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Time-varying asymmetric error correction mechanism: An application to the relationship between the oil price and economic activity

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Abstract

This study introduces a cointegration test based on an asymmetric exponential smooth transition autoregressive (AESTAR) error correction model (ECM). The proposed model based on the unit root test by Sollis (2009) employs a wild bootstrap to test for cointegration. The test has time-varying and asymmetric adjustments and is robust to heteroskedastic variances such as stochastic volatility. A Monte Carlo simulation provides evidence that the proposed test has appropriate sizes and sufficient power under stochastic volatility. The model is applied to the relationship between the oil price and economic activity, demonstrating that the proposed test supports the presence of the error correction term. This contrasts with conventional tests, which do not support this term. The empirical results indicate the usefulness of the proposed test.

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1 Introduction

The crude oil price is an important factor in economic activity. Researchers often use error correction models (ECM) to analyze the relationship between the crude oil price and economic activity. An ECM usually assumes linear adjustment. This means that the error correction mechanism is stable in the long run. However, the crude oil price and economic activity have asymmetric properties, as noted by Hamilton (1983), Mork (1989), Çatik and Önder (2013), and Ramos and Veiga (2013), among others. These results indicate that researchers should introduce asymmetry when using an ECM.

As a model with asymmetric adjustment, Enders and Siklos (2001) propose threshold cointegration tests that have an abrupt regime shift of adjustment. Their tests are based on asymptotic tests and their critical values depend on the number of variables, the deterministic terms, and the transition variables. Additionally, asymptotic cointegration tests tend to overreject the null hypothesis of no cointegration under heteroskedastic variances. Maki (2013) reports that cointegration tests allowing nonlinearity have severe size distortions in the presence of stochastic volatility, generalized autoregressive conditional heteroskedasticity (GARCH), and variance breaks. Such heteroskedastic variances often appear when we investigate energy variables. For example, Vo (2009) and Vo (2011) analyze the stochastic volatility of oil prices. Accordingly, we have to consider heteroskedastic variances when we analyze oil prices and economic activity using an ECM with asymmetry.

This study proposes testing for the null of no cointegration against the alternative of cointegration using an ECM test, allowing for asymmetry and heteroskedasticity. We also apply it to the relationship between the crude oil price and economic activity. Here, we employ an asymmetric smooth transition autoregressive (AESTAR) model introduced by Sollis (2009). As pointed out by Teräsvirta and Anderson (1992) and Skalin and Teräsvirta (2002), smooth transition models are useful because many economic agents behave differently. As a result, the economy has time-varying and asymmetric smooth regime shifts. Kapetanios et al. (2006), Kiliç (2011), and Maki (2015) introduce time-varying ESTAR-ECM. The ESTAR model has a persistent process near equilibrium, but has a strong convergence when an equilibrium error is sufficiently far from equilibrium. ESTAR models have only time-varying properties, which are useful when investigating the relationships among economic variables in the presence of various costs. However, using an AESTAR model enables us to build a model with both time-varying and asymmetry. Kiliç (2011) also developed an asymmetric error correction model using a logistic smooth transition function, with a test based on asymptotic sup-type tests. This study introduces a test using a wild bootstrap. The wild bootstrap developed by Liu (1988) can replicate resampling that does not depend on the pattern of heteroskedastic variances. In addition, the test does not need critical values that correspond to the number of variables, the deterministic terms, and the transition variables. Therefore, the proposed test can accurately investigate asymmetric error correction under stochastic volatilities.

Monte Carlo simulations demonstrate that the proposed test has appropriate size and sufficient power when compared with conventional tests under stochastic volatilities. This implies that the proposed test leads to reliable results. Then, by applying the model to the relationship between the crude oil price and economic activity, we provide evidence that the proposed test supports the presence of the error correction term, whereas conventional tests do not support this term. The empirical results indicate that the asymmetric error correction mechanism affects the short-run dynamics of economic activity.

The rest of this paper is organized as follows. Section 2 introduces the test for the AESTAR-ECM using a wild bootstrap. Section 3 presents the size and power properties of the proposed tests. Section 4 provides empirical applications to the relationship between the crude oil price and economic activity. Finally, Section 5 concludes the paper.

2 Wild bootstrap test for the AESTAR-ECM

This study introduces a test for the AESTAR-ECM using a wild bootstrap. The AESTAR-ECM allows for an asymmetric smooth transition adjustment toward the long-run equilibrium. We consider the $n \times 1$ vector of observable $I(1)$ variables $\mathbf{z}_t = (y_t, \mathbf{x}_t')'$, where y_t is a scalar value and $\mathbf{x}_t = (x_{1t}, \cdots, x_{mt})'$ is an $m \times 1$ vector. Following Kapetanios et al. (2006) and Kiliç (2011), who proposed ESTAR-ECMs based on asymptotic theories, we analyze at most one conditional cointegration relationship between y_t and $\mathbf{x}_t'$. The proposed test considers the following AESTAR-ECM and the marginal vector autoregressive (VAR) model for $\Delta \mathbf{x}_t$:

$$\Delta y_t = G_t(\gamma_1, u_{t-d})\{S_t(\gamma_2, u_{t-d})\rho_1 + (1 - S_t(\gamma_2, u_{t-d}))\rho_2\}u_{t-1} + \omega' \Delta \mathbf{x}_t + \Sigma_{i=1}^{p} \psi_i' \Delta \mathbf{z}_{t-i} + e_t, \quad (1)$$

$$\Delta \mathbf{x}_t = \Sigma_{i=1}^{p} \mathbf{\Gamma}_{xi} \Delta \mathbf{z}_{t-i} + \eta_t, \quad (2)$$

where e_t and η_t are zero-mean errors, and ω, ψ_i, and $\mathbf{\Gamma}_{xi}$ are an $m \times 1$ vector, $n \times 1$ vector, and $m \times n$ matrix, respectively. Then, $u_t = y_t - \beta' \mathbf{x}_t$ is an error correction term, with β' as the $m \times 1$ cointegrating vector. We assume that an $n \times 1$ vector $\mathbf{z}_t$ is generated by $\mathbf{z}_t = (y_t, \mathbf{x}_t')' = \mathbf{z}_{t-1} + \epsilon_t$, where ϵ_t are i.i.d. with mean zero, a positive definite variance-covariance matrix Σ, and $E|\epsilon_t|^s < \infty$ for some $s > 4$. Here, ρ_1 and ρ_2 are adjustment parameters of ECM. While a symmetric ECM has $\rho_1 = \rho_2$, $\rho_1 \neq \rho_2$ allows for an asymmetric ECM.

The transition functions $G_t(\gamma_1, u_{t-d})$ and $S_t(\gamma_2, u_{t-d})$ are given by

$$G_t(\gamma_1, u_{t-d}) = 1 - \exp(-\gamma_1 u_{t-d}^2), \gamma_1 \geq 0, \quad (3)$$

$$S_t(\gamma_2, u_{t-d}) = [1 + \exp(-\gamma_2 u_{t-d})]^{-1}, \gamma_2 \geq 0, \quad (4)$$

where u_{t-d} is a transition variable and d is a delay parameter. The AESTAR model with (3) and (4) was developed by Sollis (2009), who proposed a null hypothesis of a unit root against the AESTAR model. The AESTAR model has the properties of both an exponential function and a logistic function, and $G_t(\gamma_1, u_{t-d})$ and $S_t(\gamma_2, u_{t-d})$ take values between zero and one. Here, $G_t(\gamma_1, u_{t-d})$ is near 1 when $\gamma_1 u_{t-d}^2$ is large, and near 0 when $\gamma_1 u_{t-d}^2$ is small, and allows for a smooth transition adjustment for the error correction mechanism. The long-run dynamics affect the short-run dynamics of Δy_t when $G_t(\gamma_1, u_{t-d})$ is closer to one, but do not do so when $G_t(\gamma_1, u_{t-d})$ is closer to zero. The symmetric ESTAR-ECM developed by Kapetanios et al. (2006) and Kiliç (2011) has $\rho_1 = \rho_2$ in (1). While Kapetanios et al. (2006) used only u_{t-1} as the transition variable, Kiliç (2011) took into account u_{t-d} as the transition variable. In the model, $S_t(\gamma_2, u_{t-d})$ allows for the asymmetric adjustment of the ECM. The value of $S_t(\gamma_2, u_{t-d})$ is close to one when $u_{t-d} > 0$ and $\gamma_2 u_{t-d}$ is large, and is close to zero when $u_{t-d} < 0$ and $\gamma_2 u_{t-d}$ is small. The existence of $S_t(\gamma_2, u_{t-d})$ constitutes a logistic smooth transition between ρ_1 and ρ_2. The logistic smooth transition function nests a two-regime threshold autoregressive (TAR) model, because $S_t(\gamma_2, u_{t-d})$ with $\gamma_2 = \infty$ is an indicator function that takes only the value 0 or 1. From the properties of $G_t(\gamma_1, u_{t-d})$ and $S_t(\gamma_2, u_{t-d})$, the error correction mechanism works when $\rho_1 < 0$, $\rho_2 < 0$, and $G_t(\gamma_1, u_{t-d}) > 0$, but does not work when $\rho_1 = \rho_2 = 0$ or $G_t(\gamma_1, u_{t-d}) = 0$.

The test for the null hypothesis of no cointegration against the alternative hypothesis of the AESTAR-ECM focuses on the parameter γ_1. The null and alternative hypotheses are as follows:

$$H_0 : \gamma_1 = 0, \qquad H_1 : \gamma_1 > 0. \tag{5}$$

Here, ρ_1, ρ_2, and γ_2 are nuisance parameters under the null hypothesis and are identified under the alternative hypothesis. The solution of the identification problem is obtained using a first-order Taylor series approximation around $\gamma_1 = 0$ for (1). The approximation gives the equation

$$\begin{aligned}\Delta y_t = \rho_1 \gamma_1 u_{t-d}^2 u_{t-1} S_t(\gamma_2, u_{t-d}) + \rho_2 \gamma_1 u_{t-d}^2 u_{t-1}(1 - S_t(\gamma_2, u_{t-d}))\rho_2\} \\ +\omega' \Delta \mathbf{x}_t + \Sigma_{i=1}^p \psi_i' \Delta \mathbf{z}_{t-i} + \tilde{e}_t,\end{aligned} \tag{6}$$

where $\tilde{e}_t$ is an error term, including the remainder from the Taylor approximation. Note that γ_2 in (6) is still unidentified under the null hypothesis. Following Sollis (2009), we replace $S_t(\gamma_2, u_{t-d})$ with $S_t^*(\gamma_2, u_{t-d}) = S_t(\gamma_2, u_{t-d}) - 0.5$ and, further, take a Talyor approximation around $\gamma_2 = 0$ for (6). The result gives the equation

$$\Delta y_t = \phi_1 u_{t-d}^2 u_{t-1} + \phi_2 u_{t-d}^3 u_{t-1} + \omega' \Delta \mathbf{x}_t + \Sigma_{i=1}^p \psi_i' \Delta \mathbf{z}_{t-i} + \upsilon_t, \tag{7}$$

where υ_t is an error term. The null hypothesis for γ_1 is written as $H_0 : \phi_1 = \phi_2 = 0$. We denote θ and h_t as $\theta = (\phi_1, \phi_2, \omega', \psi_1', \cdots, \psi_p')'$ and $h_t = (u_{t-d}^2 u_{t-1}, u_{t-d}^3 u_{t-1}, \Delta \mathbf{x}_t', \Delta \mathbf{z}_{t-1}', \cdots, \Delta \mathbf{z}_{t-p}')'$. The Wald statistic for the hypothesis is given by

$$W_{AS} = \frac{1}{\hat{\sigma}^2} \hat{\phi}' \left[R \left(\sum_{t=1}^T h_t h_t' \right)^{-1} R' \right]^{-1} \hat{\phi}, \tag{8}$$

where $\hat{\phi} = (\hat{\phi}_1, \hat{\phi}_2)'$ is the ordinary least squares (OLS) estimate of ϕ_1 and ϕ_2, $\hat{\sigma}^2$ is the least squared estimate of the residual variance for (7), and R is a $2\times(2+m+np)$ matrix, such that $R\hat{\theta} = \hat{\phi}$. When we test for cointegration, the cointegrating vector is usually unknown. For this reason, we use the residual $\hat{u}_t = y_t - \hat{\beta}'\mathbf{x}_t$ instead of u_t.

If researchers employ (8), they need asymptotic critical values to test using the AESTAR-ECM. However, asymptotic critical values depend on the number of regressions and the type of deteministic terms. More importantly, tests using asymptotic values are influenced by heteroskedastic variances, even if we use the heteroskedasticity-consistent covariance matrix estimators (HCCME) proposed by White (1980). Maki (2013) reports that asymptotic cointegration tests, particularly those allowing nonlinearity, have severe size distortions in the presence of heteroskedastic variances, regardless of the use of HCCME. Therefore, we do not use the asymptotic test, but instead apply test (8) using the wild bootstrap. The test does not depend on the number of regressions, the type of deterministic terms, and heteroskedastic variances. The resample using the wild bootstrap can preserve the properties of unknown heteroskedastic variance in bootstrap samples. The algorithm of the test is as follows.

Step 1: We estimate (7) and obtain the residuals $\hat{\upsilon}_t$. Using estimated parameters and the residuals, we generate a new process under the null hypothesis of no cointegration

$$\Delta y_t^* = \hat{\omega}'\Delta\mathbf{x}_t + \Sigma_{i=1}^{p}\hat{\psi}_i'\Delta\mathbf{z}_{t-i} + \upsilon_t^*, \tag{9}$$

where $\upsilon_t^* = \epsilon_t\hat{\upsilon}_t$ and ϵ_t is such that $E(\epsilon_t) = 0$ and $E(\epsilon_t^2) = 1$. We use a Rademacher distribution, such that $\epsilon_t = 1$ and $\epsilon_t = -1$, both with a probability of 0.5. The initial observations y_0^* and y_1^* are set to zero and the sample value y_1, respectively.

Step 2: We regress y_t^* on x_t and obtain the residual. The error correction term based on the bootstrap sample is given by

$$\hat{u}_t^* = y_t^* - \hat{\beta}_b'\mathbf{x}_t, \tag{10}$$

where $\hat{\beta}_b'$ is the estimate of the cointegration vector in the bootstrap sample. We use the residuals as the error correction term for the bootstrap. When the long-run equilibrium has a constant (or both a constant and a trend), the demeaned (or demeaned and detrended) residuals are employed.

Step 3: We use the generated bootstrap sample and have the following regression:

$$\Delta y_t^* = \phi_{1b} u_{t-d}^{*2} u_{t-1}^* + \phi_{2b} u_{t-d}^{*3} u_{t-1}^* + \omega_b' \Delta \mathbf{x}_t + \Sigma_{i=1}^{p} \psi_{bi}' \Delta \mathbf{z}_{t-i} + \zeta_t, \tag{11}$$

where ζ_t is an error term.

Step 4: We compute the test statistic (8) in (11), and denote it with the bootstrap sample as W_{AS}^b.

Step 5: We repeat the bootstrap iteration from Step 1 to Step 4 a number of times. Finally, we obtain the bootstrap p-value as follows:

$$P_b(W_{AS}) = \frac{1}{B} \sum_{j=1}^{B} \mathbf{1}(W_{AS}^b > W_{AS}), \tag{12}$$

where B is the number of bootstrap iterations and $\mathbf{1}(\cdot)$ is an indicator function, such that $\mathbf{1}(\cdot)$ is 1 if $(\cdot)$ is true, and 0 otherwise. It is preferable to set the number bootstrap to more than 1,000.

3 Monte Carlo simulations

In this section, we present the size and power properties of the proposed test. We compare the performance of the test with the tests of Engle and Granger (1987) and Kiliç (2011). The test of Engle and Granger (1987) is a standard linear ECM and the test of Kiliç (2011) is an LSTAR-ECM. We denote the tests of Engle and Granger (1987), Kiliç (2011), and the wild bootstrap test of (8) as EG, KL, and AS_{WB}, respectively. For comparison, we also evaluate the performances of EG and KL using the HCCME, which are denoted as EG(W) and KL(W), respectively. All the tests employ the demeaned model and assume there is no lag, for simplicity. The nominal size of the tests is set at 0.05, and sample sizes are considered for $T = 200$ and 400. For all the experiments, the number of replications for the Monte Carlo simulations is 10,000 and the number of bootstrap replications for the wild bootstrap test is 1,000.

We investigate the rejection frequency generated from:

$$\Delta y_t = \lambda \Delta x_t + u_{1t}, \tag{13}$$

$$\Delta x_t = u_{2t}, \tag{14}$$

$$u_t = y_t - \beta x_t, \tag{15}$$

where $\lambda = 1$ and $\beta = 1$. The errors u_{1t} and u_{2t} are given by

$$\begin{pmatrix} u_{1t} \\ u_{2t} \end{pmatrix} \sim i.i.d.N \begin{pmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{pmatrix}, \tag{16}$$

where $\sigma_2^2 = 1$. Then, σ_1^2 is set to 1 for the case of homoskedastic variance with a normal error. We consider three types of stochastic volatilities for σ_1^2: stochastic volatility, markov switching stochastic volatility, and threshold stochastic volatility. The crude oil price and economic variables have those stochastic volatilities (e.g., Smith, 2002; So and Choi, 2008; Vo, 2009, 2011; and Chen, et al., 2013). Therefore, it is important to evaluate rejection frequencies under stochastic volatilities.

The u_{1t} for stochastic volatility (SV) is generated from

$$u_{1t} = \kappa_t \exp(h_t/2), \tag{17}$$

$$h_t = \delta h_{t-1} + \xi_t, \tag{18}$$

where $\kappa_t \sim i.i.d.N(0,1)$, and we set ξ_t to $\xi_t \sim i.i.d.N(0, 0.25)$. Then, SV1 and SV2 have the parameters $\delta = 0.95$ and 0.7, respectively.

For markov switching volatility (MSV), h_t is given by

$$h_t = \delta_0 h_{t-1} S_t + \delta_1 h_{t-1}(1 - S_t) + \xi_t, \tag{19}$$

where S_t is a random variable that takes a value of 0 or 1, and δ_0 and δ_1 are set to 0.95 and 0.7, respectively. The value of S_t depends on the transition probabilities, such as $P(S_{t+1} = 0|S_t = 0) = p_{00}$ and $P(S_{t+1} = 1|S_t = 1) = p_{11}$. When the transition probabilities $P(S_{t+1} = 0|S_t = 1) = p_{10} = 1 - p_{00}$ and $P(S_{t+1} = 1|S_t = 0) = p_{01} = 1 - p_{11}$ are high, h_t have frequent switches between δ_0 and δ_1. Conversely, low p_{10} and p_{01} lead to persistent switches between δ_0 and δ_1. For the transition probabilities, MSV1 and MSV2 have parameters $p_{00} = p_{11} = 0.98$ for persistent switches, and $p_{00} = p_{11} = 0.7$ for frequent switches.

For threshold stochastic volatility (TSV), h_t is replaced by

$$\text{TSV1}: \quad h_t = \delta_0 h_{t-1}\mathbf{1}\{u_{t-1} > 0\} + \delta_1 h_{t-1}\mathbf{1}\{u_{t-1} \leq 0\} + \xi_t \tag{20}$$

$$\text{TSV2}: \quad h_t = (\mu_0 + \delta_0 h_{t-1})\mathbf{1}\{u_{t-1} > 0\} + (\mu_1 + \delta_1 h_{t-1})\mathbf{1}\{u_{t-1} \leq 0\} + \xi_t, \tag{21}$$

where $\mathbf{1}\{\cdot\}$ is the indicator function and its value depends on whether $\{\cdot\}$ is true. While TSV1 has shifts only between δ_0 and δ_1, TSV2 also has shifts between constant parameters μ_0 and μ_1 in addition to δ_0 and δ_1. We set (δ_0, δ_1) and (μ_0, μ_1) to $(\delta_0, \delta_1) = (0.95, 0.7)$ and $(\mu_0, \mu_1) = (-0.5, -1)$.

Table 1: Empirical sizes

	EG	EG(W)	KL	KL(W)	AS_{WB}
Normal					
$T = 200$	0.053	0.058	0.052	0.075	0.049
$T = 400$	0.055	0.052	0.051	0.066	0.050
SV1					
$T = 200$	0.111	0.025	0.142	0.049	0.048
$T = 400$	0.102	0.022	0.144	0.037	0.050
SV2					
$T = 200$	0.057	0.049	0.062	0.065	0.053
$T = 400$	0.054	0.046	0.057	0.057	0.052
MSSV1					
$T = 200$	0.084	0.037	0.105	0.051	0.037
$T = 400$	0.080	0.028	0.105	0.045	0.050
MSSV2					
$T = 200$	0.067	0.040	0.081	0.059	0.049
$T = 400$	0.063	0.039	0.067	0.049	0.049
TSV1					
$T = 200$	0.065	0.043	0.077	0.059	0.056
$T = 400$	0.058	0.038	0.066	0.052	0.054
TSV2					
$T = 200$	0.068	0.041	0.075	0.058	0.049
$T = 400$	0.060	0.038	0.066	0.051	0.053

The rejection frequencies of the tests to compare empirical sizes are presented in Table 1. The tests other than KL(W) perform well for homoskedastic variance. The rejection frequencies of EG, EG(W), KL, and AS_{WB} are close to the nominal size, 0.05. In addition, KL(W) slightly overrejects the null hypothesis. In the presence of stochastic volatilities, EG and KL tend to have size distortions. When the error has SV1, the rejection frequencies of EG and KL are more than 0.1. The overrejection decreases for SV2. This implies that the persistence of stochastic volatility affects the empirical sizes of the asymptotic tests. Although EG(W) and KL(W) perform better than EG and KL do, they also have slight size distortions for SV1 or SV2. Unlike the asymptotic tests, AS_{WB} is not influenced by stochastic volatility. The rejection frequencies of AS_{WB} for both SV1 and SV2 are close to 0.05.

EG and KL also have size distortions when the volatility is generated by MSSV and TSV. Compared with the results between MSSV1 and MSSV2, the distortions of EG and KL for MSSV1 are larger than those for MSSV2. The persistent switches lead to overrejections for asymptotic tests and spurious cointegration. While EG(W) has small underrejections for MSSV and TSV, KL(W) has acceptable empirical sizes, particularly for $T = 400$. The empirical size of AS_{WB} does not depend on the type of volatility and AS_{WB} performs better, regardless of the sample size. The size comparison reveals that AS_{WB} leads to a reliable result.

Tables 2 and 3 illustrate the power comparison. While Table 2 presents the results under cointegration with a normal error, Table 3 reports the results under cointegration with SV1. For the data generated process (DGP) with an error correction term, (13) is replaced by

$$\Delta y_t = \lambda \Delta x_t + G_t(\gamma_1, u_{t-1})\{S_t(\gamma_2, u_{t-1})\rho_1 + (1 - S_t(\gamma_2, u_{t-1}))\rho_2\}u_{t-1} + u_{1t}, \quad (22)$$

where $G_t(\cdot)$ and $S_t(\cdot)$ are given by

$$G_t(\gamma_1, u_{t-1}) = 1 - \exp(-\gamma_1 u_{t-1}^2) \quad (23)$$

$$S_t(\gamma_2, u_{t-1}) = [1 + \exp(-\gamma_2 u_{t-1})]^{-1}. \quad (24)$$

We set adjustment parameters ρ_1 and ρ_2 to $(\rho_1, \rho_2) = \{(-0.15, -0.05)$ and $(-0.5, -0.05)\}$. Here, $(\rho_1, \rho_2) = (-0.5, -0.05)$ has stronger asymmetry than $(\rho_1, \rho_2) = (-0.15, -0.05)$. The smoothness parameters for G_t and S_t have four types: $(\gamma_1, \gamma_2) =\{(0.01, 1)$, $(0.01, 10)$, $(0.1, 1)$, and $(0.1, 10)\}$. Then, γ_1 and γ_2 determine the speed of the smooth transition of G_t and S_t, respectively. Larger γ_1 and γ_2 make the model approximately linear.

Table 2: Powers under an normal error

	EG	EG(W)	KL	KL(W)	AS_{WB}
$(\rho_1, \rho_2) = (-0.15, -0.05)$					
$(\gamma_1, \gamma_2) = (0.01, 1)$					
$T = 200$	0.127	0.135	0.104	0.140	0.126
$T = 400$	0.256	0.242	0.215	0.249	0.307
$(\gamma_1, \gamma_2) = (0.01, 10)$					
$T = 200$	0.120	0.128	0.103	0.137	0.125
$T = 400$	0.249	0.234	0.206	0.228	0.312
$(\gamma_1, \gamma_2) = (0.1, 1)$					
$T = 200$	0.337	0.339	0.292	0.348	0.359
$T = 400$	0.903	0.872	0.862	0.863	0.843
$(\gamma_1, \gamma_2) = (0.1, 10)$					
$T = 200$	0.327	0.331	0.283	0.338	0.346
$T = 400$	0.905	0.873	0.850	0.858	0.836
$(\rho_1, \rho_2) = (-0.5, -0.05)$					
$(\gamma_1, \gamma_2) = (0.01, 1)$					
$T = 200$	0.160	0.162	0.150	0.181	0.190
$T = 400$	0.405	0.365	0.347	0.358	0.535
$(\gamma_1, \gamma_2) = (0.01, 10)$					
$T = 200$	0.155	0.159	0.144	0.173	0.189
$T = 400$	0.398	0.361	0.344	0.357	0.524
$(\gamma_1, \gamma_2) = (0.1, 1)$					
$T = 200$	0.591	0.550	0.574	0.575	0.694
$T = 400$	0.987	0.973	0.985	0.977	0.983
$(\gamma_1, \gamma_2) = (0.1, 10)$					
$T = 200$	0.546	0.507	0.513	0.520	0.652
$T = 400$	0.983	0.966	0.980	0.971	0.979

In Table 2, the powers of EG(W) and KL(W) are higher than those of EG and KL because EG(W) and KL(W) overreject the null hypothesis, particularly for $T = 200$, as illustrated by Table 2. It can be observed that AS_{WB} outperforms the other tests when the speed of the smooth transition is slow for $(\rho_1, \rho_2) = (-0.15, -0.05)$. This tendency becomes clear for $(\rho_1, \rho_2) = (-0.5, -0.05)$. For example, when the error correction term has the parameters $(\rho_1, \rho_2) = (-0.5, -0.05)$ and $(\gamma_1, \gamma_2) = (0.01, 10)$ and the sample size is $T = 400$, the powers of EG, EG(W), KL, KL(W), and AS_{AB} are 0.398, 0.361, 0.341, 0.357, and 0.524, respectively. However, we cannot observe different power among the tests for $(\rho_1, \rho_2) = (-0.15, -0.05)$ and $(\gamma_1, \gamma_2) = (0.1, 1)$ and $(0.01, 10)$. These results indicate that AS_{WB} is superior to the other tests when the error correction term is asymmetrical and has a slow smooth transition.

Table 3: Powers under stochastic volatility

	EG	EG(W)	KL	KL(W)	AS_{WB}
$(\rho_1, \rho_2) = (-0.15, -0.05)$					
$(\gamma_1, \gamma_2) = (0.01, 1)$					
$T = 200$	0.300	0.074	0.327	0.113	0.153
$T = 400$	0.654	0.151	0.658	0.189	0.378
$(\gamma_1, \gamma_2) = (0.01, 10)$					
$T = 200$	0.302	0.075	0.332	0.118	0.152
$T = 400$	0.651	0.144	0.660	0.189	0.370
$(\gamma_1, \gamma_2) = (0.1, 1)$					
$T = 200$	0.526	0.161	0.530	0.216	0.259
$T = 400$	0.893	0.363	0.907	0.469	0.503
$(\gamma_1, \gamma_2) = (0.1, 10)$					
$T = 200$	0.523	0.165	0.525	0.222	0.261
$T = 400$	0.893	0.344	0.903	0.455	0.514
$(\rho_1, \rho_2) = (-0.5, -0.05)$					
$(\gamma_1, \gamma_2) = (0.01, 1)$					
$T = 200$	0.417	0.110	0.457	0.159	0.290
$T = 400$	0.798	0.239	0.799	0.314	0.604
$(\gamma_1, \gamma_2) = (0.01, 10)$					
$T = 200$	0.420	0.109	0.454	0.164	0.291
$T = 400$	0.803	0.234	0.805	0.312	0.590
$(\gamma_1, \gamma_2) = (0.1, 1)$					
$T = 200$	0.688	0.280	0.729	0.386	0.504
$T = 400$	0.933	0.508	0.954	0.690	0.732
$(\gamma_1, \gamma_2) = (0.1, 10)$					
$T = 200$	0.676	0.265	0.710	0.362	0.490
$T = 400$	0.933	0.505	0.953	0.675	0.727

When the error has SV1, as presented in Table 3, EG and KL have higher powers. This is clearly because EG and KL overreject the null hypothesis under SV1 and has size distortions. In contrast, EG(W) and KL(W) are inferior to the other tests. The inferior performances are caused by underrejecting the null hypothesis reported in Table 1. We observe that the power of AS$_{WB}$ is lower than those of EG and KL, but higher than those of EG(W) and KL(W). More importantly, AS$_{WB}$ does not have overrejections and underrejections, even in the presence of heteroskedastic variances. Therefore, AS$_{WB}$ leads to reliable results.

4 Application to the relationship between the oil price and economic activity

The crude oil price plays an important role in economic activity. Many studies, including Hamilton (1983), Mork (1989), Çatik and Önder (2013), and Ramos and Veiga (2013) have shown that the impact of the crude oil price on economic activity is asymmetric. We explore this by applying AS$_{WB}$ to the relationship between the oil price and economic activity. We use the crude oil price as the variable $\mathbf{x}_t$ in (1) and four economic indexes as the variable y_t in (1). The four economic indexes are the beverage index, industrial production index, agricultural index, and metal price index. The asymmetric response of oil prices to these variables is discussed by, for example, Meyer and Cramon-Taubadel (2004), Hammoudeh and Fattouh (2010), and Ibrahima and Chancharoenchaib (2014). The monthly data obtained from the International Monetary Fund consist of 408 observations from January 1980 to December 2013. The series codes for the crude oil price, beverage index, industrial production index, agricultural index, and metal price index in the IMF data are POILAPSP_Index, PBEVE_Index, PINDU_Index, PRAWM_Index, and PMETA_Index, respectively. All the tests include aPo constant and a trend as deterministic terms. The lag lengths are selected by the Akaike information criterion (AIC). Although we do not present the results of the unit root tests of the variables, the standard tests including Dickey-Fuller type tests provide evidence of $I(1)$.

Table 4 presents the empirical results of the cointegration tests. The p-

values were obtained by our simulation. We determined the delay parameter d of KL, KL(W), and AS_{WB} as d to minimize the p-values from $d = 1$ to $d = 12$. The p-values of EG and EG(W) are larger than 0.1, and none reject the null hypothesis. We can see different results for KL and KL(W). The p-values of KL are less than 0.05 or 0.1, except for the agricultural index. Then, KL rejects the null hypothesis of no cointegration for the other three indexes. However, the p-values of KL(W) are larger than those of KL. Thus, KL(W) rejects the null hypothesis for the industrial production index and the metal price index only at the 10% significance level. As illustrated in Section 3, KL has size distortions in the presence of heteroskedastic variances, which are reduced by KL(W). Accordingly, it appears that the difference between KL and KL(W) is caused by heteroskedastic variances. The p-values of AS_{WB} are less than 0.05

Table 4: Empirical results

	EG	EG(W)	KL	KL (W)	AS_{WB}
Beverage index	-3.208	-2.778	13.71	9.429	31.98
	(0.148)	(0.309)	(0.081)	(0.274)	(0.000)
u_{t-d}			$d = 1$	$d = 12$	$d = 9$
Industrial production index	-2.582	-1.978	17.36	14.02	25.54
	(0.397)	(0.696)	(0.021)	(0.062)	(0.018)
u_{t-d}			$d = 12$	$d = 12$	$d = 9$
Agricultural index	-2.965	-2.691	10.58	9.172	18.94
	(0.225)	(0.344)	(0.187)	(0.275)	(0.029)
u_{t-d}			$d = 7$	$d = 7$	$d = 7$
Metal price index	-2.042	-1.420	16.99	13.43	26.36
	(0.660)	(0.875)	(0.020)	(0.073)	(0.042)
u_{t-d}			$d = 1$	$d = 12$	$d = 12$

The $p-$values are in the parentheses.

and AS_{WB} strongly rejects the null hypothesis. AS_{WB} has better empirical sizes, even in the presence of heteroskedastic variances. Accordingly, the empirical results of AS_{WB} are reliable and provide evidence that the relationship between the crude oil price and economic activity has a asymmetric error correction mechanism.

In order to further investigate the asymmetric error correction mechanism, we estimate the following model:

$$\Delta y_t = G_t(\gamma_1, u_{t-d})\{S_t(\gamma_2, u_{t-d})\rho_1 + (1 - S_t(\gamma_2, u_{t-d}))\rho_2\}u_{t-1}+ \\ \omega'\Delta \mathbf{x}_t + \Sigma_{i=1}^{p}\psi_i'\Delta \mathbf{z}_{t-i} + e_t. \quad (25)$$

Table 5 reports the estimation results. The smoothness parameters γ_1 and γ_2 are determined such that the sum of squared residuals of (25) are minimized. It can be seen that the error correction terms are asymmetric. For example, the error correction term between the crude oil price and the industrial production index has adjustment parameters $(\rho_1, \rho_2) = (-0.184, -0.276)$. This indicates that, while the adjustment speed approaches -0.247 and the adjustment mechanism becomes faster if u_{t-9} is negative and small, it approaches -0.184 if u_{t-9} is positive and large. In contrast, the error correction mechanism almost never performs

Table 5: AESTAR estimates

	tv	ρ_1	ρ_2	γ_1	γ_2
Beverage index	u_{t-9}	-0.186	-0.062	0.0004	3.816
		(0.050)	(0.039)		
Industrial production index	u_{t-9}	-0.184	-0.276	0.0006	6.264
		(0.055)	(0.086)		
Agricultural index	u_{t-7}	-0.148	-0.222	0.0007	7.001
		(0.052)	(0.091)		
Metal price index	u_{t-12}	-0.088	-0.054	0.028	4.777
		(0.028)	(0.021)		

Heteroskedastic-robust standard errors are in the parentheses.

when u_{t-9} is near to zero, because $G_t(\gamma_1, u_{t-9})$ has a value near to zero. These results indicate that the error correction mechanism depends on the size of a selected transition variable, as well as its sign. As demonstrated by the Monte Carlo simulations, AS_{WB} performs better when an error correction term has strong asymmetry and a slow smooth transition. The findings from Table 5 confirm that the relationship between the crude oil price and economic activity has an asymmetric error correction mechanism.

5 Summary

This study introduced a cointegration test based on an asymmetric exponential smooth transition autoregressive (AESTAR) error correction model (ECM). The proposed test employs a wild bootstrap to test for cointegration in order to avoid size distortions in the presence of heteroskedastic variances. From the properties, the developed test has time-varying and asymmetric adjustments, and is robust to heteroskedastic variances such as stochastic volatility. In fact, the results from the Monte Carlo simulation show that the proposed test has appropriate empirical size and sufficient power, with or without stochastic volatility. When we investigated the impact of crude oil prices on economic activity, the proposed test strongly supported the presence of the error correction term. The empirical results provided evidence that the relationship between the crude oil price and economic activity has an asymmetric smooth transition error correction mechanism. Thus, the proposed test is useful when analyzing a long-run relationship with an asymmetric smooth transition adjustment under stochastic volatilities, as observed in economic variables such as commodity prices and asset prices.

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