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Hybrid model analysis on host factor of epidemics involving fuzzy Markovian chain

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Abstract

In this study, a hybrid model analysis on host factor of epidemics involving fuzzy markovian chain is presented. This hybrid model consists of stochastic approach involving markovian chain, fuzzy expert system, simulation approach and fuzzy set theory in order to obtain the system response in a more realistic manner. A stochastic model of the two state markovian-chains is applied to certain epidemics by age group. An age specific member rate and fuzzy markovian risk rate are defined and used as novel measure indices to prioritize age groups. A fuzzy expert system is used in medical diagnosis for various classifications. For simulating the process, we have taken the symptom data for the diseases from the experienced medical experts. The computation of max-min composition is assumed to describe the state of the patients (age wise) in terms of diagnosis as a fuzzy set characterized by its membership value. The analysis of age specific diagnosis of certain diseases is essential to provide the data for the planning of health services and to setting up of priorities among those services. The combination of stochastic approach and the fuzzy logic enables our computational simulation to much more faithfully portray the phenomena under study.

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1 Introduction

Mathematical modelling analysis is used extensively in the field of epidemiology of diseases. Many mathematical models are used using pure initial and boundary value conditions in solving the problems. Mathematical modelling analysis, in spite of its advantages is not well suited to handle the uncertainties associated with the problem due to lack of clear cut interpretations of the parameters. Thus there is a need to improve the existing mathematical model of the problem in order to bring its behaviour as close as possible to the actual behaviour of the system. It requires the need to study the effect of system response in the presence of uncertainties of the problem. The review of literature reveals that the deterministic approaches and stochastic approaches are used to model the response of the problem. Fuzzy set theory is used to measure uncertainties. Expert knowledge in medicine is applied using the technique of fuzzy logic. CADIAG 2 for internal medicine, EMERGE for chest pain analysis, EXPERT for ophthalmology, MYCIN foe disease diagnosis and treatment was developed. The above studies were confined to the studies of the effects of uncertainty alone. At the same time, the realistic effects of system response were not considered. There is no existing literature, which incorporates the system response and fuzzy response in the analysis of human health problem. This requires the redefinition and extension of the mathematical modelling. This is a hybrid model analysis which combines the mathematical models and fuzzy set theory.

In this paper, we consider the host factor of epidemics for analysis. The causative factors of disease are classified as agent, host and environment. The interaction of these factors is required to initiate the disease process in man. The host factors include age, sex, ethnicity and many biological factors. Here, were considered the host factor to age specification. Age is strongly related to disease than any other single host factor. Certain diseases are more frequent than certain age groups. If the attack rate of an epidemic is uniform in all the age groups it implies that all the age groups are equally susceptible and there was no pre-immunity. Many chronic and degenerative diseases show a progressive increase in prevalence with advancing age. This may reflect a persistent and cumulative exposure to causative agent or risk factors. In some chronic diseases the child prevalence are less in number or disappearance represents that the diseases are going to pass from the existence and vice-versa to be in high number.

Epidemic refers to the unusual occurrence of disease clearly in excess of expected occurrence of a community. Periodical fluctuations are the commonest characteristics of many epidemics. The seasonal variations of disease occurrence may be related to environmental conditions such as temperature, humidity, rainfall, overcrowding and life cycle of vectors which directly or indirectly favour the transmission. In this study, we choose certain diseases that are satisfying the above conditions. The analysis of age specific diagnosis is essential to provide the data for the planning of health services and to setting up of priorities among those services.

The application based on the fuzzy Markovian chain and the transmission membership function of membership in the 2 x 2 matrix is calculated. In addition a procedure is proposed for the diagnosis, which is based on the max-min composition of fuzzy relation. Moreover, a calculation of an index for medical diagnosis system is made. The objective of the hybrid model analysis is to predict the behaviour of the problem accurately. Medical analysis have caused human support, cost effective and caused diagnosis accuracy to increase. Nevertheless, the epidemiological surveillance is expensive, time consuming and require higher administrative procedures. In this study, we developed a rule based fuzzy system that uses expert knowledge and other data that mimic a real outcome.

The article is organised as follows. In section 2, we briefly outline the definition of possibility distribution function followed by fuzzy expert system designing in section 3. In section 4, we present the application of host factors of epidemics, numerical simulation and the result will follow in section 5. At the end, some conclusions are highlighted in this paper.

2 Basic Definitions

We use the stochastic approach to modelling on host factor of epidemics to facilitate age specific diagnosis analysis. Using this method, we estimate the possibility of outcomes based on available data. Here, we designed the definition of possibility distribution function. Based on the definition of possibility distribution function introduced, the fuzzy Markov chain was framed.

Definition 2.1 (Possibility Distribution Function) *Let* $\tilde{F}$ *be a fuzzy set in a universe of discourse* U*, which is characterized by its membership function* $\mu_{\tilde{F}}(u)$*, which is interpreted as the compatibility of* $u \in U$ *with the concept labeled* $\tilde{F}$*. Let* X *be a variable taking values in* U *and* $\tilde{F}$ *act as a fuzzy restriction,* $\tilde{R}(X)$*, associated with* X*. Then the proposition '*X *is* $\tilde{F}$*', which translates into* $\tilde{R}(X) = \tilde{F}$ *associates a fuzzy event,* $\tilde{\pi}$*, with* X *which is postulated to be equal to* $\tilde{R}(X)$*. The possibility distribution function,* $\pi_{\tilde{R}}(u)$*, characterizing the fuzzy event* $\tilde{\pi}$ *is defined to be numerically equal to the membership function* $\mu_{\tilde{F}}(u)$ *of* $\tilde{F}$*, that is,* $\tilde{\pi} \hat{=} \mu_{\tilde{F}}$*, that is,* $\tilde{\pi} \hat{=} \pi_{\tilde{R}}(u)$*. The symbol* $\hat{=}$ *will always stand for "denotes" or "is defined to be".*

Definition 2.2 (Finite - Fuzzy Sets) *A (finite) fuzzy set (a fuzzy event, a fuzzy relation or a fuzzy restriction,…)* $\tilde{\pi}$*, on* S*, is characterized by possibility distribution function* $\pi_{\tilde{R}}, \pi_{\tilde{R}} : S \to [0, 1]$*, that takes on a finite number of possible fuzzy sets will be denoted by* $\tilde{\pi} = \{\tilde{\pi}_n, n = 0, 1, 2, ...\}$*. The set of all fuzzy set on* S *is denoted by* $\mathcal{F}(S)$*.*

Let $\{\tilde{\pi}_n, n = 0, 1, 2, ...\}$ be a sequence of random fuzzy sets (or a discrete fuzzy parameter stochastic process). Let the possible outcomes of $\tilde{\pi}_n$ be $\tilde{i}$ $(\tilde{i} = 0, 1, 2, ...)$, where the number of outcomes may be finite (say, m) or denumerable. The possible values of $\tilde{\pi}_n$ constitute a set $S = \{0, 1, 2, ...\}$, and that the process has the fuzzy state space S. Unless otherwise stated, by fuzzy state space of a fuzzy Markov Chain, we shall imply discrete fuzzy state space (having a finite or a countably infinite number of elements); it could be $\mathcal{F}(S) = \{0, 1, 2, \ldots\}$.

Definition 2.3 (Fuzzy Markov Chain) *The fuzzy stochastic process* $\{\tilde{\pi}_n, n = 0, 1, 2, \ldots\}$ *is called a fuzzy Markov Chain, if, for* $\tilde{i}, \tilde{j}, \tilde{i}_0, \tilde{i}_1, \ldots, \tilde{i}_{n-1} \in \mathcal{F}(S)$.

$$\begin{aligned}\pi_{\tilde{R}}\{x_{n+1} = \tilde{j} \mid x_n = \tilde{i}, x_{n-1} = \tilde{i}_{n-1}, \ldots, x_1 = \tilde{i}_1, x_0 = \tilde{i}_0\} &= \pi_{\tilde{R}}\{x_{n+1} = \tilde{j} \mid x_n = \tilde{i}\} \\ &= \pi_{\tilde{i}\tilde{j}} \quad (say)\end{aligned}$$

whenever the first member is defined.

3 Fuzzy Expert System Designing

Expert system in medicine consists of medical expert and fuzzy system. We apply the fuzzy expert system in medical diagnosis. Fuzzy expert systems and various intelligent techniques help for classification. The following classifications are related to medical diagnosis: symptoms and findings, the medical knowledge, disease and diagnosis. We use the following symbols to sketch the above classification.

$\tilde{S} = (\tilde{s}_1, \ldots, \tilde{s}_m)$: set of symptoms.

$\tilde{D} = (\tilde{d}_1, \ldots, \tilde{d}_n)$: set of diseases.

$\tilde{T} = (\tilde{t}_1, \ldots, \tilde{t}_q)$: set of age groups.

All $\tilde{s}_i, \tilde{d}_j, \tilde{t}_l$ are fuzzy sets characterized by their respective membership values. The membership values of these relationships $[\tilde{T}, \tilde{S}]$ and $[\tilde{S}, \tilde{D}]$ converted into fuzzy relation $[\tilde{R}_1, \tilde{R}_2]$ are mentioned in two aspects.

(i) Occurrence of $\tilde{s}_i$ in case $\tilde{d}_j$ (infected-$\tilde{I}_1, \tilde{I}_2$),

(ii) Non-occurrence of $\tilde{s}_i$ in case $\tilde{d}_j$ (uninfected-$\tilde{U}_1, \tilde{U}_2$).

This leads to the definition of fuzzy relation. Assume two fuzzy states such as occurrence ($x_t = 1$) and non-occurrence ($x_t = 0$). The transition of possibility distribution functions of fuzzy Markov process in a 2×2 matrix is considered:

$$\Pi = \begin{Vmatrix} \pi_{00} & \pi_{01} \\ \pi_{10} & \pi_{11} \end{Vmatrix}$$

All elements in matrix Π are non-negative numbers, and $\sum_{\tilde{j}=0}^{\infty} \pi_{\tilde{i}\tilde{j}} = 1, \tilde{i} = 0, 1.$

For occurred value not re-occurred, we get $\pi_{10} = 0$, $\pi_{11} = 1$ for $\pi_{01}(t) = 1 - \pi_{00}$. $\pi_{00}(t)$ given by $\pi_{00}(t) = \pi_{\not\subset\tilde{R}}\{x_t \mid x_{t-1}\} = \pi_{\not\subset\tilde{R}}(x_t) \big/ \pi_{\not\subset\tilde{R}}(x_{t-1})$. Thus the n-step transition process can be written as

$$\pi_{00}^{(n)}(t) = \prod_{k=t}^{t+n-1} \pi_{00}(k) = \pi_{\not\subset\tilde{R}}(x_{t+n-1}) \big/ \pi_{\not\subset\tilde{R}}(x_{t-1}).$$

From the original data of the proposed age distribution, multistep transition process is calculated. Here t is the state of age group, and we get the new age-specific member rate (ASMR) between the age group t to t + n is given by

$$(ASMR)_t = 100\, \pi_{\not\subset\tilde{R}}(x_t)\, \pi_{01}^{(n)}(t) = \frac{100\ \pi_{\not\subset\tilde{R}}(x_t)\ (\pi_{\not\subset\tilde{R}}(x_{t-1}) - \pi_{\not\subset\tilde{R}}(x_{t+n-1}))}{\pi_{\not\subset\tilde{R}}(x_{t-1})}.$$

The membership functions for these (infected-$\tilde{I}_1$, $\tilde{I}_2$ and uninfected-$\tilde{U}_1$, $\tilde{U}_2$) two fuzzy states are defined to be

$$\mu_{\tilde{I}}(x) = \max_{s \in S}\{\min\,(\mu_{\tilde{I}_1}(x),\, \mu_{\tilde{I}_2}(x)\,)\}, \quad x \in X \times X$$

$$\mu_{\tilde{U}}(y) = \max_{s \in S}\{\min\,(\mu_{\tilde{U}_1}(y),\, \mu_{\tilde{U}_2}(y)\,)\}, \quad y \in Y \times Y$$

The $\tilde{s}_i$, $\tilde{d}_j$ occurrence relationships are acquired empirically from medical experts using the membership values. Other relationships such as age\symptom, symptom\disease are also defined as fuzzy sets. Possibility interpretations of relations (max-min) are used. Given an age-group\disease relationships yield fuzzy diagnostic indications that are basis for establishing occurrence and non-occurrence diagnosis. Finally, two different relations are calculated by means of fuzzy relation (fuzzy composition).

$$\tilde{I} = \tilde{I}_1 \circ \tilde{I}_2 = \{(x, \max_{s \in S}\{\min\,(\mu_{\tilde{I}_1}(x),\, \mu_{\tilde{I}_2}(x)\,)\} \mid x \in X \times X\}$$

$$\tilde{U} = \tilde{U}_1 \circ \tilde{U}_2 = \{(y, \max_{s \in S}\{\min\,(\mu_{\tilde{U}_1}(y),\, \mu_{\tilde{U}_2}(y)\,)\} \mid y \in Y \times Y\}$$

To calculate the age specific member rate for the above fuzzy sets, we use the following formulas.

$$R_t(\tilde{I}) = \frac{100\ \mu_{\not\subset \tilde{I}}(x_t)\ (\mu_{\not\subset \tilde{I}}(x_{t-1}) - \mu_{\not\subset \tilde{I}}(x_{t+n-1}))}{\mu_{\not\subset \tilde{I}}(x_{t-1})}$$

$$R_t(\tilde{U}) = \frac{100\ \mu_{\not\subset \tilde{U}}(y_t)\ (\mu_{\not\subset \tilde{U}}(y_{t-1}) - \mu_{\not\subset \tilde{U}}(y_{t+n-1}))}{\mu_{\not\subset \tilde{U}}(y_{t-1})}$$

4 Application on Host Factor of Epidemics

Applying the fuzzy expert system to host factor of epidemics is compound of expert and fuzzy system. The microbial agents that cause epidemics are numerous and include viruses, parasites and others. The data for the developed system were taken from the literature and with the help of medical experts. We consider the following set of epidemics (common disease) $\tilde{D}$ as follows:

$\tilde{d}_1$ - Leptospirosis
$\tilde{d}_2$ - Malaria
$\tilde{d}_3$ - Measles
$\tilde{d}_4$ - Influenza
$\tilde{d}_5$ - Rabies
$\tilde{d}_6$ - Filaria
$\tilde{d}_7$ - Dengue

The experienced medical experts have taken the symptom data for the diseases age wise. We consider the following set $\tilde{S}$ of common symptoms:

$\tilde{s}_1$ - Sore throat
$\tilde{s}_2$ - Lymph edema
$\tilde{s}_3$ - Head ache
$\tilde{s}_4$ - Running nose
$\tilde{s}_5$ - Fever
$\tilde{s}_6$ - Cough
$\tilde{s}_7$ - Chills

We consider the following $\tilde{T}$ of common age group $\tilde{T} = (\tilde{t}_0, \tilde{t}_1, \ldots, \tilde{t}_7)$. The membership values of these relationships $[\tilde{T}, \tilde{S}]$ and $[\tilde{S}, \tilde{D}]$ are converted into fuzzy relations and two matrices have been formed. The two matrices are

Age, symptom confirmation relationship-matrix, $[\tilde{R}_1]$

Symptom, disease confirmation relationship-matrix, $[\tilde{R}_2]$

The above matrices consist of simulation data obtained from expert are tabulated (Table 1 and Table 2). Then the computation of max-min composition $\tilde{R}_3 = \tilde{R}_1 \circ \tilde{R}_2$ is assumed to describe the state of patient (age-wise) in terms of diagnosis as a fuzzy subset $\tilde{R}_3$ of $\tilde{T} \times \tilde{D}$ characterized by its membership function

$$\mu_{\tilde{R}_3}(t, d) = \max_{s \in S}\{\min(\mu_{\tilde{R}_1}(t, s),\ \mu_{\tilde{R}_2}(s, d))\}, \quad (t, d) \in \tilde{T} \times \tilde{D}.$$

The possibility distribution function of the complement of fuzzy set $\tilde{R}_3$, $\pi_{\not\subset \tilde{R}_3}(t, d)$ is denoted by $\pi_{\not\subset \tilde{R}_3}(t, d) = 1 - \pi_{\tilde{R}_3}(t, d)$. The complement of fuzzy set $\tilde{R}_3$ is tabulated in Table 3. From $\not\subset \tilde{R}_3$, the fuzzy Markov risk rate (FMRR) is set up as a new measure of infection risk rate and tabulated in Table 4. It varies as the product of proportion of becoming infection at a given age t in two age/time steps, $\pi^{(2)}_{\tilde{U}\tilde{I}}(t, d)$. Its calculated formula is

$$FMRR\ (\tilde{T}, \tilde{D}) = 100\ \pi_{\not\subset \tilde{R}_4}(t, d)\ \pi^{(2)}_{\tilde{U}\tilde{I}}(t, d)$$

where

$$\pi^{(2)}_{\tilde{U}\tilde{I}}(t, d) = (\pi_{\not\subset \tilde{R}_4}(t-1, d) - \pi_{\not\subset \tilde{R}_4}(t+1, d)) \big/ \pi_{\not\subset \tilde{R}_4}(t-1, d),\ t \in \tilde{T},\ d \in \tilde{D}.$$

Thus, the fuzzy Markov chain model can give precise and reliable information for different age-specific prevalence and its related factors.

Table 1: $[\tilde{R}_1]$

Symptom Age (T)	S_1	S_2	S_3	S_4	S_5	S_6	S_7
Born (t_0)	0	0	0	0	0	0	0
1-5 (t_1)	.3	0	.2	.3	.1	.1	.1
6-10 (t_2)	.3	.2	.3	.4	.1	.2	.1
11-15 (t_3)	.3	.2	.3	.4	.3	.3	.1
16-35 (t_4)	.4	.3	.4	.5	.4	.3	.2
36-60 (t_5)	.4	.4	.4	.5	.5	.4	.3
61-70 (t_6)	.5	.4	.5	.6	.6	.4	.3
Above 70 (t_7)	.6	.5	.6	.6	.6	.5	.4

Table 2: [$\tilde{R}_2$]

Disease / Symptom	d_1	d_2	d_3	d_4	d_5	d_6	d_7
S_1	0	.9	0	0	.9	0	.8
S_2	0	1	0	.7	0	0	1
S_3	0	.5	.5	1	0	0	0
S_4	1	0	1	0	0	0	0
S_5	.7	.8	.9	1	.7	.7	.9
S_6	0	0	1	0	0	0	0
S_7	0	.7	0	.8	0	0	.7

Table 3: The complement of fuzzy set $\tilde{R}_3$

	d_1	d_2	d_3	d_4	d_5	d_6	d_7
t_0	1	1	1	1	1	1	1
t_1	.7	.7	.7	.8	.7	.9	.7
t_2	.6	.7	.6	.7	.7	.9	.7
t_3	.6	.6	.6	.7	.7	.9	.7
t_4	.5	.6	.5	.5	.6	.6	.6
t_5	.5	.5	.5	.5	.5	.5	.5
t_6	.4	.4	.4	.4	.4	.4	.4
t_7	.4	.4	.4	.4	.4	.4	.4

Table 4: Infection risk rate

$FMRR\ (\tilde{T}, \tilde{D})$	d_1	d_2	d_3	d_4	d_5	d_6	d_7
t_1	28.0	21.0	28.0	24.0	21.0	09.0	21.0
t_2	08.6	10.0	08.6	08.8	00.0	00.0	00.0
t_3	10.0	08.6	10.0	10.0	10.0	30.0	10.0
t_4	08.3	10.0	08.3	17.1	17.1	27.0	17.1
t_5	10.0	16.7	10.0	16.7	16.7	16.7	16.7
t_6	08.0	08.0	08.0	08.0	08.0	08.0	08.0

5 Results and Discussions

Table 4 shows that there is wide variation as the product of proportion of becoming epidemics at a given age t in two age/time steps.

From the Table 4, it is seen that the epidemics affect all ages.

The diseases d_1, d_3 and d_4 are mostly commonly present in children than adults.

The disease d_5 and d_6 shows very high prevalence up to t_4 age groups.

It is seen that the diseases of Leptospirosis, Measles and Influenza are ranking high among children. It is recommended that these age groups should be prioritized during the outbreak of epidemics. Further, it should be taken into account that the different $FMRR$ be the result of different health conditions and different exposure at different age groups. Even though all epidemics affect all age groups generally, we suggest to use $FMRR$ to target age groups to be prioritized.

6 Conclusion

In this study, a hybrid model analysis on host factor of epidemics involving fuzzy Markov-chain is proposed. The model is constructed using the conditional possibility distribution function, Markov-chain and fuzzy expert system. An age-specific member rate and Markov risk rate are defined and used as novel measure indices to prioritize age groups to identify and decide the age group that are targeted for motivation. It is concluded that due to uncertainty of the phenomena. Combination of stochastic model and fuzzy set theory enables our computational simulation to much more faithfully portray the phenomena under study.

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