

Examination of Botswana stock markets using regime switching models

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Abstract

Empirical investigations have indicated that there is presence of non-linearity in Botswana stock market. This paper therefore, examines whether the Botswana stock market displays a regime switching behavior using both the Logistic smooth transition autoregressive models (LSTAR) and Exponential smooth transition autoregressive models (ESTAR). By analyzing the residual properties, the results actually show that Logistic smooth transition autoregressive models (LSTAR) is more appropriate to model the series against linear model.

Mathematics Subject Classification: C58

Keywords: STAR; LSTAR; ESTAR; stock market; regime switching models

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1 Introduction

Studies have shown that economic and financial variables show nonlinear characteristics where the economic behaviors change when certain variables lie in different regions (Terasvirta, 1994, Anderson T.G, 2002 and Bonilla, et.al 2005). To capture these non-linear properties, we introduced, STAR models which allow nonlinear regime switching model as this will be mostly appropriate since stock rate are volatile and high period regime is usually followed by period of low regime. These properties are best capture by regime switching model. The regime switching model used in this study is Smooth Transition Autoregressive model. The particular classes of STAR model used in the study are the commonly used one, which are Exponential Smooth Transition model and Logistic Smooth Transition model. They are particularly useful in financial and macroeconomics series because of their symmetry and asymmetry properties, hence their choice in this paper. However, before applying nonlinear models such as STAR models, testing linearity against nonlinearity is an essential testing procedure when researchers wish to consider a nonlinear modeling. A unit root test in the nonlinear time series model is a joint test that under the null hypothesis testing of both unit root and linearity. It is well know that classical Dickey-Fuller (1976) tests are found of lacking power when alternative model shows non-linearity. Leybourne et al.(1998) and Harvey and Mills(2002) have discussed the situation where the model with a logistic smooth transition in intercept and trend.

2 Specification of STAR model

A STAR model is defined as

$$y_t = \pi_{10} + \pi_1' w_t + (\pi_{20} + \pi_2' w_t) F(y_{t-d}) + \mu_t \quad (1)$$

where $\pi_j = (\pi_{j1}, \dots, \pi_{jp})'$, $j = 1, 2$, $w_t = (y_{t-1}, \dots, y_{t-p})'$ $\mu_t \approx \text{nid}(0, \sigma_u^2)$

The transition function $F(y_{t-d})$ is defined as logistic function if

$$F(y_{t-d}) = \left\{1 + \exp[-\gamma(y_{t-d})]\right\}^{-1} \quad \gamma > 0 \quad (2)$$

and Exponential function if

$$F(y_{t-d}) = 1 - \exp[-\gamma(y_{t-d} - c)^2] \quad \gamma > 0 \quad (3)$$

Model in (1) with transition function (2) is known as a Logistic STAR model of order p LSTAR (p), whereas (1) with (3) is called an Exponential STAR model of order p simply put as ESTAR (p). The LSTAR model approaches a two regime threshold autoregressive model (Tong, 1990) when $\gamma_L \rightarrow \infty$ since (2) in the limit is a step function of (y_{t-d}) , the value of which changes from zero unit at c . when $\gamma_L \rightarrow 0$ the LSTAR approaches a linear $AR(p)$ model. So also for ESTAR model, Equation (1) approaches linear model as $\gamma_E \rightarrow 0$ and with probability one as $\gamma_E \rightarrow \infty$. If $C_E = \pi_{20} = 0$. The ESTAR model is identical to the Exponential autoregressive model (Haggan and Ozoki 1981).

The role of transition function (1) is that it allows the coefficients for lagged value of y_t , $(\pi_1 + \pi_2 F(y_{t-d}))'$ and $(\pi_{10} + \pi_{20} F(y_{t-d}))$ to change smoothly with y_{t-d} . This means that the dynamics of the model change with y_{t-d} . It works differently for the two STAR models. The LSTAR allows local dynamics to be different for high and low values of the transition variable y_{t-d} . This allows it or makes it possible to model non-linear effects of shock. In contrast to LSTAR case ESTAR transition function is symmetric about C_E . In the sense that the local dynamic are the same for high and for low values of y_{t-d} .

2.1 The modeling procedure for STAR models

Granger, 1993 strongly recommends employing a specific-to-general procedure when considering the use of non-linear time series models to describe the features of a particular variable. An empirical specification procedure follows this approach which consists of the following steps:

- (1) Specify an appropriate linear AR model of order p AR (p) for the time series under investigation.
- (2) Test the null hypothesis of linearity against alternative STAR type non-linearity for the models.
- (3) Estimate the parameters in the selected models
- (4) Evaluate the models using diagnostic test
- (5) Modify the models if necessary
- (6) Use the model for descriptive or forecasting purposes

2.2 Non-linearity testing

To build a non-linear model one has to find out if a linear model would be enough to adequately characterize both the statistical and economic relationship in question. This is because if a linear model would suffice there would be statistical theory available for building a reasonable model than if a non-linear model were appropriate. When testing linearity a decision has to be reached concerning the form of non-linearity in the alternative hypothesis. Furthermore, the linearity test is complicated by the fact that the model is not identified under the null hypothesis $H_0 : \gamma_i = 0, i = 1, 2, \dots$, Terasvirata (1994) shows that a score or a Lagrange Multiplier type test of linearity against an ESTAR or an LSTAR alternative can be carried out by estimating an auxiliary regression and testing hypothesis within it.

$$y_t = \beta_0 + \beta_1^1 w_t + \beta_2^1 w_t y_{t-d} + \beta_3^1 w_t y_{t-d}^2 + \beta_4^1 w_t y_{t-d}^3 + v_t \quad (4)$$

where

$$\beta_j = (\beta_{j1}, \dots, \beta_{jp})^1, j = 1, \dots, n. w_t = (y_{t-1}, \dots, y_{t-p})^1, E(v_t) = 0, \text{Var}(v_t) = \sigma_v^2 \quad (5)$$

$$\forall t \text{ and } E(v_t v_s) = 0 \text{ s } \neq t.$$

The next step is to choose transition: which is either exponential or logistic. Terasvirta (1994) proposed a sequence of hypothesis on the basis of (4) above, these hypothesis are here stated:

$$H_{04} = \beta_3 = 0$$

$$H_{03} = \beta_2 = 0 \mid \beta_3 = 0$$

$$H_{02} = \beta_1 = 0 \mid \beta_2 = \beta_3 = 0$$

If H_{04} is not rejected and H_{03} is rejected, exponential function will be chosen.

However, in all other cases logistic function has to be chosen.

3 Discussion of results

The data used in this paper is Botswana monthly stock rates from January 1987 to December 2012, covering three hundred months. The analysis began by looking at the descriptive statistics of the data, which is here presented.

Table 1 gives a brief description of the data used in the study; there is reason to reject the assumption of normality, the result obtained for jarque-Bera (very large value) statistics pointed to this conclusion.

Table 1: Summary Statistics of Botswana stock rates

Mean	Median	Std. dev.	Skewness	Kurtosis	Jarque-Bera	Observation
3.2447	9.1500	2.4693	0.4843	1.8123	43.4500 (0.0000)	300

3.1 Unit root tests

Augmented Dickey fuller test was used for two unit root tests. At level the series was not stationary, but at first difference the series appeared to be stationary, thereby paving way for the estimation of parameters of the models involved. The results of unit root tests were displayed in Tables 2 and 3 below.

Table 2: Unit root test at level.

Null Hypothesis: STOCK has a unit root

Exogenous: Constant

Lag Length: 0 (Automatic - based on SIC, maxlag=17)

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	0.201297	0.9725
Test critical values: 1% level	-3.444890	
5% level	-2.867845	
10% level	-2.570192	

*MacKinnon (1996) one-sided p-values.

Table 3: Unit root test at first difference.

Null Hypothesis: D(STOCK) has a unit root

Exogenous: Constant

Lag Length: 0 (Automatic - based on SIC, maxlag=17)

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-21.66831	0.0000
Test critical values: 1% level	-3.444923	
5% level	-2.867859	
10% level	-2.570200	

*MacKinnon (1996) one-sided p-values.

3.2 Specification of STAR model (empirical)

3.2.1. Non-linearity test

Table 4 below displayed the linearity test using F-test. From the table the null hypothesis of linearity is rejected for all variables at 1% level of significance. In all, the model suggested for retention is Logistic smooth transition model (LSTAR).

Table 4: Non-linearity test

Transition variables	F	H_{04}	H_{03}	H_{02}	Suggested model
y_{t-1}	4.1915e-06	3.3264e-02	6.8664e-04	2.0641e-03	LSTAR
y_{t-2}	4.8296e-07	4.0726e-02	1.1492e-04	7.6142e-04	LSTAR
TREND	8.0158e-03	4.7777e-01	7.1862e-02	4.1430e-03	LSTAR

3.2.2. Estimation of the parameter of the model

So far we have rejected the hypothesis of linearity of the model against non-linear STAR model, we therefore proceed to the task of estimating univariate STAR model as shown below in the Table 5.

Table 5: Estimated STAR model

PARAMETER	Estimated coefficients	Standard deviation	$t - statistic$	$p - value$
ϕ_0	0.36257	1.1249	0.3223	0.7474
ϕ_1	0.61826	0.4907	1.2600	0.2084
ϕ_2	0.24406	0.2679	0.9111	0.3628
ϕ_3	-0.04629	0.2053	-0.2255	0.8217

ϕ_4	-0.52334	0.3727	-1.4042	0.1610
ϕ_5	-0.52613	0.5161	-1.0195	0.3086
θ_0	-0.06094	0.2994	-0.2036	0.8388
θ_1	-0.42551	0.4713	-0.9028	0.3671
θ_2	0.41133	0.4904	0.8387	0.4021
θ_3	0.22657	0.3665	0.6182	0.5368
θ_4	0.87905	0.7250	1.2125	0.2260
θ_5	0.38310	0.3403	1.1256	0.2610
γ	4.48918	2.3222	1.9332	0.0539
c	7.16745	0.7606	9.4228	0.0000
AIC	-3.7643e+00			
SC	-3.4856e+00			
R^2	0.9956			

3.3 Diagnostic test

The diagnostic test used here is a test of no autocorrelation as this is based on the properties of resulting residuals. The conclusion here is based on langrange multiplier which shows the evidence of no autocorrelation as shown in theTable 6 below:

Table 6: Test of No Error Autocorrelation

Lag	$p - value$
1	0.2622
2	0.2953
3	0.3702

4	0.3787
5	0.2929
6	0.3969
7	0.3622
8	0.3894

4 Conclusion

Nonlinearity in the stock market is an area that has attracted many attentions among the academia, practitioners and investors. So many statistical tools could be used in studying their behaviour. The past study suggested that regime switching models usually out- performed linear models. This we have successfully established by using the STAR model (ESTAR and LSTAR). In the study we have successfully investigated the transition behavior of stock market in Botswana with special reference to the low and high regime volatility. The results show that nonlinear switching evidence in Botswana stock market. And finally, the results actually show that Logistic smooth transition autoregressive models (LSTAR) is more appropriate to model the series against linear model as shown in Table 4.

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