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Modeling Multivariate Time Series with Univariate Seasonal Components

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Abstract

This work focused on method of modeling multivariate time series with seasonal univariate components. Five variables representing Nigeria's Gross Domestic Products (GDP) were found to exhibit seasonal behaviours. These series were subjected to Box and Jenkins techniques and different univariate seasonal models were entertained for each component. The residuals from the fitted univariate models were cross examined. The correlation and cross correlation structures of these residuals revealed the inter-relationships among the variables, and multivariate consideration was therefore obvious. Multivariate order selection technique was employed to obtain the vector autoregressive (VAR) order of the model. A VAR (1) model was identified and developed to fit the data. Stability of the VAR process was achieved. Diagnostic checks were applied to the fitted model and the model was found to be adequate. Hence, forecasts were generated.

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1 Introduction

The analysis of time dependent variables is one of the methods designed for prediction of future events. Most variables are economical in nature and the economy of any nation partly depends on the interplay of these variables with respect to time. Indeed time series plays a vital role in planning and predicting the future economy of any nation.

Nigerian Economy is not stable over the years and as a result, the country is facing some economic crises, challenges or shocks which are internally or externally over some decades. Internally, as a result of investments and consumption pattern, as well as improper implementation of public policy and change in expectation. Externally, the crises could be as a result of population increase, revolution or war etc. Economic development of a country shows its ability to increase production of goods and services. It clearly defines increase in the Gross Domestic Product (GDP) of a country.

Macro-economic variables are instrumental in the economic performance of any country. Nigeria’s Economy has faced numerous challenges which have led to a fall in its growth rate in both Agricultural and non-Agricultural sectors which in turn affect the Gross Domestic product (GDP). It is therefore the intent of this work to study the inter-relationships among these sectors in Nigeria’s GDP. The variables under consideration are: Agriculture, Industry, Building & Construction, wholesale and retail, and Services.

2 Literature Review

A time series is a collection of observations made sequentially in time. On the other hand, analysis of time series comprises methods for analyzing time series data to extract relevant statistics and other features of the data with the aim of making future predictions.

[1] carried out a research on interest rate, Gross domestic product and inflation in the economy of Jordan using unit root test to check the integration order of the variables. The result showed that inflation causes interest rate while other variables were independent with each other. The regression result also suggested that the current interest rate has influence on growth rate and current Gross domestic.

[9] researched on inflation and economic growth in Nigeria by applying co-integration and Granger causality test. The findings suggested that there was a co-integration between inflation and economic growth. Also through empirical findings, it was discovered that inflation has no impact on growth.

[8] worked on buy-ballout modeling of Nigerian Domestic crude oil production using inverse square root transformation to make the variance stable. Quadratic trends were fitted and the error component was discovered to be normally distributed with zero mean and constant variance.

[2] examined unemployment and inflation on economic growth in Nigeria. He also applied causality test on Gross domestic product, unemployment and inflation. The study revealed that all the variables in the model were stationary. Further result indicated that unemployment and inflation possesses positive impact on economic growth.

[11] used quarterly time series data to estimate the threshold level of inflation using 13% threshold. The findings revealed that inflation has a mild effect on economic activities; and the magnitude of the negative effect of inflation in growth was higher.

[3] modeled time series using South Africa inflation data. The research was based on financial time series and autoregressive integrated moving average (ARIMA) model. Conditional heteroscedasticity (ARCH) model was fitted to the data. Box and Jenkins strategies were employed and the best fitted model was chosen from the family of models.

[4] conducted a study for 131 countries using Vector autoregressive analysis. It was discovered that higher crude oil prices were more severe for the oil importing poorer countries as compared to the developed countries. The work further revealed that with 10 Dollars per barrel increase in the price of crude oil, economic growth could decrease up to 4%.

[7] carried out a research on the inflation rate of three African Countries. The series were observed to exhibit non seasonal behavior; thus non seasonal ARIMA models were applied to each series and were adequately represented. The three series were modeled using the multivariate approach. The multivariate method was found to give adequate representation than the non seasonal linear approach.

[5] used Autocorrelation and partial Autocorrelation to identify multivariate time series model after confirming stationarity. The Akaike information criteria (AIC) and Schwartz (Bayesian) information criteria(SIC) were used to select the best model among the identified. VAR (2) multivariate model was identified as the best fitted model.

[10] compared the pared the performance between the univariate and bivariate time series models. Several tests were carried out in the comparative study. In the work, the bivariate model was found to be superior to the univariate models. The bivariate model was also found to give optimal forecasts than the univariate models. [10] concluded that if two variables are found to be interrelated, a bivariate model should be adopted rather than giving them separate univariate models.

Due to the present fall in the oil price which Nigeria so much depended on as the major source of revenue, there is need to go back to Agriculture. However, focusing attention only on Agricultural sector may not solve the problem. There is need for Government to consider other sectors that are involved in making up Nigeria's Gross Domestic products (GDP). This work intends to capture these variables along with Agricultural sector with the aim of studying their inter-relationships with respect to time and possibly develop a model for predicting the future of the various sectors under consideration.

3 Methodology

3.1 The Univariate Case

3.1.1 Stationarity

A time series is said to be stationary if the statistical property e.g. the mean and variance are constant through time. A non stationary series X_t can be made stationary by differencing. The differenced series is given as

$$Y_t = X_t - X_{t-1} \ .$$

3.1.2 Backward shift Operator

The Backward shift Operator B is defined by

$$B^m X_t = X_{t-m}$$

3.1.3 The Backward Difference Operator

The backward difference operator, ∇, is define by

$$\nabla = 1 - \mathrm{B}$$

3.1.4 Seasonal Autoregressive Integrated Moving Average

Seasonal autoregressive integrated moving average (SARIMA) model is used for time series with seasonal and non seasonal behaviour. The SARIMA multiplicative model is written as

$$SARIMA\ (p,d,q) \times (P,D,Q)_s \tag{1}$$

and this can expressed explicitly as

$$\phi_p(B)\Phi_P(B^s)\nabla^d\nabla_s^D X_t = \theta_q(B)\Theta_Q(B^s)\varepsilon_t \tag{2}$$

where

$$\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \cdots - \phi_p B^p\ ,$$

$$\Phi(B) = 1 - \Phi_{1,s}B^s - \Phi_{2,s}B^{2s} - \cdots - \Phi_{p,s}B^p,$$

$$\nabla = 1 - B,\ \nabla_s = 1 - B^s,\ \ \theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \cdots - \theta_q B^q,$$

$$\Theta(B^s) = 1 - \Theta_{1,s}B^s - \Theta_{2,s}B^{2s} - \cdots - \Theta_{Q,s}B^{QS},$$

X_t is the time series at period t, ε_t is the white noise process, s is the season,

p is the order of autoregressive components,

P is the order of seasonal autoregressive components,

d is the order of non-seasonal differencing, D is the order of seasonal differencing,

q is the order of moving average component,

Q is the order of seasonal moving average component.

3.1.5 Autocorrelation Function (acf)

This is covariance between X_t and $X_{t+k,}$ seperated by k interval of time or $lag\ k$ and is given by

$$\rho_k = \frac{Cov(X_t\, X_{t+k})}{\sqrt{Var(X_t)Var(X_{t+k})}}$$

3.1.6 Partial Autocorrelation Function

This is the correlation between X_t and X_{t+k} after mutual linear dependency in the intervening variable $X_{t+1}, X_{t+2}, \dots, X_{t+k}$ has been removed and is given by

$$\phi_{kk} = Corr(X_t\, X_{t+k}\ /X_{t+1}X_{t+3}, \dots, X_{t+k-1})$$

3.2 Multivariate Time Series

Multivariate time series is a time series that does not limit itself to the past or present of its previous information but also to the present and past information of other series.

3.2.1 White Noise Process

A white noise process $\varepsilon_t = (\varepsilon_{1t}, \dots, \varepsilon_{nt})'$ is a continuous random vector satisfying

$$E(\varepsilon_t) = 0, \Sigma_\varepsilon = E(\varepsilon_t \varepsilon_t'),\ \varepsilon_t \text{ and } \varepsilon_s \text{ are independent for } s \neq t.$$

3.2.2 Vector Autoregressive (VAR) Model

One of the models that describes the multivariate times series is the Vector Autoregressive (VAR) Model. VAR model is an independent reduced form dynamic model which involves constructing an equation that makes each endogenous variable a function of its own past values and past values of all other endogenous variables: The basic p-lag Vector autoregressive VAR(p) model has the form.

$$y_t = c + \Pi_1 y_{t-1} + \Pi_2 y_{t-2} + \cdots + \Pi_p y_{t-p} + \varepsilon_t \quad ;\ t = 0, \pm 1, \pm 2, \dots \qquad (3)$$

where

$y_t = (y_{1t}, \dots, y_{nt})'$ is an $(n \times 1)$ vector of time series variable,

Π_i are fixed $(n \times n)$ coefficient matrices,

$c = (c_1, \ldots, c_n)'$ is a fixed $(n \times 1)$ vector of intercept terms allowing for the possibility of non zero mean $E(y_t)$,

$\varepsilon_t = (\varepsilon_{1t}, \ldots, \varepsilon_{nt})'$ is an $(n \times 1)$ white noise process or innovation process. That is,

$$E(\varepsilon_t) = 0, E(\varepsilon_t \varepsilon_t') = \Sigma_\varepsilon \text{ and } E(\varepsilon_t \varepsilon_s') = 0 \text{ for } s \neq t$$

$\Sigma_\varepsilon =$ covariance matrix which is assume to be non singular if not otherwise stated.

The model can be written in the matrix form as

$$\begin{pmatrix} y_{1t} \\ y_{2t} \\ . \\ . \\ y_{nt} \end{pmatrix} = \begin{pmatrix} c_1 \\ c_2 \\ . \\ . \\ c_n \end{pmatrix} + \begin{pmatrix} \pi_{11}^1 & \pi_{12}^1 & . & . & \pi_{1n}^1 \\ \pi_{21}^1 & \pi_{22}^1 & . & . & \pi_{2n}^1 \\ . & . & . & . & . \\ . & . & . & . & . \\ \pi_{n1}^1 & \pi_{n2}^1 & . & . & \pi_{nn}^1 \end{pmatrix} \begin{pmatrix} y_{1t-1} \\ y_{2t-1} \\ . \\ . \\ y_{nt-1} \end{pmatrix} +$$

$$\begin{pmatrix} \pi_{11}^2 & \pi_{12}^2 & . & . & \pi_{1n}^2 \\ \pi_{21}^2 & \pi_{22}^2 & . & . & \pi_{2n}^2 \\ . & . & . & . & . \\ . & . & . & . & . \\ \pi_{n1}^2 & \pi_{n2}^2 & . & . & \pi_{nn}^2 \end{pmatrix} \begin{pmatrix} y_{1t-2} \\ y_{2t-2} \\ . \\ . \\ y_{nt-2} \end{pmatrix}$$

$$+ \cdots + \begin{pmatrix} \pi_{11}^p & \pi_{12}^p & . & . & \pi_{1n}^p \\ \pi_{21}^p & \pi_{22}^p & . & . & \pi_{2n}^p \\ . & . & . & . & . \\ . & . & . & . & . \\ \pi_{n1}^p & \pi_{n2}^p & . & . & \pi_{nn}^p \end{pmatrix} \begin{pmatrix} y_{1t-p} \\ y_{2t-p} \\ . \\ . \\ y_{nt-p} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ . \\ . \\ \varepsilon_{nt} \end{pmatrix} \quad (4)$$

3.2.3 Stationary Process

A stochastic process is said to be stationary if its first and second moments are time invariant. In other words, a stochastic process y_t is stationary, if

$E(y_t) = \mu$ for all t

and

$E[(y_t - \mu)(y_{t-k} - \mu)'] = \Gamma_y(k) = \Gamma_y(-k)'$ for all t and $k = 0,1,2 \ldots$

3.2.4 Stable VAR (p) Processes

The process (3) is stable if its reverse characteristic polynomial of the VAR(p) has no roots in and on the complex unit circle. Formally $\mathrm{y_t}$ is stable if

$$\det\,(I_n - \Pi_1 z - \cdots - \Pi_p z^p) \neq 0 \text{ for } |z| \leq 1. \tag{5}$$

A stable VAR(p) process $y_t, t = 0, \pm 1, \pm 2, \ldots$, is stationary.

3.2.5 *Autocovariances of a Stable VAR(p) Process*

For a vector autoregressive process of order p [$VAR(p)$], we have

$$y_t - \mu = \pi_1(y_{t-1} - \mu) + \cdots + \pi_p\big(y_{t-p} - \mu\big) + \varepsilon_t\,, \tag{6}$$

Post multiplying both sides by $(y_{t-k} - \mu)'$ and taking expectation, we have for k= 0 using $\Gamma_{\mathrm{y}}(i) = \Gamma_{\mathrm{y}}(-i)'$

$$\Gamma_{\mathrm{y}}(0) = \pi_1(y_{t-1} - \mu) + \cdots + \pi_p\big(y_{t-p} - \mu\big) + \Sigma_\varepsilon$$

$$= \pi_1\Gamma_{\mathrm{y}}(1)' + \cdots + \pi_p\Gamma_{\mathrm{y}}(p)' + \Sigma_\varepsilon \tag{7}$$

If $h > 0$

$$\Gamma_{\mathrm{y}}(k) = \pi_1\Gamma_{\mathrm{y}}(k-1) + \cdots + \pi_p\Gamma_{\mathrm{y}}(k-p) \tag{8}$$

These equations can be used to compute the autocovariance functions $\Gamma_{\mathrm{y}}(\mathrm{k})$ for $k \geq p$, if $\pi_1, \ldots, \pi_p$ and $\Gamma_{\mathrm{y}}(\mathrm{p}-1), \ldots, \Gamma_{\mathrm{y}}(0)$ are known.

3.2.6 Autocorrelation of a Stable VAR(p) Process

For a stable VAR (p) process, the autocorrelations are given by

$$R_y(k) = D^{-1}\Gamma_{\mathrm{y}}(\mathrm{k})D^{-1} \tag{9}$$

Here D is a diagonal matrix with the standard deviation of the component of y_t on the main diagonal. Thus,

$$D^{-1} = \begin{bmatrix} \frac{1}{\sqrt{\gamma_{11}(0)}} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \frac{1}{\sqrt{\gamma_{nn}(0)}} \end{bmatrix} \qquad (10)$$

and the correlation between $y_{i,t}$ and $y_{j,t-k}$ is

$$\rho_{ij}(k) = \frac{\gamma_{ij}(k)}{\sqrt{\gamma_{ii}(0)}\sqrt{\gamma_{jj}(0)}} \qquad (11)$$

which is just the $ij -$ th element of $R_y(k)$.

3.2.7 VAR Order Selection

This work considers three basic ways usually called model selection criteria for determining the order p of the VAR process. The criteria are:

(i) *Akaike Information Criterion*

This is given by

$$AIC(p) = In\left|\tilde{\Sigma}_\varepsilon(p)\right| + \frac{2}{N}(\text{number of estimated parameter})$$

$$= In\left|\tilde{\Sigma}_\varepsilon(p)\right| + \frac{2pn^2}{N}$$

The estimate $(\widehat{AIC})$ for p is chosen so that this criterion is minimized.

(ii) *Hannan-Quin Criterion*

This is given as

$$HQ(p) = In\left|\tilde{\Sigma}_\varepsilon(p)\right| + \frac{2InIn}{N}(freely\ estimated\ parameters)$$

$$= In\left|\tilde{\Sigma}_\varepsilon(p)\right| + \frac{2InInN}{N}pn^2$$

The estimate $(\widehat{HQ})$ is the order that minimizes $HQ\ (p)$ for $p = 0,1,\ldots,P$

(iii) *Schwarz Criterion*

This is given by

$$SC(p) = In\left|\tilde{\Sigma}_\varepsilon(p)\right| + \frac{InN}{N}(freely\ estimated\ parameters)$$

$$= In\left|\tilde{\Sigma}_\varepsilon(p)\right| + \frac{2InN}{N}pn^2$$

The estimate $(\widehat{SC})$ is chosen so as to minimize the value of the criterion;

where p is the VAR order,

$\tilde{\Sigma}_\varepsilon$ is the estimate of white noise covariance matrix Σ_ε

n is the number of time series components of the vector time series

N is the sample size.

3.3 Diagnostic Checks

After fitting the model, we need to examine whether the model is adequate or not. One of the ways of checking the adequacy of the model is by examining the behaviour of the residuals matrices. This is simply to examine whether it follows a white noise process or not. According to [6] ; if $\rho_{uv}(i)$ is the true correlation coefficients corresponding to the $r_{uv}(i)$, then we have the following hypothesis test at 5% level to check whether or not a given multivariate series follows a white noise process or not. The hypothesis states:

$$H_0: \rho_{uv}(i) = 0$$

Against

$$H_1: \rho_{uv}(i) \neq 0$$

Decision

Reject H_0 if $\left|\sqrt{N} r_{uv,i}\right| > 2$ or

Equivalently

$$\left|r_{uv,i}\right| > \frac{2}{\sqrt{N}}$$

Thus in practical sense, we compute the correlation of the series to be tested (possibly after some stationary transformation) and compare their absolute value with $\frac{2}{\sqrt{N}}$.

3.4 Forecasting

Suppose that $y_t = (y_{1t}, \ldots, y_{nt})'$ is an $n-$ dimensional stable process $VAR(p)$. Then the minimum MSE predictor for forecast h at forecast origin time t is the conditional expected value given as:

$$E(y_{t+h}) = E\left(y_{t+h} \,\middle|\, \Omega_t\right) = E\left(y_{t+h} \,\middle|\, \{y_s \,|\, s \leq t\}\right);$$

and by recursion, $VAR(1)$ process gives

$$E_t(y_{t+h}) = \left(I_n + \pi_1 + \cdots + \pi_1^{h-1}\right)c + \pi_1^h y_t.$$

4 Data Analysis and Results

The data used in this work is a quarterly data obtained from Nigerian National bureau of Statistics (NNBS) for the period of 1981-2013. The five GDP variables of interest are Agriculture (y_{1t}), Industry (y_{2t}), Building & Construction (y_{3t}), Wholesale & Retail (y_{4t}), and Services (y_{5t}).

4.1 Raw Data Plots

The raw data plots of the five variables are shown below in Figure 1 below. The above plots reveal that the series are not stationary and were all differenced to obtain stationarity.

4.2 Modeling of the Univariate component

Figure 1 clearly shows that each component series exhibits seasonal behaviour. Since the major aim of this work is to build a multivariate (vector) model, we might not delve deep into univariate preambles. However, employing the univariate techniques in section 3.1, the following seasonal ARIMA models were fitted and found adequate for the five variables representing the sectors of Nigeria's Gross domestic products and the residuals were obtained for further analysis.

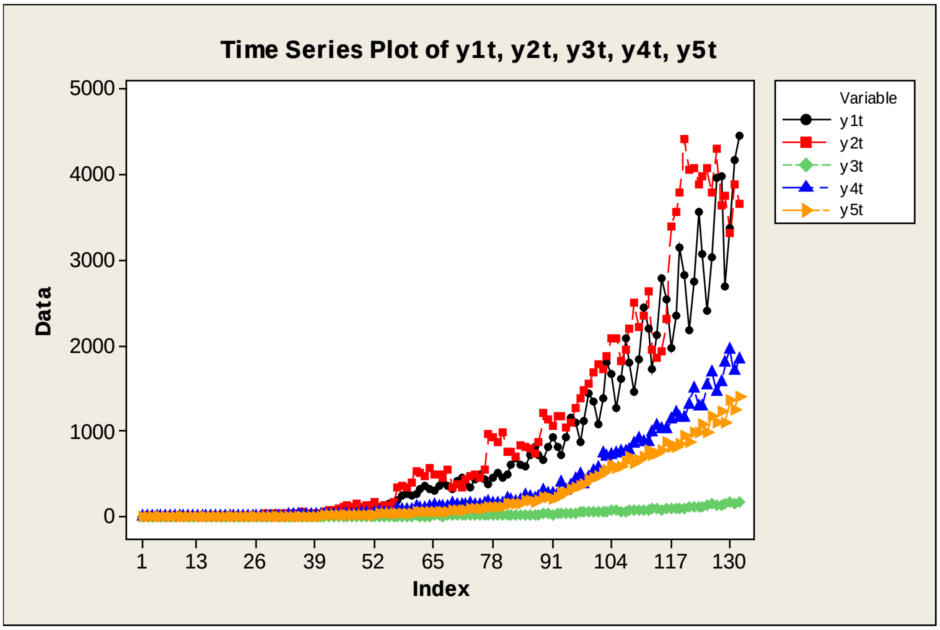

Figure 1: Series plots of the sectors in Nigeria's GDP

(i) *Agriculture* (y_{1t}) : SARIMA (1,1,1)(1,0,1)$_4$

(ii) *Industry* (y_{2t}) : SARIMA (0,1,1)(1,0,1)$_4$

(iii) *Building & Const*. (y_{3t}): SARIMA (1,1,1)(1,0,1)$_4$

(iv) *Wholesale & Retail* (y_{4t}) : SARIMA (1,1,0)(2,0,1)$_4$

(v) *Services* (y_{5t}): SARIMA (1,1,1)(2,0,1)$_4$

4.3 Residual Correlation and Cross Correlation

The residual correlations and cross correlations of the fitted univariate components are shown in the tables below:

Table 1: Residual Correlation Table of the Differenced Series

	$y_{1t}.$	y_{2t}	y_{3t}	y_{4t}	y_{5t}
$y_{it}.$	1				
y_{2t}	0.954784	1			
y_{3t}	0.666825	0.664304	1		
y_{4t}	0.767769	0.864995	0.695682	1	
y_{5t}	0.682775	0.667073	0.794183	0.892509	1

Table 2: Residual Cross Correlation Table of the Differenced Series

	$y_{1t}.$	y_{2t}	y_{3t}	y_{4t}	y_{5t}
$y_{1t}.$		0.855	0.767	0.768	0.683
y_{2t}			0.764	0.765	0.667
y_{3t}				0.696	0.794
y_{4t}					0.993
y_{5t}					

As seen above, the raw correlations and cross correlations are quite high; suggesting strong relationship among the variables. Thus, multivariate consideration is obvious.

4.4 VAR Order Selection

Using the obtained data for this work, the values of the three model selection criteria were computed using *gretl* software and are displayed in Table 3. It is clearly seen in the table that the three model selection criteria attain their minimum at *lag* 1 as indicated by the values with the asterisk. Thus, the selected model is $VAR(1)$.

Table 3: Model selection criteria table

$lags$	$loglik$	$p(LR)$	AIC	BIC	HQC
1	464.07333		-11.2345553^{*}	-12.53768^{*}	-12.951552^{*}
2	531.21692	0.00000	-10.936949	-10.659348	-10.418109
3	589.49804	0.00000	-8.491634	-6.633306	-7.736958
4	674.85161	0.00000	-9.497527	-7.058472	-8.507015
5	768.61450	0.00000	-10.643575	-7.623792	-9.417227
6	788.63349	0.02890	-10.560558	-6.960048	-9.098374
7	802.20315	0.34896	-10.370052	-6.188815	-8.672033
8	819.69297	0.08857	-10.244883	-5.482918	-8.311027
9	858.89334	0.00000	-10.481556	-5.138863	8.311863
10	881.00490	0.01024	-10.433415	-4.509995	-8.027887
11	896.66187	0.17883	-10.277696	-3.773550	-7.636334
12	918.5338	0.01158	-10.225564	-3.140689	-7.348363

4.5 Final model with the significant parameters

Examining Table 4 below, we observe that some parameters of the above model (4) are not significant and thus have to be removed from the expression. Hence, the final model of the $VAR(1)$ process becomes:

$$\begin{pmatrix} y_{1t} \\ y_{2t} \\ y_{3t} \\ y_{4t} \\ y_{5t} \end{pmatrix} = \begin{pmatrix} 18.2057 \\ 32.4675 \\ 0.0272 \\ -3.2784 \\ -1.51173 \end{pmatrix} +$$
$$+\begin{pmatrix} 0.6268 & -0.0125 & 0.3028 & 2.0804 & -3.1373 \\ 0.0106 & 0.7796 & 0.0000 & 0.0000 & -0.8521 \\ 0.007 & 0.0051 & 0.5229 & 0.0127 & 0.0044 \\ -0.0143 & 0.0636 & -0.9170 & 0.6270 & 0.0000 \\ 0.0000 & 0.0217 & -0.1356 & 0.0000 & 0.2351 \end{pmatrix} \begin{pmatrix} y_{1t-1} \\ y_{2t-1} \\ y_{3t-1} \\ y_{4t-1} \\ y_{5t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \\ \varepsilon_{4t} \\ \varepsilon_{5t} \end{pmatrix} \quad (12)$$

This can be expressed explicitely as:

$$y_{1t} = 18.2057 + 0.6268y_{1t-1} - 0.0125y_{2t-2} + 13.3028y_{3t-1} + 2.0804y_{4t-1} - 3.1373y_{5t-1}$$

$$y_{2t} = 32.4675 + 0.0106y_{1t-1} + 0.7796y_{2t-2} - 0.8521y_{5t-1}$$

$$y_{3t} = 0.0272 + 0.007y_{1t-1} + 0.0051y_{2t-2} + 0.5229y_{3t-1} + 0.0127y_{4t-1} + 0.0044y_{5t-1}$$

$$y_{4t} = -3.2784 - 0.0143y_{1t-1} + 0.0636y_{2t-1} - 0.917y_{3t-1} + 0.627y_{4t-1}$$

$$y_{5t} = -1.51173 + 0.0217y_{2t-2} - 0.1356y_{3t-1} + 0.2351y_{5t-1}$$

Table 4: Estimated Parameters for the VAR(1) model

$Model$	$Coefficients\ (P-values)$	$Significant\ parameters$
y_{1t}	$c_1 = 18.2052(0.0311)$ $\pi_{11} = 18.2052(0.0020)$ $\pi_{12} = -0.0125(0.0010$ $\pi_{13} = 0.3028(0.00410)$ $\pi_{14} = -3.1373(0.00010)$ $\pi_{15} = (0.00002)$	$\pi_{11}, \pi_{12}, \pi_{13}, \pi_{14}, \pi_{15}$

y_{2t}	c_2= 32.475 (0.16139) $\pi_{21} = 0.1064(0.0045)$ $\pi_{22} = 0.7796(0.00001)$ $\pi_{23} = -0.1223(0.98150)$ $\pi_{24} = -0.2514(0.48450)$ $\pi_{25} = 0.8521(0.00091)$	$\pi_{21}, \pi_{22}, \pi_{25}$
y_{3t}	c_3= 0.0272 (0.9704) $\pi_{31} = 0.007(0.00450)$ $\pi_{32} = 0.0051(0.00001)$ $\pi_{33} = 0.5229(0.00030)$ $\pi_{34} = 0.0127(0.00221)$ $\pi_{35} = 0.044(0.00251))$	$\pi_{31}, \pi_{32}, \pi_{33}, \pi_{34}, \pi_{35}$
y_{4t}	c_4= -3.2784 (0.65485) $\pi_{41} = -0.0143(0.00450)$ $\pi_{42} = 0.0636(0.00001)$ $\pi_{43} = -0.9170(0.00030)$ $\pi_{44} = 0.6270(0.00221)$ $\pi_{45} = 0.4784(0.00681))$	$\pi_{11}, \pi_{12}, \pi_{13}, \pi_{14}$
y_{5t}	c_5= -1.51173 (0.75384) $\pi_{51} = 0.0668(0.04610)$ $\pi_{52} = 0.00217(0.00362)$ $\pi_{53} = 0.1356(0.00081)$ $\pi_{54} = 0.6270(0.03501)$ $\pi_{55} = 0.2351(0.0041))$	$\pi_{52}, \pi_{53}, \pi_{55}$

4.6 Stability of the VAR (1) Process

Using expression (5), the roots of $I_n - \Pi_1 z - \cdots - \Pi_p z^p = 0$ are

$$z_1 = 5.23,\ z_2 = -7.41,\ z_3 = -1.57,\ z_4 = 11.2,\ z_5 = 8.11.$$

Since $|z_j| > 1 \ \ \forall\, j$, the process is stable. In other words, it is stable since all the roots of $I_n - \Pi_1 z - \cdots - \Pi_p z^p = 0$ lie outside the unit circle.

5 Diagnosis

After obtaining the above model, the next step is to carry out diagnostic checks to ascertain whether the above VAR(1) model is adequate or not. This is achieved by following the hypothesis stated in section 3.3 of the methodology. Thus we have

$$H_0: \rho_{uv}(i) = 0$$

Against

$$H_1: \rho_{uv}(i) \neq 0$$

Since

$$N = 132 \Rightarrow \frac{2}{\sqrt{132}} = 0.1741$$

Then

$$H_0 \text{ is rejected if } |r_{uv,i}| > \frac{2}{\sqrt{N}} = 0.1741.$$

Now, examining the residual correlation matrices at different lags in Appendix A; it clearly shows that none of the residual autocorrelations $|r_{uv,i}|$ is greater than 0.1741. In other words, the residuals follow a white noise process. This shows that the fitted model is adequate.

6 Forecasts

Since the obtained model is adequate, it can now be used for prediction. The quarterly forecasts generated for the next seven years are displayed in Table 5.

Table 5: Forecasts

$Year$	y_{1t}	y_{2t}	y_{3t}	y_{4t}	y_{5t}
2014	4472.11	3761.21	172.03	1922.23	1612.56
	5112.28	3928.16	181.35	1969.35	1689.63
	5321.27	4110.23	193.42	1023.44	1813.69
	5621.88	4203.21	209.12	1213.99	2005.23
2015	4235.21	3845.15	175.44	1434.23	2100.43
	4623.44	3925.23	198.11	1623.19	2325.92
	4324.92	4070.53	232.52	1925.32	2372.14
	4428.10	4061.29	275.17	1710.92	2410.11
2016	4312.43	4010.53	297.26	1525.20	2472.43
	4305.25	4100.20	310.19	1395.5	2602.42
	4295.92	4325.19	390.75	2010.59	2825.59
	5100.52	4591.75	400.2	2125.52	3617.29
2017	5105.29	4479.56	426.79	2159.70	3721.52
	5279.22	4505.62	446.26	2295.44	3961.28
	5295.59	4579.16	487.15	2515.66	3995.21
	5362.61	4756.49	425.96	3385.18	4509.17
2018	5235.77	4778.21	451.34	3481.42	4762.17
	5305.60	4942.26	467.86	3976.33	5162.32
	5385.62	5100.29	440.96	4222.76	5351.86
	5499.02	5202.49	439.56	4317.28	5561.78
2019	5602.88	5293.62	459.67	5526.86	5418.66
	5756.24	5372.66	486.34	5716.37	5321.75
	5861.16	5511.17	495.05	5962.73	4930.86
	5802.74	5434.67	498.67	5612.34	4741.97
2020	5995.78	5657.14	501.34	5534.78	5601.46
	6100.56	5854.67	520.58	5345.77	5695.90
	6025.89	5802.46	540.67	5788.91	5789.62
	6378.43	6002.31	598.62	6100.09	5886.67

7 Discussion and Conclusion

This research focused on building a multivariate time series model for the variables representing the different sectors of the Nigeria's Gross Domestic products (Agriculture, Industries, Building & Construction, wholesale & Retail and Services). It is interesting to note that modeling a univariate time series without considering the influence of other variables could be misleading. This was noted by [10]. In line with [10], this work has covered a more general case where five variables are interrelated. Besides; unlike [7], this work has also addressed a situation where the multivariate components exhibit seasonal behavior. Hence, multivariate approach can also be applied to series with periodic nature.

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Appendix A

Residual Correlation Matrices(R_i Matrix for $i = 0, 1, 2, \ldots, 10$)

$$R_0 = \begin{pmatrix} 1.000 & & & & \\ 0.012 & 1.000 & & & \\ 0.144 & 0.130 & 1.000 & & \\ -0.137 & 0.033 & -0.012 & 1.000 & \\ 0.074 & 0.023 & -0.054 & 0.110 & 1.000 \end{pmatrix}$$

$$R_1 = \begin{pmatrix} 0.112 & -0.024 & 0.061 & 0.124 & 0.007 \\ 0.021 & 0.134 & -0.005 & -0.038 & -0.102 \\ -0.042 & 0.008 & -0.071 & 0.003 & 0.151 \\ -0.137 & 0.025 & -0.012 & 0.011 & 0.112 \\ 0.074 & 0.131 & -0.054 & 0.113 & 0.005 \end{pmatrix}$$

$$R_2 = \begin{pmatrix} 0.152 & 0.031 & 0.003 & 0.013 & -0.113 \\ 0.012 & 0.142 & -0.005 & 0.004 & 0.042 \\ 0.019 & 0.002 & -0.026 & 0.003 & 0.151 \\ -0.137 & 0.033 & -0.012 & 0.011 & 0.112 \\ 0.074 & 0.131 & -0.054 & 0.110 & 0.071 \end{pmatrix}$$

$$R_3 = \begin{pmatrix} 0.016 & 0.024 & 0.014 & 0.090 & -0.143 \\ 0.023 & 0.009 & -0.034 & 0.041 & 0.136 \\ 0.004 & 0.132 & -0.141 & 0.161 & 0.122 \\ -0.155 & 0.164 & -0.021 & 0.006 & 0.115 \\ 0.135 & 0.016 & -0.113 & 0.041 & 0.009 \end{pmatrix}$$

$$R_4 = \begin{pmatrix} 0.130 & 0.007 & 0.143 & 0.001 & -0.134 \\ 0.003 & 0.143 & -0.119 & 0.061 & 0.139 \\ 0.138 & 0.121 & -0.121 & 0.005 & 0.002 \\ -0.013 & 0.033 & -0.012 & 0.011 & 0.112 \\ 0.074 & 0.146 & -0.008 & 0.110 & 0.071 \end{pmatrix}$$

$$R_5 = \begin{pmatrix} 0.150 & 0.141 & 0.163 & -0.006 & 0.116 \\ 0.001 & 0.142 & -0.144 & 0.128 & 0.120 \\ 0.004 & 0.146 & -0.019 & 0.090 & 0.010 \\ -0.080 & 0.043 & -0.022 & 0.053 & 0.006 \\ 0.132 & 0.007 & -0.160 & 0.154 & 0.121 \end{pmatrix}$$

$$R_6 = \begin{pmatrix} 0.025 & 0.043 & 0.002 & 0.146 & 0.031 \\ 0.012 & 0.142 & -0.005 & 0.004 & 0.042 \\ 0.034 & 0.130 & -0.002 & 0.003 & 0.151 \\ -0.137 & 0.033 & -0.012 & 0.111 & 0.112 \\ 0.074 & 0.118 & -0.054 & 0.110 & -0.103 \end{pmatrix}$$

$$R_7 = \begin{pmatrix} 0.110 & 0.005 & 0.002 & 0.146 & -0.031 \\ 0.047 & 0.038 & 0.019 & 0.088 & 0.014 \\ 0.122 & 0.135 & -0.135 & 0.158 & 0.160 \\ -0.155 & 0.033 & -0.012 & 0.011 & 0.112 \\ 0.113 & 0.071 & -0.008 & 0.107 & 0.071 \end{pmatrix}$$

$$R_8 = \begin{pmatrix} 0.004 & -0.138 & 0.142 & 0.031 & 0.124 \\ 0.009 & 0.132 & -0.118 & -0.153 & -0.142 \\ -0.153 & 0.002 & -0.101 & 0.119 & 0.051 \\ -0.153 & 0.002 & -0.101 & 0.119 & 0.052 \\ 0.062 & 0.073 & -0.099 & 0.128 & 0.159 \end{pmatrix}$$

$$R_9 = \begin{pmatrix} 0.150 & 0.052 & 0.012 & 0.003 & -0.123 \\ 0.031 & 0.132 & -0.136 & 0.122 & 0.110 \\ 0.160 & 0.023 & -0.005 & 0.153 & 0.129 \\ -0.037 & 0.118 & -0.012 & 0.011 & 0.112 \\ 0.030 & 0.008 & -0.133 & 0.117 & 0.026 \end{pmatrix}$$

$$R_{10} = \begin{pmatrix} 0.151 & 0.103 & 0.163 & -0.006 & 0.116 \\ 0.022 & 0.017 & -0.144 & 0.128 & 0.120 \\ 0.136 & 0.109 & -0.116 & 0.010 & 0.110 \\ -0.136 & 0.131 & -0.022 & 0.053 & 0.124 \\ 0.116 & 0.138 & -0.054 & 0.160 & 0.003 \end{pmatrix}$$