=\frac{1}{2}r\vec{B}^2\,. \tag{29}$$

Observe that in this field, mass transforms into length:

$$m\rightarrow r\,. \tag{30}$$

Let the scalar potential be φ:

$$\varphi=-\frac{1}{2}r\theta^2\,. \tag{31}$$

The gradient of φ is given as:

$$\vec{\nabla}\varphi=\frac{\partial\varphi}{\partial r}\hat{r}+\frac{1}{r}\frac{\partial\varphi}{\partial\theta}\hat{\theta}+\frac{\partial\varphi}{\partial z}\hat{z}\,. \tag{32}$$

$$\vec{\nabla}\varphi=-\theta\hat{\theta}=-\vec{E}\,. \tag{33}$$

The vector potential $|\vec{A}|$ is given by the equation:

$$\vec{B}=\vec{\nabla}x\vec{A}\,. \tag{34}$$

Therefore, we must have:

$$\vec{B}=\frac{\hat{r}}{r}\left(\frac{\partial A_\theta}{\partial\theta}-\frac{\partial(rA_\theta)}{\partial z}\right)-\hat{\theta}\left(\frac{\partial A_z}{\partial r}-\frac{\partial A_r}{\partial z}\right)+\frac{\hat{z}}{r}\left(\frac{\partial(rA_\theta)}{\partial r}-\frac{\partial A_r}{\partial\theta}\right). \tag{35}$$

Equation (35) reduces to:

$$\vec{B}=-\hat{\theta}\left(\frac{\partial A_z}{\partial r}-\frac{\partial A_r}{\partial z}\right). \tag{36}$$

Since r is constant, we must have:

$$\vec{B} = \frac{\partial A_r}{\partial z}\hat{\theta}. \tag{37}$$

or

$$A_r = \int \frac{v}{n} dz = \frac{vz}{n}$$

from which we can deduce:

$$\vec{A} = \frac{vz}{n}\hat{r}. \tag{38}$$

Therefore,

$$\vec{E} = -\frac{\partial \vec{A}}{\partial t} - \vec{\nabla}\varphi. \tag{39}$$

We see that:

$$\vec{\nabla}\cdot\vec{A} = \frac{1}{r}\frac{\partial}{\partial r}\left(rA_r\right) + \frac{1}{r}\frac{\partial A_\theta}{\partial \theta} + \frac{\partial A_z}{\partial z} = 0. \tag{40}$$

This is a Coulomb gauge [26].

The quantum mechanical wave function of the de Broglie wave packet is the phase wave:

$$\psi = \sqrt{\rho}e^{-iE}. \tag{41}$$

This function is normalized since

$$\int_0^{2\pi} \psi^* \psi r \, d\theta = 1. \tag{42}$$

The traditional Schrodinger equation [27] is:

$$i\hbar\frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + U\psi. \tag{43}$$

However, for this field, mass transforms into length. Therefore, the equation becomes:

$$i\tilde{\hbar}\frac{\partial \psi}{\partial t} = -\frac{\tilde{\hbar}^2}{2r}\nabla^2\psi + U\psi, \tag{44}$$

where $\square = \left(\frac{v}{n}\right) r^2$. The energy eigenvalue derives from:

$$i\tilde{\hbar}\frac{\partial \psi}{\partial t} = \tilde{\hbar}\left(\frac{\omega}{n}\right)\psi \ . \tag{45}$$

Consequently, the total energy of the system is the Hamiltonian H :

$$H = \tilde{\hbar}\left(\frac{\omega}{n}\right) = r\left(\frac{v}{n}\right)^2 .$$

or

$$H = r\vec{B}^2 . \tag{46}$$

$$\nabla^2\psi = \frac{1}{r^2}\frac{\partial^2\psi}{\partial\theta^2} = -\frac{1}{r^2}\psi \ . \tag{47}$$

Therefore,

$$-\frac{\tilde{\hbar}^2}{2r}\nabla^2\psi = \left(\frac{1}{r^2}\right)\left(\frac{(v/n)^2}{2}r^3\right)\psi = \left(\frac{1}{2}r\left(\frac{v}{n}\right)^2\right)\psi \ .$$

or

$$-\frac{\tilde{\hbar}^2}{2r}\nabla^2\psi = \left(\frac{1}{2}r\vec{B}^2\right)\psi \ . \tag{48}$$

The potential energy, U is the magnetic energy stored in the field: $U_B = \frac{1}{2}r\vec{B}^2$.

Therefore,

$$U\psi = \left(\frac{1}{2}r\vec{B}^2\right)\psi \tag{49}$$

and the Schrodinger equation (44) is satisfied. We can show that the wave functions are orthonormal. Let

$$\psi_1 = \sqrt{\rho}e^{-i\frac{\omega}{n_1}t} \quad \text{and} \quad \psi_2 = \sqrt{\rho}e^{-i\frac{\omega}{n_2}t} \ . \tag{50}$$

Manipulation of the Schrodinger equation yields:

$$-\frac{\tilde{\hbar}^2}{2r}\left(\psi_1\nabla^2\psi_2^* - \psi_2^*\nabla^2\psi_1\right) = \left(U_2 - U_1\right)\psi_1\psi_2^* , \tag{51}$$

where $U_1 = \frac{1}{2} r \left(\frac{v}{n_1} \right)^2$ and $U_2 = \frac{1}{2} r \left(\frac{v}{n_2} \right)^2$. If we integrate equation (51) over a volume, it becomes:

$$-\frac{\tilde{\hbar}^2}{2r} \int_V \left(\psi_1 \nabla^2 \psi_2^* - \psi_2^* \nabla^2 \psi_1 \right) dV = \left(U_2 - U_1 \right) \int_V \psi_1 \psi_2^* dV \ . \tag{52}$$

Application of Green's theorem yields:

$$-\frac{\tilde{\hbar}^2}{2r} \int_S \left(\psi_1 \vec{\nabla} \psi_2^* - \psi_2^* \vec{\nabla} \psi_1 \right) \cdot d\vec{a} = \left(U_2 - U_1 \right) \int_V \psi_1 \psi_2^* dV \ . \tag{53}$$

The left side vanishes and we have:

$$0 = \left(U_2 - U_1 \right) \int_V \psi_1 \psi_2^* dV \ . \tag{54}$$

Thus, the wave functions are orthonormal.

We can calculate the field tensor [28]:

$$F^{\alpha\beta} = \partial^\alpha A^\beta - \partial^\beta A^\alpha \ , \tag{55}$$

where $\partial^\alpha = \left(\partial / \partial x_0 - \vec{\nabla} \right)$ and $\left(x_0, x_1, x_2, x_3 \right) = \left(t, r, \theta, z \right)$. This results in:

$$F^{\alpha\beta} = \begin{pmatrix} 0 & 0 & -E_\theta & 0 \\ 0 & 0 & 0 & -B_\theta \\ E_\theta & 0 & 0 & 0 \\ 0 & B_\theta & 0 & 0 \end{pmatrix} . \tag{56}$$

The covariant form of Maxwell's equations are as follows. The inhomogeneous equations are:

$$\partial_\alpha F^{\alpha\beta} = \mu_0 J^\beta \ . \tag{57}$$

The homogeneous equations are:

$$\partial^\alpha F^{\beta\gamma} + \partial^\beta F^{\gamma\alpha} + \partial^\gamma F^{\alpha\beta} = 0 \ . \tag{58}$$

The scalar curvature R can be written as

$$R = \mu_0 \rho \ . \tag{59}$$

The contravariant metric tensor [29] is

$$g^{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & \infty & 0 & 0 \\ 0 & 0 & \mu_0^2\rho^2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{60}$$

The diagonal components of the Ricci curvature tensor [30] $R_{\mu\nu}$ can be derived from the equation:

$$R = g^{\mu\nu}R_{\mu\nu}. \tag{61}$$

Therefore we can write:

$$R_{\mu\nu} = \begin{pmatrix} -\mu_0\rho & R_{01} & R_{02} & R_{03} \\ R_{10} & 0 & R_{12} & R_{13} \\ R_{20} & R_{21} & \in_0/\rho & R_{23} \\ R_{30} & R_{31} & R_{32} & \mu_0\rho \end{pmatrix}. \tag{62}$$

The contravariant stress tensor [31] is derived from the following equations:

$$T^{00} = \frac{1}{4\pi}\left(\vec{E}^2 + \vec{B}^2\right) + \frac{1}{2\pi}\vec{\nabla}\cdot\left(\varphi\vec{E}\right).$$

$$T^{0i} = \frac{1}{2\pi}\left(\vec{E}x\vec{B}\right)_i + \frac{1}{2\pi}\vec{\nabla}\cdot\left(A_i\vec{E}\right).$$

$$T^{i0} = \frac{1}{2\pi}\left(\vec{E}x\vec{B}\right)_i + \frac{1}{2\pi}\left[\left(\vec{\nabla}x\varphi\vec{B}\right)_i - \frac{\partial}{\partial x_0}\left(\varphi\vec{E}_i\right)\right]. \tag{63}$$

The covariant form is derived from

$$T_{\mu\nu} = T^{\alpha\beta}g_{\alpha\mu}g_{\beta\nu}. \tag{64}$$

The covariant metric tensor is

$$g_{\alpha\beta} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \in_0^2/\rho^2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{65}$$

Equations (62)-(65) can be used to determine:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + g_{\mu\nu}\Lambda = \frac{8\pi G}{c^4}T_{\mu\nu}, \tag{66}$$

the Einstein equations [32] with Λ and G as the cosmological and gravitational constants, respectively.

3 Conclusion

In summary, we have shown that the slow de Broglie matter field in the rest frame of an orbiting mass, based on the de Broglie condition, is a solution to the Maxwell-Schrodinger-Einstein equations. This result is a unified field configuration that has, until now, been unsuccessfully sought by theoretical physicists. We believe that the success of our approach derives from a redirection into non-relativistic, extremely slow wave fields.

Conflict of Interest. There is no conflict of interest for this paper.

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