Theoretical Mathematics & Applications, vol.4, no.1, 2014, 71-76
ISSN: 1792-9687 (print), 1792-9709 (online)
Scienpress Ltd, 2014

The geometry of polynomial diagrams

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Abstract

In this paper we introduce the concept of polynomial diagrams and its area for special polynomials. We study the properties of polynomial area diagrams. The formula for the area of an arbitrary polynomial diagrams.

Mathematics Subject Classification: 51F05; 26A09; 26A06
Keywords: Polynomial diagrams; area of polynomial diagrams

1 Introduction

Consider the polynomial of a special form (1), that we will use throughout the paper, where $q \in \mathbb{Z}^+$ and $q \neq 0$, k=degP(x).

$$P(x) = \sum_{i=0}^{k} q^{n+i} x^{k-i} \tag{1}$$

Let the map $\mathfrak{F}$ take each monomial $q^{n+i}x^{k-i}$ to integer point $(q^{n+i}; k-i)$ namely

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Article Info: *Received* : February 3, 2014. *Revised* : March 27, 2014.
Published online : March 31, 2014.

$\mathfrak{F} : q^{n+i}x^{k-i} \longrightarrow (q^{n+i}x^{k-i}; k-i)$, where $i = 0, 1, \dots, n$. This construction is reminiscent of the Newton diagram, see for example [1, 2]. Now we introduce the following concept.

2 Preliminary Notes

Definition 1. *The flat polygon passing through the vertices at integer points* $A(q^n; 0)$ *and* $A_i(q^{n+i}; k-i)$ *is called Polynomial diagrams and is denoted by* $\mathfrak{pd}$.

Now we shall give the following definition.

Definition 2. *Area* $\mathfrak{pd}$ *is called area of planar polygon that specifies the polynomial diagram and is denoted by* $S^q(\mathfrak{pd})$.

This polinomials diagrams is well defined from (1), take $k = 2$ and $n \in \mathbb{N}$. It follows from (1) that $P(x) = q^n x^2 + q^{n+1}x + q^{n+2}$, while we concider $P(x)$ in this form if you do not agreed to and reverse.

3 Main Results

Theorem 1. *The area* $S^q(\mathfrak{pd})$ *is calculated using the following formula*

$$S^q(\mathfrak{pd}) = \frac{q^n(q+3)(q-1)}{2}. \tag{2}$$

Proof. Polynomial diagram for (2) a flat the polygon passes through the vertices $A(q^n; 0), A_0(q^n; 2), A_1(q^{n+1}; 1), A_2(q^{n+2}; 0)$ and consider the point $\hat{A}_1(q^{n+1}; 0)$. We divide a $\mathfrak{pd}$ on two polygons namely: $AA_0A_1\hat{A}_1$ - rectangular trapezoid and $A_1A_2\hat{A}_2$ - right triangle. Then $S^q(\mathfrak{pd})$ has the following form

$$S^q(\mathfrak{pd}) = S(AA_0A_1\hat{A}_1) + S(A_1A_2\hat{A}_2)$$

We claim that $S(AA_0A_1\hat{A}_1) = \frac{3(q^{n+1}-q^n)}{2}$, and $S(A_1A_2\hat{A}_2) = \frac{q^{n+2}-q^{n+1}}{2}$.

The result is

$$S^q(\mathfrak{p}\mathfrak{d}) = \frac{3(q^{n+1} - q^n)}{2} + \frac{q^{n+2} - q^{n+1}}{2}. \qquad (3)$$

Now, after spending the expression (3) elementary algebraic manipulations we obtain (2). This completes the proof of Theorem 1. □

Lemma 2. *If* $k = 2$ *and* $n = 0$, *then*

$$S^q(\mathfrak{p}\mathfrak{d}) = \frac{(q+3)(q-1)}{2}. \qquad (4)$$

Proof. For $n = 0$, the result follows immediately from (2). □

It is interesting to note that if we consider (4), for different values q, the area ratio approaches unity. Let is make these observations in the Table 1. Now we shall give the following theorem.

Theorem 3. *For any* $q \in \mathbb{Z}^+$ *and* $q \neq 0$

$$\lim_{q\to\infty} \frac{S^{q+1}(\mathfrak{p}\mathfrak{d})}{S^q(\mathfrak{p}\mathfrak{d})} = 1.$$

Proof. Taking into account Lemma 2, we obtain

$$\lim_{q\to\infty} \frac{S^{q+1}(\mathfrak{p}\mathfrak{d})}{S^q(\mathfrak{p}\mathfrak{d})} = \lim_{q\to\infty} \frac{q(q+4)}{(q+3)(q-1)} = \lim_{q\to\infty} \frac{q^2+4q}{q^2+2q-3} = 1.$$

The theorem is proved. □

By definition, put formula

$$\Delta^2 S^q(\mathfrak{p}\mathfrak{d}) \overset{\text{def}}{=} S^{q+2}(\mathfrak{p}\mathfrak{d}) - 2S^{q+1}(\mathfrak{p}\mathfrak{d}) + S^q(\mathfrak{p}\mathfrak{d}). \qquad (5)$$

Again there is a desire to consider the values of pairwise differences. As a result, we obtain that it is constant for all values of one. We reflect these observations in the Table 2.

Now we shall give the following theorem.

Theorem 4. *For any* $q \in \mathbb{Z}^+$, *and* $q \neq 0$,

$$\Delta^2 S^q(\mathfrak{p}\mathfrak{d}) = 1.$$

Proof. Taking into account Lemma 2, we obtain

$$S^{q+1}(\mathfrak{pd}) = \frac{q(q+4)}{2} = \frac{q^2+4q}{2}$$

and

$$S^{q+2}(\mathfrak{pd}) = \frac{(q+5)(q+1)}{2} = \frac{q^2+6q+5}{2}.$$

Now substituting these formulas and (4) to (5).

$\Delta^2 S^q(\mathfrak{pd}) = \frac{q^2+6q+5}{2} - \frac{2q^2+8q}{2} + \frac{q^2+2q-3}{2} = \frac{2}{2} = 1.$

This completes the proof of Theorem 4. □

Note that the basis of the results set out in the first part of this work were obtained purely an experimental way. The motivation of this study was the work [3]. It is also interesting to note that the result of Theorem 3, can not be generalized to the general case, it can be seen from numerical calculations. After the above considerations we may formulate the main result of this work.

Consider the polynomial in the form (1).

Theorem 5. *If* $\mathfrak{pd}$ *-polynomial diagram for (1), than*

$$S^q(\mathfrak{pd}) = \sum_{m=0}^{k-2} \frac{q^{n+m}(q-1)(2k-2m+1)}{2} + \frac{q^{n+k}-q^{n+k-1}}{2}.$$

Proof. The proof is completely analogous to that of Theorem 1. The basic idea is to divide a polynomial chart on $k-1$ rectangular trapezoids and one right-angled triangle and then only remains to summarize the data space. In fact, we used the method of trapezoids.

Now give the proof in more detail.

We fix a number m satisfies the condition $0 \leqslant m \leqslant k-2$ .Divide a polynomial diagram on $k-1$ rectangular trapezoid. Consider the trapezium with vertices at points $A(q^{n+m}; k-m); B(q^{n+m}; 0); C(q^{n+m+1}; k-m+1); D(q^{n+m+1}; 0)$.

Then its area is calculated by the following formula:

$$S(ABCD) = \frac{(q^{n+m+1}-q^{n+m})(2k-2m+1)}{2} \tag{6}$$

Now it is necessary to take into account that such trapezoids is $k-1$, but it should be noted that the summation index is not taking up to $k-1$ to $k-2$

as well as we need to extreme tip of the trapezoid $k-1$ coincides with the vertex of a right triangle. Then we have $m+1 = k-1$, then $m = k-2$.

It remains only to sum equation (6) and add to the resulting area of a right triangle, and hold small algebraic operations.

$$S^q(\mathfrak{p}\mathfrak{d}) = \sum_{m=0}^{k-2} \frac{q^{n+m}(q-1)(2k-2m+1)}{2} + \frac{q^{n+k}-q^{n+k-1}}{2}$$

□

4 Tables

This section contains tables used in this article. That data presented in these tables is a primary tool for the results obtained in this work. Note that numerical experimentation greatly helped in the task.

Table 1:

q	$S^q(\mathfrak{p}\mathfrak{d})$	$\frac{S^{q+1}(\mathfrak{p}\mathfrak{d})}{S^q(\mathfrak{p}\mathfrak{d})}$
2	2.5	2.4
3	6	1.75
4	10.5	1.52
5	16	1.4
6	22.5	1.3
...	...	...
16	142.5	1.12

Table 2:

q	$S^q(\mathfrak{pd})$	$\frac{S^{q+1}(\mathfrak{pd})}{S^q(\mathfrak{pd})}$
2	2.5	2.4
3	6	1.75
4	10.5	1.52
5	16	1.4
6	22.5	1.3
...	...	...
16	142.5	1.12

References

[1] V.I. Arnold, Random permutations and Young Diagrams, *Mathematical Enlightenment*, **6**, (2002), 67-72.

[2] V.I. Arnold, Experimenting with random permutations of a large number of elements, *Mathematical Enlightenment*, **7**, (2011), 107-122.

[3] V.I. Arnold, Statistics Fibonacci cycles automorphisms, *Mathematical Enlightenment*, **3**, (2001), 13-23.