Theoretical Mathematics & Applications, vol.3, no.1, 2013, 183-198
ISSN: 1792- 9687 (print), 1792-9709 (online)
Scienpress Ltd, 2013

Sub-compatible maps and common fixed point theorem in non-Archimedean Menger PM-space

Arihant Jain[1], Bijendra Singh[2] and Mridula Jain[3]

Abstract

The object of this paper is to establish common fixed point theorems for four self maps using the concepts of compatible maps, sub-sequential continuous maps, sub-compatible maps, reciprocally continuous maps and conditionally reciprocally continuous maps in a non-Archimedean Menger PM-space. Our results extend and generalize the result of Bouhadjera et. al. [1] from metric space to N.A. Menger PM-space. We also furnish an example in support of our result.

Mathematics Subject Classification: 47H10, 54H25

Keywords: Non-Archimedean Menger probabilistic metric space; Common fixed points; Compatible maps; Sub-compatible maps; Sub-sequential continuous maps; Reciprocal continuous maps and Conditionally reciprocal continuous maps

---

[1] Department of Applied Mathematics, Shri Guru Sandipani Institute of Technology and Science, Ujjain (M.P.) – 456650, e-mail : arihant2412@gmail.com
[2] School of Studies in Mathematics, Vikram University, Ujjain (M.P.) – 456010
[3] Department of Mathematics, Govt. Madhav Science, P.G. College, Ujjain (M.P.)-456010

Article Info: *Received* : December 3, 2012. *Revised* : February 26, 2013
Published online : April 15, 2013

1 Introduction

There have been a number of generalizations of metric space. One such generalization is Menger space initiated by Menger [12]. It is a probabilistic generalization in which we assign to any two points x and y, a distribution function $F_{x,y}$. Schweizer and Sklar [15] studied this concept and gave some fundamental results on this space.

The notion of compatible mapping in a Menger space has been introduced by Mishra [13]. Using the concept of compatible mappings of type (A), Jain et. al. [5, 6] proved some interesting fixed point theorems in Menger space. Afterwards, Jain et. al. [7] proved the fixed point theorem using the concept of weak compatible maps in Menger space.

The notion of non-Archimedean Menger space has been established by Istrӑtescu and Crivat [11]. The existence of fixed point of mappings on non-Archimedean Menger space has been given by Istrӑtescu [10]. This has been the extension of the results of Sehgal and Bharucha - Reid [16] on a Menger space. In the sequel, Hadzic [8], Chang [2] and Cho. et. al. [3] proved a common fixed point theorem for compatible mappings in non-Archimedean Menger PM-space. Afterwards, Singh et. al. [17, 18, 19, 20] proved interesting results on N.A. Menger PM-space.

In an interesting article, Bouhadjera et. al. [1] introduced notions of subcompatibility and subsequential continuity, and utilized them to prove several common fixed point theorems in metric space. Afterwards in 2011, Imdad et. al. [9] in his interesting article improved the theorems coined by Bouhadjera et. al. [1]. In the sequel, Pant et. al. [14] introduced the new notion of conditional reciprocal continuity which unifies the approaches of three well known notions – reciprocal continuity, subsequential continuity and conditional commutativity and thus, generalized the results of Bouhadjera et. al. [1].

Motivated by the results of Imdad et. al. [9] and Pant et. al. [14], we establish common fixed point theorems for four self maps using the concepts of compatible maps, sub-sequential continuous maps, sub-compatible maps, reciprocally continuous maps and conditionally reciprocally continuous maps in a non-Archimedean Menger PM-space. Our results extend and generalize the result of Bouhadjera et. al. [1] from metric space to N.A. Menger PM-space. We also furnish an example in support of our result.

For the sake of completeness, we recall some definitions and known results in non-Archimedean Menger probabilistic metric space.

2 Preliminaries

For terminologies, notations and properties of probabilistic metric spaces, refer to [4], [13] and [16].

First, we start with the definition of non-Archimedean probabilistic metric space given by Cho et. al. [3].

Definition 2.1 [3] Let X be a non-empty set and $\mathcal{D}$ be the set of all left-continuous distribution functions. An ordered pair (X, $\mathcal{F}$) is called a non-Archimedean probabilistic metric space (briefly, a N.A. PM-space) if $\mathcal{F}$ is a mapping from X×X into $\mathcal{D}$ satisfying the following conditions (the distribution function $\mathcal{F}(x,y)$ is denoted by $F_{x,y}$ for all $x,y \in X$) :

(PM-1) $F_{u,v}(x) = 1$, for all $x > 0$, if and only if $u = v$;

(PM-2) $F_{u,v} = F_{v,u}$;

(PM-3) $F_{u,v}(0) = 0$;

(PM-4) If $F_{u,v}(x) = 1$ and $F_{v,w}(y) = 1$ then $F_{u,w}(\max\{x, y\}) = 1$,

for all $u, v, w \in X$ and $x, y > 0$.

The following definition deals with the t-norm given by Cho et. al. [3].

Definition 2.2 [3] A t-norm is a function $\Delta : [0,1] \times [0,1] \rightarrow [0,1]$ which is associative, commutative, non-decreasing in each coordinate and $\Delta(a,1) = a$ for every $a \in [0,1]$.

The following definitions explains the N.A. Menger PM-space given by Cho et. al. [3].

Definition 2.3 [3] A *N.A. Menger PM-space* is an ordered triple $(X, \mathcal{F}, \Delta)$, where $(X, \mathcal{F})$ is a non-Archimedean PM-space and Δ is a t-norm satisfying the following condition:

(PM-5) $F_{u,w}(\max\{x,y\}) \geq \Delta(F_{u,v}(x), F_{v,w}(y))$, for all $u, v, w \in X$ and $x, y \geq 0$.

Cho et. al. [3] also gave the following definitions of type $(C)_g$ and type $(D)_g$.

Definition 2.4 [3] A PM-space $(X, \mathcal{F})$ is said to be of type $(C)_g$ if there exists a $g \in \Omega$ such that

$$g(F_{x,y}(t)) \leq g(F_{x,z}(t)) + g(F_{z,y}(t))$$

for all $x, y, z \in X$ and $t \geq 0$, where $\Omega = \{g \mid g : [0,1] \rightarrow [0, \infty)$ is continuous, strictly decreasing, $g(1) = 0$ and $g(0) < \infty\}$.

Definition 2.5 [3] A N.A. Menger PM-space $(X, \mathcal{F}, \Delta)$ is said to be of type $(D)_g$ if there exists a $g \in \Omega$ such that

$$g(\Delta(s,t)) \leq g(s) + g(t)$$

for all $s, t \in [0,1]$.

Remark 2.1 [3]

(1) If a N.A. Menger PM-space $(X, \mathcal{F}, \Delta)$ is of type $(D)_g$ then $(X, \mathcal{F}, \Delta)$ is of type $(C)_g$.

(2) If a N.A. Menger PM-space $(X, \mathcal{F}, \Delta)$ is of type $(D)_g$, then it is metrizable, where the metric d on X is defined by

$$d(x,y) = \int_0^1 g\left(F_{x,y}(t)\right) d(t) \text{ for all } x, y \in X. \qquad (*)$$

Throughout this paper, suppose $(X, \mathcal{F}, \Delta)$ be a complete N.A. Menger PM-space of type $(D)_g$ with a continuous strictly increasing t-norm Δ.

Let $\phi : [0,+\infty) \to [0,+\infty)$ be a function satisfied the condition (Φ) :

(Φ) ϕ is upper-semicontinuous from the right and $\phi(t) < t$ for all $t > 0$.

Lemma 2.1 [3] If a function $\phi : [0,+\infty) \to [0,+\infty)$ satisfies the condition (Φ), then we have

(1) For all $t \geq 0$, $\lim_{n\to\infty} \phi^n(t) = 0$, where $\phi^n(t)$ is n^{th} iteration of $\phi(t)$.

(2) If $\{t_n\}$ is a non-decreasing sequence of real numbers and $t_{n+1} \leq \phi(t_n)$, $n = 1, 2, \ldots$ then $\lim_{n\to\infty} t_n = 0$. In particular, if $t \leq \phi(t)$ for all $t \geq 0$, then $t = 0$.

The following definition explains the compatible self maps in N.A. Menger PM-space.

Definition 2.6 [3] Let $A, S : X \to X$ be mappings. A and S are said to be compatible if

$\lim_{n\to\infty} g(F_{ASx_n,SAx_n}(t)) = 0$ for all $t > 0$, whenever $\{x_n\}$ is a sequence in X such that

$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$ for some z in X.

Definition 2.7 Two mappings A and S of a N.A. Menger PM-space $(X, \mathcal{F}, \Delta)$ are called reciprocally continuous if $ASx_n \to Az$ and $SAx_n \to Sz$, whenever $\{x_n\}$ is a sequence in X such that $Ax_n, Sx_n \to z$ for some z in X.

If A and S are both continuous, then they are obviously reciprocally continuous but converse is not true. Moreover, in the setting of common fixed point theorems for compatible pair of mappings satisfying contractive conditions, continuity of one of the mappings A and S implies their reciprocal continuity but not conversely.

Definition 2.8 [19] Self maps A and S of a N.A. Menger PM-space $(X, \mathcal{F}, \Delta)$ are said to be weakly compatible (or coincidentally commuting) if they commute at their coincidence points, i.e. if Ap = Sp for some $p \in X$ then ASp = SAp.

It is easy to see that two compatible maps are weakly compatible but converse is not true.

Motivated by Bouhadjera et. al. [1] in metric space, we introduce the notion of subcompatibility in N.A. Menger PM-space as follows :

Definition 2.9 Self mappings A and S of a N.A. Menger PM-space $(X, \mathcal{F}, \Delta)$ are said to be subcompatible if and only if there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z,\ z \in X \text{ and satisfy } \lim_{n\to\infty} g(F_{ASx_n,SAx_n}(t)) = 0.$$

We now give an example which explains the Definition 2.9.

Example 2.1 Let $(X, \mathcal{F}, \Delta)$ be the N.A. Menger PM-space, where $X = [0, \infty)$ and the metric d on X is defined in condition (*) of Remark 2.1. Define self maps A and S as follows :

$$Ax = \begin{cases} 2+x, & \text{if} \quad 0 \le x \le 2, \\ 3x-1, & \text{if} \quad 2 < x < \infty, \end{cases} \quad \text{and} \quad Sx = \begin{cases} 2-x, & \text{if} \quad 0 \le x < 2, \\ 3x-2, & \text{if} \quad 2 \le x < \infty. \end{cases}$$

Consider the sequence $x_n = \frac{2}{n}$ in X.

We have $\lim_{n\to\infty} A(x_n) = \lim_{n\to\infty}\left(2+\frac{2}{n}\right) = 2$

and $\lim_{n\to\infty} S(x_n) = \lim_{n\to\infty}\left(2-\frac{2}{n}\right) = 2$.

Next, $\lim_{n\to\infty} AS(x_n) = \lim_{n\to\infty} A\left(2-\frac{2}{n}\right) = \lim_{n\to\infty}\left\{2+\left(2-\frac{2}{n}\right)\right\} = 4$

and $\lim_{n\to\infty} SA(x_n) = \lim_{n\to\infty} S\left(2+\frac{2}{n}\right) = \lim_{n\to\infty}\left\{3\left(2+\frac{2}{n}\right)-2\right\} = 4.$

Now, $\lim_{n\to\infty} g(F_{ASx_n,SAx_n}(t)) = 0$.

Therefore, the pair (A, S) is sub-compatible.

Definition 2.10 Self mappings A and S of a N.A. Menger PM-space $(X, \mathcal{F}, \Delta)$ are said to be reciprocally continuous if and only if $\lim_{n\to\infty} ASx_n = At$ and $\lim_{n\to\infty} SAx_n = St$ whenever there is a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = t$, $t \in X$.

Clearly, any continuous pair is reciprocally continuous but the converse is not true in general.

Definition 2.11 Self mappings A and S of a N.A. Menger PM-space $(X, \mathcal{F}, \Delta)$ are said to be subsequentially continuous if and only if there exists a sequence $\{x_n\}$ in X such that

$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = t$, $t \in X$ and satisfy $\lim_{n\to\infty} ASx_n = At$ and $\lim_{n\to\infty} SAx_n = St$.

Clearly, if A and S are both continuous or reciprocally continuous then they are obviously subsequentially continuous.

The following example explains the Definition 2.11.

Example 2.2 Let (X, $\mathcal{F}$, Δ) be the N.A. Menger PM-space, where X = $[0, \infty)$ and the metric d on X is defined in condition (*) of Remark 2.1. Define self maps A and S as follows :

$$Ax = \begin{cases} 2+x, & \text{if} \quad 0 \le x \le 2, \\ 3x-1, & \text{if} \quad 2 < x < \infty, \end{cases} \quad \text{and} \quad Sx = \begin{cases} 2-x, & \text{if} \quad 0 \le x < 2, \\ 4x-2, & \text{if} \quad 2 \le x < \infty. \end{cases}$$

Consider the sequence $x_n = \frac{2}{n}$ in X.

We have $\lim_{n\to\infty} A(x_n) = \lim_{n\to\infty}\left(2+\frac{2}{n}\right) = 2$

and $\lim_{n\to\infty} S(x_n) = \lim_{n\to\infty}\left(2-\frac{2}{n}\right) = 2$.

Next, $\lim_{n\to\infty} AS(x_n) = \lim_{n\to\infty} A\left(2-\frac{2}{n}\right) = \lim_{n\to\infty}\left\{2+\left(2-\frac{2}{n}\right)\right\} = 4 = A(2)$

and $\lim_{n\to\infty} SA(x_n) = \lim_{n\to\infty} S\left(2+\frac{2}{n}\right) = \lim_{n\to\infty}\left\{4\left(2+\frac{2}{n}\right)-2\right\} = 6 = S(2).$

Therefore, the pair of mappings (A, S) is sub-sequential continuous.

Motivated by Pant et. al.[14], we now give the following definition :

Definition 2.12 Self mappings A and S of a N.A. Menger PM-space (X, $\mathcal{F}$, Δ) are said to be conditionally reciprocal continuous (CRC) if whenever the set of sequences $\{x_n\}$ satisfying $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n$ is non-empty, there exists a sequence $\{y_n\}$ satisfying $\lim_{n\to\infty} Ay_n = \lim_{n\to\infty} Sy_n = t$ (say) such that $\lim_{n\to\infty} ASy_n = At$ and $\lim_{n\to\infty} SAy_n = St$.

If A and S are either continuous or reciprocally continuous or subsequentially continuous then they are obviously conditionally reciprocally continuous but converse is not true as shown in the following example.

Example 2.3 Let (X, $\mathcal{F}$, Δ) be the N.A. Menger PM-space, where X = [2, 20] and the metric d on X is defined in condition (*) of Remark 2.1. Define self maps A and S as follows :

$$Ax = \begin{cases} 2 & \text{if} \quad x = 2 \\ 2 & \text{if} \quad x > 5 \\ 6 & \text{if} \quad 2 < x \le 5 \end{cases} \quad \text{and} \quad Sx = \begin{cases} 2 & \text{if} \quad x = 2 \\ \dfrac{x+1}{3} & \text{if} \quad x > 5 \\ 12 & \text{if} \quad 2 < x \le 5 \end{cases}.$$

Consider the constant sequence given by $x_n = 2$.

Then $\lim_{n\to\infty} Ax_n = 2 = \lim_{n\to\infty} Sx_n$.

Now, $\lim_{n\to\infty} ASx_n = \lim_{n\to\infty} A(2) = 2 = A(2)$ and $\lim_{n\to\infty} SAx_n = \lim_{n\to\infty} S(2) = 2 = S(2)$.

Hence, A and S are conditionally reciprocally continuous mappings.

Suppose $\{y_n\}$ be the sequence in X given by $y_n = 5 + \varepsilon_n$ where $\varepsilon_n \to 0$ as $n \to \infty$.

Then $\lim_{n\to\infty} Ay_n = 2$, $\lim_{n\to\infty} Sy_n = 2$, $\lim_{n\to\infty} AS(y_n) = \lim_{n\to\infty} A\left(2 + \frac{\varepsilon_n}{3}\right) = 6 \ne A(2)$

and $\lim_{n\to\infty} SA(y_n) = \lim_{n\to\infty} S(2) = 2 = S(2)$.

Thus, $\lim_{n\to\infty} SA(y_n) = S(2)$ and $\lim_{n\to\infty} AS(y_n) \ne A(2)$.

Hence, A and S are not reciprocally continuous mappings.

3 Main Result

Theorem 3.1 Let A, B, S and T be four self mappings of N.A. Menger PM-space (X, $\mathcal{F}$, Δ). If the pairs (A, S) and (B, T) are compatible and subsequentially continuous, then

(a) A and S have a coincidence point.

(b) B and T have a coincidence point.

Further, let for all x, y in X

(c) $g(F_{Ax,By}(t)) \leq \phi(\max\{g(F_{Sx,Ty}(t)), g(F_{Sx,Ax}(t)), g(F_{Ty,By}(T)),$

$$\tfrac{1}{2}(g(F_{Sx,By}(T)) + g(F_{Ty,Ax}(t)))\})$$

for all t > 0, where a function $\phi : [0,+\infty) \to [0,+\infty)$ satisfies the condition (Φ).

Then A, B, S and T have a unique common fixed point in X.

Proof : Since the pairs (A, S) and (B, T) are compatible and subsequentially continuous, then there exists two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z,\ z \in X$$

and satisfy

$$\lim_{n\to\infty} g(F_{ASx_n,SAx_n}(t)) = g(F_{Az,Sz}(t)) = 0;\ \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = z',\ z' \in X$$

and which satisfy

$$\lim_{n\to\infty} g(F_{BTy_n,TBy_n}(t)) = g(F_{Bz',Tz'}(t)) = 0.$$

Therefore, Az = Sz and Bz' = Tz'; that is, z is a coincidence point of A and S and z' is a coincidence point of B and T.

Now, we prove z = z'.

Put $x = x_n$ and $y = y_n$ in inequality (c), we get

$$g(F_{Ax_n,By_n}(t)) \leq \phi(\max\{g(F_{Sx_n,Ty_n}(t)), g(F_{Sx_n,Ax_n}(t)), g(F_{Ty_n,By_n}(T)),$$
$$\tfrac{1}{2}(g(F_{Sx_n,By_n}(T)) + g(F_{Ty_n,Ax_n}(t)))\})$$

Letting $n \to \infty$, we get

$$g(F_{z,z'}(t)) \leq \phi(\max\{g(F_{z,z'}(t)), g(F_{z,z}(t)), g(F_{z',z'}(t)),$$
$$\tfrac{1}{2}(g(F_{z,z'}(t)) + g(F_{z',z}(t)))\})$$
$$= \phi(g(F_{z,z'}(t)))$$

which implies that $g(F_{z,z'}(t)) = 0$ by Lemma 2.1 and so we have z = z'.

Again, we claim that Az = z.

Substitute $x = z$ and $y = y_n$ in inequality (c), we get

$$g(F_{Az,By_n}(t)) \leq \phi(\max\{g(F_{Sz,Ty_n}(t)), g(F_{Sz,Az}(t)), g(F_{Ty_n,By_n}(T)), \tfrac{1}{2}(g(F_{Sz,By_n}(T)) + g(F_{Ty_n,Az}(t)))\}).$$

Taking the limit as $n \to \infty$, we get

$$g(F_{Az,z'}(t)) \leq \phi(\max\{g(F_{Az,z'}(t)), g(F_{z,z}(t)), g(F_{z',z'}(T)), \tfrac{1}{2}(g(F_{Az,z'}(T)) + g(F_{z',Az}(t)))\})$$
$$= \phi(g(F_{Az,z'}(t)))$$

which implies that $g(F_{Az,z'}(t)) = 0$ by Lemma 2.1 and so we have $Az = z' = z$.

Again, we claim that $Bz = z$.

Substitute $x = z$ and $y = z$ in inequality (c), we get

$$g(F_{Az,Bz}(t)) \leq \phi(\max\{g(F_{Sz,Tz}(t)), g(F_{Sz,Az}(t)), g(F_{Tz,Bz}(T)), \tfrac{1}{2}(g(F_{Sz,Bz}(T)) + g(F_{Tz,Az}(t)))\})$$

$$g(F_{z,Bz}(t)) \leq \phi(\max\{g(F_{z,Bz}(t)), g(F_{Az,Az}(t)), g(F_{Bz,Bz}(T)), \tfrac{1}{2}(g(F_{z,Bz}(T)) + g(F_{Bz,z}(t)))\})$$
$$= \phi(g(F_{z,Bz}(t)))$$

which implies that $g(F_{z,Bz}(t)) = 0$ by Lemma 2.1 and so we have $z = Bz = Tz$.

Therefore, $z = Az = Bz = Sz = Tz$; that is z is common fixed point of A, B, S and T.

Uniqueness : Let w be another common fixed point of A, B, S and T. Then $Aw = Bw = Sw = Tw = w$.

Put $x = z$ and $y = w$ in inequality (c), we get

$$g(F_{Az,Bw}(t)) \leq \phi(\max\{g(F_{Sz,Tw}(t)), g(F_{Sz,Az}(t)), g(F_{Tw,Bw}(T)), \tfrac{1}{2}(g(F_{Sz,Bw}(T)) + g(F_{Tw,Az}(t)))\})$$

or, $$g(F_{z,w}(t)) \leq \phi(\max\{g(F_{z,w}(t)), g(F_{z,z}(t)), g(F_{w,w}(T)), \tfrac{1}{2}(g(F_{z,w}(T)) + g(F_{w,z}(t)))\})$$
$$= \phi(g(F_{z,w}(t))),$$

which implies that $g(F_{z,w}(t)) = 0$ by Lemma 2.1 and so we have $z = w$.

Therefore, uniqueness follows.

Theorem 3.2 The conclusions of Theorem 3.1 remains valid if we replace compatibility with subcompatibility and subsequential continuity with reciprocally continuity, besides retaining the rest of the hypotheses.

Corollary 3.1 Let A, B, S and T be four self mappings of N.A. Menger PM-space (X, $\mathcal{F}$, Δ). If the pairs (A, S) and (B, T) are compatible and conditionally reciprocal continuous, then

(a) A and S have a coincidence point.

(b) B and T have a coincidence point.

Further, let for all x, y in X

(c) $g(F_{Ax,By}(t)) \leq \phi(\max\{g(F_{Sx,Ty}(t)), g(F_{Sx,Ax}(t)), g(F_{Ty,By}(T)),$

$$\tfrac{1}{2}(g(F_{Sx,By}(T)) + g(F_{Ty,Ax}(t)))\})$$

for all $t > 0$, where a function $\phi : [0,+\infty) \to [0,+\infty)$ satisfies the condition (Φ). Then A, B, S and T have a unique common fixed point in X.

Proof : Since subsequential continuity implies conditionally reciprocal continuity, so the proof follows from Theorem 3.1.

Corollary 3.2 Let A, B, S and T be four self mappings of N.A. Menger PM-space (X, $\mathcal{F}$, Δ). If the pairs (A, S) and (B, T) are sub-compatible and conditionally reciprocal continuous, then

(a) A and S have a coincidence point.

(b) B and T have a coincidence point.

Further, let for all x, y in X

(c) $g(F_{Ax,By}(t)) \leq \phi(\max\{g(F_{Sx,Ty}(t)), g(F_{Sx,Ax}(t)), g(F_{Ty,By}(T)),$

$$\tfrac{1}{2}(g(F_{Sx,By}(T)) + g(F_{Ty,Ax}(t)))\})$$

for all $t > 0$, where a function $\phi : [0,+\infty) \to [0,+\infty)$ satisfies the condition (Φ). Then A, B, S and T have a unique common fixed point in X.

Proof : Since reciprocal continuity implies conditionally reciprocal continuity, so the proof follows from Theorem 3.2.

Now we give an example in support of our Theorem 3.1.

Example 3.1 Let (X, $\mathcal{F}$, Δ) be the N.A. Menger PM-space, where X = [0, ∞) and the metric d on X is defined in condition (*) of Remark 2.1. Define self maps A and B as follows :

$$Ax = \begin{cases} x/4, & \text{if} \quad 0 \le x \le 2, \\ 3x-4, & \text{if} \quad 2 < x < \infty, \end{cases} \quad \text{and} \quad Bx = \begin{cases} x/6, & \text{if} \quad 0 \le x \le 2, \\ 2x-2, & \text{if} \quad 2 < x < \infty. \end{cases}$$

Consider the sequence $x_n = \frac{1}{n}$ in X.

We have $\lim_{n\to\infty} A(x_n) = \lim_{n\to\infty} \frac{1}{4n} = 0 = \lim_{n\to\infty} \frac{1}{6n} = \lim_{n\to\infty} B(x_n)$.

Next, $\lim_{n\to\infty} AB(x_n) = \lim_{n\to\infty} A\left(\frac{1}{6n}\right) = \lim_{n\to\infty}\left(\frac{1}{24n}\right) = 0 = A(0)$

$$\lim_{n\to\infty} BA(x_n) = \lim_{n\to\infty} B\left(\frac{1}{4n}\right) = \lim_{n\to\infty}\left(\frac{1}{24n}\right) = 0 = B(0)$$

and $\lim_{n\to\infty} g(F_{ABx_n,BAx_n}(t)) = 0$.

Consider another sequence $x_n = \left(2+\frac{1}{n}\right)$. Then

$$\lim_{n\to\infty} A(x_n) = \lim_{n\to\infty} A\left(2+\frac{1}{n}\right) = \lim_{n\to\infty}\left\{3\left(2+\frac{1}{n}\right)-4\right\} = 2$$

$$\lim_{n\to\infty} B(x_n) = \lim_{n\to\infty} B\left(2+\frac{1}{n}\right) = \lim_{n\to\infty}\left\{2\left(2+\frac{1}{n}\right)-2\right\} = 2.$$

Also, $\lim_{n\to\infty} AB(x_n) = \lim_{n\to\infty} A\left(2+\frac{2}{n}\right) = \lim_{n\to\infty}\left\{3\left(2+\frac{2}{n}\right)-4\right\} = 2 \ne A(2)$

$$\lim_{n\to\infty} BA(x_n) = \lim_{n\to\infty} B\left(2+\frac{3}{n}\right) = \lim_{n\to\infty}\left\{2\left(2+\frac{3}{n}\right)-2\right\} = 2 \ne B(2).$$

But $\lim_{n\to\infty} g(F_{ABx_n,BAx_n}(t)) = 0$.

Therefore, the pair (A, B) is compatible as well as subsequential continuous, but not reciprocally continuous. Therefore all the conditions of Theorem 3.1 are satisfied. Also, 0 is the coincidence as well as the unique common fixed point of the pair (A, B).

Acknowledgement. The authors are thankful to the referee for his valuable comments.

References

[1] H. Bouhadjera and C. Godet-Thobie, Common fixed point theorems for pairs of subcompatible maps, arXiv:0906.3159v1 [math.FA] 17 June 2009.

[2] S.S. Chang, Fixed point theorems for single-valued and multi-valued mappings in non-Archimedean Menger probabilistic metric spaces, *Math. Japonica*, **35**(5), (1990), 875-885.

[3] Y.J. Cho, K.S. Ha and S.S. Chang, Common fixed point theorems for compatible mappings of type (A) in non-Archimedean Menger PM-spaces, *Math. Japonica*, **48**(1), (1997), 169-179.

[4] Y.J. Cho, K.S. Park and S.S. Chang, Fixed point theorems in metric spaces and probabilistic metric spaces, *Internat. J. Math. & Math. Sci.*, **19**(2), (1996), 243-252.

[5] Arihant Jain and Bijendra Singh, Common fixed point theorem in Menger space through compatible maps of type (A), *Chh. J. Sci. Tech.*, **2**, (2005), 1-12.

[6] Arihant Jain and Bijendra Singh, A fixed point theorem in Menger space through compatible maps of type (A), *V.J.M.S.*, **5**(2), (2005), 555-568.

[7] Arihant Jain and Bijendra Singh, Common fixed point theorem in Menger Spaces, *The Aligarh Bull. of Math.*, **25**(1), (2006), 23-31.

[8] O. Hadzic, A note on Istrăṫescu's fixed point theorems in non-Archimedean Menger spaces, *Bull. Math. Soc. Sci. Math. Rep. Soc. Roum. T.*, **24**(72), 3(1980), 277-280.

[9] M. Imdad, Javid Ali and M. Tanveer, Remarks on some recent metrical common fixed point theorems, *Applied Mathematics Letters*, **24**, (2011), 1165-1169.

[10] V.I. Istrătescu, Fixed point theorems for some classes of contraction mappings on nonarchimedean probabilistic metric space, *Publ. Math. (Debrecen)*, **25**, (1978), 29-34.

[11] V.I. Istrătescu and N. Crivat, On some classes of nonarchimedean Menger spaces, *Seminar de spatii Metrice probabiliste, Univ. Timisoara*, **12**, (1974).

[12] K. Menger, Statistical metrics, *Proc. Nat. Acad. Sci., USA*, **28**, (1942), 535 -537.

[13] S.N. Mishra, Common fixed points of compatible mappings in PM-spaces, *Math. Japon*., **36**(2), (1991), 283-289.

[14] R.P. Pant and R.K. Bisht, Common fixed point theorems under a new continuity condition, *Ann. Univ. Ferrara*, **58**, (2012), 127-141.

[15] B. Schweizer and A. Sklar, Statistical metric spaces, *Pacific J. Math*., **10**, (1960), 313-334.

[16] V.M. Sehgal and A.T. Bharucha-Reid, Fixed points of contraction maps on probabilistic metric spaces, *Math. System Theory*, **6**, (1972), 97- 102.

[17] Bijendra Singh, Arihant Jain and Pallavi Agarwal, Semi-compatibility in non-Archimedean Menger PM-space, *Commentationes Mathematicae*, **49**(1), (2009), 15-25.

[18] Bijendra Singh, Arihant Jain and Pallavi Agarwal, Common fixed point of coincidentally commuting mappings in non-Archimedean Menger PM-

spaces, *Italian Journal of Pure and Applied Mathematics*, **25**, (2009), 213-218.

[19] Bijendra Singh, Arihant Jain and Pallavi Agarwal, Weak compatibility in non-Archimedean Menger PM-space, *Journal of Indian Acad. Math*., **31**(2), (2009), 335-350.

[20] Bijendra Singh, Arihant Jain and Mridula Jain, Compatible maps and fixed points in non-Archimedean Menger PM-spaces, *Int. J. Contemp. Math. Sciences*, **6**(38), (2011), 1895-1905.