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Robust Estimation of the Memory Parameter

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Abstract

Recent studies have found indications of long-range dependence in financial time series and used conventional, non-robust estimates of the memory parameter, which measures the degree of long-range dependence, for the calculation of buy and sell signals. In this paper, new robust estimators are proposed which are possibly more appropriate for financial data. The new estimators are compared with various robust and non-robust competitors by means of extensive simulations. In addition to additive outliers and heavy-tailed distributions, also conditional heteroscedasticity is considered. The results show that the robust estimators do not generally deliver better results than the conventional estimators but only in special cases, the existing robust estimators with respect to the root-mean-square error and the new robust estimators with respect to the bias. Finally, the different estimators are used to investigate possible long-range dependence both in developed and developing stock markets. The results of this empirical study suggest that long-range dependence is present only in the volatility and is therefore of no use for directional forecasting and trading.

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1. Introduction

Reviewing all articles dealing with the short-term directional forecasting of the prices of stocks, commodities, and currencies, which have been published in 2016 by one of the 96 journals covered by the Social Sciences Citation Index in the category “Business, Finance”, Reschenhofer et al. [1] found hardly any evidence of economically significant predictability. Among the few exceptions were pronounced periodic patterns in Turkish equity returns [2] and the high autocorrelation of returns on the ASE20 Greek stock index [3]. A further feature of interest was long-range dependence. However, the results were conflicting. Using sectoral data from the Amman Stock Exchange, Al-Shboula and Anwar [4] found no indications of long-range dependence. In contrast, Auer [5] used a more extensive set of data from different emerging markets and observed time-varying predictability based on the Hurst coefficient, which is a simple measure of long-range dependence. Earlier research on this topic is also inconclusive. For example, Cajueiro and Tabak [6] observed decreasing Hurst coefficients in emerging markets returns whereas Hull and McGroarty [7] found no evidence of a trend.

Clearly, the presence of long-range dependence implies only that returns are predictable and not that markets are inefficient. To prove the latter statement, Auer [5] and Batten et al. [8] constructed automatic rules for the trading of precious metals, which can in principle also be used for stocks. They used estimates $\widehat{H}$ of the Hurst coefficient H for the calculation of buy and sell signals, which makes only sense if there is indeed long-range dependence and it varies over time. Batten et al. [8] used the values 0.4, 0.6 as thresholds for $\widehat{H}$ and Auer [5] the values 0.3, 0.7, where $H = 0.5$ indicates rapidly (exponentially) decaying autocorrelations (short memory) and $H < 0.5$ / $H > 0.5$ indicates slowly (hyperbolically) decaying negative / positive autocorrelations (long memory). However, Mangat and Reschenhofer [9] showed that it is virtually impossible to distinguish between close neighboring values such as −0.2, 0, 0.2 let alone −0.1, 0, 0.1 when testing is conducted in a rolling analysis with a small to medium window length (22, 66: [8]; 240: [10]). Constructing new tests, which are highly robust against conditional heteroscedasticity and heavy tails, and applying them to the same datasets, Reschenhofer and Mangat [11] were able to corroborate their earlier findings.

In the non-classical case, where deviations from normality and homo-scedasticity are possible, estimating is an equally demanding task as testing. It is therefore the goal of this paper to introduce new robust estimators. Particular emphasis is placed on the small-sample properties of these estimators. Section 2 reviews existing estimators and describes the new estimators. In Section 3, the results of a simulation study are presented, which compares the new estimators both with conventional, non-robust estimators and robust estimators that have been designed for the case of additive outliers [12] [13]. Section 4 uses both the robust and the non-robust estimators to investigate the long-run properties of selected stock market indices. Section 5 concludes.

2. Methods

In this paper, the fractional differencing parameter d is used instead of the Hurst coefficient H. These two measures of long-range dependence are related via $H = d + 0.5$. In Subsection 2.1, we briefly review conventional estimators for d before we turn to robust estimators in Subsection 2.2. In the latter subsection, we introduce new robust estimators.

2.1 Conventional estimators

Using a suitable time series model, such as the autoregressive fractionally integrated moving average (ARFIMA) model

$$y_t = \left(1 - \phi_1 L - \cdots - \phi_p L^p\right)^{-1} (1 - L)^{-d} (1 - \theta_1 L - \cdots - \theta_q L^q) u_t \tag{1}$$

[14] [15] with spectral density

$$f(\omega) = \frac{\sigma^2}{2\pi} \left|1 - e^{-i\omega}\right|^{-2d} \left|1 - \sum_{j=1}^{q} \theta_j e^{-i\omega j}\right|^2 \left|1 - \sum_{j=1}^{p} \phi_j e^{-i\omega j}\right|^{-2} \tag{2}$$

for the estimation of the fractional differencing parameter d requires the correct specification of the model dimension, which is unknown in practice. The uncertainty regarding the model dimension can be avoided by adopting Geweke and Porter-Hudak's [16] approach. Their semiparametric estimator is obtained by regressing the log periodogram

$$\log I(\omega_k) = \log \frac{1}{2\pi n} \left|\sum_{t=1}^{n} y_t e^{-i\omega_k t}\right|^2 \tag{3}$$

of the observations $y_1, \dots, y_n$ on the deterministic regressor

$$-\log\left|1 - e^{-i\omega_j}\right|^2 = -2\log\left(2\sin(\omega_j/2)\right), \tag{4}$$

where $\omega_j = 2\pi j/n$, $j = 1, .., K \ll [n/2]$, are the first K Fourier frequencies. Hurvich et al. [17] showed that the slope of this simple linear regression is a consistent estimator of the memory parameter d when only Fourier frequencies in the neighborhood of frequency zero are used, more precisely when $K = o(n^{4/5})$ and $\log^2(n) = o(K)$. For possible improvement of the log periodogram estimator $\hat{d}_{GPH}$ of Geweke and Porter-Hudak [16], Robinson [18] proposed to trim out the contributions from the lowest frequencies, which have a non-standard asymptotic distribution (see also [19] [20]), and Hassler [21], Peiris and Court [22], Reisen [23] proposed to replace the periodogram ordinates $I(\omega_k)$, $j = 1, \dots, K$, occurring in the log periodogram regression by the lag-window estimates

$$\hat{f}\left(\omega_j\right) = \frac{1}{2\pi} \sum_{s=-m}^{m} w(s/m) \hat{\gamma}(s) e^{-i\omega_j s},\ j = 1, \dots, K, \tag{5}$$

where $\hat{\gamma}(s)$ denotes the sample autocovariance at lag s and the lag window $w: [0,1] \to \Re$ satisfies $w(0) = 1$, $|w(s)| \le 1$, and $w(-s) = w(s)$. A popular example is the Parzen window

$$w(z) = \begin{cases} 1 - 6z^2 + 6|z|^3, & |z| < \frac{1}{2}, \\ 2(1 - |z|)^3, & \frac{1}{2} \le |z| \le 1. \end{cases} \quad (6)$$

In contrast to these smoothed periodogram estimators, which depend not only on the number K of included frequencies but also on the choice of the lag window and the specification of the truncation point m, the narrow-band Whittle-likelihood estimator [24] [25] and the goodness-of-fit estimator of Reschenhofer et al. [26] depend only on K but are still very competitive as has been shown through extensive simulations [26]. The former estimator, $\hat{d}_W$, is obtained by minimization of the Whittle-likelihood in the neighborhood of frequency zero over a set D of possible values of d and the latter estimator, $\hat{d}_{KS}$, by minimization of the test statistic of the two-sided Kolmogorov-Smirnov goodness-of-fit test applied to the cumulative standardized periodogram in the neighborhood of frequency zero (for a related hypothesis test see [9]).

2.2 Robust estimators

Molinares et. al. [12] introduced a variant of the smoothed periodogram estimator that is robust against additive outliers. Their robust estimator $\hat{d}_{rAC}$ is obtained by using the rectangular window $w(z) = 1, |z| \le 1$, specifying the truncation point as $m = \lfloor n^{\beta} \rfloor$ with $\beta = 0.7$, and replacing the sample autocovariances occurring in (5) by the robust sample autocovariances of Ma and Genton [27]. Using the established approach of reducing the problem of estimating the covariance to that of estimating the variance by means of the identity

$$cov(x, y) = \frac{1}{4}(var(x + y) - var(x - y)) \quad (7)$$

[28] [29], the only thing Ma and Genton [27] had to do was to select an appropriate estimator for the variance. They chose the highly robust estimator Q_n^2 of Croux and Rousseeuw [30] and Rousseeuw and Croux [31]. For a sample $z_1, \ldots, z_n$ of size n, Q_n is approximately the lower sample quartile of the $\binom{n}{2}$ interpoint distances $|z_i - z_j|$, $i \ne j$, multiplied by the constant $c = 2.2191$ in order to achieve consistency in the Gaussian case.

Alternatively, a robust estimator of the memory parameter d may be based on a robust version of the periodogram. Li's [32] Laplace periodogram is obtained by replacing the least squares (LS) criterion with the least absolute deviations (LAD) criterion in the harmonic regression procedure that produces the ordinary periodogram. To allow more flexibility for making the tradeoff between efficiency and robustness, Li [33] proposed to use the more general L_p-norm criterion, which contains the LS criterion and the LAD criterion as special cases for $p = 2$ and $p = 1$, respectively. Similarly, Reisen et al. [13] generalized the ordinary periodogram

$$I(\omega_k) = \frac{n}{8\pi}(A_k^2 + B_k^2) = \frac{n}{8\pi}\left(\left(\frac{\sum_{t=1}^{n} y_t \cos(\omega_k t)}{\sum_{t=1}^{n} \cos^2(\omega_k t)}\right)^2 + \left(\frac{\sum_{t=1}^{n} y_t \sin(\omega_k t)}{\sum_{t=1}^{n} \sin^2(\omega_k t)}\right)^2\right) \tag{8}$$

by replacing the identity function id occurring in

$$\begin{aligned} 0 &= A_k \sum_{t=1}^{n} \cos^2(\omega_k t) - \sum_{t=1}^{n} y_t \cos(\omega_k t) \\ &= \sum_{t=1}^{n} \cos(\omega_k t)(A_k \cos(\omega_k t) - y_t) \\ &= \sum_{t=1}^{n} \cos(\omega_k t)\, id(A_k \cos(\omega_k t) - y_t) \end{aligned} \tag{9}$$

and

$$\begin{aligned} 0 &= B_k \sum_{t=1}^{n} \sin^2(\omega_k t) - \sum_{t=1}^{n} y_t \sin(\omega_k t) \\ &= \sum_{t=1}^{n} \sin(\omega_k t)(B_k \sin(\omega_k t) - y_t) \\ &= \sum_{t=1}^{n} \sin(\omega_k t)\, id(B_k \sin(\omega_k t) - y_t) \end{aligned} \tag{10}$$

by a suitable function which is less sensitive to outliers than the identity function, e.g., the Huber function

$$\psi(z) = \begin{cases} z, & |z| \le C, \\ sign(z)C, & |z| > C, \end{cases} \tag{11}$$

and solving the resulting equations for A_k and B_k, respectively.

Reschenhofer and Mangat [11] used ratios

$$\frac{J(\omega_j)}{J(\omega_k)} = \frac{I(\omega_j)}{f(\omega_j)} \Big/ \frac{I(\omega_k)}{f(\omega_k)} \tag{12}$$

of normalized periodogram ordinates in order to test hypotheses about the fractional differencing parameter d. Noticing that for large n the normalized periodogram ordinates $J(\omega_j)$ are, under suitable assumptions [19], approximately independent and standard exponentially distributed, which implies that the ratios $R_{j,k}$ approximately have an F distribution with 2 numerator degrees of freedom and 2 denominator degrees, they proposed to truncate the ratios to the interval [0,1] in order to remedy the problem of non-existing moments. The mean and the variance of the truncated $F(2,2)$-distribution are given by

$$\mu_{tF} = 2\log(2) - 1 \sim 0.3862944 \tag{13}$$

and

$$\sigma_{tF}^2 = 2 - 4\log^2(2) \sim 0.07818794, \tag{14}$$

respectively [34] [11]. If the spectral density $f(\omega)$ can in the neighborhood of frequency zero be approximated by

$$f(\omega) \sim c(4\sin^2(\omega/2))^{-d}, \tag{15}$$

the expected value of the sample mean of those normalized ratios

$$R_{j,k}(d) = \frac{I(\omega_j)}{I(\omega_k)}\left(\frac{\sin^2(\omega_j/2)}{\sin^2(\omega_k/2)}\right)^d \tag{16}$$

that fall into the interval $[0,1]$ will approximately be equal to μ_{tF}. Noticing that the median of an $F(2,2)$-distribution is 1, we propose to estimate the unknown parameter d by minimizing

$$\left(\sum_{\substack{j<k \\ R_{j,k}<1}} R_{j,k}(d) - \frac{(K-2)(K-1)}{4}\,\mu_{tF}\right)^2. \tag{17}$$

The minimization is performed over non-positive values of d if the number of ratios

$$\frac{I(\omega_j)}{I(\omega_k)} \in [0,1],\ j<k, \tag{18}$$

is greater than $(K-2)(K-1)/4$ and over non-negative values of d otherwise. The resulting estimator is denoted by $\hat{d}_{trF}$.

When we consider the median of the normalized ratios $R_{j,k}(d)$ instead of their mean, there is no need for truncation. We have

$$median\left(\frac{I(\omega_j)}{I(\omega_k)}\left(\frac{\sin^2(\omega_j/2)}{\sin^2(\omega_k/2)}\right)^d\right) \approx 1, \tag{19}$$

and

$$median\left(\frac{I(\omega_j)}{I(\omega_k)}\right) \approx \left(\frac{\sin^2(\omega_j/2)}{\sin^2(\omega_k/2)}\right)^{-d}. \tag{20}$$

Taking logs of both sides gives

$$median\left(-\frac{\log(I(\omega_j))-\log(I(\omega_k))}{\log(\sin^2(\omega_j/2))-\log(\sin^2(\omega_k/2))}\right) \approx d, \tag{21}$$

which can be used for the robust estimation of d. Noting that the resulting estimator is just the median of the slopes of the lines crossing all $n(n-1)/2$ pairs of points

$$\left(\log(\sin^2(\omega_j/2)), \log(I(\omega_j))\right), \tag{22}$$

we see that it is identical to the Theil-Sen estimator $\hat{d}_{TS}$ [35] [36] applied for the estimation of the slope in Geweke and Porter-Hudak's [16] log periodogram regression.

3. Simulations

In this section, we compare the performance of the different robust and non-robust estimators discussed in Section 2 by means of simulations. Of particular interest are the new estimators $\hat{d}_{trF}$, which is based on the truncated $F(2,2)$ distribution, and $\hat{d}_{TS}$, which is an estimator of the Theil-Sen type. The other competing estimators are the robust estimators $\hat{d}_{rAC}$ (robust autocorrelations with $m = [n^{\beta}]$, where $\beta = 0.7$) and $\hat{d}_H$ (robust periodogram obtained with the Huber function) and the non-robust estimators $\hat{d}_{GPH}$ (ordinary log periodogram regression), $\hat{d}_{GPH}^{tr}$

(trimming), $\hat{d}_W$ (narrow-band Whittle likelihood), $\hat{d}_{KS}$ (Kolmogorov-Smirnov goodness-of-fit test), $\hat{d}_{smP}^{0.9}$ (smoothing with Parzen window and truncation point $m = [n^\beta]$, where $\beta = 0.9$), $\hat{d}_{smP}^{0.5}$ (smoothing with Parzen window and $\beta = 0.5$). Three sample sizes, namely $n = 100, 300, 1000$, and three types of data generating models are used. The first model is an ARFIMA(1,d,0) model with standard normal innovations. Varying the AR parameter ϕ_1 in this model allows to examine the robustness of the different estimators against short-range dependence. The second model is obtained from the first by contaminating it with additive outliers w_t of size 3 occurring with probability 0.1, i.e., $P(w_t = 0) = 0.9$ and $P(w_t = -3) = P(w_t = 3) = 0.05$. The third model is the ARFIMA(1,d,0)-GARCH(1,1) model

$$(1-L)^d(1-\phi_1 L)y_t = (1-\theta_1 L)u_t, \tag{23}$$

where

$$u_t = \sigma_t z_t,$$
$$z_t\sqrt{\nu/(\nu-2)} \text{ i.i.d. } t(\nu),$$
$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta_1 \sigma_{t-1}^2.$$

In this model, the frequency of outliers increases as the degrees of freedom of the t-distribution decreases and volatility fluctuations become more prominent as the sum $\alpha_1 + \beta_1$ moves closer to 1. By choosing $\nu = 5$ and $\alpha_0 = 1$, $\alpha_1 = 0.1$, $\beta_1 = 0.8999$, we ensure that both the non-normality and the conditional heteroscedasticity are highly visible.

For each model and each combination of $d = -0.4, -0.2, 0, 0.2, 0.4$ and $\phi_1 = -0.75, -0.5, -0.25, 0, 0.25, 0.5, 0.75$, 10,000 realizations of length $n = 100, 300, 1000$ are generated with the help of the R-package ‘fracdiff’ and estimates of d are obtained from each realization. The highest frequency used for the estimation is given by $K = [n^\alpha]$ with $\alpha = 0.7$. For the computation of $\hat{d}_{GPH}^{tr}$, only the first Fourier frequency is omitted. For the computation of $\hat{d}_W$ and $\hat{d}_{KS}$, the set $D = \{-1 + j/100 | j = 0, 1, \dots, 200\}$ is used. For the computation of $\hat{d}_{rAC}$ and $\hat{d}_H$, the R package ’tsqn’ is used. All computations are carried out with the free statistical software R [37]. The performance of the competing estimators is evaluated in terms of bias and root-mean-square error (RMSE). The results for the first, second, and third model are given in Tables 1-6, 7-12, and 13-18, respectively. In each case, the first two tables show the bias and the RMSE for $n = 100$, the third and fourth tables show the bias and the RMSE for $n = 300$, and the fifth and sixth tables show the bias and the RMSE for $n = 1000$. The information contained in these tables is made available in a clearer and more comprehensible way by plotting boxplots of the bias and the RMSE of the estimates obtained with the competing forecasts (see Figures 1-3). This is done separately for each sample size and each model type. For the construction of each boxplot, 35 different values are used which correspond to the 35 different combinations of d and ϕ_1. In general, the widely used estimators $\hat{d}_{smP}^{\beta}$, which are based on the smoothed periodogram, perform quite well in terms of RMSE when the degree of smoothing is reduced (by increasing β) as the sample size n increases. Only in the case of ARFIMA-GARCH models with t-distributed innovations and large n, they are outperformed by the robust estimators $\hat{d}_{rAC}$ and $\hat{d}_H$ (see Figures 2.f and 3.f). However, this out-performance comes with a cost, namely a huge bias (see Figures 2.c and 3.c). Although the squared bias is usually small relative to the variance, there are applications, such as the empirical study in the next section, where only the bias matters. The new robust estimators $\hat{d}_{trF}$ and $\hat{d}_{TS}$ have a small bias in some cases (see Figure 2.b and Figures 2.c, 3.c, respectively) but a large one in other cases. It is therefore important to have a rough idea which models are most plausible in a concrete application.

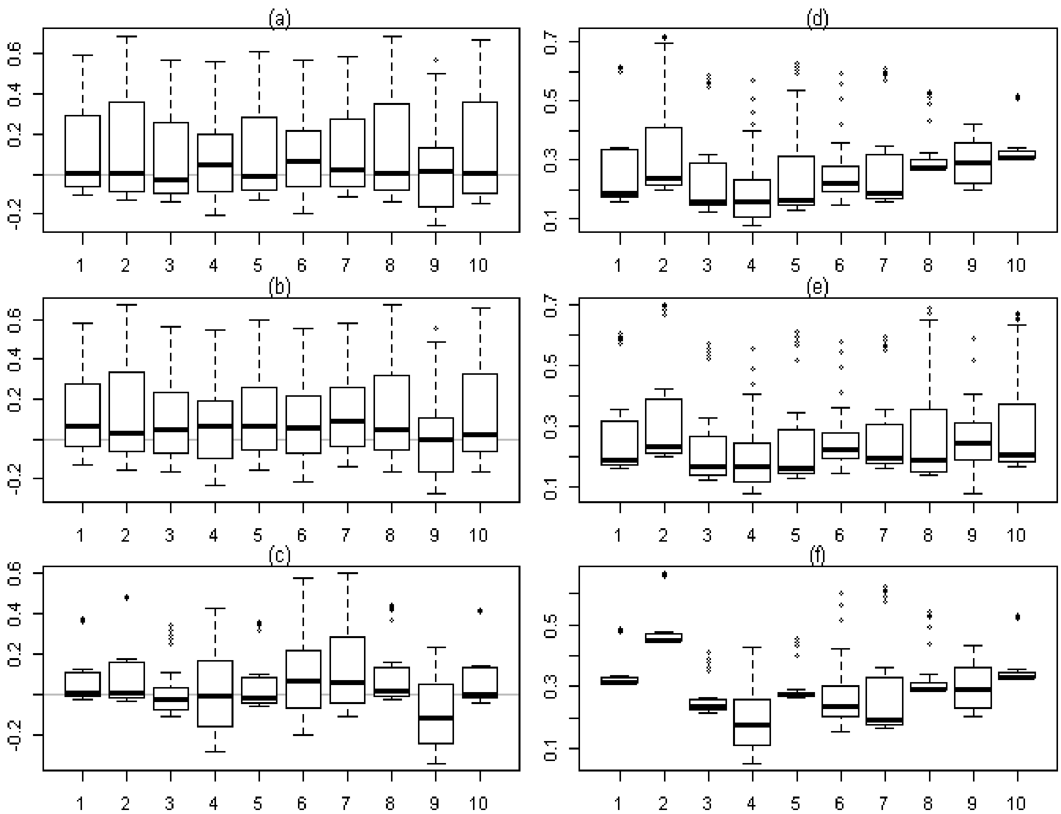

Figure 1: Boxplots of bias (a-c) and RMSE (d-f) of estimates obtained with (1) $\hat{d}_{GPH}$ (log periodogram regression), (2) $\hat{d}_{GPH}^{tr}$ (trimming), (3) $\hat{d}_{smP}^{0.9}$ (smoothing with Parzen window and $\beta = 0.9$), (4) $\hat{d}_{smP}^{0.5}$ (smoothing with Parzen window and $\beta = 0.5$), (5) $\hat{d}_W$ (narrow-band Whittle likelihood), (6) $\hat{d}_{rAC}$ (robust autocorrelations), (7) $\hat{d}_H$ (robust periodogram obtained with the Huber function), (8) $\hat{d}_{KS}$ (goodness-of-fit testing), (9) $\hat{d}_{trF}$ (trimmed F-distribution), (10) $\hat{d}_{TS}$

(Theil-Sen) for $n = 100$ and 35 different combinations of d and ϕ_1 ($d = -0.4, -0.2, 0, 0.2, 0.4$, $\phi_1 = -0.75, -0.5, -0.25, 0, 0.25, 0.5, 0.75$) in 3 different scenarios (a, d: ARFIMA, b, e: additive outliers, c,f: ARFIMA-GARCH).

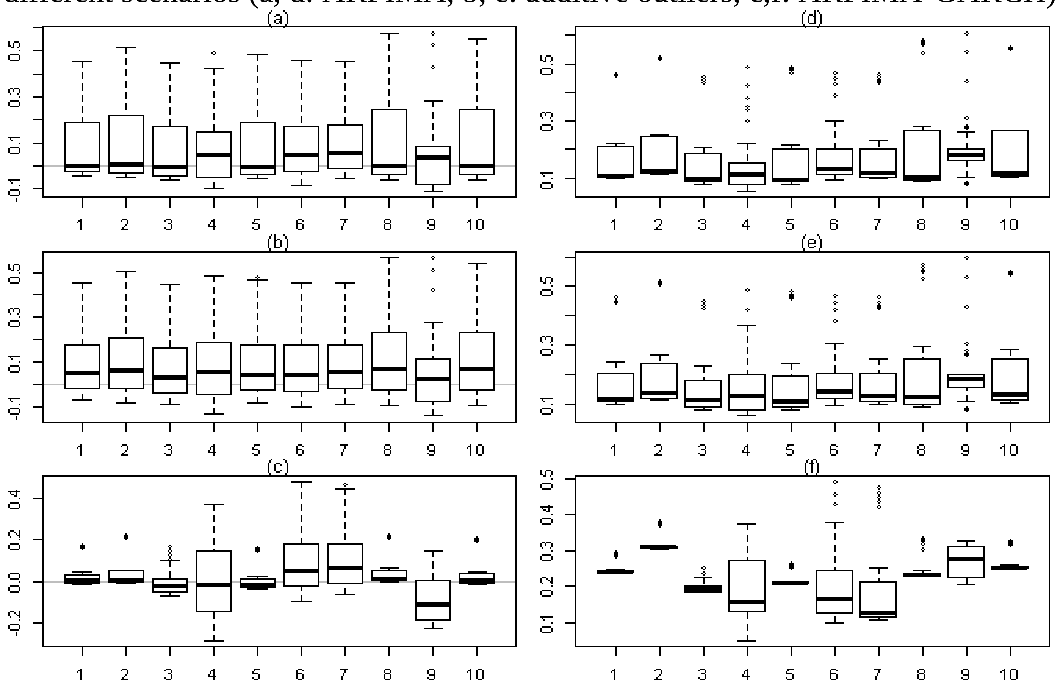

Figure 2: Boxplots of bias (a-c) and RMSE (d-f) of estimates obtained with (1) $\hat{d}_{GPH}$ (log periodogram regression), (2) $\hat{d}_{GPH}^{tr}$ (trimming), (3) $\hat{d}_{smP}^{0.9}$ (smoothing with Parzen window and $\beta = 0.9$), (4) $\hat{d}_{smP}^{0.5}$ (smoothing with Parzen window and $\beta = 0.5$), (5) $\hat{d}_W$ (narrow-band Whittle likelihood), (6) $\hat{d}_{rAC}$ (robust autocorrelations), (7) $\hat{d}_H$ (robust periodogram obtained with the Huber function), (8) $\hat{d}_{KS}$ (goodness-of-fit testing), (9) $\hat{d}_{trF}$ (trimmed F-distribution), (10) $\hat{d}_{TS}$ (Theil-Sen) for $n = 300$ and 35 different combinations of d and ϕ_1 ($d = -0.4, -0.2, 0, 0.2, 0.4$, $\phi_1 = -0.75, -0.5, -0.25, 0, 0.25, 0.5, 0.75$) in 3 different scenarios (a, d: ARFIMA, b, e: additive outliers, c,f: ARFIMA-GARCH).

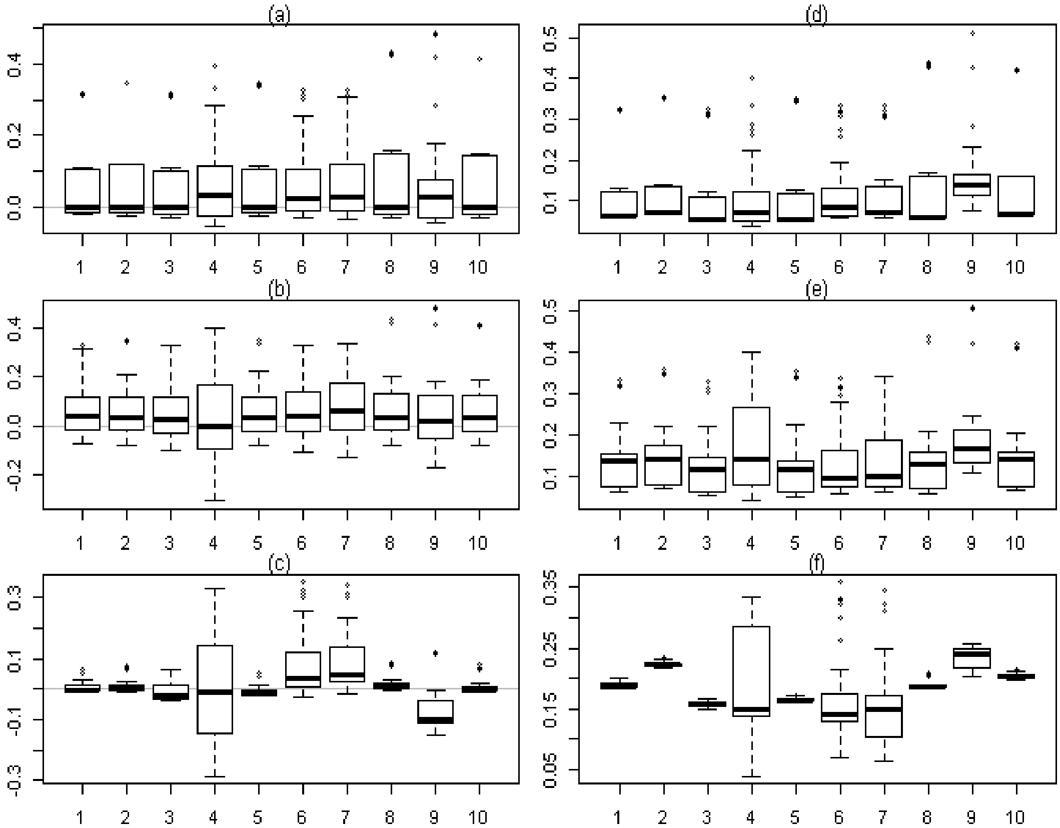

Figure 3: Boxplots of bias (a-c) and RMSE (d-f) of estimates obtained with (1) $\hat{d}_{GPH}$ (log periodogram regression), (2) $\hat{d}_{GPH}^{tr}$ (trimming), (3) $\hat{d}_{smP}^{0.9}$ (smoothing with Parzen window and $\beta = 0.9$), (4) $\hat{d}_{smP}^{0.5}$ (smoothing with Parzen window and $\beta = 0.5$), (5) $\hat{d}_{W}$ (narrow-band Whittle likelihood), (6) $\hat{d}_{rAC}$ (robust autocorrelations), (7) $\hat{d}_{H}$ (robust periodogram obtained with the Huber function), (8) $\hat{d}_{KS}$ (goodness-of-fit testing), (9) $\hat{d}_{trF}$ (trimmed F-distribution), (10) $\hat{d}_{TS}$ (Theil-Sen) for $n = 1000$ and 35 different combinations of d and ϕ_1 ($d = -0.4, -0.2, 0, 0.2, 0.4$, $\phi_1 = -0.75, -0.5, -0.25, 0, 0.25, 0.5, 0.75$) in 3 different scenarios (a, d: ARFIMA, b, e: additive outliers, c,f: ARFIMA-GARCH).

4. Empirical Results

Investigating daily index returns of both developed and developing stock markets, Reschenhofer et al. [26] found indications of long-range dependence only in absolute returns but not in signed returns. In the following we reanalyze their data using additionally also the robust estimators $\hat{d}_{rAC}$ (robust autocorrelations), $\hat{d}_{H}$ (robust periodogram), $\hat{d}_{trF}$ (trimmed F-distribution), and $\hat{d}_{TS}$ (Theil-Sen), which should be particularly appropriate for this task because of their robustness to deviations from Gaussianity and (conditional) homoscedasticity. Six major world indices were downloaded from Yahoo Finance, namely S&P 500 (30.12.1927-

31.01.2020), CCA40 (01.03.1990-31.01.2020), Nikkei 225 (05.01.1965-31.01.2020), Bovespa Index (27.04.1993-31.01.2020), BIST 100 (14.12.1992-09.02.2018), and Hang Seng Index (31.12.1986-31.01.2020). Figures 4 and 5 show cumulative plots of the estimates obtained with a rolling window of $n = 300$ days and $K = 54$. For ease of interpretation of the last values in these plots, all estimates are divided by the total number of subsamples. It appears from a first look at Figures 4 and 5 that the robust estimators $\hat{d}_{rAC}$, $\hat{d}_H$, and $\hat{d}_{trF}$ suggest a more extreme long-term behavior than the other estimators. In the case of signed returns, there are indications of a non-zero value of d ($\hat{d}_{rAC}$, $\hat{d}_{trF}$: negative, $\hat{d}_H$: positive). However, the size of the non-zero estimates is not large enough to be of any economic significance. In the case of absolute returns, the estimates obtained with $\hat{d}_{rAC}$ and $\hat{d}_H$ are much larger than those obtained with the other estimators and the estimates obtained with $\hat{d}_{trF}$ are smaller. However, in case of extreme deviations from the standard assumptions, the estimates $\hat{d}_{rAC}$ and $\hat{d}_{trF}$ are only superior with respect to RMSE (see Figure 2.f) but not with respect to bias (see Figure 2.c). While the variance is usually much greater than the squared bias and is therefore the decisive factor in the RMSE, it is the other way round in our rolling analysis. When we are dealing with averages of estimates obtained from different subsamples, the variance decreases as the sample size increases but the bias remains fixed. According to Figure 2.c and Table 15, the bias of $\hat{d}_{rAC}$ and $\hat{d}_{trF}$ is often much greater than that of the other estimators, which explains the striking discrepancies in Figure 5.

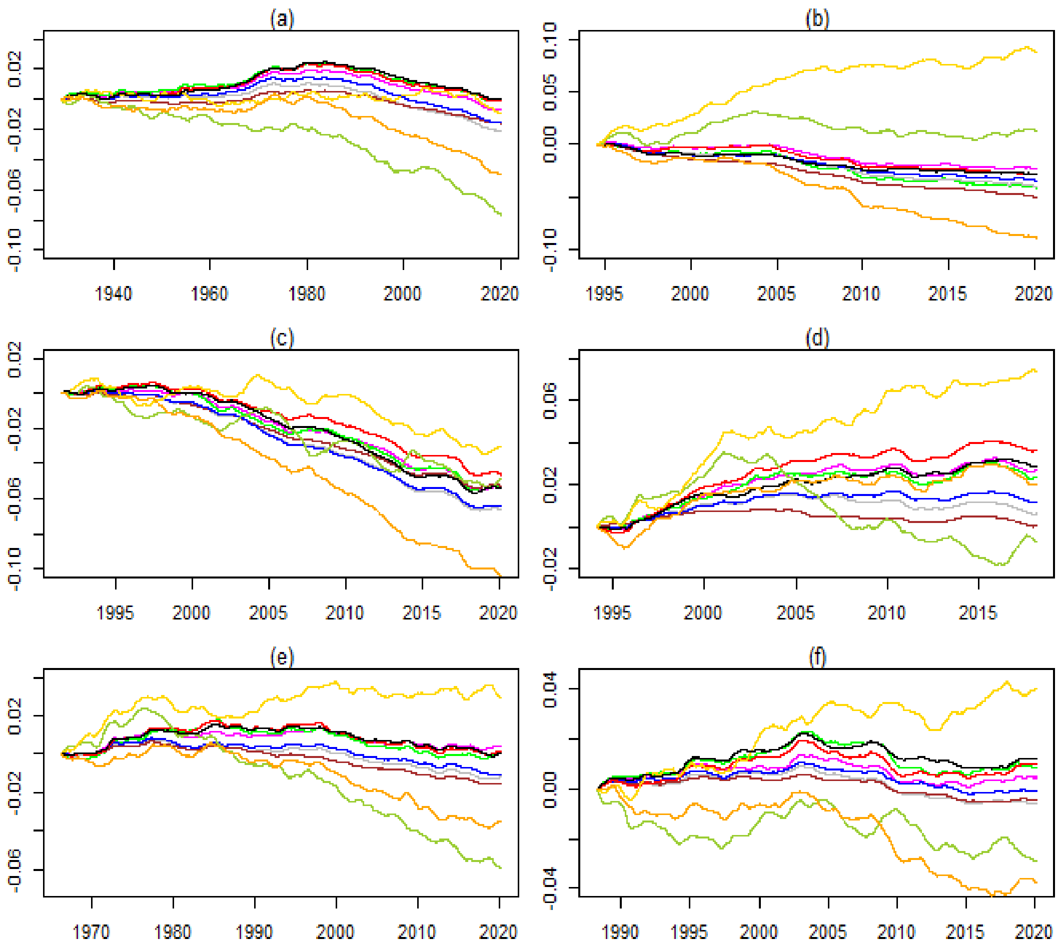

Figure 4: Cumulative plots of the estimates obtained by applying $\hat{d}_{GPH}$ (magenta), $\hat{d}_{GPH}^{tr}$ (green), $\hat{d}_{smP}^{0.9}$ (gray), $\hat{d}_{smP}^{0.5}$ (brown), $\hat{d}_{W}$ (blue), $\hat{d}_{rAC}$ (yellowgreen), $\hat{d}_{H}$ (gold), $\hat{d}_{KS}$ (black), $\hat{d}_{trF}$ (orange), $\hat{d}_{TS}$ (red) to the daily log returns of (a) S&P 500, (b) Ibovespa, (c) CCA 40 (d) BIST 100, (e) Nikkei 225, (f) Hang Seng Index with a rolling window of $n = 300$ days and $K = 54$.

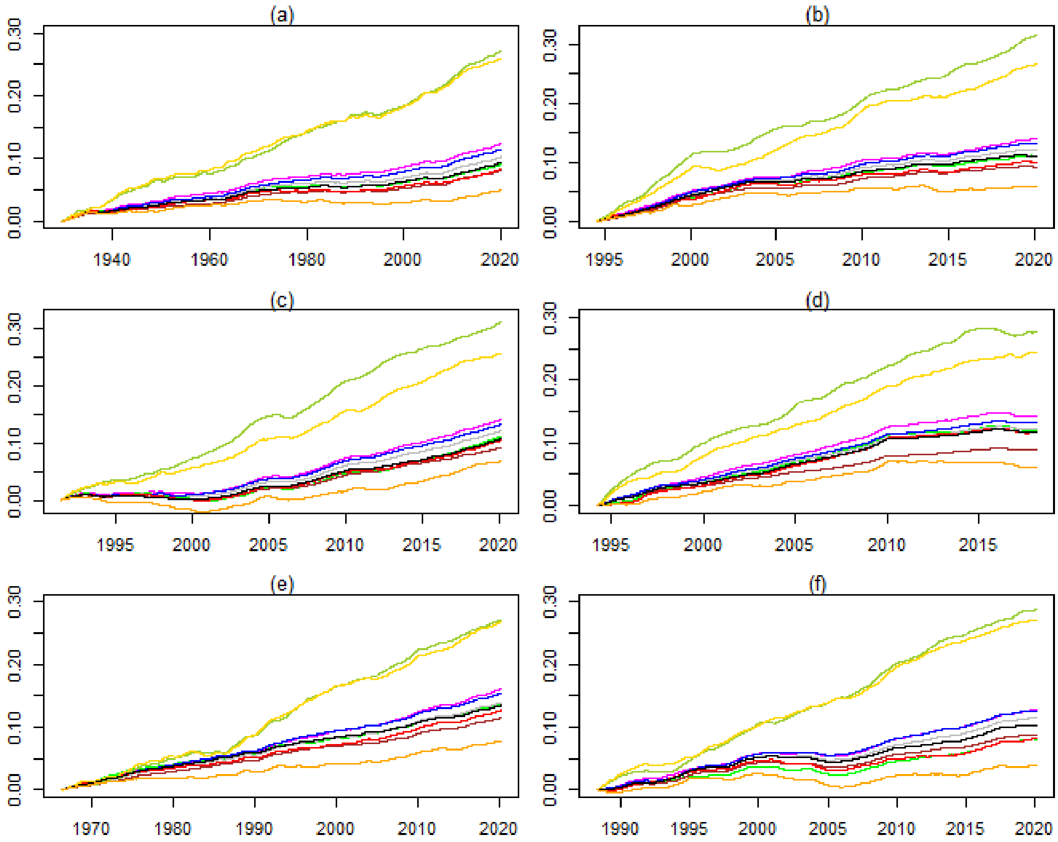

Figure 5: Cumulative plots of the estimates obtained by applying $\hat{d}_{GPH}$ (magenta), $\hat{d}^{tr}_{GPH}$ (green), $\hat{d}^{0.9}_{smP}$ (gray), $\hat{d}^{0.5}_{smP}$ (brown), $\hat{d}_{W}$ (blue), $\hat{d}_{rAC}$ (yellowgreen), $\hat{d}_{H}$ (gold), $\hat{d}_{KS}$ (black), $\hat{d}_{trF}$ (orange), $\hat{d}_{TS}$ (red) to the log absolute daily returns of (a) S&P 500, (b) Ibovespa, (c) CCA 40 (d) BIST 100, (e) Nikkei 225, (f) Hang Seng Index with a rolling window of $n = 300$ days and $K = 54$.

5. Discussion

In this paper, we have proposed new robust estimators for the memory parameter and compared them with existing robust and non-robust estimators. For this comparison, we have carried out an extensive simulation study with different data generating models and different sample sizes. The results of this simulation study are mixed. In the case of ARFIMA-GARCH models with t-distributed innovations, which are probably the most suitable models for financial applications, the new robust estimators usually have a smaller bias and a larger RMSE than the existing robust estimators. In this case, the latter estimators outperform also all non-robust estimators in terms of RMSE, provided that the sample size is not too small.

In the simulation study, we have set the bandwidth parameter α to 0.7. However, this value (which has also been used in [12]) is quite large, particularly in case of small and medium sample sizes. Indeed, when we used the same setting in a rolling analysis of international index return series with window size $n = 300$, we obtained estimates of the memory parameter that are considerably smaller than those obtained in a previous study with $\alpha = 0.5$ [26]. This may be an indication that the frequency range used in the log periodogram regression is too large to ensure the approximate linearity of the log periodogram. The tables containing the detailed results of the simulation study are therefore only of limited use for the interpretation of the empirical findings. At least, there is a large agreement between the different estimators. Only the estimators based on the robust periodogram and robust autocorrelations, respectively, deviate substantially, which can in principle be explained by their large bias, even though the exact size of this bias is not known. Overall, the empirical results clearly confirm that there is long-range dependence only in the absolute index returns but not in the signed index returns.

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Table 1: Bias of the estimators obtained from 10,000 realizations of length $n = 100$ of ARFIMA(1,*d*,0) processes with standard normal innovations and parameters $d = -0.4,\ -0.2,\ 0,\ 0.2, 0.4$ and $\phi_1 = -0.75, -0.5,\ -0.25, 0,\ 0.25, 0.5, 0.75$ (estimation is based on the first $\mathrm{K} = 25$ Fourier frequencies).

d	ϕ_1	$\hat{d}_{GPH}$	$\hat{d}_{GPH}^{tr}$	$\hat{d}_{smP}^{0.9}$	$\hat{d}_{smP}^{0.5}$	$\hat{d}_{W}$	$\hat{d}_{rAC}$	$\hat{d}_{H}$	$\hat{d}_{KS}$	$\hat{d}_{trF}$	$\hat{d}_{TS}$
-0.4	-0.75	-0.045	-0.084	-0.048	0.044	-0.053	0.216	0.017	-0.067	0.125	-0.091
	-0.5	-0.060	-0.095	-0.062	0.047	-0.063	0.148	0.019	-0.071	0.131	-0.100
	-0.25	-0.038	-0.061	-0.043	0.078	-0.040	0.098	0.032	-0.040	0.116	-0.065
	0	0.019	0.011	0.013	0.137	0.016	0.096	0.066	0.033	0.100	0.011
	0.25	0.122	0.143	0.115	0.229	0.121	0.161	0.147	0.165	0.116	0.146
	0.5	0.303	0.364	0.293	0.365	0.311	0.310	0.311	0.389	0.267	0.365
	0.75	0.594	0.689	0.573	0.560	0.609	0.568	0.586	0.689	0.567	0.670
-0.2	-0.75	-0.088	-0.119	-0.104	-0.057	-0.097	0.039	-0.059	-0.107	-0.108	-0.129
	-0.5	-0.083	-0.110	-0.100	-0.044	-0.091	0.001	-0.051	-0.097	-0.103	-0.118
	-0.25	-0.055	-0.074	-0.073	-0.011	-0.063	-0.022	-0.028	-0.061	-0.099	-0.080
	0	0.006	0.007	-0.014	0.051	-0.003	0.008	0.022	0.016	-0.062	0.003
	0.25	0.114	0.141	0.094	0.145	0.108	0.100	0.118	0.151	0.038	0.141
	0.5	0.296	0.359	0.274	0.285	0.298	0.269	0.293	0.374	0.267	0.358
	0.75	0.590	0.690	0.559	0.497	0.598	0.536	0.576	0.677	0.504	0.669
0	-0.75	-0.103	-0.132	-0.131	-0.134	-0.117	-0.105	-0.097	-0.128	-0.223	-0.141
	-0.5	-0.090	-0.115	-0.117	-0.116	-0.102	-0.104	-0.082	-0.110	-0.212	-0.123
	-0.25	-0.060	-0.076	-0.089	-0.081	-0.075	-0.088	-0.056	-0.075	-0.180	-0.083
	0	0.001	0.000	-0.029	-0.020	-0.014	-0.032	0.001	0.003	-0.104	-0.002
	0.25	0.110	0.135	0.080	0.075	0.098	0.074	0.106	0.141	0.037	0.138
	0.5	0.291	0.357	0.261	0.220	0.288	0.246	0.282	0.362	0.230	0.355
	0.75	0.587	0.686	0.550	0.448	0.589	0.485	0.566	0.664	0.403	0.665
0.2	-0.75	-0.106	-0.133	-0.140	-0.185	-0.125	-0.181	-0.113	-0.137	-0.247	-0.143
	-0.5	-0.086	-0.112	-0.123	-0.164	-0.108	-0.139	-0.092	-0.118	-0.225	-0.120
	-0.25	-0.058	-0.074	-0.095	-0.130	-0.080	-0.105	-0.065	-0.081	-0.188	-0.082
	0	0.002	0.002	-0.035	-0.071	-0.020	-0.042	-0.007	-0.005	-0.109	-0.002
	0.25	0.113	0.140	0.077	0.027	0.095	0.065	0.103	0.136	0.011	0.140
	0.5	0.294	0.361	0.256	0.172	0.282	0.217	0.277	0.353	0.156	0.357
	0.75	0.584	0.682	0.544	0.410	0.577	0.398	0.560	0.636	0.248	0.662
0.4	-0.75	-0.095	-0.125	-0.135	-0.207	-0.123	-0.196	-0.112	-0.138	-0.255	-0.135
	-0.5	-0.079	-0.103	-0.119	-0.187	-0.108	-0.137	-0.094	-0.119	-0.230	-0.113
	-0.25	-0.053	-0.070	-0.091	-0.155	-0.080	-0.099	-0.066	-0.084	-0.204	-0.078
	0	0.007	0.007	-0.031	-0.097	-0.020	-0.043	-0.005	-0.007	-0.137	0.004
	0.25	0.120	0.145	0.079	-0.003	0.091	0.041	0.103	0.130	-0.046	0.146
	0.5	0.298	0.361	0.258	0.143	0.278	0.154	0.277	0.342	0.033	0.359
	0.75	0.572	0.667	0.532	0.378	0.525	0.269	0.547	0.551	0.052	0.646

Table 2: RMSE of the estimators obtained from 10,000 realizations of length $n = 100$ of ARFIMA(1,d,0) processes with standard normal innovations and parameters $d = -0.4,\ -0.2,\ 0,\ 0.2, 0.4$ and $\phi_1 = -0.75, -0.5,\ -0.25, 0,\ 0.25, 0.5, 0.75$ (estimation is based on the first $\mathrm{K} = 25$ Fourier frequencies).

d	ϕ_1	$\hat{d}_{GPH}$	$\hat{d}_{GPH}^{tr}$	$\hat{d}_{smP}^{0.9}$	$\hat{d}_{smP}^{0.5}$	$\hat{d}_W$	$\hat{d}_{rAC}$	$\hat{d}_H$	$\hat{d}_{KS}$	$\hat{d}_{trF}$	$\hat{d}_{TS}$
-0.4	-0.75	0.178	0.220	0.131	0.085	0.149	0.317	0.169	0.277	0.221	0.316
	-0.5	0.177	0.225	0.136	0.089	0.148	0.271	0.166	0.274	0.223	0.308
	-0.25	0.169	0.210	0.129	0.109	0.138	0.238	0.167	0.272	0.227	0.311
	0	0.163	0.200	0.123	0.158	0.132	0.222	0.177	0.274	0.226	0.311
	0.25	0.203	0.248	0.168	0.242	0.179	0.243	0.219	0.282	0.227	0.310
	0.5	0.344	0.418	0.318	0.374	0.338	0.357	0.350	0.325	0.232	0.339
	0.75	0.616	0.718	0.587	0.567	0.624	0.592	0.608	0.528	0.327	0.519
-0.2	-0.75	0.187	0.236	0.161	0.095	0.164	0.222	0.175	0.275	0.198	0.310
	-0.5	0.184	0.230	0.161	0.091	0.161	0.204	0.171	0.277	0.199	0.312
	-0.25	0.172	0.214	0.143	0.080	0.144	0.196	0.166	0.274	0.198	0.309
	0	0.163	0.201	0.126	0.095	0.130	0.179	0.162	0.274	0.199	0.306
	0.25	0.198	0.244	0.156	0.166	0.168	0.199	0.200	0.280	0.202	0.310
	0.5	0.338	0.412	0.301	0.297	0.326	0.316	0.334	0.321	0.218	0.335
	0.75	0.612	0.719	0.574	0.505	0.614	0.557	0.598	0.523	0.302	0.514
0	-0.75	0.191	0.240	0.180	0.157	0.173	0.221	0.189	0.272	0.297	0.306
	-0.5	0.184	0.230	0.171	0.142	0.164	0.209	0.181	0.275	0.296	0.307
	-0.25	0.175	0.214	0.155	0.117	0.150	0.197	0.170	0.275	0.296	0.306
	0	0.163	0.202	0.131	0.087	0.132	0.167	0.162	0.275	0.293	0.306
	0.25	0.196	0.242	0.150	0.114	0.162	0.175	0.193	0.279	0.287	0.306
	0.5	0.333	0.410	0.290	0.237	0.317	0.292	0.325	0.315	0.273	0.331
	0.75	0.610	0.715	0.566	0.459	0.605	0.506	0.590	0.515	0.268	0.516
0.2	-0.75	0.194	0.241	0.189	0.206	0.180	0.253	0.198	0.273	0.379	0.311
	-0.5	0.184	0.231	0.178	0.188	0.169	0.219	0.188	0.271	0.372	0.306
	-0.25	0.172	0.213	0.160	0.160	0.153	0.194	0.174	0.271	0.375	0.306
	0	0.160	0.198	0.134	0.119	0.132	0.163	0.162	0.271	0.366	0.305
	0.25	0.198	0.244	0.151	0.099	0.161	0.166	0.193	0.275	0.347	0.307
	0.5	0.335	0.412	0.287	0.199	0.313	0.264	0.321	0.308	0.309	0.331
	0.75	0.607	0.712	0.559	0.424	0.591	0.421	0.584	0.487	0.223	0.510
0.4	-0.75	0.189	0.237	0.189	0.233	0.180	0.260	0.199	0.268	0.419	0.309
	-0.5	0.180	0.225	0.177	0.216	0.169	0.210	0.187	0.270	0.416	0.310
	-0.25	0.173	0.214	0.162	0.190	0.155	0.184	0.175	0.268	0.414	0.309
	0	0.162	0.198	0.135	0.146	0.131	0.155	0.162	0.268	0.407	0.308
	0.25	0.203	0.247	0.155	0.112	0.160	0.149	0.192	0.269	0.383	0.307
	0.5	0.339	0.413	0.289	0.182	0.308	0.204	0.320	0.299	0.326	0.334
	0.75	0.596	0.697	0.548	0.397	0.533	0.302	0.572	0.434	0.217	0.509

Table 3: Bias of the estimators obtained from 10,000 realizations of length $n = 300$ of ARFIMA(1,d,0) processes with standard normal innovations and parameters $d = -0.4,\ -0.2,\ 0,\ 0.2, 0.4$ and $\phi_1 = -0.75, -0.5,\ -0.25, 0,\ 0.25, 0.5, 0.75$ (estimation is based on the first $\mathrm{K} = 54$ Fourier frequencies).

d	ϕ_1	$\hat{d}_{GPH}$	$\hat{d}_{GPH}^{tr}$	$\hat{d}_{smP}^{0.9}$	$\hat{d}_{smP}^{0.5}$	$\hat{d}_W$	$\hat{d}_{rAC}$	$\hat{d}_H$	$\hat{d}_{KS}$	$\hat{d}_{trF}$	$\hat{d}_{TS}$
-0.4	-0.75	-0.008	-0.025	-0.008	0.098	-0.012	0.264	0.086	-0.022	0.082	-0.034
	-0.5	-0.018	-0.030	-0.019	0.096	-0.021	0.173	0.091	-0.027	0.084	-0.040
	-0.25	-0.011	-0.022	-0.013	0.105	-0.013	0.107	0.082	-0.016	0.078	-0.028
	0	0.012	0.007	0.009	0.132	0.010	0.075	0.079	0.018	0.072	0.007
	0.25	0.071	0.076	0.069	0.188	0.071	0.103	0.110	0.098	0.089	0.087
	0.5	0.196	0.223	0.192	0.294	0.201	0.213	0.211	0.263	0.211	0.248
	0.75	0.453	0.511	0.447	0.488	0.483	0.458	0.451	0.575	0.577	0.550
-0.2	-0.75	-0.037	-0.046	-0.045	0.010	-0.042	0.075	-0.002	-0.048	-0.083	-0.056
	-0.5	-0.036	-0.043	-0.045	0.012	-0.041	0.017	0.003	-0.045	-0.083	-0.052
	-0.25	-0.024	-0.029	-0.033	0.026	-0.028	-0.005	0.006	-0.027	-0.070	-0.036
	0	0.002	0.001	-0.007	0.052	-0.002	0.001	0.019	0.008	-0.040	0.001
	0.25	0.063	0.071	0.053	0.108	0.059	0.055	0.072	0.088	0.037	0.081
	0.5	0.189	0.218	0.179	0.218	0.193	0.182	0.190	0.257	0.236	0.244
	0.75	0.448	0.508	0.436	0.423	0.478	0.439	0.440	0.571	0.524	0.548
0	-0.75	-0.043	-0.050	-0.058	-0.054	-0.050	-0.050	-0.042	-0.056	-0.107	-0.061
	-0.5	-0.040	-0.046	-0.056	-0.051	-0.048	-0.056	-0.038	-0.052	-0.100	-0.056
	-0.25	-0.029	-0.033	-0.044	-0.038	-0.035	-0.044	-0.026	-0.036	-0.085	-0.041
	0	-0.001	-0.001	-0.016	-0.010	-0.008	-0.019	-0.002	0.001	-0.040	-0.002
	0.25	0.060	0.070	0.045	0.047	0.055	0.044	0.059	0.084	0.060	0.080
	0.5	0.188	0.218	0.172	0.158	0.189	0.172	0.182	0.252	0.225	0.243
	0.75	0.446	0.507	0.429	0.373	0.473	0.420	0.432	0.563	0.431	0.547
0.2	-0.75	-0.044	-0.052	-0.063	-0.095	-0.055	-0.086	-0.057	-0.062	-0.091	-0.064
	-0.5	-0.039	-0.046	-0.057	-0.088	-0.049	-0.060	-0.049	-0.054	-0.082	-0.055
	-0.25	-0.027	-0.032	-0.045	-0.075	-0.036	-0.045	-0.033	-0.038	-0.067	-0.038
	0	0.002	0.003	-0.016	-0.047	-0.008	-0.014	-0.004	0.000	-0.033	0.000
	0.25	0.063	0.073	0.043	0.009	0.053	0.046	0.056	0.080	0.038	0.082
	0.5	0.190	0.219	0.170	0.120	0.186	0.167	0.178	0.247	0.170	0.245
	0.75	0.448	0.508	0.428	0.345	0.471	0.378	0.428	0.559	0.279	0.547
0.4	-0.75	-0.037	-0.046	-0.056	-0.100	-0.050	-0.074	-0.056	-0.060	-0.111	-0.057
	-0.5	-0.033	-0.041	-0.052	-0.096	-0.046	-0.052	-0.048	-0.053	-0.103	-0.050
	-0.25	-0.019	-0.024	-0.039	-0.083	-0.033	-0.036	-0.031	-0.037	-0.090	-0.032
	0	0.008	0.007	-0.012	-0.056	-0.006	-0.011	-0.003	-0.001	-0.060	0.005
	0.25	0.066	0.075	0.048	-0.002	0.054	0.042	0.055	0.079	-0.007	0.084
	0.5	0.194	0.221	0.174	0.109	0.187	0.139	0.176	0.246	0.065	0.247
	0.75	0.447	0.507	0.427	0.332	0.464	0.278	0.421	0.530	0.085	0.544

Table 4: RMSE of the estimators obtained from 10,000 realizations of length $n = 300$ of ARFIMA(1,d,0) processes with standard normal innovations and parameters $d = -0.4,\ -0.2,\ 0,\ 0.2, 0.4$ and $\phi_1 = -0.75, -0.5,\ -0.25, 0,\ 0.25, 0.5, 0.75$ (estimation is based on the first $\mathrm{K} = 54$ Fourier frequencies).

d	ϕ_1	$\hat{d}_{GPH}$	$\hat{d}_{GPH}^{tr}$	$\hat{d}_{smP}^{0.9}$	$\hat{d}_{smP}^{0.5}$	$\hat{d}_W$	$\hat{d}_{rAC}$	$\hat{d}_H$	$\hat{d}_{KS}$	$\hat{d}_{trF}$	$\hat{d}_{TS}$
-0.4	-0.75	0.110	0.123	0.080	0.111	0.088	0.300	0.135	0.095	0.199	0.116
	-0.5	0.106	0.119	0.081	0.109	0.087	0.227	0.137	0.094	0.203	0.113
	-0.25	0.103	0.117	0.079	0.118	0.083	0.180	0.130	0.090	0.196	0.108
	0	0.102	0.114	0.079	0.142	0.082	0.154	0.129	0.092	0.184	0.103
	0.25	0.123	0.136	0.105	0.195	0.107	0.158	0.149	0.134	0.178	0.133
	0.5	0.221	0.251	0.208	0.299	0.216	0.240	0.234	0.280	0.261	0.268
	0.75	0.464	0.523	0.454	0.491	0.491	0.471	0.462	0.584	0.609	0.560
-0.2	-0.75	0.106	0.122	0.090	0.054	0.090	0.152	0.100	0.100	0.169	0.118
	-0.5	0.107	0.122	0.091	0.055	0.090	0.131	0.099	0.099	0.168	0.116
	-0.25	0.103	0.118	0.086	0.060	0.085	0.123	0.101	0.093	0.161	0.110
	0	0.100	0.113	0.078	0.075	0.078	0.112	0.101	0.089	0.151	0.102
	0.25	0.119	0.134	0.096	0.122	0.100	0.119	0.123	0.128	0.161	0.130
	0.5	0.214	0.245	0.195	0.224	0.210	0.208	0.215	0.275	0.309	0.264
	0.75	0.459	0.521	0.443	0.427	0.486	0.450	0.452	0.580	0.542	0.558
0	-0.75	0.108	0.123	0.098	0.078	0.094	0.125	0.108	0.104	0.194	0.120
	-0.5	0.108	0.123	0.098	0.076	0.094	0.124	0.108	0.103	0.190	0.118
	-0.25	0.103	0.117	0.091	0.068	0.087	0.116	0.103	0.095	0.186	0.110
	0	0.099	0.113	0.081	0.058	0.079	0.101	0.100	0.089	0.177	0.102
	0.25	0.117	0.133	0.092	0.074	0.097	0.107	0.116	0.125	0.190	0.129
	0.5	0.213	0.246	0.189	0.168	0.206	0.197	0.208	0.270	0.276	0.264
	0.75	0.457	0.520	0.436	0.379	0.482	0.430	0.444	0.573	0.438	0.558
0.2	-0.75	0.109	0.125	0.103	0.114	0.097	0.137	0.115	0.107	0.187	0.122
	-0.5	0.108	0.123	0.099	0.109	0.094	0.118	0.112	0.103	0.183	0.118
	-0.25	0.103	0.117	0.092	0.098	0.087	0.106	0.105	0.095	0.174	0.109
	0	0.101	0.114	0.084	0.080	0.081	0.096	0.101	0.090	0.160	0.103
	0.25	0.119	0.135	0.092	0.065	0.096	0.104	0.115	0.122	0.152	0.130
	0.5	0.214	0.246	0.188	0.137	0.203	0.191	0.204	0.266	0.203	0.264
	0.75	0.459	0.521	0.435	0.352	0.479	0.388	0.440	0.569	0.280	0.557
0.4	-0.75	0.109	0.125	0.102	0.128	0.096	0.127	0.117	0.107	0.177	0.121
	-0.5	0.106	0.121	0.099	0.125	0.094	0.111	0.112	0.103	0.170	0.116
	-0.25	0.101	0.116	0.092	0.114	0.087	0.101	0.105	0.095	0.159	0.107
	0	0.101	0.114	0.083	0.096	0.080	0.092	0.101	0.089	0.136	0.102
	0.25	0.121	0.136	0.096	0.080	0.097	0.099	0.115	0.122	0.101	0.132
	0.5	0.218	0.248	0.192	0.135	0.204	0.164	0.203	0.264	0.080	0.267
	0.75	0.458	0.519	0.435	0.342	0.471	0.293	0.434	0.536	0.086	0.555

Table 5: Bias of the estimators obtained from 10,000 realizations of length $n = 1{,}000$ of ARFIMA(1,d,0) processes with standard normal innovations and parameters $d = -0.4,\ -0.2,\ 0,\ 0.2, 0.4$ and $\phi_1 = -0.75, -0.5,\ -0.25, 0, 0.25, 0.5, 0.75$ (estimation is based on the first $\mathrm{K} = 125$ Fourier frequencies).

d	ϕ_1	$\hat{d}_{GPH}$	$\hat{d}_{GPH}^{tr}$	$\hat{d}_{smP}^{0.9}$	$\hat{d}_{smP}^{0.5}$	$\hat{d}_W$	$\hat{d}_{rAC}$	$\hat{d}_H$	$\hat{d}_{KS}$	$\hat{d}_{trF}$	$\hat{d}_{TS}$
-0.4	-0.75	0.001	-0.005	0.002	0.108	0.000	0.257	0.136	-0.006	0.040	-0.013
	-0.5	-0.004	-0.009	-0.004	0.106	-0.004	0.165	0.139	-0.009	0.041	-0.015
	-0.25	-0.002	-0.006	-0.002	0.110	-0.003	0.107	0.125	-0.006	0.039	-0.011
	0	0.009	0.007	0.009	0.122	0.008	0.068	0.104	0.010	0.043	0.006
	0.25	0.038	0.038	0.037	0.150	0.037	0.060	0.097	0.051	0.064	0.047
	0.5	0.113	0.121	0.112	0.220	0.115	0.122	0.141	0.157	0.158	0.149
	0.75	0.320	0.348	0.319	0.397	0.346	0.326	0.326	0.432	0.480	0.415
-0.2	-0.75	-0.015	-0.018	-0.020	0.032	-0.017	0.067	0.029	-0.021	-0.033	-0.025
	-0.5	-0.014	-0.017	-0.019	0.032	-0.017	0.016	0.028	-0.019	-0.033	-0.023
	-0.25	-0.012	-0.014	-0.016	0.036	-0.013	-0.004	0.024	-0.014	-0.028	-0.018
	0	0.001	0.000	-0.004	0.049	-0.001	-0.005	0.023	0.003	-0.014	-0.001
	0.25	0.032	0.035	0.027	0.079	0.031	0.023	0.044	0.046	0.030	0.043
	0.5	0.109	0.119	0.104	0.150	0.110	0.101	0.114	0.153	0.146	0.147
	0.75	0.316	0.345	0.311	0.331	0.342	0.315	0.313	0.428	0.488	0.413
0	-0.75	-0.020	-0.022	-0.027	-0.024	-0.022	-0.028	-0.019	-0.025	-0.033	-0.028
	-0.5	-0.018	-0.020	-0.026	-0.023	-0.021	-0.029	-0.017	-0.024	-0.029	-0.026
	-0.25	-0.013	-0.015	-0.021	-0.017	-0.016	-0.023	-0.012	-0.017	-0.020	-0.018
	0	0.000	0.000	-0.008	-0.004	-0.003	-0.009	0.000	0.001	-0.001	-0.001
	0.25	0.032	0.035	0.024	0.026	0.029	0.023	0.031	0.044	0.065	0.043
	0.5	0.107	0.117	0.099	0.096	0.107	0.100	0.104	0.150	0.177	0.146
	0.75	0.315	0.345	0.306	0.283	0.340	0.311	0.306	0.426	0.418	0.412
0.2	-0.75	-0.019	-0.021	-0.028	-0.053	-0.024	-0.030	-0.033	-0.028	-0.030	-0.028
	-0.5	-0.017	-0.019	-0.026	-0.050	-0.021	-0.021	-0.027	-0.024	-0.027	-0.025
	-0.25	-0.011	-0.012	-0.020	-0.045	-0.016	-0.016	-0.017	-0.017	-0.019	-0.016
	0	0.001	0.001	-0.008	-0.033	-0.004	-0.005	-0.005	0.000	-0.005	0.000
	0.25	0.032	0.035	0.022	-0.003	0.027	0.026	0.026	0.042	0.036	0.043
	0.5	0.109	0.119	0.099	0.069	0.107	0.104	0.102	0.150	0.139	0.148
	0.75	0.316	0.345	0.306	0.259	0.339	0.304	0.300	0.423	0.283	0.412
0.4	-0.75	-0.014	-0.017	-0.023	-0.044	-0.021	-0.020	-0.035	-0.026	-0.045	-0.025
	-0.5	-0.013	-0.015	-0.022	-0.043	-0.019	-0.016	-0.026	-0.023	-0.044	-0.022
	-0.25	-0.007	-0.009	-0.016	-0.037	-0.013	-0.010	-0.016	-0.015	-0.035	-0.013
	0	0.006	0.005	-0.003	-0.025	0.000	0.002	-0.003	0.001	-0.022	0.004
	0.25	0.036	0.038	0.027	0.004	0.030	0.030	0.025	0.044	0.011	0.047
	0.5	0.112	0.122	0.102	0.075	0.109	0.097	0.099	0.150	0.067	0.150
	0.75	0.318	0.347	0.309	0.266	0.340	0.247	0.298	0.423	0.090	0.414

Table 6: RMSE of the estimators obtained from 10,000 realizations of length $n = 1{,}000$ of ARFIMA(1,d,0) processes with standard normal innovations and parameters $d = -0.4,\ -0.2,\ 0,\ 0.2, 0.4$ and $\phi_1 = -0.75, -0.5,\ -0.25, 0, 0.25, 0.5, 0.75$ (estimation is based on the first $\mathrm{K} = 125$ Fourier frequencies).

d	ϕ_1	$\hat{d}_{GPH}$	$\hat{d}_{GPH}^{tr}$	$\hat{d}_{smP}^{0.9}$	$\hat{d}_{smP}^{0.5}$	$\hat{d}_W$	$\hat{d}_{rAC}$	$\hat{d}_H$	$\hat{d}_{KS}$	$\hat{d}_{trF}$	$\hat{d}_{TS}$
-0.4	-0.75	0.067	0.070	0.052	0.114	0.055	0.275	0.150	0.058	0.155	0.068
	-0.5	0.065	0.070	0.050	0.112	0.052	0.196	0.153	0.057	0.158	0.067
	-0.25	0.063	0.067	0.049	0.115	0.050	0.151	0.140	0.055	0.154	0.065
	0	0.064	0.067	0.050	0.127	0.050	0.118	0.121	0.057	0.149	0.064
	0.25	0.073	0.077	0.062	0.155	0.062	0.104	0.115	0.077	0.149	0.078
	0.5	0.129	0.138	0.123	0.223	0.125	0.141	0.154	0.167	0.202	0.162
	0.75	0.326	0.354	0.323	0.399	0.350	0.333	0.332	0.437	0.512	0.420
-0.2	-0.75	0.064	0.070	0.053	0.048	0.052	0.111	0.068	0.060	0.116	0.069
	-0.5	0.064	0.069	0.053	0.049	0.052	0.090	0.069	0.059	0.114	0.069
	-0.25	0.064	0.069	0.052	0.051	0.051	0.083	0.066	0.057	0.114	0.067
	0	0.062	0.067	0.050	0.061	0.049	0.076	0.066	0.056	0.109	0.063
	0.25	0.070	0.075	0.057	0.087	0.058	0.071	0.076	0.074	0.110	0.077
	0.5	0.125	0.136	0.115	0.155	0.121	0.119	0.130	0.165	0.201	0.160
	0.75	0.322	0.352	0.315	0.333	0.346	0.320	0.319	0.433	0.509	0.418
0	-0.75	0.065	0.070	0.056	0.044	0.053	0.079	0.064	0.060	0.139	0.069
	-0.5	0.064	0.069	0.056	0.044	0.053	0.077	0.064	0.060	0.141	0.068
	-0.25	0.063	0.068	0.053	0.041	0.051	0.069	0.063	0.057	0.140	0.066
	0	0.062	0.066	0.050	0.038	0.049	0.062	0.062	0.056	0.143	0.063
	0.25	0.069	0.075	0.055	0.046	0.056	0.063	0.068	0.072	0.170	0.076
	0.5	0.123	0.135	0.110	0.104	0.118	0.115	0.121	0.161	0.230	0.159
	0.75	0.321	0.351	0.311	0.286	0.344	0.317	0.312	0.431	0.426	0.417
0.2	-0.75	0.064	0.070	0.057	0.068	0.054	0.070	0.070	0.061	0.125	0.070
	-0.5	0.064	0.069	0.056	0.066	0.053	0.064	0.068	0.060	0.123	0.068
	-0.25	0.063	0.068	0.054	0.062	0.052	0.060	0.064	0.058	0.121	0.066
	0	0.061	0.066	0.051	0.054	0.049	0.057	0.061	0.056	0.120	0.063
	0.25	0.070	0.076	0.055	0.044	0.056	0.062	0.068	0.071	0.122	0.076
	0.5	0.125	0.137	0.111	0.082	0.118	0.118	0.119	0.161	0.174	0.160
	0.75	0.322	0.351	0.310	0.263	0.343	0.309	0.306	0.428	0.284	0.417
0.4	-0.75	0.064	0.068	0.056	0.072	0.054	0.065	0.072	0.061	0.103	0.068
	-0.5	0.064	0.069	0.056	0.072	0.053	0.060	0.067	0.060	0.102	0.068
	-0.25	0.063	0.068	0.054	0.068	0.051	0.059	0.065	0.058	0.098	0.065
	0	0.062	0.066	0.051	0.062	0.049	0.057	0.062	0.055	0.090	0.063
	0.25	0.072	0.077	0.058	0.058	0.058	0.064	0.069	0.072	0.077	0.079
	0.5	0.128	0.139	0.114	0.094	0.120	0.112	0.117	0.161	0.079	0.162
	0.75	0.324	0.353	0.313	0.273	0.344	0.256	0.305	0.428	0.090	0.420

Table 7: Bias of the estimators obtained from 10,000 contaminated (with additive outliers) realizations of length $n = 100$ of ARFIMA(1,d,0) processes with standard normal innovations and parameters $d = -0.4,\ -0.2,\ 0,\ 0.2, 0.4$ and $\phi_1 = -0.75, -0.5,\ -0.25, 0,\ 0.25, 0.5, 0.75$ (estimation is based on the first K $=$ 25 Fourier frequencies).

d	ϕ_1	$\hat{d}_{GPH}$	$\hat{d}_{GPH}^{tr}$	$\hat{d}_{smP}^{0.9}$	$\hat{d}_{smP}^{0.5}$	$\hat{d}_W$	$\hat{d}_{rAC}$	$\hat{d}_H$	$\hat{d}_{KS}$	$\hat{d}_{trF}$	$\hat{d}_{TS}$
-0.4	-0.75	0.095	0.051	0.077	0.121	0.085	0.231	0.120	0.068	0.092	0.045
	-0.5	0.068	0.024	0.052	0.115	0.062	0.175	0.108	0.048	0.091	0.022
	-0.25	0.064	0.029	0.047	0.128	0.057	0.129	0.103	0.051	0.088	0.027
	0	0.092	0.075	0.077	0.170	0.085	0.127	0.120	0.095	0.094	0.077
	0.25	0.163	0.176	0.151	0.245	0.159	0.182	0.180	0.197	0.124	0.181
	0.5	0.315	0.370	0.301	0.367	0.318	0.316	0.320	0.390	0.269	0.370
	0.75	0.582	0.672	0.561	0.549	0.594	0.556	0.576	0.670	0.552	0.652
-0.2	-0.75	-0.016	-0.047	-0.037	-0.008	-0.026	0.057	0.003	-0.035	-0.088	-0.053
	-0.5	-0.023	-0.050	-0.044	-0.005	-0.033	0.019	-0.004	-0.038	-0.089	-0.057
	-0.25	-0.011	-0.029	-0.033	0.016	-0.021	0.000	0.006	-0.021	-0.082	-0.037
	0	0.030	0.029	0.009	0.065	0.021	0.023	0.041	0.038	-0.048	0.026
	0.25	0.122	0.147	0.101	0.148	0.115	0.106	0.126	0.156	0.041	0.145
	0.5	0.291	0.350	0.267	0.279	0.290	0.261	0.288	0.362	0.254	0.347
	0.75	0.572	0.667	0.541	0.482	0.578	0.520	0.558	0.652	0.488	0.644
0	-0.75	-0.084	-0.108	-0.113	-0.115	-0.098	-0.099	-0.082	-0.104	-0.207	-0.115
	-0.5	-0.075	-0.097	-0.103	-0.102	-0.088	-0.096	-0.070	-0.093	-0.198	-0.103
	-0.25	-0.052	-0.066	-0.082	-0.075	-0.068	-0.083	-0.050	-0.066	-0.171	-0.074
	0	0.003	0.003	-0.028	-0.020	-0.013	-0.030	0.002	0.004	-0.106	-0.001
	0.25	0.101	0.124	0.072	0.068	0.089	0.066	0.100	0.130	0.026	0.128
	0.5	0.276	0.337	0.245	0.207	0.271	0.232	0.268	0.340	0.213	0.333
	0.75	0.565	0.657	0.528	0.430	0.564	0.473	0.546	0.634	0.392	0.635
0.2	-0.75	-0.117	-0.142	-0.152	-0.190	-0.136	-0.188	-0.124	-0.145	-0.257	-0.150
	-0.5	-0.098	-0.122	-0.135	-0.170	-0.120	-0.150	-0.104	-0.127	-0.235	-0.130
	-0.25	-0.070	-0.086	-0.106	-0.138	-0.091	-0.114	-0.075	-0.091	-0.196	-0.092
	0	-0.012	-0.012	-0.047	-0.081	-0.033	-0.054	-0.019	-0.018	-0.122	-0.015
	0.25	0.097	0.122	0.062	0.014	0.079	0.053	0.089	0.117	-0.001	0.122
	0.5	0.274	0.336	0.237	0.156	0.261	0.203	0.259	0.327	0.141	0.331
	0.75	0.560	0.652	0.519	0.391	0.551	0.391	0.537	0.607	0.244	0.629
0.4	-0.75	-0.125	-0.156	-0.165	-0.231	-0.153	-0.215	-0.138	-0.166	-0.275	-0.164
	-0.5	-0.105	-0.132	-0.145	-0.208	-0.133	-0.159	-0.117	-0.144	-0.251	-0.139
	-0.25	-0.074	-0.093	-0.113	-0.173	-0.101	-0.120	-0.085	-0.105	-0.221	-0.100
	0	-0.013	-0.017	-0.050	-0.114	-0.039	-0.059	-0.024	-0.028	-0.153	-0.018
	0.25	0.099	0.120	0.058	-0.021	0.071	0.027	0.084	0.105	-0.062	0.121
	0.5	0.276	0.334	0.236	0.126	0.255	0.146	0.257	0.313	0.026	0.330
	0.75	0.547	0.634	0.508	0.360	0.507	0.265	0.523	0.534	0.053	0.611

Table 8: RMSE of the estimators obtained from 10,000 contaminated (with additive outliers) realizations of length $n = 100$ of ARFIMA(1,d,0) processes with standard normal innovations and parameters $d = -0.4,\ -0.2,\ 0,\ 0.2, 0.4$ and $\phi_1 = -0.75, -0.5,\ -0.25, 0,\ 0.25, 0.5, 0.75$ (estimation is based on the first $\mathrm{K} = 25$ Fourier frequencies).

d	ϕ_1	$\hat{d}_{GPH}$	$\hat{d}_{GPH}^{tr}$	$\hat{d}_{smP}^{0.9}$	$\hat{d}_{smP}^{0.5}$	$\hat{d}_{W}$	$\hat{d}_{rAC}$	$\hat{d}_{H}$	$\hat{d}_{KS}$	$\hat{d}_{trF}$	$\hat{d}_{TS}$
-0.4	-0.75	0.190	0.211	0.146	0.144	0.159	0.322	0.205	0.157	0.192	0.184
	-0.5	0.176	0.203	0.134	0.139	0.146	0.277	0.195	0.147	0.194	0.178
	-0.25	0.174	0.202	0.131	0.149	0.142	0.246	0.192	0.148	0.191	0.175
	0	0.187	0.214	0.144	0.187	0.154	0.228	0.203	0.169	0.192	0.184
	0.25	0.231	0.267	0.195	0.258	0.206	0.258	0.243	0.247	0.213	0.245
	0.5	0.355	0.422	0.327	0.375	0.345	0.361	0.359	0.421	0.350	0.407
	0.75	0.605	0.701	0.575	0.556	0.610	0.581	0.598	0.690	0.591	0.674
-0.2	-0.75	0.164	0.209	0.127	0.077	0.132	0.217	0.165	0.143	0.191	0.182
	-0.5	0.163	0.206	0.132	0.080	0.135	0.202	0.161	0.146	0.192	0.181
	-0.25	0.164	0.201	0.127	0.081	0.131	0.185	0.164	0.142	0.192	0.175
	0	0.167	0.201	0.126	0.104	0.132	0.182	0.167	0.148	0.189	0.169
	0.25	0.202	0.247	0.160	0.168	0.172	0.200	0.204	0.214	0.215	0.218
	0.5	0.333	0.404	0.296	0.291	0.319	0.309	0.330	0.395	0.342	0.385
	0.75	0.594	0.697	0.556	0.490	0.594	0.543	0.582	0.672	0.515	0.667
0	-0.75	0.182	0.229	0.168	0.141	0.162	0.219	0.183	0.174	0.284	0.206
	-0.5	0.178	0.222	0.162	0.132	0.156	0.204	0.177	0.166	0.279	0.199
	-0.25	0.173	0.213	0.152	0.113	0.148	0.192	0.172	0.158	0.266	0.187
	0	0.162	0.203	0.130	0.087	0.130	0.166	0.163	0.144	0.246	0.168
	0.25	0.191	0.235	0.146	0.110	0.157	0.172	0.190	0.198	0.225	0.209
	0.5	0.320	0.393	0.276	0.225	0.302	0.281	0.313	0.375	0.283	0.373
	0.75	0.588	0.687	0.544	0.442	0.581	0.495	0.570	0.653	0.408	0.658
0.2	-0.75	0.200	0.247	0.199	0.210	0.188	0.260	0.205	0.201	0.339	0.229
	-0.5	0.190	0.235	0.187	0.194	0.177	0.226	0.194	0.188	0.326	0.215
	-0.25	0.177	0.219	0.167	0.166	0.159	0.199	0.179	0.169	0.298	0.195
	0	0.162	0.200	0.138	0.124	0.134	0.166	0.163	0.146	0.249	0.168
	0.25	0.190	0.234	0.143	0.096	0.152	0.162	0.185	0.188	0.190	0.205
	0.5	0.319	0.392	0.270	0.185	0.294	0.252	0.306	0.363	0.199	0.371
	0.75	0.583	0.682	0.536	0.406	0.567	0.413	0.562	0.623	0.254	0.652
0.4	-0.75	0.205	0.255	0.211	0.254	0.201	0.275	0.214	0.217	0.349	0.239
	-0.5	0.193	0.240	0.196	0.233	0.187	0.226	0.200	0.201	0.327	0.221
	-0.25	0.180	0.222	0.175	0.205	0.167	0.197	0.184	0.177	0.299	0.200
	0	0.164	0.200	0.141	0.157	0.136	0.163	0.165	0.145	0.235	0.169
	0.25	0.191	0.232	0.146	0.113	0.149	0.147	0.182	0.182	0.156	0.203
	0.5	0.320	0.390	0.270	0.169	0.288	0.200	0.303	0.347	0.093	0.370
	0.75	0.571	0.666	0.525	0.380	0.518	0.298	0.549	0.542	0.080	0.635

Table 9: Bias of the estimators obtained from 10,000 contaminated (with additive outliers) realizations of length $n = 300$ of ARFIMA(1,d,0) processes with standard normal innovations and parameters $d = -0.4,\ -0.2,\ 0,\ 0.2, 0.4$ and $\phi_1 = -0.75, -0.5,\ -0.25, 0,\ 0.25, 0.5, 0.75$ (estimation is based on the first $\mathrm{K} = 54$ Fourier frequencies).

d	ϕ_1	$\hat{d}_{GPH}$	$\hat{d}_{GPH}^{tr}$	$\hat{d}_{smP}^{0.9}$	$\hat{d}_{smP}^{0.5}$	$\hat{d}_{W}$	$\hat{d}_{rAC}$	$\hat{d}_{H}$	$\hat{d}_{KS}$	$\hat{d}_{trF}$	$\hat{d}_{TS}$
-0.4	-0.75	0.170	0.148	0.158	0.200	0.165	0.278	0.199	0.147	0.113	0.134
	-0.5	0.143	0.120	0.131	0.182	0.139	0.207	0.183	0.121	0.094	0.108
	-0.25	0.121	0.099	0.111	0.173	0.118	0.152	0.160	0.103	0.087	0.090
	0	0.110	0.094	0.100	0.177	0.107	0.123	0.143	0.103	0.082	0.093
	0.25	0.135	0.130	0.126	0.214	0.131	0.137	0.157	0.147	0.112	0.139
	0.5	0.222	0.241	0.215	0.302	0.223	0.225	0.232	0.278	0.224	0.265
	0.75	0.451	0.505	0.444	0.483	0.478	0.455	0.450	0.565	0.566	0.542
-0.2	-0.75	0.039	0.027	0.027	0.060	0.034	0.088	0.057	0.026	-0.025	0.018
	-0.5	0.026	0.017	0.014	0.053	0.021	0.044	0.047	0.015	-0.033	0.008
	-0.25	0.023	0.016	0.012	0.055	0.018	0.024	0.041	0.017	-0.033	0.010
	0	0.034	0.032	0.022	0.070	0.028	0.023	0.045	0.036	-0.016	0.030
	0.25	0.077	0.084	0.066	0.116	0.074	0.066	0.084	0.099	0.049	0.093
	0.5	0.191	0.218	0.180	0.216	0.194	0.182	0.192	0.254	0.234	0.242
	0.75	0.438	0.497	0.426	0.415	0.467	0.431	0.431	0.555	0.511	0.534
0	-0.75	-0.034	-0.040	-0.049	-0.045	-0.042	-0.047	-0.033	-0.044	-0.093	-0.049
	-0.5	-0.033	-0.038	-0.050	-0.044	-0.041	-0.051	-0.032	-0.044	-0.092	-0.047
	-0.25	-0.024	-0.027	-0.040	-0.034	-0.031	-0.042	-0.022	-0.031	-0.078	-0.034
	0	0.000	0.000	-0.016	-0.010	-0.008	-0.020	-0.001	0.001	-0.038	-0.002
	0.25	0.057	0.065	0.041	0.043	0.051	0.039	0.055	0.077	0.055	0.075
	0.5	0.180	0.209	0.164	0.150	0.180	0.166	0.176	0.240	0.215	0.232
	0.75	0.435	0.494	0.417	0.363	0.460	0.411	0.422	0.546	0.423	0.531
0.2	-0.75	-0.065	-0.073	-0.083	-0.111	-0.075	-0.100	-0.074	-0.081	-0.110	-0.083
	-0.5	-0.056	-0.063	-0.074	-0.102	-0.065	-0.075	-0.063	-0.070	-0.099	-0.072
	-0.25	-0.040	-0.046	-0.058	-0.087	-0.050	-0.056	-0.046	-0.051	-0.082	-0.052
	0	-0.009	-0.009	-0.027	-0.057	-0.018	-0.024	-0.014	-0.012	-0.043	-0.012
	0.25	0.053	0.062	0.034	0.000	0.043	0.037	0.046	0.068	0.029	0.070
	0.5	0.180	0.207	0.160	0.111	0.176	0.159	0.169	0.234	0.160	0.231
	0.75	0.437	0.495	0.416	0.335	0.457	0.372	0.416	0.542	0.277	0.530
0.4	-0.75	-0.069	-0.081	-0.087	-0.130	-0.081	-0.099	-0.084	-0.093	-0.139	-0.092
	-0.5	-0.058	-0.068	-0.077	-0.120	-0.071	-0.074	-0.070	-0.080	-0.127	-0.078
	-0.25	-0.039	-0.046	-0.058	-0.101	-0.052	-0.054	-0.050	-0.058	-0.107	-0.054
	0	-0.007	-0.010	-0.027	-0.070	-0.020	-0.024	-0.017	-0.018	-0.074	-0.013
	0.25	0.054	0.061	0.035	-0.013	0.042	0.032	0.044	0.064	-0.017	0.069
	0.5	0.183	0.208	0.163	0.100	0.175	0.133	0.166	0.230	0.061	0.232
	0.75	0.436	0.494	0.416	0.324	0.451	0.275	0.411	0.518	0.085	0.528

Table 10: RMSE of the estimators obtained from 10,000 contaminated (with additive outliers) realizations of length $n = 300$ of ARFIMA(1,d,0) processes with standard normal innovations and parameters $d = -0.4,\ -0.2,\ 0,\ 0.2, 0.4$ and $\phi_1 = -0.75, -0.5,\ -0.25, 0,\ 0.25, 0.5, 0.75$ (estimation is based on the first K $=$ 54 Fourier frequencies).

d	ϕ_1	$\hat{d}_{GPH}$	$\hat{d}_{GPH}^{tr}$	$\hat{d}_{smP}^{0.9}$	$\hat{d}_{smP}^{0.5}$	$\hat{d}_W$	$\hat{d}_{rAC}$	$\hat{d}_H$	$\hat{d}_{KS}$	$\hat{d}_{trF}$	$\hat{d}_{TS}$
-0.4	-0.75	0.198	0.188	0.176	0.207	0.184	0.307	0.224	0.171	0.185	0.172
	-0.5	0.175	0.166	0.153	0.190	0.162	0.247	0.208	0.149	0.174	0.151
	-0.25	0.159	0.152	0.136	0.181	0.144	0.203	0.189	0.134	0.171	0.140
	0	0.149	0.147	0.127	0.185	0.134	0.175	0.174	0.134	0.169	0.138
	0.25	0.167	0.172	0.148	0.220	0.153	0.179	0.187	0.172	0.185	0.172
	0.5	0.244	0.267	0.229	0.306	0.237	0.250	0.253	0.294	0.271	0.284
	0.75	0.462	0.518	0.451	0.486	0.485	0.467	0.461	0.574	0.598	0.552
-0.2	-0.75	0.107	0.117	0.083	0.081	0.087	0.153	0.116	0.091	0.147	0.106
	-0.5	0.104	0.116	0.080	0.076	0.082	0.132	0.111	0.088	0.149	0.104
	-0.25	0.103	0.115	0.080	0.077	0.082	0.121	0.108	0.089	0.149	0.103
	0	0.106	0.118	0.082	0.089	0.084	0.115	0.110	0.096	0.148	0.107
	0.25	0.126	0.141	0.104	0.128	0.109	0.124	0.131	0.136	0.167	0.138
	0.5	0.215	0.245	0.196	0.223	0.210	0.208	0.216	0.272	0.307	0.263
	0.75	0.450	0.510	0.434	0.419	0.475	0.442	0.443	0.565	0.531	0.544
0	-0.75	0.105	0.120	0.093	0.072	0.089	0.124	0.105	0.098	0.187	0.114
	-0.5	0.106	0.119	0.094	0.072	0.090	0.123	0.106	0.098	0.187	0.113
	-0.25	0.102	0.116	0.089	0.066	0.085	0.115	0.102	0.093	0.185	0.108
	0	0.099	0.113	0.081	0.058	0.080	0.103	0.100	0.089	0.178	0.102
	0.25	0.114	0.130	0.089	0.071	0.094	0.105	0.114	0.120	0.188	0.125
	0.5	0.206	0.238	0.182	0.161	0.198	0.191	0.202	0.259	0.268	0.253
	0.75	0.446	0.507	0.425	0.369	0.468	0.421	0.434	0.556	0.431	0.542
0.2	-0.75	0.119	0.136	0.116	0.127	0.109	0.146	0.125	0.119	0.199	0.133
	-0.5	0.115	0.130	0.110	0.120	0.103	0.126	0.119	0.112	0.192	0.126
	-0.25	0.107	0.122	0.099	0.107	0.093	0.111	0.109	0.101	0.181	0.114
	0	0.101	0.114	0.086	0.086	0.082	0.099	0.102	0.091	0.164	0.103
	0.25	0.113	0.129	0.087	0.065	0.090	0.100	0.110	0.115	0.151	0.123
	0.5	0.206	0.236	0.179	0.129	0.193	0.184	0.196	0.253	0.196	0.252
	0.75	0.448	0.508	0.424	0.343	0.465	0.383	0.429	0.551	0.279	0.540
0.4	-0.75	0.123	0.141	0.121	0.152	0.115	0.142	0.133	0.128	0.199	0.140
	-0.5	0.116	0.133	0.114	0.143	0.108	0.122	0.123	0.119	0.190	0.131
	-0.25	0.106	0.121	0.101	0.128	0.096	0.108	0.111	0.105	0.173	0.115
	0	0.101	0.114	0.087	0.105	0.082	0.095	0.103	0.090	0.147	0.104
	0.25	0.115	0.129	0.091	0.081	0.091	0.097	0.110	0.113	0.108	0.123
	0.5	0.208	0.237	0.182	0.128	0.193	0.159	0.195	0.250	0.080	0.253
	0.75	0.447	0.507	0.424	0.334	0.459	0.291	0.424	0.524	0.085	0.538

Table 11: Bias of the estimators obtained from 10,000 contaminated (with additive outliers) realizations of length $n = 1{,}000$ of ARFIMA(1,d,0) processes with standard normal innovations and parameters $d = -0.4,\ -0.2,\ 0,\ 0.2, 0.4$ and $\phi_1 = -0.75, -0.5,\ -0.25, 0,\ 0.25, 0.5, 0.75$ (estimation is based on the first $\mathrm{K} = 125$ Fourier frequencies).

d	ϕ_1	$\hat{d}_{GPH}$	$\hat{d}^{tr}_{GPH}$	$\hat{d}^{0.9}_{smP}$	$\hat{d}^{0.5}_{smP}$	$\hat{d}_W$	$\hat{d}_{rAC}$	$\hat{d}_H$	$\hat{d}_{KS}$	$\hat{d}_{trF}$	$\hat{d}_{TS}$
-0.4	-0.75	0.220	0.208	0.213	0.240	0.220	0.280	0.250	0.199	0.183	0.190
	-0.5	0.194	0.181	0.187	0.221	0.194	0.220	0.233	0.174	0.158	0.163
	-0.25	0.165	0.152	0.159	0.202	0.166	0.172	0.206	0.146	0.131	0.137
	0	0.139	0.127	0.133	0.188	0.140	0.133	0.177	0.124	0.112	0.117
	0.25	0.127	0.118	0.122	0.191	0.126	0.113	0.154	0.124	0.115	0.119
	0.5	0.159	0.161	0.155	0.238	0.158	0.150	0.175	0.187	0.180	0.183
	0.75	0.328	0.352	0.326	0.397	0.349	0.331	0.333	0.431	0.475	0.415
-0.2	-0.75	0.068	0.064	0.061	0.087	0.066	0.090	0.087	0.059	0.040	0.055
	-0.5	0.054	0.050	0.048	0.078	0.052	0.053	0.075	0.046	0.027	0.042
	-0.25	0.043	0.038	0.036	0.070	0.040	0.032	0.062	0.035	0.017	0.032
	0	0.038	0.035	0.032	0.071	0.036	0.023	0.051	0.036	0.018	0.033
	0.25	0.053	0.054	0.047	0.090	0.051	0.040	0.061	0.063	0.045	0.061
	0.5	0.115	0.124	0.110	0.152	0.116	0.107	0.119	0.156	0.150	0.151
	0.75	0.312	0.341	0.307	0.327	0.337	0.311	0.310	0.421	0.482	0.406
0	-0.75	-0.015	-0.016	-0.023	-0.019	-0.018	-0.025	-0.014	-0.020	-0.025	-0.022
	-0.5	-0.015	-0.016	-0.023	-0.020	-0.019	-0.027	-0.015	-0.020	-0.023	-0.022
	-0.25	-0.011	-0.012	-0.019	-0.015	-0.014	-0.022	-0.010	-0.014	-0.016	-0.016
	0	0.000	0.000	-0.007	-0.004	-0.002	-0.009	0.000	0.001	0.002	0.000
	0.25	0.030	0.033	0.022	0.024	0.027	0.022	0.029	0.042	0.061	0.041
	0.5	0.103	0.113	0.095	0.093	0.103	0.097	0.101	0.145	0.175	0.141
	0.75	0.310	0.339	0.301	0.278	0.334	0.306	0.301	0.418	0.412	0.404
0.2	-0.75	-0.041	-0.044	-0.050	-0.073	-0.046	-0.049	-0.052	-0.051	-0.051	-0.051
	-0.5	-0.034	-0.036	-0.043	-0.066	-0.038	-0.038	-0.042	-0.042	-0.044	-0.043
	-0.25	-0.024	-0.025	-0.033	-0.057	-0.028	-0.028	-0.029	-0.031	-0.032	-0.030
	0	-0.009	-0.010	-0.018	-0.042	-0.013	-0.014	-0.013	-0.011	-0.015	-0.011
	0.25	0.025	0.027	0.015	-0.010	0.020	0.019	0.020	0.034	0.027	0.035
	0.5	0.000	0.001	-0.030	-0.138	-0.011	0.076	0.074	0.006	-0.076	0.000
	0.75	0.054	0.066	0.024	-0.110	0.045	0.272	0.270	0.075	-0.016	0.068
0.4	-0.75	-0.068	-0.079	-0.099	-0.304	-0.079	-0.105	-0.129	-0.076	-0.172	-0.081
	-0.5	-0.054	-0.063	-0.087	-0.299	-0.067	-0.073	-0.097	-0.062	-0.162	-0.066
	-0.25	-0.037	-0.045	-0.070	-0.293	-0.050	-0.054	-0.074	-0.043	-0.145	-0.047
	0	-0.017	-0.022	-0.051	-0.287	-0.031	-0.037	-0.050	-0.020	-0.128	-0.025
	0.25	-0.008	-0.013	-0.040	-0.282	-0.020	-0.007	-0.013	-0.009	-0.118	-0.014
	0.5	0.011	0.012	-0.022	-0.275	-0.001	0.066	0.067	0.014	-0.105	0.009
	0.75	0.062	0.074	0.030	-0.256	0.052	0.231	0.269	0.082	-0.059	0.076

Table 12: RMSE of the estimators obtained from 10,000 contaminated (with additive outliers) realizations of length $n = 1000$ of ARFIMA(1,*d*,0) processes with standard normal innovations and parameters $d = -0.4,\ -0.2,\ 0,\ 0.2, 0.4$ and $\phi_1 = -0.75, -0.5,\ -0.25, 0,\ 0.25, 0.5, 0.75$ (estimation is based on the first $\text{K} = 125$ Fourier frequencies).

d	ϕ_1	$\hat{d}_{GPH}$	$\hat{d}^{tr}_{GPH}$	$\hat{d}^{0.9}_{smP}$	$\hat{d}^{0.5}_{smP}$	$\hat{d}_W$	$\hat{d}_{rAC}$	$\hat{d}_H$	$\hat{d}_{KS}$	$\hat{d}_{trF}$	$\hat{d}_{TS}$
-0.4	-0.75	0.229	0.219	0.218	0.243	0.225	0.294	0.258	0.206	0.213	0.202
	-0.5	0.203	0.193	0.193	0.224	0.200	0.238	0.241	0.181	0.194	0.176
	-0.25	0.177	0.166	0.166	0.205	0.173	0.195	0.215	0.156	0.174	0.152
	0	0.153	0.144	0.142	0.191	0.148	0.158	0.187	0.135	0.160	0.133
	0.25	0.141	0.136	0.131	0.194	0.136	0.138	0.166	0.135	0.164	0.135
	0.5	0.171	0.174	0.163	0.240	0.166	0.165	0.186	0.196	0.215	0.193
	0.75	0.334	0.358	0.329	0.399	0.353	0.337	0.339	0.436	0.508	0.420
-0.2	-0.75	0.093	0.093	0.079	0.095	0.082	0.121	0.107	0.081	0.110	0.085
	-0.5	0.082	0.083	0.069	0.086	0.072	0.099	0.097	0.072	0.106	0.077
	-0.25	0.075	0.077	0.061	0.079	0.063	0.086	0.088	0.065	0.106	0.071
	0	0.073	0.076	0.059	0.080	0.060	0.076	0.080	0.066	0.105	0.071
	0.25	0.081	0.086	0.069	0.098	0.071	0.078	0.087	0.085	0.114	0.088
	0.5	0.131	0.141	0.121	0.157	0.126	0.124	0.135	0.167	0.206	0.163
	0.75	0.318	0.347	0.311	0.329	0.342	0.317	0.316	0.426	0.503	0.411
0	-0.75	0.064	0.069	0.054	0.042	0.052	0.076	0.063	0.058	0.140	0.067
	-0.5	0.064	0.069	0.055	0.042	0.052	0.076	0.063	0.058	0.140	0.067
	-0.25	0.063	0.068	0.053	0.041	0.051	0.069	0.062	0.057	0.142	0.065
	0	0.062	0.067	0.050	0.038	0.049	0.063	0.063	0.056	0.146	0.063
	0.25	0.068	0.073	0.054	0.045	0.056	0.062	0.068	0.071	0.166	0.074
	0.5	0.120	0.131	0.107	0.101	0.114	0.112	0.118	0.156	0.229	0.154
	0.75	0.316	0.345	0.305	0.281	0.338	0.311	0.307	0.423	0.421	0.409
0.2	-0.75	0.074	0.080	0.071	0.084	0.067	0.081	0.081	0.074	0.133	0.081
	-0.5	0.070	0.076	0.066	0.079	0.062	0.071	0.075	0.069	0.129	0.077
	-0.25	0.066	0.072	0.060	0.071	0.057	0.064	0.068	0.063	0.125	0.071
	0	0.062	0.067	0.054	0.060	0.051	0.058	0.063	0.057	0.122	0.065
	0.25	0.067	0.073	0.053	0.045	0.053	0.060	0.065	0.066	0.121	0.072
	0.5	0.137	0.166	0.114	0.140	0.113	0.095	0.097	0.128	0.204	0.141
	0.75	0.147	0.178	0.113	0.113	0.123	0.278	0.277	0.151	0.179	0.156
0.4	-0.75	0.153	0.182	0.149	0.305	0.137	0.123	0.144	0.144	0.243	0.163
	-0.5	0.147	0.175	0.142	0.300	0.132	0.094	0.115	0.139	0.234	0.156
	-0.25	0.143	0.171	0.132	0.294	0.124	0.079	0.096	0.132	0.219	0.149
	0	0.140	0.167	0.124	0.288	0.119	0.068	0.079	0.130	0.205	0.145
	0.25	0.140	0.167	0.120	0.283	0.116	0.057	0.064	0.126	0.197	0.143
	0.5	0.138	0.163	0.114	0.276	0.113	0.086	0.092	0.127	0.185	0.140
	0.75	0.152	0.182	0.116	0.257	0.126	0.239	0.276	0.154	0.148	0.159

Table 13: Bias of the estimators obtained from 10,000 realizations of length $n = 100$ of ARFIMA(1,*d*,0)-GARCH(1,1) processes with *t*-distributed innovations and ARFIMA parameters $d = -0.4,\ -0.2,\ 0,\ 0.2, 0.4$ and $\phi_1 = -0.75, -0.5, -0.25, 0,\ 0.25, 0.5, 0.75$ (estimation is based on the first $\mathrm{K} = 25$ Fourier frequencies).

d	ϕ_1	$\hat{d}_{GPH}$	$\hat{d}_{GPH}^{tr}$	$\hat{d}_{smP}^{0.9}$	$\hat{d}_{smP}^{0.5}$	$\hat{d}_W$	$\hat{d}_{rAC}$	$\hat{d}_H$	$\hat{d}_{KS}$	$\hat{d}_{trF}$	$\hat{d}_{TS}$
-0.4	-0.75	0.023	0.010	0.015	0.233	-0.006	0.219	0.048	0.025	0.142	0.009
	-0.5	0.007	-0.010	-0.001	0.223	-0.020	0.156	0.056	0.012	0.144	-0.007
	-0.25	0.007	-0.004	-0.002	0.227	-0.019	0.111	0.060	0.014	0.142	-0.006
	0	0.019	0.017	0.008	0.241	-0.007	0.113	0.086	0.027	0.141	0.011
	0.25	0.045	0.054	0.033	0.272	0.019	0.179	0.161	0.059	0.145	0.045
	0.5	0.127	0.171	0.111	0.325	0.101	0.325	0.317	0.158	0.148	0.142
	0.75	0.375	0.488	0.342	0.423	0.357	0.577	0.600	0.443	0.229	0.420
-0.2	-0.75	-0.008	-0.019	-0.042	0.090	-0.037	0.038	-0.042	-0.005	-0.052	-0.019
	-0.5	-0.006	-0.015	-0.044	0.088	-0.037	0.003	-0.032	-0.003	-0.053	-0.018
	-0.25	-0.002	-0.002	-0.038	0.094	-0.032	-0.015	-0.019	0.005	-0.047	-0.009
	0	0.006	0.009	-0.029	0.107	-0.022	0.014	0.026	0.017	-0.047	0.001
	0.25	0.032	0.045	-0.004	0.135	0.003	0.106	0.124	0.047	-0.043	0.033
	0.5	0.116	0.165	0.080	0.188	0.091	0.276	0.302	0.149	-0.009	0.133
	0.75	0.370	0.479	0.313	0.281	0.352	0.539	0.587	0.433	0.153	0.414
0	-0.75	-0.020	-0.031	-0.077	-0.041	-0.051	-0.105	-0.091	-0.019	-0.190	-0.031
	-0.5	-0.017	-0.024	-0.075	-0.040	-0.048	-0.109	-0.079	-0.015	-0.187	-0.029
	-0.25	-0.014	-0.020	-0.072	-0.035	-0.046	-0.091	-0.051	-0.012	-0.185	-0.023
	0	-0.001	-0.001	-0.058	-0.021	-0.031	-0.029	0.009	0.005	-0.171	-0.005
	0.25	0.032	0.045	-0.027	0.004	0.002	0.076	0.115	0.047	-0.156	0.031
	0.5	0.118	0.169	0.055	0.052	0.087	0.249	0.298	0.144	-0.096	0.134
	0.75	0.368	0.484	0.294	0.138	0.350	0.490	0.581	0.429	0.085	0.414
0.2	-0.75	-0.031	-0.039	-0.103	-0.166	-0.063	-0.190	-0.111	-0.028	-0.279	-0.041
	-0.5	-0.024	-0.030	-0.098	-0.164	-0.056	-0.149	-0.091	-0.023	-0.272	-0.031
	-0.25	-0.016	-0.016	-0.091	-0.158	-0.047	-0.103	-0.059	-0.012	-0.262	-0.022
	0	-0.004	-0.002	-0.078	-0.148	-0.035	-0.039	0.002	0.001	-0.256	-0.010
	0.25	0.029	0.039	-0.047	-0.125	-0.001	0.068	0.114	0.040	-0.232	0.028
	0.5	0.111	0.159	0.033	-0.086	0.082	0.217	0.286	0.137	-0.166	0.129
	0.75	0.370	0.488	0.274	-0.011	0.344	0.398	0.566	0.414	0.006	0.415
0.4	-0.75	-0.027	-0.032	-0.113	-0.291	-0.059	-0.206	-0.114	-0.027	-0.344	-0.038
	-0.5	-0.021	-0.023	-0.112	-0.289	-0.056	-0.139	-0.090	-0.025	-0.340	-0.031
	-0.25	-0.019	-0.017	-0.106	-0.284	-0.051	-0.096	-0.055	-0.019	-0.336	-0.024
	0	-0.004	-0.004	-0.093	-0.276	-0.038	-0.041	0.005	-0.006	-0.327	-0.011
	0.25	0.036	0.054	-0.056	-0.258	0.002	0.040	0.110	0.041	-0.294	0.038
	0.5	0.111	0.160	0.018	-0.228	0.078	0.149	0.279	0.129	-0.239	0.129
	0.75	0.361	0.485	0.254	-0.170	0.318	0.269	0.549	0.371	-0.120	0.412

Table 14: RMSE of the estimators obtained from 10,000 realizations of length $n = 100$ of ARFIMA(1,d,0)-GARCH(1,1) processes with t-distributed innovations and ARFIMA parameters $d = -0.4,\ -0.2,\ 0,\ 0.2, 0.4$ and $\phi_1 = -0.75, -0.5, -0.25, 0,\ 0.25, 0.5, 0.75$ (estimation is based on the first $\mathrm{K} = 25$ Fourier frequencies).

d	ϕ_1	$\hat{d}_{GPH}$	$\hat{d}_{GPH}^{tr}$	$\hat{d}_{smP}^{0.9}$	$\hat{d}_{smP}^{0.5}$	$\hat{d}_{W}$	$\hat{d}_{rAC}$	$\hat{d}_{H}$	$\hat{d}_{KS}$	$\hat{d}_{trF}$	$\hat{d}_{TS}$
-0.4	-0.75	0.315	0.448	0.217	0.242	0.265	0.330	0.183	0.289	0.230	0.330
	-0.5	0.314	0.448	0.216	0.233	0.266	0.304	0.183	0.289	0.232	0.332
	-0.25	0.309	0.449	0.217	0.237	0.262	0.271	0.181	0.287	0.230	0.327
	0	0.316	0.451	0.221	0.250	0.268	0.252	0.188	0.295	0.230	0.328
	0.25	0.319	0.457	0.227	0.279	0.271	0.270	0.232	0.302	0.235	0.332
	0.5	0.335	0.477	0.248	0.332	0.289	0.377	0.359	0.340	0.242	0.354
	0.75	0.487	0.663	0.409	0.427	0.452	0.603	0.621	0.542	0.332	0.531
-0.2	-0.75	0.312	0.449	0.223	0.112	0.269	0.237	0.176	0.290	0.204	0.329
	-0.5	0.307	0.444	0.224	0.111	0.267	0.229	0.170	0.287	0.202	0.321
	-0.25	0.310	0.449	0.228	0.116	0.273	0.220	0.168	0.295	0.205	0.329
	0	0.311	0.448	0.222	0.126	0.267	0.202	0.168	0.294	0.205	0.329
	0.25	0.309	0.448	0.223	0.150	0.268	0.213	0.207	0.297	0.209	0.326
	0.5	0.331	0.474	0.237	0.198	0.285	0.326	0.346	0.333	0.222	0.349
	0.75	0.481	0.655	0.387	0.287	0.445	0.563	0.611	0.529	0.306	0.522
0	-0.75	0.310	0.447	0.235	0.078	0.269	0.238	0.192	0.285	0.298	0.326
	-0.5	0.309	0.450	0.236	0.079	0.270	0.234	0.183	0.290	0.298	0.326
	-0.25	0.309	0.450	0.237	0.077	0.271	0.213	0.175	0.292	0.298	0.328
	0	0.311	0.451	0.235	0.069	0.271	0.181	0.166	0.292	0.291	0.328
	0.25	0.312	0.453	0.229	0.066	0.269	0.186	0.201	0.299	0.289	0.327
	0.5	0.331	0.476	0.234	0.081	0.285	0.299	0.341	0.329	0.276	0.349
	0.75	0.486	0.665	0.376	0.149	0.446	0.513	0.606	0.523	0.275	0.529
0.2	-0.75	0.308	0.448	0.253	0.180	0.276	0.273	0.201	0.290	0.384	0.325
	-0.5	0.312	0.447	0.253	0.178	0.276	0.235	0.190	0.291	0.378	0.329
	-0.25	0.312	0.449	0.251	0.173	0.276	0.203	0.178	0.291	0.373	0.328
	0	0.315	0.450	0.250	0.164	0.277	0.168	0.166	0.294	0.370	0.334
	0.25	0.309	0.454	0.236	0.142	0.267	0.172	0.202	0.292	0.356	0.327
	0.5	0.331	0.475	0.237	0.106	0.283	0.265	0.331	0.321	0.312	0.349
	0.75	0.482	0.661	0.362	0.053	0.433	0.422	0.591	0.495	0.227	0.525
0.4	-0.75	0.312	0.448	0.264	0.299	0.276	0.277	0.206	0.287	0.435	0.330
	-0.5	0.315	0.449	0.264	0.297	0.277	0.219	0.192	0.287	0.431	0.332
	-0.25	0.311	0.445	0.261	0.292	0.274	0.189	0.177	0.285	0.427	0.328
	0	0.314	0.451	0.259	0.284	0.275	0.162	0.169	0.285	0.421	0.328
	0.25	0.312	0.454	0.245	0.266	0.267	0.154	0.199	0.285	0.393	0.327
	0.5	0.330	0.471	0.239	0.235	0.277	0.207	0.326	0.304	0.343	0.346
	0.75	0.476	0.658	0.349	0.177	0.398	0.305	0.576	0.436	0.224	0.523

Table 15: Bias of the estimators obtained from 10,000 realizations of length $n = 300$ of ARFIMA(1,*d*,0)-GARCH(1,1) processes with *t*-distributed innovations and ARFIMA parameters $d = -0.4,\ -0.2,\ 0,\ 0.2, 0.4$ and $\phi_1 = -0.75, -0.5, -0.25, 0,\ 0.25, 0.5, 0.75$ (estimation is based on the first $\mathrm{K} = 54$ Fourier frequencies).

d	ϕ_1	$\hat{d}_{GPH}$	$\hat{d}_{GPH}^{tr}$	$\hat{d}_{smP}^{0.9}$	$\hat{d}_{smP}^{0.5}$	$\hat{d}_{W}$	$\hat{d}_{rAC}$	$\hat{d}_{H}$	$\hat{d}_{KS}$	$\hat{d}_{trF}$	$\hat{d}_{TS}$
-0.4	-0.75	0.029	0.021	0.025	0.266	0.006	0.281	0.164	0.028	0.129	0.017
	-0.5	0.011	0.005	0.011	0.264	-0.009	0.218	0.164	0.014	0.133	0.002
	-0.25	0.013	0.009	0.011	0.264	-0.006	0.174	0.150	0.019	0.130	0.005
	0	0.013	0.007	0.008	0.268	-0.008	0.146	0.137	0.017	0.130	0.004
	0.25	0.019	0.018	0.017	0.278	0.001	0.148	0.146	0.030	0.126	0.017
	0.5	0.045	0.052	0.041	0.303	0.026	0.237	0.231	0.063	0.125	0.048
	0.75	0.174	0.223	0.167	0.372	0.160	0.478	0.465	0.225	0.149	0.209
-0.2	-0.75	-0.002	-0.008	-0.024	0.119	-0.023	0.087	0.035	0.002	-0.058	-0.010
	-0.5	0.000	0.003	-0.023	0.120	-0.022	0.053	0.035	0.006	-0.055	-0.005
	-0.25	0.001	0.004	-0.021	0.122	-0.019	0.029	0.036	0.008	-0.055	-0.003
	0	0.000	0.003	-0.021	0.125	-0.019	0.022	0.039	0.009	-0.056	-0.002
	0.25	0.010	0.017	-0.013	0.134	-0.011	0.065	0.081	0.019	-0.052	0.009
	0.5	0.039	0.051	0.016	0.158	0.019	0.186	0.199	0.055	-0.038	0.045
	0.75	0.170	0.218	0.144	0.223	0.155	0.443	0.447	0.220	0.059	0.204
0	-0.75	-0.009	-0.008	-0.044	-0.017	-0.028	-0.051	-0.033	0.001	-0.150	-0.012
	-0.5	-0.007	-0.006	-0.045	-0.017	-0.028	-0.053	-0.028	0.001	-0.150	-0.011
	-0.25	-0.007	-0.007	-0.045	-0.015	-0.028	-0.044	-0.017	0.000	-0.149	-0.012
	0	-0.003	-0.003	-0.041	-0.013	-0.023	-0.017	0.009	0.003	-0.145	-0.008
	0.25	0.003	0.008	-0.033	-0.004	-0.015	0.044	0.065	0.015	-0.140	0.004
	0.5	0.032	0.047	-0.005	0.017	0.013	0.172	0.188	0.051	-0.110	0.040
	0.75	0.164	0.214	0.124	0.076	0.151	0.415	0.436	0.217	0.026	0.199
0.2	-0.75	-0.013	-0.012	-0.059	-0.150	-0.032	-0.099	-0.057	-0.005	-0.190	-0.015
	-0.5	-0.012	-0.009	-0.059	-0.149	-0.031	-0.062	-0.042	-0.003	-0.189	-0.014
	-0.25	-0.008	-0.003	-0.055	-0.148	-0.028	-0.037	-0.021	0.001	-0.186	-0.009
	0	-0.006	0.000	-0.054	-0.145	-0.025	-0.006	0.007	0.006	-0.181	-0.007
	0.25	0.005	0.013	-0.042	-0.138	-0.013	0.052	0.066	0.018	-0.174	0.007
	0.5	0.033	0.050	-0.015	-0.121	0.014	0.167	0.185	0.052	-0.141	0.041
	0.75	0.160	0.211	0.108	-0.074	0.145	0.365	0.423	0.210	-0.022	0.195
0.4	-0.75	-0.012	-0.005	-0.068	-0.286	-0.033	-0.088	-0.063	-0.005	-0.228	-0.014
	-0.5	-0.013	-0.007	-0.069	-0.285	-0.034	-0.045	-0.042	-0.006	-0.229	-0.015
	-0.25	-0.009	-0.002	-0.064	-0.284	-0.029	-0.023	-0.021	-0.001	-0.229	-0.012
	0	-0.011	-0.006	-0.065	-0.283	-0.030	-0.003	0.004	-0.001	-0.227	-0.012
	0.25	0.003	0.009	-0.052	-0.277	-0.017	0.045	0.061	0.013	-0.218	0.003
	0.5	0.033	0.054	-0.024	-0.264	0.013	0.133	0.174	0.051	-0.192	0.042
	0.75	0.160	0.216	0.101	-0.229	0.145	0.263	0.406	0.206	-0.096	0.197

Table 16: RMSE of the estimators obtained from 10,000 realizations of length $n = 300$ of ARFIMA(1,d,0)-GARCH(1,1) processes with t-distributed innovations and ARFIMA parameters $d = -0.4,\ -0.2,\ 0,\ 0.2, 0.4$ and $\phi_1 = -0.75, -0.5, -0.25, 0,\ 0.25, 0.5, 0.75$ (estimation is based on the first $\mathrm{K} = 54$ Fourier frequencies).

d	ϕ_1	$\hat{d}_{GPH}$	$\hat{d}_{GPH}^{tr}$	$\hat{d}_{smP}^{0.9}$	$\hat{d}_{smP}^{0.5}$	$\hat{d}_W$	$\hat{d}_{rAC}$	$\hat{d}_H$	$\hat{d}_{KS}$	$\hat{d}_{trF}$	$\hat{d}_{TS}$
-0.4	-0.75	0.248	0.313	0.189	0.271	0.213	0.329	0.205	0.236	0.228	0.260
	-0.5	0.246	0.311	0.187	0.269	0.213	0.298	0.202	0.233	0.229	0.259
	-0.25	0.244	0.312	0.187	0.269	0.212	0.264	0.190	0.235	0.228	0.258
	0	0.244	0.310	0.185	0.273	0.210	0.227	0.177	0.231	0.227	0.257
	0.25	0.241	0.311	0.186	0.282	0.209	0.209	0.183	0.235	0.224	0.255
	0.5	0.245	0.313	0.190	0.307	0.213	0.269	0.254	0.245	0.224	0.259
	0.75	0.297	0.381	0.251	0.374	0.266	0.491	0.477	0.333	0.254	0.326
-0.2	-0.75	0.240	0.307	0.187	0.129	0.210	0.191	0.121	0.230	0.211	0.254
	-0.5	0.239	0.305	0.186	0.130	0.208	0.183	0.117	0.230	0.211	0.253
	-0.25	0.242	0.310	0.186	0.132	0.209	0.165	0.115	0.232	0.210	0.254
	0	0.241	0.308	0.187	0.134	0.210	0.143	0.114	0.232	0.210	0.253
	0.25	0.240	0.308	0.186	0.143	0.209	0.141	0.135	0.233	0.212	0.253
	0.5	0.239	0.310	0.186	0.166	0.207	0.218	0.226	0.238	0.215	0.254
	0.75	0.293	0.379	0.238	0.228	0.265	0.457	0.460	0.331	0.257	0.323
0	-0.75	0.240	0.310	0.194	0.053	0.210	0.158	0.115	0.233	0.286	0.256
	-0.5	0.238	0.304	0.192	0.053	0.209	0.152	0.110	0.228	0.284	0.251
	-0.25	0.238	0.308	0.194	0.052	0.209	0.137	0.106	0.232	0.286	0.255
	0	0.239	0.309	0.194	0.052	0.210	0.117	0.106	0.232	0.283	0.255
	0.25	0.235	0.307	0.188	0.049	0.204	0.114	0.124	0.229	0.279	0.248
	0.5	0.240	0.312	0.188	0.051	0.208	0.201	0.216	0.240	0.274	0.254
	0.75	0.288	0.376	0.225	0.086	0.260	0.428	0.449	0.327	0.257	0.321
0.2	-0.75	0.236	0.304	0.199	0.158	0.207	0.162	0.122	0.227	0.317	0.247
	-0.5	0.238	0.306	0.200	0.157	0.210	0.134	0.115	0.229	0.318	0.252
	-0.25	0.242	0.308	0.200	0.156	0.212	0.115	0.108	0.233	0.315	0.254
	0	0.239	0.310	0.200	0.153	0.210	0.103	0.107	0.232	0.312	0.254
	0.25	0.240	0.309	0.197	0.146	0.209	0.113	0.125	0.234	0.309	0.251
	0.5	0.241	0.310	0.193	0.130	0.209	0.194	0.214	0.240	0.288	0.255
	0.75	0.285	0.372	0.219	0.084	0.256	0.378	0.437	0.318	0.224	0.315
0.4	-0.75	0.238	0.305	0.206	0.290	0.211	0.149	0.127	0.229	0.326	0.251
	-0.5	0.243	0.310	0.209	0.290	0.213	0.116	0.115	0.231	0.328	0.257
	-0.25	0.242	0.311	0.207	0.288	0.212	0.105	0.110	0.230	0.329	0.257
	0	0.238	0.308	0.205	0.287	0.209	0.100	0.109	0.228	0.325	0.251
	0.25	0.237	0.306	0.201	0.280	0.207	0.108	0.124	0.227	0.319	0.251
	0.5	0.241	0.312	0.196	0.268	0.210	0.165	0.208	0.235	0.293	0.253
	0.75	0.286	0.375	0.218	0.232	0.253	0.282	0.423	0.305	0.205	0.316

Table 17: Bias of the estimators obtained from 10,000 realizations of length $n = 1{,}000$ of ARFIMA(1,d,0)-GARCH(1,1) processes with t-distributed innovations and ARFIMA parameters $d = -0.4,\ -0.2,\ 0,\ 0.2, 0.4$ and $\phi_1 = -0.75, -0.5, -0.25, 0,\ 0.25, 0.5, 0.75$ (estimation is based on the first $\mathrm{K} = 125$ Fourier frequencies).

d	ϕ_1	$\hat{d}_{GPH}$	$\hat{d}_{GPH}^{tr}$	$\hat{d}_{smP}^{0.9}$	$\hat{d}_{smP}^{0.5}$	$\hat{d}_{W}$	$\hat{d}_{rAC}$	$\hat{d}_{H}$	$\hat{d}_{KS}$	$\hat{d}_{trF}$	$\hat{d}_{TS}$
-0.4	-0.75	0.028	0.023	0.029	0.284	0.013	0.301	0.234	0.028	0.113	0.020
	-0.5	0.015	0.011	0.017	0.282	0.001	0.258	0.228	0.019	0.117	0.009
	-0.25	0.016	0.014	0.017	0.283	0.003	0.217	0.208	0.021	0.116	0.011
	0	0.012	0.008	0.012	0.283	-0.002	0.168	0.178	0.015	0.118	0.006
	0.25	0.010	0.009	0.011	0.285	-0.002	0.133	0.154	0.016	0.119	0.007
	0.5	0.022	0.023	0.023	0.294	0.010	0.161	0.176	0.031	0.114	0.022
	0.75	0.067	0.078	0.065	0.331	0.054	0.351	0.341	0.090	0.116	0.080
-0.2	-0.75	-0.002	0.000	-0.014	0.134	-0.015	0.105	0.076	0.005	-0.055	-0.004
	-0.5	-0.006	-0.005	-0.017	0.133	-0.018	0.079	0.071	0.002	-0.056	-0.007
	-0.25	-0.005	-0.004	-0.016	0.133	-0.017	0.050	0.058	0.002	-0.057	-0.007
	0	0.000	0.001	-0.013	0.134	-0.014	0.023	0.045	0.006	-0.054	-0.002
	0.25	0.000	0.000	-0.012	0.137	-0.012	0.034	0.056	0.008	-0.056	-0.002
	0.5	0.007	0.010	-0.005	0.146	-0.005	0.107	0.119	0.018	-0.047	0.008
	0.75	0.053	0.065	0.040	0.179	0.042	0.322	0.315	0.079	-0.004	0.068
0	-0.75	-0.005	-0.002	-0.027	-0.008	-0.018	-0.025	-0.017	0.003	-0.102	-0.006
	-0.5	-0.010	-0.009	-0.031	-0.009	-0.022	-0.025	-0.013	-0.001	-0.104	-0.011
	-0.25	-0.006	-0.005	-0.030	-0.008	-0.020	-0.022	-0.009	0.001	-0.103	-0.008
	0	-0.005	-0.002	-0.027	-0.007	-0.017	-0.010	0.003	0.004	-0.101	-0.006
	0.25	-0.004	-0.001	-0.026	-0.005	-0.017	0.022	0.030	0.004	-0.097	-0.004
	0.5	0.002	0.007	-0.019	0.002	-0.008	0.103	0.107	0.017	-0.089	0.005
	0.75	0.054	0.071	0.031	0.033	0.043	0.316	0.304	0.081	-0.025	0.071
0.2	-0.75	-0.005	-0.004	-0.034	-0.145	-0.018	-0.013	-0.002	0.002	-0.115	-0.007
	-0.5	-0.007	-0.005	-0.037	-0.145	-0.020	0.001	0.012	0.002	-0.119	-0.009
	-0.25	-0.005	-0.004	-0.035	-0.145	-0.018	0.008	0.021	0.002	-0.113	-0.006
	0	-0.004	-0.001	-0.033	-0.144	-0.016	0.008	0.025	0.006	-0.113	-0.004
	0.25	-0.003	-0.001	-0.033	-0.142	-0.016	0.007	0.025	0.005	-0.113	-0.004
	0.5	0.005	0.009	-0.025	-0.137	-0.008	0.011	0.030	0.017	-0.101	0.008
	0.75	0.054	0.070	0.023	-0.112	0.042	0.056	0.074	0.081	-0.046	0.071
0.4	-0.75	-0.006	-0.003	-0.040	-0.284	-0.020	0.030	0.022	0.000	-0.151	-0.007
	-0.5	-0.005	-0.002	-0.038	-0.284	-0.017	0.038	0.036	0.004	-0.149	-0.006
	-0.25	-0.006	-0.002	-0.039	-0.284	-0.019	0.034	0.037	0.002	-0.149	-0.006
	0	-0.008	-0.005	-0.040	-0.283	-0.020	0.032	0.035	0.001	-0.149	-0.007
	0.25	-0.002	0.000	-0.035	-0.282	-0.015	0.030	0.033	0.006	-0.145	-0.002
	0.5	0.007	0.013	-0.026	-0.278	-0.005	0.033	0.037	0.019	-0.136	0.012
	0.75	0.051	0.066	0.018	-0.261	0.041	0.064	0.073	0.076	-0.098	0.067

Table 18: RMSE of the estimators obtained from 10,000 realizations of length $n = 1,000$ of ARFIMA(1,d,0)-GARCH(1,1) processes with t-distributed innovations and ARFIMA parameters $d = -0.4, -0.2, 0, 0.2, 0.4$ and $\phi_1 = -0.75, -0.5, -0.25, 0, 0.25, 0.5, 0.75$ (estimation is based on the first $\mathrm{K} = 125$ Fourier frequencies).

d	ϕ_1	$\hat{d}_{GPH}$	$\hat{d}_{GPH}^{tr}$	$\hat{d}_{smP}^{0.9}$	$\hat{d}_{smP}^{0.5}$	$\hat{d}_{W}$	$\hat{d}_{rAC}$	$\hat{d}_{H}$	$\hat{d}_{KS}$	$\hat{d}_{trF}$	$\hat{d}_{TS}$
-0.4	-0.75	0.198	0.227	0.158	0.287	0.169	0.332	0.249	0.190	0.220	0.209
	-0.5	0.193	0.224	0.155	0.285	0.168	0.299	0.242	0.188	0.224	0.205
	-0.25	0.195	0.227	0.156	0.286	0.169	0.263	0.222	0.190	0.222	0.207
	0	0.191	0.221	0.153	0.285	0.166	0.215	0.193	0.185	0.224	0.203
	0.25	0.193	0.227	0.155	0.288	0.167	0.176	0.170	0.187	0.224	0.206
	0.5	0.190	0.222	0.153	0.297	0.164	0.185	0.189	0.188	0.222	0.202
	0.75	0.199	0.235	0.165	0.333	0.173	0.359	0.347	0.209	0.220	0.216
-0.2	-0.75	0.186	0.221	0.152	0.140	0.163	0.168	0.107	0.185	0.206	0.201
	-0.5	0.189	0.223	0.154	0.139	0.165	0.152	0.102	0.185	0.204	0.202
	-0.25	0.186	0.221	0.152	0.139	0.163	0.127	0.091	0.184	0.205	0.201
	0	0.185	0.218	0.150	0.140	0.161	0.105	0.082	0.183	0.203	0.198
	0.25	0.186	0.222	0.152	0.142	0.162	0.091	0.087	0.185	0.206	0.201
	0.5	0.188	0.222	0.153	0.151	0.164	0.129	0.136	0.186	0.204	0.201
	0.75	0.193	0.230	0.157	0.183	0.168	0.328	0.322	0.205	0.215	0.211
0	-0.75	0.186	0.219	0.156	0.040	0.164	0.108	0.071	0.186	0.253	0.200
	-0.5	0.188	0.222	0.158	0.041	0.165	0.099	0.069	0.185	0.257	0.202
	-0.25	0.187	0.222	0.158	0.041	0.164	0.088	0.066	0.186	0.255	0.202
	0	0.185	0.220	0.155	0.040	0.162	0.074	0.065	0.185	0.253	0.202
	0.25	0.185	0.219	0.154	0.039	0.162	0.070	0.073	0.184	0.251	0.199
	0.5	0.186	0.220	0.154	0.039	0.161	0.121	0.126	0.186	0.250	0.202
	0.75	0.194	0.231	0.158	0.049	0.171	0.322	0.311	0.208	0.239	0.213
0.2	-0.75	0.187	0.221	0.160	0.150	0.163	0.175	0.149	0.185	0.254	0.201
	-0.5	0.185	0.220	0.158	0.150	0.162	0.156	0.149	0.183	0.252	0.199
	-0.25	0.185	0.219	0.157	0.150	0.161	0.144	0.146	0.182	0.249	0.200
	0	0.189	0.225	0.161	0.149	0.166	0.138	0.149	0.189	0.251	0.205
	0.25	0.187	0.222	0.160	0.148	0.163	0.134	0.149	0.187	0.250	0.202
	0.5	0.186	0.220	0.157	0.142	0.162	0.134	0.149	0.186	0.243	0.200
	0.75	0.192	0.229	0.156	0.118	0.169	0.145	0.164	0.206	0.216	0.209
0.4	-0.75	0.188	0.222	0.163	0.287	0.165	0.150	0.151	0.185	0.245	0.202
	-0.5	0.187	0.223	0.163	0.286	0.164	0.141	0.155	0.186	0.245	0.203
	-0.25	0.187	0.221	0.163	0.286	0.165	0.138	0.156	0.186	0.244	0.202
	0	0.186	0.220	0.163	0.286	0.164	0.137	0.156	0.183	0.243	0.199
	0.25	0.186	0.221	0.161	0.284	0.163	0.138	0.157	0.183	0.240	0.200
	0.5	0.185	0.220	0.159	0.280	0.163	0.140	0.157	0.186	0.231	0.199
	0.75	0.194	0.233	0.159	0.263	0.171	0.149	0.172	0.203	0.202	0.213