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Empirical analysis of asymmetries and long memory among international stock market returns: A Multivariate FIAPARCH-DCC approach

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Abstract

This study examines the interdependence of four stock prices namely (KOSPI, NIKKEI225, SSE and MSCI). The aim of this paper is to examine how the dynamics of correlations between the major stock prices evolved from January 01, 2000 to December10, 2013. To this end, we adopt a dynamic conditional correlation (DCC) model into a multivariate Fractionally Integrated Asymmetric Power ARCH (FIAPARCH) framework, which accounts for long memory, power effects, leverage terms and time varying correlations. The empirical findings indicate the evidence of time-varying comovement, a high persistence of the conditional correlation and the dynamic correlations revolve around a constant

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level and the dynamic process appears to be mean reverting. Moreover, the univariate FIAPARCH models are particularly useful in forecasting market risk exposure for synthetic portfolios of stocks and currencies.

JEL classification: C13 ; C22; C32 ; C52

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1 Introduction

Modeling volatility is an important issue of research in financial markets. Leptokurtosis and volatility clustering are common observation in financial time series (Mandelbrot, 1963). It is well known that financial returns have non-normal distribution which tends to have fat-tailed. Mandelbrot (1963) strongly rejected normal distribution for data of asset returns, conjecturing that financial return processes behave like non-Gaussian stable processes (commonly referred to as “Stable Paretian” distributions).

Many high-frequency financial time series have been shown to exhibit the property of long-memory and Financial time series are often available at a higher frequency than the other time series (Harris &Sollis, 2003).The long range dependence or the long memory implies that the present information has a persistent impact on future counts. Note that the long memory property is related to the sampling frequency of a time series.

To circumvent the drawbacks of this literature, recent research on stock market returns linkages focuses on their dynamic conditional correlations in a time-varying GARCH framework (see Engle et Sheppard, 2001; Tse etTsui, 2002; Engle, 2002). The dynamic conditional correlation (DCC) GARCH approach provides a robust analysis of time-varying linkages by allowing conditional asymmetries in both volatilities and correlations, while investigates the second order moments dynamics of financial time-series and overcomes the heteroskedasticity problem (see Perez-Rodriguez, 2006; Kitamura, 2010; Antonakakis, 2012). Other sophisticated techniques, which avoid the limitations of the standard approaches, are regime switching models (see Boyer et al., 2006), copulas with and without regime-switching (see Patton, 2006; Boero et al., 2011) and nonparametric approaches (see Rodriquez, 2007; Kenourgios et al., 2011).

In this paper, we empirically investigate the time-varying linkages of four daily stock prices, namely KOSPI composite index (Korea), NIKKEI225 (Japan), SSE composite index (Chine) and MSCI word index (MSCI) from January 01, 2000 until December 10, 2013. We use a DCC model into a multivariate fractionally integrated APARCH framework (FIAPARCH-DCC model), which provides the tools to understand how financial volatilities move together over time and across markets. Conrad et al. (2011) applied a multivariate fractionally integrated asymmetric power ARCH (FIAPARCH) model that combines long memory, power transformations of the conditional variances, and leverage effects with constant conditional correlations (CCC) on eight national stock market indices returns. The long-range volatility dependence, the power transformation of returns and the asymmetric response of volatility to positive and negative shocks are three features that improve the modeling of the volatility process of asset returns. We extend their model by estimating time varying conditional correlations among the stock prices.

The flexibility feature represents the key advantage of the FIAPARCH model of Tse (1998)since it includes a large number of alternative GARCH specifications. Specifically, it increases the flexibility of the conditional variance specification by allowing an asymmetric response of volatility to positive and negative shocks and long-range volatility dependence. In addition, it allows the data to determine the power of returns for which the predictable structure in the volatility pattern is the strongest (see Conrad et al., 2011). Although many studies use various multivariate GARCH models in order to estimate DCCs among markets during financial crises (see Chiang et al., 2007; Celic, 2012; Kenourgios et al., 2011), the forecasting superiority of FIAPARCH on other GARCH models is supported by Conrad et al. (2011), Chkili et al. (2012) and Dimitriou and Kenourgios (2013).

The present study investigate dynamics correlations among stock prices from January 01, 2000 until December 10, 2013. We provide a robust analysis of dynamic linkages among stock markets that goes beyond a simple analysis of correlation breakdowns. The time-varying DCCs are captured from a multivariate student-t-FIAPARCH-DCC model which takes into account long memory behavior, speed of market information, asymmetries and leverage effects.

The rest of the paper is organized as follows. Section 2 presents the econometric methodology. Section 3 provides the data and a preliminary analysis. Section 4 displays and discusses the empirical findings and their interpretation, while section 5 provides our conclusions.

2 Econometric methodology

2.1 Univariate FIAPARCH model

The AR(1) process represents one of the most common models to describe a time series r_t of stock returns. Its formulation is given as

$$(1-\xi L)r_t = c + \varepsilon_t,\ \ t \in \mathbb{N} \tag{1}$$

with

$$\varepsilon_t = z_t\sqrt{h_t} \tag{2}$$

where $|c| \in [0, +\infty[$, $|\xi| < 1$ and $\{z_t\}$ are independently and identically distributed $(i.i.d.)$ random variables with $E(z_t) = 0$. The variance h_t is positive with probability equal to unity and is a measurable function of Σ_{t-1}, which is the σ −algebra generated by $\{r_{t-1}, r_{t-2}, ...\}$. Therefore, h_t denotes the conditional variance of the returns $\{r_t\}$, that is:

$$E[r_t/\Sigma_{t-1}] = c + \xi r_{t-1} \tag{3}$$

$$Var[r_t/\Sigma_{t-1}] = h_t \tag{4}$$

Tse (1998) uses a FIAPARCH(1,d,1) model in order to examine the conditional heteroskedasticity of the yen-dollar exchange rate. Its specification is given as

$$(1-\beta L)\left(h_t^{\delta/2} - \omega\right) = [(1-\beta L) - (1-\phi L)(1-L)^d](1+\gamma s_t)|\varepsilon_t|^\delta \tag{5}$$

where $\omega \in [0, \infty[$, $|\beta| < 1$, $|\phi| < 1$, $0 \leq d \leq 1$, $s_t = 1$ if $\varepsilon_t < 0$ and 0 otherwise, $(1-L)^d$ is the financial differencing operator in terms of a hypergeometric function (see Conrad et al., 2011), γ is the leverage coefficient, and δ is the power term parameter (a Box-Cox transformation) that takes (finite) positive values. A sufficient condition for the conditional variance h_t to be positive almost surely for all t is that $\gamma > -1$ and the parameter combination (ϕ, d, β) satisfies the inequality constraints provided in Conrad et Haag (2006) and Conrad (2010).When $\gamma > 0$, negative shocks have more impact on volatility than positive shocks.

The advantage of this class of models is its flexibility since it includes a large number of alternative GARCH specifications. When $d = 0$, the process in Eq. (5) reduces to the APARCH(1,1) oneof Ding et al. (1993), which nests two major classes of ARCH models. In particular, a Taylor/Schwert type of formulation (Taylor, 1986;Schwert, 1990)is specified when $\delta = 1$, and a Bollerslev(1986) type is specified when $\delta = 2$.When $\gamma = 0$and $\delta = 2$, the process in Eq. (5) reduces to the $FIGARCH(1, d, 1)$ specification (see Baillie et al., 1996;Bollerslev and Mikkelsen, 1996) which includes Bollerslev's (1986) GARCH model (when $d = 0$) and the IGARCH specification (when $d = 1$) as special cases.

2.2 Multivariate FIAPARCH model with dynamic conditional correlations

In what follow, we introduce the multivariate FIAPARCH process (M-FIAPARCH) taking into account the dynamic conditional correlation (DCC) hypothesis (see Dimitriou et al., 2013) advanced by Engle (2002). This approach generalizes the Multivariate Constant Conditional Correlation (CCC) FIAPARCH model of Conrad et al.(2011). The multivariate DCC model of Engle (2002) and Tse and Tsui (2002) involves two stages to estimate the conditional covariance matrix H_t. In the first stage, we fit a univariate FIAPARCH(1,d,1) model in order to obtain the estimations of $\sqrt{h_{iit}}$. The daily stock returns are assumed to be generated by a multivariate AR(1) process of the following form:

$$Z(L)r_t = \mu_0 + \varepsilon_t \tag{6}$$

where

- $\mu_0 = [\mu_{0,i}]_{i=1,\dots,n}$: the $N-$dimensional column vector of constants;
- $|\mu_{0,i}| \in [0, \infty[$;
- $Z(L) = diag\{\psi(L)\}$: an $N \times N$ diagonal matrix ;
- $\psi(L) = [1 - \psi_i L]_{i=1,\dots,n}$;
- $|\psi_i| < 1$;
- $r_t = [r_{i,t}]_{i=1,\dots,N}$: the $N-$dimensional column vector of returns;
- $\varepsilon_t = [\varepsilon_{i,t}]_{i=1,\dots,N}$: the$N-$dimensional column vector of residuals.

The residual vector is given by

$$\varepsilon_t = z_t \odot {h_t}^{\wedge 1/2} \tag{7}$$

where

- $\odot$: the Hadamard product;
- $\wedge$: the elementwise exponentiation.

$h_t = [h_{it}]_{i=1,\dots,N}is\Sigma_{t-1}$ measurable and the stochastic vector $z_t = [z_{it}]_{i=1,\dots,N}$ is independent and identically distributed with mean zero and positive definite covariance matrix $\rho = [\rho_{ijt}]_{i,j=1,\dots,N}$ with $\rho_{ij} = 1$ for $i = j$ .Note that

$E(\varepsilon_t/\mathcal{F}_{t-1}) = 0$and $H_t = E(\varepsilon_t\varepsilon_t'/\mathcal{F}_{t-1}) = diag(h_t^{\wedge 1/2})\,\rho\,diag(h_t^{\wedge 1/2})$. $h_t$is the vector of conditional variances and $\rho_{i,j,t} = h_{i,j,t}/\sqrt{h_{i,t}h_{j,t}}\forall\, i,j = 1,\dots,N$ are the dynamic conditional correlations.

The multivariate FIAPARCH(1,d,1) is given by

$$B(L)\left(h_t^{\wedge\delta/2} - \omega\right) = [B(L) - \Delta(L)\Phi(L)][\mathrm{I}_N + \Gamma_t]|\varepsilon_t|^{\wedge\delta} \tag{8}$$

where$|\varepsilon_t|$ is the vector ε_t with elements stripped of negative values.

Besides, $B(L) = diag\{\beta(L)\}$ with $\beta(L) = [1-\beta_i L]_{i=1,\dots,N}$ and $|\beta_i| < 1$. Moreover, $\Phi(L) = diag\{\phi(L)\}$ with $\phi(L) = [1-\phi_i L]_{i=1,\dots,N}$and $|\phi_i| < 1$. In addition, $\omega = [\omega_i]_{i=1,\dots,N}$ with $\omega_i \in [0,\infty[$ and $\Delta(L) = diag\{d(L)\}$ with $d(L) = [(1-L)^{d_i}]_{i=1,\dots,N}$ $\forall\, 0 \le d_i \le 1$. Finally, $\Gamma_t = diag\{\gamma \odot s_t\}$ with $\gamma = [\gamma_i]_{i=1,\dots,N}$ and $s_t = [s_{it}]_{i=1,\dots,N}$ where $s_{it} = 1$ if $\varepsilon_{it} < 0$ and 0 otherwise.

In the second stage, we estimate the conditional correlation using the transformed stock return residuals, which are estimated by their standard deviations from the first stage. The multivariate conditional variance is specified as follows:

$$H_t = D_t R_t D_t \tag{9}$$

where $D_t = diag\left(h_{11t}^{1/2},\dots,h_{NNt}^{1/2}\right)$ denotes the conditional variance derived from the univariate AR(1)-FIAPARCH(1,d,1) model and $R_t = (1-\theta_1-\theta_2)R + \theta_1\psi_{t-1} + \theta_2 R_{t-1}$ is the conditional correlation matrix[4].

In addition, θ_1 and θ_2 are the non-negative parameters satisfying $(\theta_1 + \theta_2) < 1$, $R = \{\rho_{ij}\}$ is a time-invariant symmetric $N\times N$ positive definite parameter matrix with $\rho_{ii} = 1$ and ψ_{t-1} is the $N\times N$ correlation matrix of ε_τ

[4] Engle (2002) derives a different form of DCC model. The evolution of the correlation in DCC is given by: $Q_t = (1-\alpha-\beta)\bar{Q} + \alpha z_{t-1} + \beta Q_{t-1}$, where $Q = (q_{ijt})$ is the $N\times N$ time-varying covariance matrix of z_t, $\bar{Q} = E[z_t z_t']$ denotes the $n\times n$ unconditional variance matrix of z_t, while α and β are nonnegative parameters satisfying $(\alpha+\beta) < 1$. Since Q_t does not generally have units on the diagonal, the conditional correlation matrix R_t is derived by scaling Q_t as follows: $R_t = (diag(Q_t))^{-1/2}Q_t(diag(Q_t))^{-1/2}$.

for $\tau = t - M, t - M + 1, \ldots, t - 1$. The $i, j - th$ element of the matrix ψ_{t-1} is given as follows:

$$\psi_{ij,t-1} = \frac{\sum_{m=1}^{M} z_{i,t-m} z_{j,t-m}}{\sqrt{\left(\sum_{m=1}^{M} z_{i,t-m}^2\right)\left(\sum_{m=1}^{M} z_{j,t-m}^2\right)}}, \quad 1 \leq i \leq j \leq N \tag{10}$$

where$z_{it} = \varepsilon_{it}/\sqrt{h_{iit}}$ is the transformed stock return residuals by their estimated standard deviations taken from the univariate AR(1)-FIAPARCH(1,d,1) model.

The matrix ψ_{t-1} could be expressed as follows:

$$\psi_{t-1} = B_{t-1}^{-1} L_{t-1} L_{t-1}' B_{t-1}^{-1} \tag{11}$$

where B_{t-1} is a $N \times N$ diagonal matrix with $i - th$ diagonal element given by $\left(\sum_{m=1}^{M} z_{i,t-m}^2\right)$ and $L_{t-1} = (z_{t-1}, \ldots, z_{t-M})$ is a $N \times N$ matrix, with $z_t = (z_{1t}, \ldots, z_{Nt})'$.

To ensure the positivity of ψ_{t-1} and therefore of R_t, a necessary condition is that $M \leq N$.Then, R_t itself is a correlation matrix if $R_{t-1}$is also a correlation matrix. The correlation coefficient in a bivariate case is given as:

$$\rho_{12,t} = (1 - \theta_1 - \theta_2)\rho_{12} + \theta_2 \rho_{12,t} + \theta_1 \frac{\sum_{m=1}^{M} z_{1,t-m} z_{2,t-m}}{\sqrt{(\sum_{m=1}^{M} z_{1,t-m}^2)(\sum_{m=1}^{M} z_{2,t-m}^2)}} \tag{12}$$

3 Data and preliminary analyses

The data comprises daily stock indexes: KOSPI (Korea), NIKKEI225 (Japan), SSE (China) and MSCI (Morgan Stanley Capital International). MSCI market classification consists of following three criteria: size and liquidity, market accessibility and economic development. All data are sourced from the (http//www.econstats.com). The sample covers a period from January 01, 2000 until December 10, 2013, leading to a sample size of 3639 observations. For each indexes, the continuously compounded return is computed as $r_t = 100 \times ln(p_t/p_{t-1})$ for $t = 1,2, \ldots, T$, where $p_t$is the price on day t.

Summary statistics for the stock market returns are displayed in Table 1(Panel

A). From these tables, KOSPI is the most volatile, as measured by the standard deviation of 1.6544%, while MSCI is the least volatile with a standard deviation of 1.4641%. Besides; we observe that NIKKEI225 has the highest level of excess kurtosis, indicating that extreme changes tend to occur more frequently for the stock price. In addition, all stock market returns exhibit high values of excess kurtosis. To accommodate the existence of “fat tails”, we assume student-t distributed innovations. Furthermore, the Jarque-Bera statistic rejects normality at the 1% level for all stock prices. Moreover, all stock market return series are stationary, I(0), and thus suitable for long memory tests. Finally, they exhibit volatility clustering, revealing the presence of heteroskedasticity and strong ARCH effects.

In order to detect long-memory process in the data, we use the log-periodogram regression (GPH) test of Geweke and Porter-Hudak (1983) on two proxies of volatility, namely squared returns and absolute returns. The test results are displayed in Table 1 (Panel D). Based on these tests’ results, we reject the null hypothesis of no long-memory for absolute and squared returns at 1% significance level. Subsequently, all volatilities proxies seem to be governed by a fractionally integrated process. Thus, FIAPARCH seem to be an appropriate specification to capture volatility clustering, long-range memory characteristics and asymmetry.

Figure 1 illustrates the evolution of stock indexes during the period from January 1, 2000 until December 10, 2013. The figure shows significant variations in the levels during the turmoil, especially at the time of Lehman Brothers failure (September 15, 2008). Specifically, when the global financial crisis triggered, there was a decline for all prices. Figure 2 plots the evolution of stock market returns over time. The figure shows that all stock indexes trembled since 2008 with different intensity during the global financial and European sovereign debt crises. Moreover, the plot shows a clustering of larger return volatility around and after 2008. This means that foreign exchange markets are characterized by

Table 1

Summary statistics and long memory test's results.

	KOSPI	NIKKEI225	SSE	MSCI
Panel A: descriptive statistics				
Mean	1.81E-02	-0.0053	0.0135	-1.77E-05
Maximum	11.284	13.235	9.4008	6.5246
Minimum	-12.805	-12.111	-9.2562	-9.936
Std. Deviation	1.6544	1.5304	1.5456	1.4641
Skewness	-0.54142***	-0.4348***	-0.0887***	-0.241***
	(0,0000)	(0,0000)	-0.0287	(0,0000)
ExcessKurtosis	5.7577***	6.8355***	4.7723***	3.0688***
	(0,0000)	(0,0000)	(0,0000)	(0,0000)
Jarque-Bera	5204.3***	7199.2***	3458***	1463.2***
	(0,0000)	(0,0000)	(0,0000)	(0,0000)
Panel B: Serial correlation and LM-ARCH tests				
$LB(20)$	31.6153**	14.4001	44.7177***	72.6072***
	(0.0475)	(0.8096)	(0.0012)	(0,0000)
$LB^2(20)$	1339.54***	3792.44***	695.483***	1433.72***
	(0,0000)	(0,0000)	(0.0000)	(0,0000)
ARCH 1-10	48.134***	141.66***	25.233***	44.144***
	(0,0000)	(0,0000)	(0,0000)	(0,0000)
Panel C: Unit Root tests				
ADF test statistic	-35.3164***	-36.819***	-33.7277***	-33.1275***
	(-1.9409)	(-1.9409)	(-1.9409)	(-1.9409)
Panel D: long memory tests (GPH test$-d$ estimates)				
Squared returns				
$m=T^{0.5}$	0.4238	0.2687	0.4593	0.5946
	[0.0698]	[0.0573]	[0.0813]	[0.0900]
$m=T^{0.6}$	0.3486	0.4649	0.3690	0.3955
	[0.0464]	[0.0498]	[0.0620]	[0.0580]
Absolute returns				
$m=T^{0.5}$	0.5047	0.3403	0.4781	0.5623
	[0.0742]	[0.0812]	[0.0838]	[0.1050]
$m=T^{0.6}$	0.4157	0.4487	0.37002	0.4381
	[0.0509]	[0.0570]	[0.0568]	[0.0697]

Notes: Stock market returns are in daily frequency. r^2 and $|r|$ are squared log return and absolute log return, respectively. m denotes the bandwith for the Geweke and Porter-Hudak's (1983) test. Observations for all series in the whole sample period are 3639. The numbers in brackets are t-statistics and numbers in parentheses are p-values. ***, **, and * denote statistical significance at 1%, 5% and 10% levels, respectively. $LB(20)$ and $LB^2(20)$ are the 20th order Ljung-Box tests for serial correlation in the standardized and squared standardized residuals, respectively.

volatility clustering, i.e., large (small) volatility tends to be followed by large (small) volatility, revealing the presence of heteroskedasticity. This market phenomenon has been widely recognized and successfully captured by ARCH/GARCH family models to adequately describe stock market returns dynamics. This is important because the econometric model will be based on the interdependence of the stock markets in the form of second moments by modeling the time varying variance-covariance matrix for the sample.

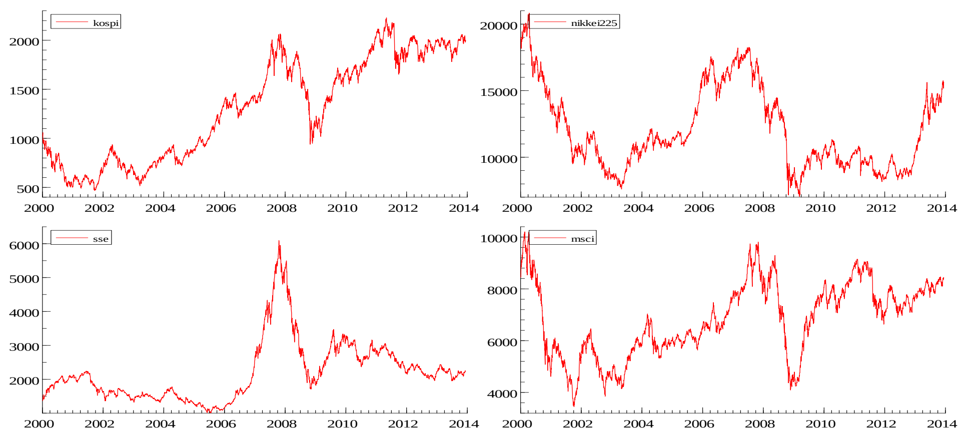

Figure 1: Stock prices behavior over time

4 Empirical results

4.1 The univariate FIAPARCH estimates

In order to take into account the serial correlation and the GARCH effects observed in our time series data, and to detect the potential long range dependence in volatility, we estimate the student[5]-t-AR(0)-FIAPARCH(1,d,1)[6] model defined

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[5] The z_t random variable is assumed to follow a student distribution (see Bollerslev, 1987) with $\upsilon > 2$ degrees of freedom and with a density given by:

$$D(z_t, \upsilon) = \frac{\Gamma(\upsilon+\frac{1}{2})}{\Gamma(\frac{\upsilon}{2})\sqrt{\pi(\upsilon-2)}}(1+\frac{z_t^2}{\upsilon-2})^{\frac{1}{2}-\upsilon}$$

by Eqs. (1) and (5). Table 2 reports the estimation results of the univariate FIAPARCH(1,d,1) model for each stock market return series of our sample.

The estimates of the constants in the mean are statistically significant at 1% level or better for all the series except for the NIKKEI225. Besides, the constants in the variance are significant except for KOSPI and MSCI currencies. In addition, for all currencies, the estimates of the leverage term (γ) are statistically significant, indicating an asymmetric response of volatilities to positive and negative shocks. This finding confirms the assumption that there is negative correlation between returns and volatility. According to Patton (2006), such asymmetric effects could be explained by the asymmetric behavior of central banks in their currency interventions. In other words, Patton (2006) argues that when central banks emphasize on competitiveness over price stability, the exchange rates may display higher volatility during periods of depreciation compared to periods of appreciation.

Moreover, the estimates of the power term (δ) are highly significant for all currencies and ranging from 1.4582 to 1.9252. Conrad et al. (2011) show that when the series are very likely to follow a non-normal error distribution, the superiority of a squared term $(\delta = 2)$ is lost and other powertransformations may be more appropriate. Thus, these estimates support the selection of

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where $\Gamma(v)$ is the gamma function and v is the parameter that describes the thickness of the distribution tails. The Student distribution is symmetric around zero and, for $v > 4$, the conditional kurtosis equals $3(v-2)/(v-4)$, which exceeds the normal value of three. For large values of v, its density converges to that of the standard normal.
For a Student-*t* distribution, the log-likelihood is given as: $L_{Student} = T\left\{log\Gamma\left(\frac{v+1}{2}\right) - log\Gamma\left(\frac{v}{2}\right) - \frac{1}{2}log[\pi(v-2)]\right\} - \frac{1}{2}\sum_{t=1}^{T}\left[\log(h_t) + (1+v)log\left(1+\frac{z_t^2}{v-2}\right)\right]$
where T is the number of observations, v is the degrees of freedom, $2 < v \leq \infty$ and $\Gamma(.)$ is the gamma function.

[6] The lag orders$(1, d, 1)$and $(0,0)$ for FIAPARCH and ARMA models, respectively, are selected by Akaike (AIC) and Schwarz (SIC) information criteria. The results are available from the author upon request.

FIAPARCH model for modeling conditional variance of stock market returns. Besides, all stock indexes display highly significant differencing fractional parameters(d), indicating a high degree of persistence behavior. This implies that the impact of shocks on the conditional volatility of stock market' returns consistently exhibits a hyperbolic rate of decay. Interestingly, the highest power term is obtained for NIKKEI225 stock index, one is characterized by the highest degree of persistence. In all cases, the estimated degrees of freedom parameter (v) is highly significant and leads to an estimate of the Kurtosis which is equal to $3(v-2)/(v-4)$ and is also different from three.

In addition, all the ARCH parameters (ϕ) satisfy the set of conditions which guarantee the positivity of the conditional variance. Moreover, according to the values of the Ljung-Box tests for serial correlation in the standardized and squared standardized residuals, there is no statistically significant evidence, at the 1% level, of misspecification in almost all cases except for the MSCI stock index.

Numerous studies have documented the persistence of volatility in stock and exchange rate returns (see Ding et al., 1993; Ding et Granger, 1996, among others).The majority of these studies have shown that the volatility process is well approximated by an IGARCH process. Nevertheless, from the FIAPARCH estimates reported in Table 2, it appears that the long-run dynamics are better modeled by the fractional differencing parameter.

Table 2

Univariate FIAPARCH(1,d,1) models (MLE).

	KOSPI		NIKKEI225		SSE		MSCI	
	Coefficient	t-prob	Coefficient	t-prob	Coefficient	t-prob	Coefficient	t-prob
Estimate								
c	0.0652***	0.0004	0.0269	0.1760	0.0291*	0.0946	0.0483***	0.0046
ω	0.0334	0.4514	0.1353***	0.0014	0.2771***	0.0080	0.0450	0.2037
d	0.2359***	0.0024	0.4102***	0.0000	0.3146***	0.0000	0.3132***	0.0000
ϕ	0.1152	0.1572	0.1116**	0.0368	-0.1097	0.3816	0.1731***	0.0091
β	0.3142***	0.0134	0.4919***	0.0000	0.1428	0.3486	0.4571***	0.0000
γ	0.8930	0.0235	0.4465***	0.0010	0.3323***	0.0000	0.5574***	0.0032
δ	1.5594***	0.0000	1.4582***	0.0000	1.9252***	0.0000	1.6832***	0.0000
v	5.8608***	0.0000	8.2601***	0.0000	3.6846***	0.0000	6.1827***	0.0000
Diagnostics								
$LB(20)$	19.2243	0.5072	11.7653	0.9239	53.5749***	0.0000	45.0142***	0.0010
$LB^2(20)$	22.5678	0.2077	31.2876**	0.0266	10.6958	0.9068	29.9101**	0.0383

Notes: For each of the five exchange rates, Table 2 reports the Maximum Likelihood Estimates (MLE) for the student-t-FIAPARCH(1,d,1) model. $\boldsymbol{LB(20)}$ and $\boldsymbol{LB^2(20)}$ indicate the Ljung-Box tests for serial correlation in the standardized and squared standardized residuals, respectively. $\boldsymbol{v}$ denotes the the t-student degrees of freedom.parameter ***, ** and * denote statistical significance at 1%, 5% and 10% levels, respectively.

To test for the persistence of the conditional heteroskedasticity models, we examine the Likelihood Ratio (LR) statistics for the linear constraints $d = 0$ (APARCH(1,1) model) and $d \neq 0$ (FIAPARCH(1,d,1) model). We construct a series of LR tests in which the restricted case is the APARCH(1,1) model $(d = 0)$ of Ding et al. (1993). Let l_0 be the log-likelihood value under the null hypothesis that the true model is APARCH(1,1) and l the log-likelihood value under the alternative that the true model is FIAPARCH(1,d,1). Then, the LR test,$2(l - l_0)$, has a chi-squared distribution with 1 degree of freedom when the null hypothesis is true.

For reasons of brevity, we omit the table with the test results, which are available from the author upon request. In summary, the LR tests provide a clear rejection of the APARCH(1,1) model against the FIAPARCH(1,d,1) one for all stock prices. Thus, purely from the perspective of searching for a model that best describes the volatility in the stock price series, the FIAPARCH(1,d,1) model appears to be the most satisfactory representation. This finding is important since the time series behavior of volatility could affect asset prices through the risk premium (see Christensen and Nielsen, 2007; Christensen et al., 2010;Conrad et al., 2011).

With the aim of checking for the robustness of the LR testing results discussed above, we apply the Akaike (AIC), Schwarz (SIC), Shibata (SHIC) or Hannan-Quinn (HQIC) information criteria to rank the ARCH type models. According to these criteria, the optimal specification (i.e., APARCH or FIAPARCH) for all stock prices is the FIAPARCH one. The two common values of the power term (δ) imposed throughout much of the GARCH literature are $\delta = 2$ (Bollerslev's model) and $\delta = 1$ (the Taylor/Schwert specification). According to Brooks et al. (2000), the invalid imposition of a particular value for the power term may lead to sub-optimal modeling and forecasting performance. For that reason, we test whether the estimated power terms are significantly different from unity or two using Wald tests (results not reported).

We find that all five estimated power coefficients are significantly different from unity. Furthermore, each of the power terms is significantly different from two. Hence, on the basis of these findings, support is found for the (asymmetric) power fractionally integrated model, which allows an optimal power transformation term to be estimated. The evidence obtained from the Wald tests is reinforced by the model ranking provided by the four model selection criteria (values not reported). This is a noteworthy result since He and Teräsvirta (1999) emphasized that if the standard Bollerlsev type of model is augmented by the 'heteroskedasticity' parameter, the estimates of the ARCH and GARCH coefficients almost certainly change. More importantly, Karanasos and Schurer (2008) show that, in the univariate GARCH-in-mean level formulation, the significance of the in-mean effect is sensitive to the choice of the power term.

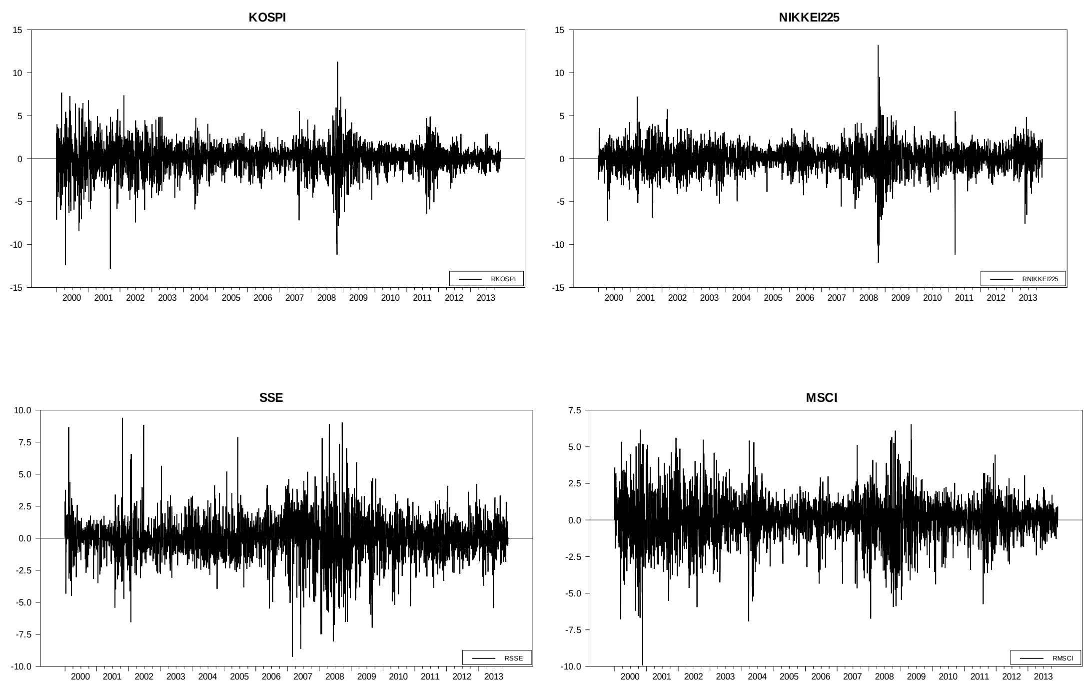


Figure 2: Stock market returns behavior over time

4.2 The bivariate FIAPARCH(1,d,1)-DCC estimates

The analysis above suggests that the FIAPARCH specification describes the conditional variances of the four stock prices well. Therefore, the multivariate FIAPARCH model seems to be essential for enhancing our understanding of the relationships between the (co)volatilities of economic and financial time series.

In this section, within the framework of the multivariate DCC model, we analyze the dynamic adjustments of the variances for the four stock prices. Overall, we estimate six bivariate specifications for our analysis. Table 3(Panels A and B) reports the estimation results of the bivariate student-t-FIAPARCH(1,d,1)-DCC model. The ARCH and GARCH parameters (a and b) of the DCC(1,1) model capture, respectively, the effects of standardized lagged shocks and the lagged dynamic conditional correlations effects on current dynamic conditional correlation. They are statistically significant at the 5% level, except for the ARCH parameter between (KOSPI-SSE) and (KOSPI-MSCI),indicating the existence of time-varying correlations. Moreover, they are non-negative, justifying the appropriateness of the FIAPARCH model. When $a = 0$and $b = 0$, we obtain the Bollerslev’s (1990) Constant Conditional Correlation (CCC) model. As shown in Table 3, the estimated coefficients a and b are significantlypositive and satisfy the inequality $a + b < 1$ in each of the pairs of stock prices. Besides, the t-student degrees of freedom parameter (v)is highly significant, supporting the choice of this distribution.

The statistical significance of the DCC parameters ($a$and b) reveals a considerable time-varying comovement and thus a high persistence of the conditional correlation. The sum of these parameters is close to unity. This implies that the volatility displays a highly persistent fashion. Since $a + b < 1$, the dynamic correlations revolve around a constant level and the dynamic process appears to be mean reverting. The multivariate FIAPARCH-DCC model is so important to consider in our analysis since it has some key advantages. First, it captures the long range dependence property. Second, it allows obtaining all possible pair-wise conditional correlation coefficients for the stock market returns in the sample. Third, it’s possible to investigate their behavior during periods of particular interest, such as periods of the global financial and European sovereign debt crises. Fourth, the model allows looking at possible financial contagion effects between international foreign exchange markets.

Finally, it is crucial to check whether the selected stock price series display evidence of multivariate Long Memory ARCH effects and to test ability of the Multivariate FIAPARCH specification to capture the volatility linkages among stock prices. Kroner and Ng (1998) have confirmed the fact that only few diagnostic tests are kept to the multivariate GARCH-class models compared to the diverse diagnostic tests devoted to univariate counterparts. Furthermore, Bauwens et al. (2006) have noted that the existing literature on multivariate diagnostics is sparse compared to the univariate case. In our study, we refer to the most broadly used diagnostic tests, namely the Hosking's and Li and McLeod's Multivariate Portmanteau statistics on both standardized and squared standardized residuals. According to Hosking (1980), Li and McLeod (1981) and McLeod and Li (1983) autocorrelation test results reported in Table 3 (Panel B), the multivariate diagnostic tests allow accepting the null hypothesis of no serial correlation on both standardized and squared standardized residuals and thus there is no evidence of statistical misspecification.

Figure 3 illustrates the evolution of the estimated dynamic conditional correlations dynamics among international stock markets. Compared to the pre-crises period, the estimated DCCs show a decline during the post-crises period. Such evidence is in contrast with the findings of previous research on foreign exchange and stock markets, which show increases in correlations during periods of financial turmoil (see Kenourgios et al., 2011; Dimitriou et al., 2013; Dimitriou and Kenourgios, 2013). Nevertheless, the different path of the estimated DCCs displays fluctuations for all pairs of stock prices across the phases of the global financial and European sovereign debt crises, suggesting that the assumption of constant correlation is not appropriate. The above findings motivate a more extensive analysis of DCCs, in order to capture contagion dynamics during different phases of the two crises

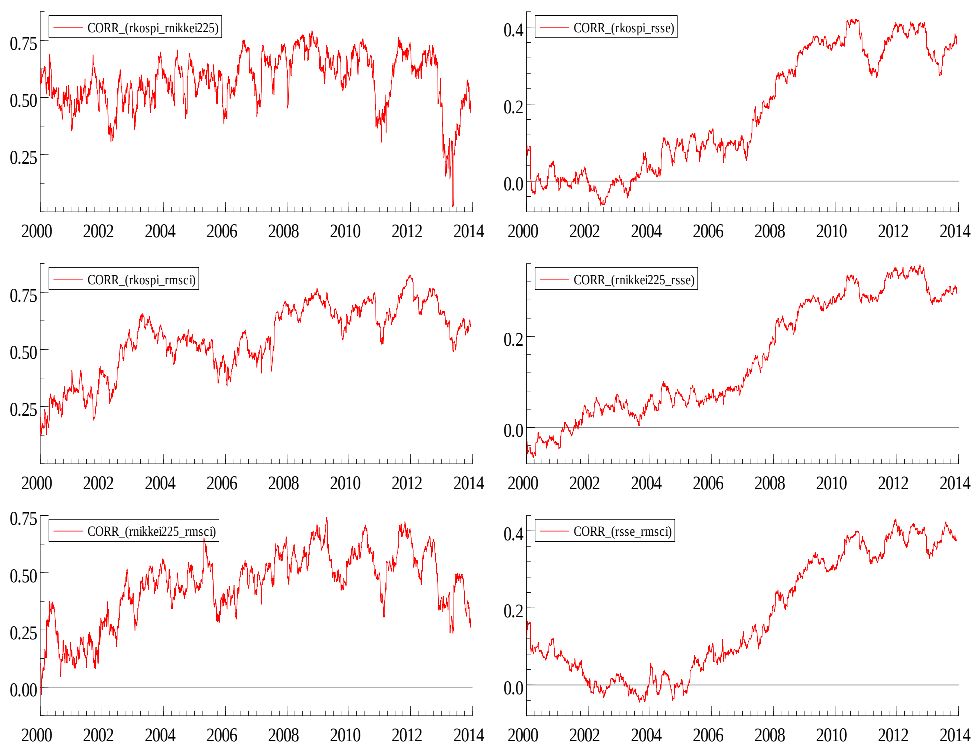


Figure3: The DCC behavior over time.

Nevertheless, the different path of the estimated DCCs displays fluctuations for all pairs of stock prices across the phases of the global financial and European sovereign debt crises, suggesting that the assumption of constant correlation is not appropriate. The above findings motivate a more extensive analysis of DCCs, in order to capture contagion dynamics during different phases of the two crise

Table 3

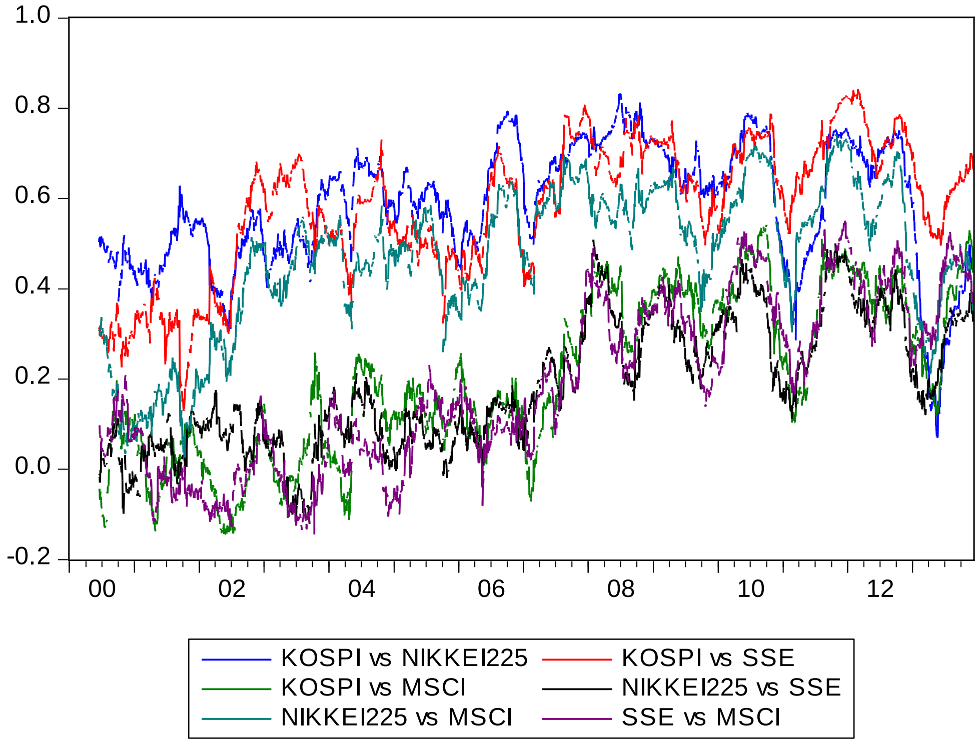
Estimation results from the bivariate FIAPARCH(1,d,1)-DCC model.

	KOSPI-NIKKEI225		KOSPI-SSE		KOSPI-MSCI		NIKKEI225-SSE		NIKKEI225-MSCI		SSE-MSCI	
	coefficient	t-prob	coefficient	t-prob	coefficient	t-prob	coefficient	t-prob	Coefficient	t-prob	coefficient	t-prob
Panel A: Estimates of Multivariate DCC												
a	0.0248***	0.0000	0.0042	0.1068	0.0124	0.1353	0.0030***	0.0003	0.0163**	0.0471	0.0040**	0.0462
b	0.9682***	0.0000	0.9956***	0.0000	0.9875***	0.0000	0.9969***	0.0000	0.9833***	0.0000	0.9959***	0.0000
v	8.1989***	0.0000	5.4434***	0.0000	6.4155***	0.0000	6.3523***	0.0000	8.2822***	0.0000	5.5042***	0.0000
Panel B : Diagnostic tests												
$Hosking(20)$	79.0740	0.5082	122.379***	0.0016	123.804***	0.0012	108.057**	0.0200	100.927*	0.0569	133.382***	0.0001
$Hosking^2(20)$	85.0790	0.2730	81.4598	0.3721	127.368***	0.0003	68.4734	0.7710	85.6401	0.2592	127.745***	0.0003
$Li-McLeod(20)$	79.0597	0.5087	122.266***	0.0016	123.739***	0.0012	107.993**	0.0202	100.849*	0.0576	133.274***	0.0001
$Li-McLeod^2(20)$	85.0561	0.2736	81.4995	0.3709	127.317***	0.0003	68.4901	0.7705	85.6269	0.2595	127.549***	0.0003

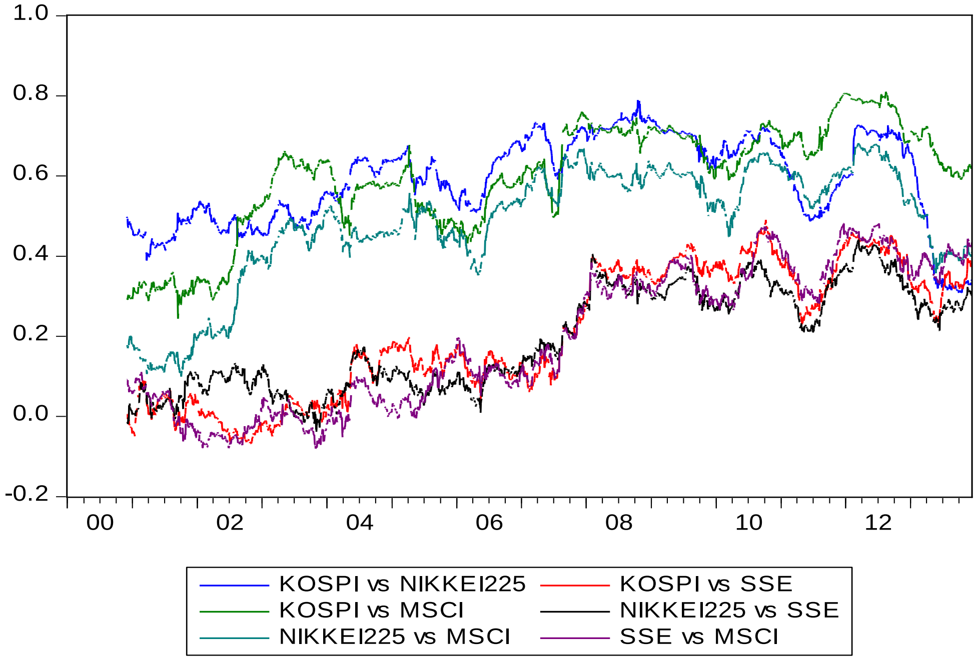
Notes: The superscripts ***, ** and * denote the statistical significance at 1%, 5% and 10% levels, respectively.$v$indicates the student's distribution's degrees of freedom. $Hosking\ (20)$and$Hosking^2(20)$ denote the Hosking's Multivariate Portmanteau Statistics on both standardized and squared standardized Residuals. $Li-McLeod\ (20)$ and $Li-McLeod^2(20)$ indicate the Li and McLeod's Multivariate Portmanteau Statistics on both Standardized and squared standardized Residuals.

In Figure 4, we plot the rolling correlations between each pair of stock prices with time spans of four months, eight months, one year, two years and four years, respectively. Interestingly, we find more fluctuations of the rolling correlations in downward directions between each pair, particularly after 2007, regardless of the selected time spans. Moreover, we mainly detect sharp decreases in the correlations between each pair since 2010.

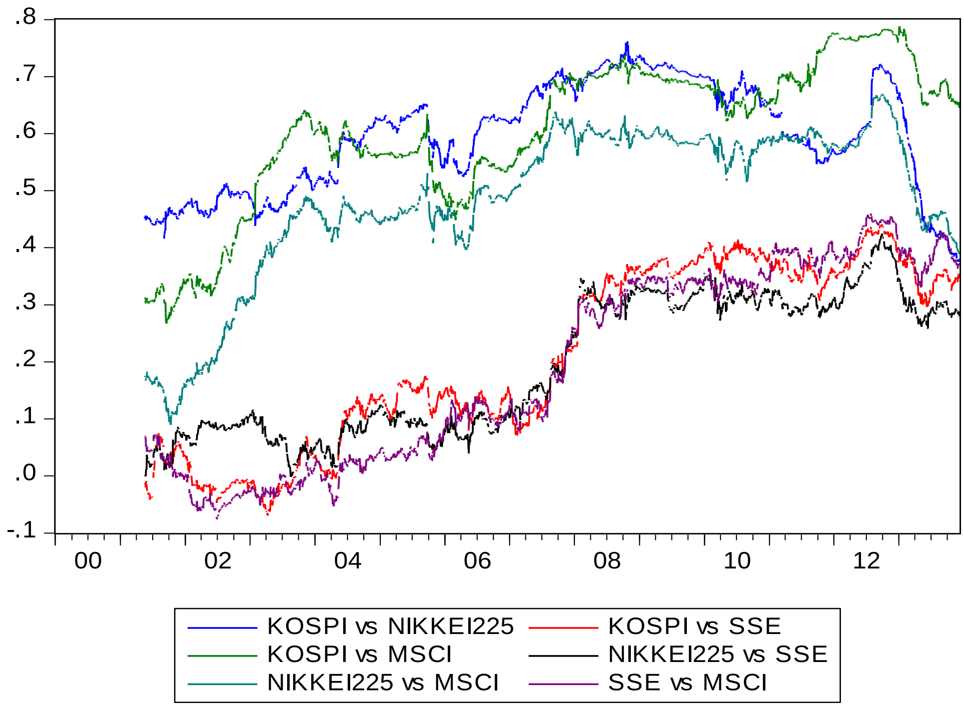
(a) Four-month rolling correlation



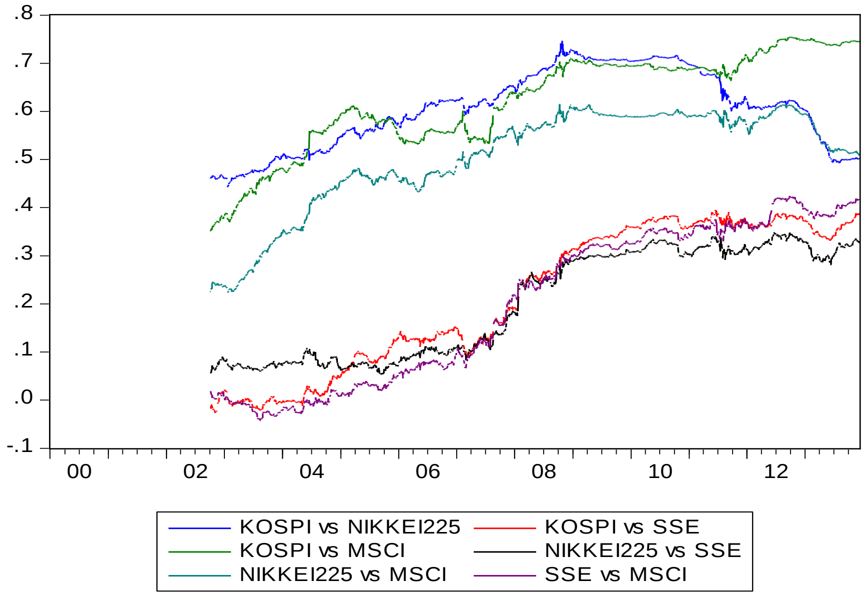
(b) Eight-month rolling correlation



(c) Two-year rolling correlation



(d) Two-year rolling correlation



(e) Four-year rolling correlation

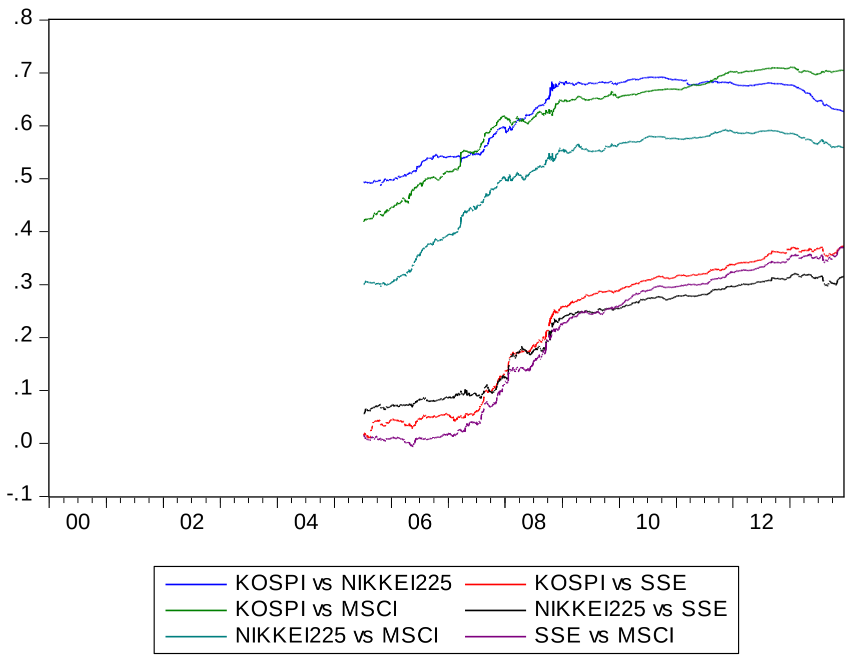


Figure 4: Rolling correlations between stock index pair. (a) Four-month rolling correlation. (b) Eight-month rolling correlation. (c) Two-year rolling correlation. (d) Two-year rolling correlation. (e) Four-year rolling correlation.

4 Conclusion

The present study examines the dynamic correlations among international stock prices namely KOSPI, NIKKEI225, SSE and MSCI. Specifically, we employ a multivariate FIAPARCH-DCC model, during the period from January 01, 2000 to December 10, 2013, focusing on the estimated dynamic conditional correlations among the stock markets. This approach allows investigating the second order moments dynamics of stock prices taking into account long range dependence behavior, asymmetries and leverage effects.

The FIAPARCH model is identified as the best specification for modeling the conditional heteroscedasticity of individual time series. We then extended the above univariate GARCH models to a bivariate framework with dynamic conditional correlation parameterization in order to investigate the interaction between stock markets. Our results document strong evidence of time-varying comovement, a high persistence of the conditional correlation (the volatility displays a highly persistent fashion) and the dynamic correlations revolve around a constant level and the dynamic process appears to be mean reverting.

More interestingly, the univariate FIAPARCH models are particularly useful in forecasting market risk exposure for synthetic portfolios of stocks and currencies. Our out-of-sample analysis confirms the superiority of the univariate FIAPARCH model and the bivariate DCC-FIAPARCH model over the competing specifications in almost all cases.

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