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Riesel and Sierpiński problems are solved

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Abstract

In 1956, Riesel (1929-2014) proved that there exists infinitely many positive odd numbers k such that the quantities $Q_m = k2^m\text{-}1$ are composite for every m≥1.
In 1960, Sierpiński (1882-1969) proved that there exists infinitely many positive odd numbers k such that the quantities $Q_m = k2^m+1$ are composite for every m≥1.
The main contribution of this paper is to present a new approach to the present conjectures which wrongly state that the smallest Riesel number is R=509203 and that the smallest Sierpiński number is 78557. The key idea of this new approach is that both problems can be solved by using congruences only.
With this approach which avoids the burden of tracking a prime value in Q_m values, the elementary proofs are given that the smallest Riesel number is R=31859 and that the smallest Sierpiński number is S=22699.

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1 Introduction

In 1956, Riesel proved [1] that there exists infinitely many positive odd numbers k such that the quantities $Q_m = k2^m$-1 are composite for every m≥1. In other words, when k is a Riesel number R, all members of the following set are composite: $\{R\ 2^m\ -1 : m \in \mathbb{N}\}$. The conjecture is now that the smallest Riesel number is R=509203. This problem is still open in 2015.
In 1960, Sierpińnski proved [4] that there exists infinitely many positive odd numbers k such that the quantities $Q_m = k2^m$+1 are composite for every m≥1. In other words, when k is a Sierpińnski number S, all members of the following set are composite: $\{S\ 2^m\ +1 : m \in \mathbb{N}\}$. In 1967, Sierpińnski and Selfridge conjectured that the smallest Sierpińnski number is S=78557. This problem is still open in 2015.

2 Preliminary notes

2.1 Riesel and Sierpińnski numbers can only be odd

This is due to the fact that if the even values $R=k_r2^\alpha$ or $S=k_s2^\alpha$ with odd k_r and k_s exist, the quantities:

$$Q_r = R2^m\text{-}1 = (k_r2^\alpha)2^m\ \text{-}1 = k_r2^{m+\alpha}\ \text{-}1$$
$$Q_s = S2^m\text{+}1 = (k_s2^\alpha)2^m\ \text{+}1 = k_s2^{m+\alpha}\ \text{+}1$$

are no more dealing with R or S but with k_r or k_s, which is not the purpose.

2.2 A method to characterize Riesel and Sierpińnski numbers

In particular parts of this section, only Riesel numbers are dealt with, even if the result is also valid for Sierpińnski numbers.

According to the fundamental theorem of arithmetics and to the convention that the number 1 is not prime, each composite natural number greater than 1 can be factorized in only one way by powers of increasing primes.

A consequence of it is that any natural number N greater than 1 is either a prime (2 or an odd prime) or a multiple qp of any prime p of the factorization of N.

This is particularly true for the quantities Q_m=k2^m-1 and Q_m=k2^m+1 which are used to characterize Riesel and Sierpiński numbers, so that we can write for Riesel numbers by instance, with p_m being a prime and q's being prime or not:

$$Q_m = R2^m\text{-}1 = q_m\ p_m$$
$$Q_{m+\alpha} = R2^{m+\alpha}\text{-}1 = q_{m+\alpha}\ p_{m+\alpha}$$

Now, we can look for the conditions on p_m and α which make that both Q_m and $Q_{m+\alpha}$ are multiples of p_m. When it is the case, we have:

$$p_{m+\alpha} = p_m$$

so that:

$$Q_m = R2^m\text{-}1 = q_m\ p_m$$
$$Q_{m+\alpha} = R2^{m+\alpha}\text{-}1 = q_{m+\alpha}\ p_m$$

and:

$$p_m = Q_{m+\alpha}\text{-}Q_m\ /\ (q_{m+\alpha}\text{-}q_m)$$

and also:

$$Q_{m+\alpha} = R2^{m+\alpha}\text{-}1 = 2^\alpha R2^m\text{+}1$$
$$Q_{m+\alpha} = (2^\alpha\text{-}1)(R2^m) + R2^m\text{+}1$$
$$Q_{m+\alpha} = (2^\alpha\text{-}1)(R2^m) + Q_m$$
$$Q_{m+\alpha}\ \text{-}\ Q_m = (2^\alpha\text{-}1)R(2^m)$$
$$(q_{m+\alpha}\ \text{-}\ q_m)p_m = (2^\alpha\text{-}1)R(2^m)$$

and:

$$(q_{m+\alpha}\ \text{-}\ q_m) = (2^\alpha\text{-}1)R(2^m)/p_m$$

As the left quantity has to be an integer, so does the right one and we find the partial condition (for two indexes):

Partial condition (for two indexes):
If Q_m and $Q_{m+\alpha}$ share a common odd prime divisor d=p_m,
this divisor has to divide either R or 2^α-1.

Now, by definition, for a Riesel number R, all the quantities $Q_{m+\alpha} = R2^{m+\alpha}$-1 for any m⩾0 and any α >0 are always divisible by an odd divisor d. We can then write the complete condition:

Complete condition (for all indexes):
For a Riesel number R,
m has to be a covering set of the set ℕ of natural numbers,
and:
for any m>0 and α >0, all the quantities $Q_{m+\alpha}$ have to always be divisible by an odd divisor d=p_m that divides either R or 2^{α}-1.

The difficulty here is to find a practical method that handles both parts of the complete condition.
Starting with the second part of the condition, we know that multiples αn of n are in the congruence n+αn (or 0 mod n). But the first difficulty that arises is that there exists no formula of direct factorization for $Q_m = k2^m$-1 as, by instance, the well known identity a^2-b^2=(a-b)(a+b). So, the only possible reference for the factorization of each Q_m is the infinite table of factorization of all natural numbers, whose existence is proved by the fundamental theorem of arithmetics, but which cannot entirely exist due to its infinite dimension.
For the first part of the condition, we know that if a relation is true for the set of congruences i={1, ..., μ} mod μ, indeed the relation is true for all i's, so that this set of congruences is a covering set of the set of natural numbers ℕ, and that a finite set of divisors d_j exists for all Q_i values, this set being used repeatedly, infinitely many times in a periodic manner.
So, the practical method will be to find a module μ such that the relation:

Q_i is always divisible by an odd divisor d_j >1

is true for a set of congruences i={1, ..., μ} mod μ.

2.3 The number 2293 is not a Riesel number

Without tracking prime Q values, the detailed calculations are given here which prove that 2293 is not a Riesel number, just to show what happens when a number k is not a Riesel number.

Proof. To study the number 2293, we first look at the factorizations of Q_i=2293×2^i-1 for i varying from 1 to 21:

Table 1. Factorizations of Q_i=2293×2^i-1 for i=1,21

Q=2293×2^1-1 =	4585 = 5×7×131
Q=2293×2^2-1 =	9171 = 3^2×1019
Q=2293×2^3-1 =	18343 = 13×17×83
Q=2293×2^4-1 =	36687 = 3×7×1747
Q=2293×2^5-1 =	73375 = 5^3×587
Q=2293×2^6-1 =	146751 = 3×11×4447
Q=2293×2^7-1 =	293503 = 7×23×1823
Q=2293×2^8-1 =	587007 = 3^4×7247
Q=2293×2^9-1 =	1174015 = 5×234803
Q=2293×2^{10}-1 =	2348031 = 3×7^2×15973
Q=2293×2^{11}-1 =	4696063 = 17×276239
Q=2293×2^{12}-1 =	9392127 = 3×67×46727
Q=2293×2^{13}-1 =	18784255 = 5×7×19×47×601
Q=2293×2^{14}-1 =	37568511 = 3^2×307×13597
Q=2293×2^{15}-1 =	75137023 = 13×193×29947
Q=2293×2^{16}-1 =	150274047 = 3×7×11×650537
Q=2293×2^{17}-1 =	300548095 = 5×5407×11117
Q=2293×2^{18}-1 =	601096191 = 3×23×37×235447
Q=2293×2^{19}-1 =	1202192383 = 7×17×1669×6053
Q=2293×2^{20}-1 =	2404384767 = 3^2×503×531121
Q=2293×2^{21}-1 =	4808769535 = 5×733×1312079

which proves that:

$$\text{when } i(<22) = 1+4\alpha,\ Q_i=2293\times 2^i+1 = 5K$$
$$\text{when } i(<22) = 2+4\alpha,\ Q_i=2293\times 2^i+1 = 3K$$
$$\text{when } i(<22) = 4+4\alpha,\ Q_i=2293\times 2^i+1 = 3K$$

which cover:

$$i(<22) = \{1,2,4\}+4\alpha$$

but not:

$$i(<22) = 3+4\alpha$$

So, for a better understanding of what happens when i(<22) = 3+4α, the last table has to be extended as in Table 2 for i=3+4α with p being the last and big adequate prime factor of the Q_i values.

Table 2. Values of Q_i=2293×2^i-1 for i=3+4α

$Q=2293\times2^{3}-1=$	$13\times17\times83$
$Q=2293\times2^{7}-1=$	$7\times23\times1823$
$Q=2293\times2^{11}-1=$	17×276239
$Q=2293\times2^{15}-1=$	$13\times193\times29947$
$Q=2293\times2^{19}-1=$	$7\times17\times1669\times6053$
$Q=2293\times2^{23}-1=$	2017×9536479
$Q=2293\times2^{27}-1=$	$13^3\times17\times29\times149\times1907$
$Q=2293\times2^{31}-1=$	$7^2\times19\times p$
$Q=2293\times2^{35}-1=$	$17^2\times613\times p$
$Q=2293\times2^{39}-1=$	$13\times p$
$Q=2293\times2^{43}-1=$	$7\times17\times107\times167\times281\times p$
$Q=2293\times2^{47}-1=$	$19913\times693409\times p$
$Q=2293\times2^{51}-1=$	$13\times17\times23\times439\times p$
$Q=2293\times2^{55}-1=$	$7\times29\times3600761\times p$
$Q=2293\times2^{59}-1=$	$17\times47\times137\times p$
$Q=2293\times2^{63}-1=$	$13\times601\times p$
...	...
$Q=2293\times2^{99}-1=$	$13\times17\times2917\times p$

which proves that:

$$\text{when } i(<99) = 7+12\alpha,\ Q_i=2293\times2^i-1 = 7K$$
$$\text{when } i(<99) = 3+12\alpha,\ Q_i=2293\times2^i-1 = 13K$$
$$\text{when } i(<99) = 3+8\alpha,\ Q_i=2293\times2^i-1 = 17K$$

to which, we have to add the already found congruences:

$$\text{when } i(<22) = 1+4\alpha,\ Q_i=2293\times2^i+1 = 5K$$
$$\text{when } i(<22) = 2+4\alpha,\ Q_i=2293\times2^i+1 = 3K$$
$$\text{when } i(<22) = 4+4\alpha,\ Q_i=2293\times2^i+1 = 3K$$

The last six congruences in i, extended and rewritten with the module μ=24 which is the least common multiple of their modules, respectively cover:

$$i(<99) = \{7,19\} \bmod 24$$
$$i(<99) = \{3,15\} \bmod 24$$
$$i(<99) = \{3,11,19\} \bmod 24$$

$$i(<99) = \{1,5,9,13,17,21\} \bmod 24$$
$$i(<99) = \{2,6,10,14,18,22\} \bmod 24$$
$$i(<99) = \{4,8,12,16,20,24\} \bmod 24$$
but not:
$$i(<99) = 23 \bmod 24$$

where the Q_i values are coprimes (do not share a common divisor).

So, we cannot say that i = {1,...,24} mod 24 is a covering set of the set ℕ of natural integers. As, when the above 99 limit for i is replaced by infinity, the congruence i(not limited) = 23 mod 24 generates infinitely many coprime Q_i values, it proves that for all m's, the set of divisors of these values is not finite, which finally proves that 2293 is not a Riesel number. □

This method also proves that the numbers 9221 and 23669 are not Riesel numbers.

3 Main Result 1: Proof that R=31859 is the smallest Riesel number

According to the distributed computing project Primegrid [2] cited in [3], the last facts that would establish the proof that 509203 is the smallest Riesel number, are the proofs that the 50 numbers k:
2293, 9221, 23669, 31859, 38473, 46663, 67117, 74699, 81041, 93839, 97139, 107347, 121889, 129007, 143047, 146561, 161669, 192971, 206039, 206231, 215443, 226153, 234343, 245561, 250027, 273809, 315929, 319511, 324011, 325123, 327671, 336839, 342847, 344759, 362609, 363343, 364903, 365159, 368411, 371893, 384539, 386801, 397027, 409753, 444637, 470173, 474491, 477583, 485557, 494743
are not Riesel numbers, these proofs being based upon the fact that all of these numbers would generate some prime Q value.
Without tracking prime Q values, the detailed calculations are given here which prove that 31859 is a Riesel number.

Proof. To study the number 31859, we first look at the factorizations of $Q_i=31859\times2^i-1$ for i varying from 1 to 21:

Table 3. Factorizations of Q_i=31859×2^i-1 for i=1,21

Q=31859×2^1-1 =	63717 = 3×67×317
Q=31859×2^2-1 =	127435 = 5×7×11×331
Q=31859×2^3-1 =	254871 = 3^2×28319
Q=31859×2^4-1 =	509743 = 13×113×347
Q=31859×2^5-1 =	1019487 = 3×7×43×1129
Q=31859×2^6-1 =	2038975 = 5^2×81559
Q=31859×2^7-1 =	4077951 = 3×19×29×2467
Q=31859×2^8-1 =	8155903 = 7^2×17×9791
Q=31859×2^9-1 =	16311807 = 3^3×23×26267
Q=31859×2^{10}-1 =	32623615 = 5×569×11467
Q=31859×2^{11}-1 =	65247231 = 3×7×p
Q=31859×2^{12}-1 =	130494463 = 11×p
Q=31859×2^{13}-1 =	260988927 = 3×61×p
Q=31859×2^{14}-1 =	521977855 = 5×7×97×p
Q=31859×2^{15}-1 =	1043955711 = 3^2×p
Q=31859×2^{16}-1 =	2087911423 = 13×17^2×p
Q=31859×2^{17}-1 =	4175822847 = 3×7×479×p
Q=31859×2^{18}-1 =	8351645695 = 5×p
Q=31859×2^{19}-1 =	16703291391 = 3×41×43×p
Q=31859×2^{20}-1 =	33406582783 = 7×23×239×p
Q=31859×2^{21}-1 =	66813165567 = 3^2×79×1723×54539

which proves that:

when i(<22) = 1+2α, Q_i=31859×2^i-1 = 3K
when i(<22) = 2+4α, Q_i=31859×2^i-1 = 5K
when i(<22) = 2+3α, Q_i=31859×2^i-1 = 7K
when i(<22) = 4+12α, Q_i=31859×2^i-1 = 13K
when i(<22) = 8α, Q_i=31859×2^i-1 = 17K
when i(<22) = 9+11α, Q_i=31859×2^i-1 = 23K
when i(<22) = 5+14α, Q_i=31859×2^i-1 = 43K
which cover:
i(<22) = {1,2,3,4,5,6,7,8,9,10,11,-,13,14,15,16,17,18,19,20,21}
but not:
i(<22) = {12}

So, for a better understanding of what happens for 12, the last table has to be extended as in Table 4 where p, q and r are big primes.

Table 4. Factorizations of Q_i for i=4α

Q=31859×2^4-1 =	13×113×347
Q=31859×2^8-1 =	7^2×17×9791
Q=31859×2^{12}-1 =	11×p
Q=31859×2^{16}-1 =	13×17^2×p
Q=31859×2^{20}-1 =	7×23×239×p
Q=31859×2^{24}-1 =	17×397×p
Q=31859×2^{28}-1 =	13×617×p
Q=31859×2^{32}-1 =	7×11×17×113×331×p
Q=31859×2^{36}-1 =	2273×p
Q=31859×2^{40}-1 =	13×17×191×p×q
Q=31859×2^{44}-1 =	7×311×607×p
Q=31859×2^{48}-1 =	17×4217×p
Q=31859×2^{52}-1 =	11×13×p×q
Q=31859×2^{56}-1 =	7×17×5573×p
Q=31859×2^{60}-1 =	79×113×14321×p
Q=31859×2^{64}-1 =	13×17×23×p×q
Q=31859×2^{68}-1 =	7×397×p
Q=31859×2^{72}-1 =	11×17×p
Q=31859×2^{76}-1 =	13×89819×p×q
Q=31859×2^{80}-1 =	7×17×p×q
Q=31859×2^{84}-1 =	1916249×p
Q=31859×2^{88}-1 =	13×17×113×p
Q=31859×2^{92}-1 =	7×11×331×p
Q=31859×2^{96}-1 =	17×p
Q=31859×2^{100}-1 =	13×881×p×q
Q=31859×2^{104}-1 =	7×17×211×p×q
Q=31859×2^{108}-1 =	23^2×p×q×r

This proves that:

$$\text{when } i(<109) = 12+20\alpha,\ Q_i=11K$$

so that the overall covering congruences are:

when i(<109) = 1+2α, Q_i=31859×2^i-1 = 3K
when i(<109) = 2+4α, Q_i=31859×2^i-1 = 5K
when i(<109) = 2+3α, Q_i=31859×2^i-1 = 7K
when i(<109) = 12+20α, Q_i=31859×2^i-1 = 11K
when i(<109) = 4+12α, Q_i=31859×2^i-1 = 13K
when i(<109) = 8+8α, Q_i=31859×2^i-1 = 17K
when i(<109) = 9+11α, Q_i=31859×2^i-1 = 23K
when i(<109) = 5+14α, Q_i=31859×2^i-1 = 43K

which, extended and rewritten with the module μ=120*77=9240 which is the least common multiple of the modules of the last eight congruences in i, sum up to:

when i(<9240) =
{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,...,9240} mod 9240

So, as we can say that i = {1,...,9240} mod 9240 is a covering set of the set ℕ of natural integers, the above 9240 limit for i can be replaced by infinity. This finally proves that for all m's, the set of divisors of all Q_m=31859×2^m-1 values is the finite set:

{3,5,7,11,13,17,23,43}

which proves that 31859 is a Riesel number. Finally, as from the Primegrid project, the remaining numbers to test have been considered in the increasing order, and as 2293 as well as 9221 and 23669 were found, in section 2.3, not to be Riesel numbers, this proves that 31859 is the smallest Riesel number.

□

4 Main result 2: Proof that S=22699 is the smallest Sierpiński number

According to the distributed computing project Seventeen or Bust [5] that is cited in [6], the last facts that would establish the proof that 78557 is the smallest Sierpiński number, are the proofs that the six numbers k = 10223, 21181, 22699, 24737, 55459, and 67607 are not Sierpiński numbers, these proofs being based upon the fact that all of these numbers would generate some prime Q value.

Without tracking prime Q values, the detailed calculations are given here which prove that 22699 is a Sierpińiski number.

Proof. To study the number 22699, we first look at the factorizations of Q_i=22699×2^i+1 for i varying from 1 to 21:

Table 5. Factorizations of Q_i=22699×2^i+1 for i=1,21

$Q=22699\times2^{1}+1=$	$45399=3\times37\times409$
$Q=22699\times2^{2}+1=$	$90797=7^2\times17\times109$
$Q=22699\times2^{3}+1=$	$181593=3^2\times20177$
$Q=22699\times2^{4}+1=$	$363185=5\times19\times3823$
$Q=22699\times2^{5}+1=$	$726369=3\times7\times34589$
$Q=22699\times2^{6}+1=$	$1452737=11\times13\times10159$
$Q=22699\times2^{7}+1=$	$2905473=3\times73\times13267$
$Q=22699\times2^{8}+1=$	$5810945=5\times7\times166027$
$Q=22699\times2^{9}+1=$	$11621889=3^2\times1291321$
$Q=22699\times2^{10}+1=$	$23243777=17\times23\times59447$
$Q=22699\times2^{11}+1=$	$46487553=3\times7\times83\times149\times179$
$Q=22699\times2^{12}+1=$	$92975105=5\times18595021$
$Q=22699\times2^{13}+1=$	$185950209=3\times431\times143813$
$Q=22699\times2^{14}+1=$	$371900417=7\times53\times1002427$
$Q=22699\times2^{15}+1=$	$743800833=3^3\times1259\times21881$
$Q=22699\times2^{16}+1=$	$1487601665=5\times11\times73\times370511$
$Q=22699\times2^{17}+1=$	$2975203329=3\times7\times113\times233\times5381$
$Q=22699\times2^{18}+1=$	$5950406657=13\times17\times26924917$
$Q=22699\times2^{19}+1=$	$11900813313=3\times10133\times391487$
$Q=22699\times2^{20}+1=$	$23801626625=5^3\times7\times2293\times11863$
$Q=22699\times2^{21}+1=$	$47603253249=3\times23\times9973\times23059$

which proves that:

when i(<22) = {1,3}+4α, Q_i=22699×2^i+1 = 3K
when i(<22) = 4+4α, Q_i=22699×2^m+1 = 5K
which cover:
when i(<22) = {1,3,4}+4α
but not:
when i(<22) = 2+4α

So, for a better understanding of what happens in that case, the last table has to be extended as in Table 6 where p and q are big primes.

Table 6. Factorizations of Q_i for $i=2+4\alpha$

$Q=22699\times2^{2}+1=$	$7^2\times17\times109$
$Q=22699\times2^{6}+1=$	$11\times13\times10159$
$Q=22699\times2^{10}+1=$	$17\times23\times59447$
$Q=22699\times2^{14}+1=$	$7\times53\times1002427$
$Q=22699\times2^{18}+1=$	$13\times17\times26924917$
$Q=22699\times2^{22}+1=$	$19\times47\times1721\times61949$
$Q=22699\times2^{26}+1=$	$7\times11\times17\times1163715893$
$Q=22699\times2^{30}+1=$	$13\times173\times63841\times169753$
$Q=22699\times2^{34}+1=$	$17\times73\times2711\times p$
$Q=22699\times2^{38}+1=$	$7\times109\times3539\times p$
$Q=22699\times2^{42}+1=$	$13\times17\times67\times107\times24443\times p$
$Q=22699\times2^{46}+1=$	$11\times233\times1213\times5507\times6329\times14741$
$Q=22699\times2^{50}+1=$	$7\times17\times8269\times p$
$Q=22699\times2^{54}+1=$	$13\times23\times59\times2269\times p$
$Q=22699\times2^{58}+1=$	$17^2\times19\times3467\times p$
$Q=22699\times2^{62}+1=$	$7\times p$
$Q=22699\times2^{66}+1=$	$11\times13\times17\times53\times p\times q$
$Q=22699\times2^{70}+1=$	$73\times239\times3884047\times p$
$Q=22699\times2^{74}+1=$	$7\times17\times109\times p\times q$
$Q=22699\times2^{78}+1=$	$13\times p\times q$
$Q=22699\times2^{82}+1=$	$17\times p\times q$
$Q=22699\times2^{86}+1=$	$7^2\times11\times p\times q$
$Q=22699\times2^{90}+1=$	$13\times17\times5741\times5857\times p\times q$
$Q=22699\times2^{94}+1=$	$19\times34613\times p$
$Q=22699\times2^{98}+1=$	$7\times17\times23\times1086731\times p$
$Q=22699\times2^{102}+1=$	$13\times p\times q$

This proves that:

$$\text{when } i(<103) = 2+12\alpha,\ Q_i=7K$$
$$\text{when } i(<103) = 6+20\alpha,\ Q_i=11K$$
$$\text{when } i(<103) = 6+12\alpha,\ Q_i=13K$$
$$\text{when } i(<103) = 2+8\alpha,\ Q_i=17K,$$

$$\text{when i}(<103) = 22+36\alpha,\ Q_i=19K,$$
$$\text{when i}(<103) = \{14,\ 66\}+72\alpha,\ Q_i=53K$$
$$\text{when i}(<103) = 34+36\alpha,\ Q_i=73K$$

to which we must add the already found congruences:

$$\text{when i}(<22) = \{1,\ 3\}+4\alpha,\ Q_i=3K$$
$$\text{when i}(<22) = 4+4\alpha,\ Q_i=5K$$

The last nine congruences in i, extended and rewritten with the module μ=360 which is the least common multiple of all their modules, respectively cover:

$$\text{when i}(<360) = \{1,5,9,13,...,353,357\}+360\alpha,\ Q_i=3K$$
$$\text{when i}(<360) = \{3,7,11,15,...,355,359\}+360\alpha,\ Q_i=3K$$
$$\text{when i}(<360) = \{4,8,12,16,...,356,360\}+360\alpha,\ Q_i=5K$$
$$\text{when i}(<360) = \{2,14,26,38,50,...,350\}+360\alpha,\ Q_i=7K$$
$$\text{when i}(<360) = \{6,26,46,66,86,...,346\}+360\alpha,\ Q_i=11K$$
$$\text{when i}(<360) = \{6,18,30,42,..,346,354\}+360\alpha,\ Q_i=13K$$
$$\text{when i}(<360) = \{2,10,18,26,..,346,354\}+360\alpha,\ Q_i=17K$$
$$\text{when i}(<360) = \{22,58,94,130,...,346\}+360\alpha,\ Q_i=19K$$
$$\text{when i}(<360) = \{14,66,86,138,...,354\}+360\alpha,\ Q_i=53K$$
$$\text{when i}(<360) = \{34,70,106,...,358\}+360\alpha,\ Q_i=73K$$

which sums up to:

$$\text{i}(<360) = \{1,...,360\} \bmod 360$$

So, as we can say that i = {1,...,360} modulo 360 is a covering set of the set $\mathbb{N}$ of natural integers, the above 360 limit for i can be replaced by infinity. This finally proves that for all m's, the set of divisors of all $Q_m=22699\times2^m+1$ values is the finite set:

$$\{3,5,7,11,13,17,19,53,73\}$$

which proves that 22699 is a Sierpiński number.

Finally, as from the Primegrid project, the remaining numbers to test have been considered (out of this article) in the increasing order, and as 10223 and 21181 were found not to be Sierpiński numbers, this proves that 22699 is the smallest Sierpiński number.

□

Remark. A secondary result is that we can now understand that the different divisors d_j that constitute the finite set of divisors, are the different modules d_j of the different congruences that are necessary to cover the set $\mathbb{N}$ of natural integers, and that the number of these divisors is the number of these different congruences.

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