Death from stroke during the Danish malnutrition period 1999-2007

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Abstract

Data from the Danish Health and Medicines Authority (Statens Serum Institut) show that Denmark had a problem around 2003 with malnutrition among old people. Malnutrition is classified as a cause of death; however, it also indirectly causes an increasing number of deaths due to other diseases that it provokes. Based on Danish experiences from 1994 to 2012, this article shows a close association between the death rate from malnutrition and the death rate from stroke (apoplexy) in the same period. The death rate from malnutrition during the Danish malnutrition period 1999 to 2007 thus underestimates the total effect of malnutrition on the death rate. Using the death rate from malnutrition as indicator, the article shows the association between malnutrition and the death rate from stroke depending on age group, gender, and time (medical development).

Mathematical Subject Classification: Statistics; numerical analysis; special functions

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1 Introduction

Since 1995, the literature has overflowed with articles connecting nutrition to health. The Dutch famine, which occurred from October 1944 to May 1945, during the German occupation, has opened a new discussion on the association between prenatal exposure to famine and health in later life; see Abeelen et al. [1], Yarde et al. [27], de Rooij [8], de Rooij and Roseboom [9], and Ekamper et al. [10].

The Dutch famine, included the entire population in a given geographic area. Close studies on children conceived or born during this period show that children exposed to famine during pregnancy have an increased risk of, among others, being hospitalized for COPD and asthma, and of developing diabetes.

Hardly anyone would connect Denmark with starvation or malnutrition. Nevertheless, that was the case in Denmark between 1999 and 2007. (Sparre-Sørensen and Kristensen [24]). See also appendix Table A1.

Rasmussen et al. [22] gave an early warning in a questionnaire-based investigation on inadequate clinical nutrition. Mowe et al. [19], who based their study on a questionnaire from 2004 noted that “nutritional support ranks low on the list of treatment and evaluation priorities. This is in contrast to the high prevalence of under-nutrition and inadequate nutritional treatment among hospitalized patients.”

Lindorff-Larsen et al. [18] concluded that “although significant positive changes had thus occurred, the main barriers against implementation of good nutrition continued to be lack of knowledge, interest and responsibility, in combination with difficulties in making a nutrition plan.” Wengler et al. [21] and Mowe et al. [20] supported this point of view.

Johansson et al. [12] underlined that the problem of undernutrition has not been solved, and a pessimistic Beck and Hansen [2] concluded that “meals prepared for
residents in Danish nursing homes and for those receiving Meals-on-Wheels do not consistently offer adequate nutritional content.”

The present study, in contrast to the Dutch famine studies, only includes malnutrition among people above 55 years.

The actual level of malnutrition is unknown. Therefore, we apply the death rate from malnutrition as a proxy for malnutrition. However, because the same malnutrition hits harder the older someone is, the death rate cycle from malnutrition has a larger amplitude for the age group 85+ than for younger age groups.

The FOOD Trial Collaboration [11] discussed the association between malnutrition and stroke. “Of 275 undernourished patients, 102 (37%) were dead by final follow-up compared with only 445 (20%) of 2194 patients of normal nutritional status.”

More recent studies have shown that malnutrition not only increases all-cause mortality, but also that pre-stroke underweight increases post-stroke mortality among older women (Bell et al. [3]).

Skolarus et al. [23] note that “after adjustment for demographics, stroke severity, and stroke mortality risk factors, the relationship between BMI and mortality was U shaped. The lowest mortality risk was observed among patients with an approximate BMI of 35 kg/m², whereas those with lower or higher BMI had higher mortality risk. Malnutrition may even be the actual cause of death in some stroke deaths, as acute thiamin deficiency, a possible side effect of malnutrition, can be difficult to distinguish from acute ischemic stroke; see Blum et al. [5]. It is therefore relevant to examine the effect of the Danish malnutrition period on the number deaths caused by stroke.

The purpose of this article is to relate the malnutrition period in Denmark 1999-2007 to the increase in the death rate from apoplexy in the same period.
2 Data

Definition: The death rate is the number of deaths from a certain cause per 100000 persons in a considered group (for instance the number of deaths from stroke per 100 000 women in the age group 80-84). The Danish data on the death rate from malnutrition and apoplexy are taken from The Danish Health and Medicines Authority (Statens Serum Institut): Malnutrition, B-040, and death from Haemorrhagia and Apoplexia Cerebralis, B-061, in everyday speech “stroke”.

Based on the total dataset for deaths and death rates from malnutrition and stroke (apoplexy) in Denmark 1994-2012, the included variables are:

Variables:


(Strw – for women, Strm for men).

NStr number of deaths from stroke.


(Dmalw – for women, Dmalm – for men).

StMa death rate from stroke associated with deaths from malnutrition.

NStMa number of deaths from stroke (apoplexy) provoked by malnutrition.

T the period (or year), 1977 = 1. Indicates technical progress, which reduces diseases with a certain percentage every year.

TD T/100 applied to omit too high values, to avoid overflow in forecasts.

Age age at death. Applied to 5-year age groups. Diseases tend to grow almost exponentially over age.

CohYear cohort dummy; see appendix Table 5A.

B variable created by estimated cohort dummy coefficients.

Figure 1 (below) shows the death rate from malnutrition during the years 1994 to 2012. Figure 2 shows the death rate from stroke from 1994 to 2012. As shown in
Figure 2, during the discussed span of years, stroke is rapidly declining as a cause of death. We see that the death rate from stroke is quite high, although declining, while the death rate from malnutrition is very low.

Over age, the death rate is clearly increasing for malnutrition as well as stroke. The death rate from stroke is generally rapidly declining over the period. However, in the Danish malnutrition period 1999 to 2007, we see an increased death rate from stroke above the declining trend.

Figure 1: The death rate from malnutrition shown by data for Danish men and women for each age group from 65-69 to 85+ during the period 1994 to 2012.
Figure 2: The death rate from stroke shown by data for Danish men and women for each age group from 65-69 to 85+ during the period 1994 to 2012.

### 3 Method

Initially, the death rate from malnutrition in Denmark from 1994 to 2012 is described without including any theoretical explanation.

Second, the expansion method is applied to the discussion of the association between malnutrition and an induced higher death rate from stroke.

The expansion methodology suggests the construction of models by first defining a core relationship (the initial model) and then extending it to encompass relevant contexts. The expansions are carried out by redefining the letter parameters of the initial model into functions of the contextual variables. Pioneering in the field is Casetti [6, 7]. The present Expansion Model could be further developed if the dataset was bigger; see for example: Kristensen and Tkocz [13], and Tkocz and Kristensen [26].

Third, the Age-Period-Cohort method was included to improve the estimations. Kristensen [15, 16, and 17] modeled the death rate from stroke by the Age-Period-
Cohort Method. The Age-Period-Cohort Method further develops the expanded model by including the cohort effects.

### 3.1 A simple description of Stroke and Malnutrition

Figure 3 shows a simple model of the association between malnutrition and stroke. The visual impression, given by data for age groups from 75-79 to 85+ over the period 1994-2012, is that malnutrition has bent the stroke curve upwards for men. A similar comparison can be made for women, as can be seen from Figures 1 and 2.

![Figure 3: The death rate from stroke and malnutrition (multiplied by 40) for men.](image)

The visual impression alone supports the association between the death rate from malnutrition and the death rate from stroke.

We notice the following developments, which each could form individual (initial) models:
The death rate from stroke declines over \textbf{time}.

The death rate from stroke is associated with \textbf{malnutrition}.

The death rate from stroke is increasingly associated with malnutrition at increasing \textbf{ages}.

The \textit{effect} of malnutrition on stroke changes with time, that is, with the medical development.

Not seen is the death rate from stroke associated with a cohort effect \textbf{B}, as will be discussed below.

\subsection*{3.2 The Expansion Method}

The Expansion Method shows how to combine a set of \textit{individual} models into one. It is quite easy to make an \textit{individual} model for the death rate from stroke for a given age group. Such a model will of course not include age and be calculated as:

\begin{align*}
\log(\text{Str}_{85}) &= \alpha_{1.85} + \alpha_{2.85}TD^2 + (\alpha_{3.85} + \alpha_{4.85}TD^2) \text{ Dmal} \\
&+ (\alpha_{5.85} + \alpha_{6.85}TD^2)\text{Dmal(-1)} \\
\log(\text{Str}_{80}) &= \alpha_{1.80} + \alpha_{2.80}TD^2 + (\alpha_{3.80} + \alpha_{4.80}TD^2) \text{ Dmal} \\
&+ (\alpha_{5.80} + \alpha_{5.80}TD^2)\text{Dmal(-1)} \\
\vdots \\
\log(\text{Str}_{55}) &= \alpha_{1.55} + \alpha_{2.55}TD^2 + (\alpha_{3.55} + \alpha_{4.55}TD^2) \text{ Dmal} \\
&+ (\alpha_{5.55} + \alpha_{6.55}TD^2)\text{Dmal(-1)} \\
\end{align*}

\begin{equation}
\text{(1)}
\end{equation}

Omitted were, for simplicity, the residuals in the true relationships, and the classical OLS assumptions are assumed to be fulfilled. This model was calculated for men and women for the age groups 55-59, 60-64, 65-69, 70-74, 75-79, 80-84, and 85+.
The model thus explains the development of the death rate from stroke for a given age group by time and by the death rate from malnutrition. However, the coefficients of the individual models develop uniformly with age.

Therefore,

\[ \alpha_{j, \text{Age}} = f(\text{Age}) \]  

\[ \alpha_{1, \text{Age}} = \beta_1 + \beta_2 \text{Age} + \beta_3 \text{Age}^2 \]  

As \( \alpha_{3, \text{Age}} \) and \( \alpha_{4, \text{Age}} \) are functions of Age, it is seen that the effect on malnutrition on stroke varies with age, and medical development (time).

In principle, we (after the logic of the Expansion Method) could write an all-over model as:

\[
\log(\text{Str}) = (\beta_1 + \beta_2 \text{Age} + \beta_3 \text{Age}^2) + (\beta_4 + \beta_5 \text{Age} + \beta_6 \text{Age}^2) \text{TD}^2 \\
+ [(\beta_7 + \beta_8 \text{Age} + \beta_9 \text{Age}^2) + (\beta_{10} + \beta_{11} \text{Age} + \beta_{12} \text{Age}^2) \text{TD}^2] \text{Dmal} \\
+ [(\beta_{13} + \beta_{14} \text{Age} + \beta_{15} \text{Age}^2) + (\beta_{16} + \beta_{17} \text{Age} + \beta_{18} \text{Age}^2) \text{TD}^2] \text{Dmal}(-1)
\]  

We estimate the model and remove coefficients which do not increase the adjusted \( R^2 \) by backward selection. Besides, we remove multicorrelated variables. See Appendix 3.

### 3.2.1 The Malnutrition Cycle and the declining Death Rate from Stroke

The estimated model is used to create calculated values (forecasts) of stroke, as dependent of, among others, malnutrition. The estimated values of \( \beta_j \) are indicated by \( b_j \).

Using Latin letters for the estimated coefficients in (4) the separate effect of malnutrition (using the death rate from malnutrition as an indicator) on the death rate from stroke is calculated as:
\[ Str_{Age} = \exp \left[ (b_1 + b_2 Age) + (b_4 + b_5 Age + b_6 Age^2) TD^2 \right] \\
+ (b_7 + b_8 Age) D_{mal} + [(b_{16} + b_{17} Age + b_{18} Age^2)TD^2]D_{mal}(-1) \] (5)

Simulations of the changing level of the death rate from malnutrition are now made with inserted values for \( Age \) and \( TD \). We now call the downward time trend in the death rate from stroke the *positive development in the medical level.*

![Calculated death rate from stroke related to malnutrition. Women, 1996-level.](image)

Figure 4: The association between the death rate from malnutrition and the death rate from stroke simulated on a 1996 medical level for three age groups. Women.

We fix the medical level to the 1996 level; that is, \( TD \) is fixed to 1996 in Figures 4 and 5.
In Figures 4 and 5 the “time” is fixed to 1996. The malnutrition cycles are the same for all three age groups (1994 to 2012), but the death rate cycles from malnutrition differ. With the increasing death rate from malnutrition ($D_{mal}$) the death rate from stroke increases.

3.2.2 The “net” effect of Malnutrition on the Death Rate from Stroke

“StMa” indicates how much the (calculated) death rate from stroke increases when the death rate from malnutrition increases. The indirect effect on the death rate from stroke is quite impressive; however, the model is very simple.

$$StMa_{Age} = \exp\left[ (b_1 + b_2 \text{Age}) + (b_4 + b_5 \text{Age} + b_6 \text{Age}^2)TD^2 \right]$$

\begin{align*}
+ (b_7 + b_8 \text{Age})D_{mal} + [(b_{16} + b_{17} \text{Age} + b_{18} \text{Age}^2)TD^2]D_{mal(-1)} \\
- \exp\left[ (b_1 + b_2 \text{Age}) + (b_4 + b_5 \text{Age} + b_6 \text{Age}^2)TD^2 \right]
\end{align*}

(6)
Figures 7 and 8 below show the net effect with focus on the age group 85+ for women and men indicated by “no cohort” calculated with (6).

### 3.3 The Age-Period-Cohort Model

The Age-Period-Cohort model includes the positive dummies for cohorts. To omit multicollinearity, age, period, and cohort must not be linearly connected.

*Creation of the cohort variable*

The available time series for the death rate from stroke goes from 1977 to 2012. Because stroke affects people from a relatively young age, we have no trouble including the age groups from age 55. Deaths from malnutrition are later in life but age 55 is, nevertheless, functional.

Using 1977, we can go back to a year of birth for a 85-year-old person:

\[ 1977 - 85 = 1892. \]

As the data stop by 2012 we can from the age of 55 find the birth year:

\[ 2012 - 55 = 1957. \]

The data for Dnal is from 1994 to 2012. Filling out with the lowest level of malnutrition, we can have data going back to 1977 for both datasets. Actually we only used the data as far back as to 1983 – 85 = 1898.

The applied age groups are 55-60 to 85+. The period is initially 1977 to 2012. The technical details are described in Appendix 3.
Stage 1. Adding Cohort Dummies
Unlike the Expansion Method we here start with the overall model for stroke extended by the cohort effect. The assumption behind the cohort dummies (see Kristensen [15]) is that each cohort throughout life remain in the same health group, e.g., the 85-year-old in 1983 is in the same health group as the 80-year-old in 1978.

\[
\log(\text{Str}) = [\alpha_1 + \alpha_2 \text{Age}] + [\alpha_4 + \alpha_5 \text{Age} + \alpha_6 \text{Age}^2] TD^2 \\
+ [\alpha_7 + \alpha_8 \text{Age}] D\text{mal} + [(\alpha_{16} + \alpha_{17} \text{Age} + \alpha_{18} \text{Age}^2)TD^2] D\text{mal}(-1) \\
+ \beta_1 \text{Coh1899} + \beta_2 \text{Coh1900} + \ldots + \beta_{54} \text{Coh1952}
\]

(7)

Equation (7) is transformed in order to save degrees of freedom and to get a function that is easier to deal with. As shown by Kristensen [15], the estimated cohort coefficients can form (column) vectors and be applied as variables related to each age group:

Figure 6: The beta coefficients related to the birth year of men and women, 1899 to 1952.
\[ B_{85} = [\beta_{11} \beta_{12} \beta_{13} \ldots \beta_{29}] \]
\[ B_{80} = [\beta_{16} \beta_{17} \beta_{18} \ldots \beta_{34}] \]
\[ \vdots \]
\[ B_{55} = [\beta_{41} \beta_{42} \beta_{43} \ldots \beta_{59}] \]  

**Stage 2. For each individual age group**

With the B vectors created from (7) the cohort effects can be included in the models for the individual age groups, e.g.:

\[
\log(\text{Str}_{85}) = \alpha_{1,85} + \alpha_{2,85} TD^2 + (\alpha_{3,85} + \alpha_{4,85} TD^2) \text{Dmal} \\
+ (\alpha_{5,85} + \alpha_{6,85} TD^2) \text{Dmal}(-1) + \gamma_{85} B_{85}
\]  

**Stage 3. The final Age-Period-Cohort Model**

The vectors in (7) can again be combined in one overall (column) vector with 7*18 = 126 observation, which is a variable to replace the cohort dummies in equation (6).

\[ B_{Age} = [B_{85}B_{80}B_{75}B_{70}B_{65}B_{60}B_{55}] \]  

Stage 3 gives (7) on a new form with the cohort effect included.

\[
\log(\text{Str}) = [\alpha_1 + \alpha_2 \text{Age}] \\
+ [\alpha_4 + \alpha_5 \text{Age} + \alpha_6 \text{Age}^2] TD^2 \\
+ [\alpha_7 + \alpha_8 \text{Age}] \text{Dmal} \\
+ [(\alpha_{16} + \alpha_{17} \text{Age} + \alpha_{18} \text{Age}^2)TD^2] \text{Dmal}(-1) \\
+ \gamma_1 B_{Age}
\]  

The age group 85+ belongs to generations who were relatively healthy in relation to stroke. Therefore, the simple expansion model underestimates the negative effect of malnutrition on this age group, as seen from Figures 7 and 8. Empirical examples are given in Kristensen [16, 17].
Figure 7: The cohort effect on the evaluation of the effect of malnutrition. Women.

Figure 8: The cohort effect on the evaluation of the effect of malnutrition. Men.
4. The excess mortality from stroke associated with malnutrition

We can now calculate the malnutrition-provoked excess number of deaths from stroke in the Danish malnutrition period 1999-2007. The share of cases of stroke provoked by malnutrition is

\[
\text{ShareStMa}_{\text{Age}} = \frac{\text{StMa}_{\text{Age}}}{\text{Stroke}_{\text{Age}}} \tag{12}
\]

The number of deaths from stroke (apoplexy) provoked by malnutrition are calculated as

\[
\text{NStMa}_{\text{Age}} = \text{NoStroke} \times \text{ShareStMa}_{\text{Age}} \tag{13}
\]

The periods 1995-1998 and 2008-2012 are seen as “normal” in relation to deaths from stroke associated with malnutrition. The small disturbances between the three periods due to the time lag are disregarded.

The calculated number of extra deaths from stroke associated with malnutrition, 1999 to 2007, rounded, is shown in Table 1. Table 1: shows that about 2500 deaths from stroke were connected to malnutrition in the malnutrition period in Denmark 1999 - 2007.

Table 1: Calculated number of extra deaths from stroke associated with malnutrition, 1999 to 2007

<table>
<thead>
<tr>
<th></th>
<th>Age-Period-Cohort Method</th>
<th>Expansion Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Men</td>
<td>969.2</td>
<td>918.0</td>
</tr>
<tr>
<td>Women</td>
<td>1638.4</td>
<td>1549.5</td>
</tr>
<tr>
<td>Total</td>
<td>2607.6</td>
<td>2467.5</td>
</tr>
</tbody>
</table>
5 Discussion

Models composed by straight lines, parabolas, third degree polynomials, etc., cannot tell the “true” story about biological and human aspects. Therefore, simplicity have advantages, and so has advanced model building. Here we study the connection between malnutrition and the death rate of stroke. Malnutrition is expressed by the death rate from malnutrition. Thus, deaths from stroke are partly related to malnutrition.

Three reasons can be given for the association between stroke and malnutrition: First, doctors wrote “stroke” instead of “malnutrition” on the death certificate. Second, malnutrition pushed the stroke patients into death. And third, malnutrition caused death from stroke.

The death rate from stroke declined rapidly during the period 1994-2012. Factors, which increase the (natural) death rate from stroke, are consequently systematically underestimated. The death rate from malnutrition and malnutrition-related deaths from stroke are only indicators of bigger underlying problems. Deaths from stroke might disappear, but the invalidating aspects of malnutrition will continue. So will the low quality of life. A weakness in the model is that the malnutrition death rate data only cover the years 1994 to 2012.

6 Conclusion

The period from 1999 to 2007 was characterized by a significantly increased death rate from malnutrition for people from the age of 55 years in Denmark.

The literature draws a generally negative picture of the development in the food situation for people dependent on food from an institution. Here the death rate from malnutrition confirms by number of deaths what the literature describes as low quality.
Among persons who suffer from malnutrition, those who do not die from malnutrition have higher risks of dying from other diseases brought on by malnutrition.

This is a parallel to the Dutch experience of prenatal exposure to famine in 1944-1945, but here based on the Danish experience in the period 1999-2007.

References


# Appendix 1. Number of Deaths.

Table 1A: Number of Deaths from Malnutrition related to age, 1994-2012.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>55-59</td>
<td>1</td>
<td>6</td>
<td>1</td>
<td>0</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>60-64</td>
<td>1</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>65-69</td>
<td>1</td>
<td>5</td>
<td>1</td>
<td>1</td>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td>70-74</td>
<td>1</td>
<td>19</td>
<td>2</td>
<td>1</td>
<td>16</td>
<td>1</td>
</tr>
<tr>
<td>75-79</td>
<td>0</td>
<td>16</td>
<td>1</td>
<td>5</td>
<td>20</td>
<td>6</td>
</tr>
<tr>
<td>80-84</td>
<td>4</td>
<td>27</td>
<td>5</td>
<td>6</td>
<td>40</td>
<td>9</td>
</tr>
<tr>
<td>85+</td>
<td>8</td>
<td>67</td>
<td>7</td>
<td>8</td>
<td>178</td>
<td>20</td>
</tr>
<tr>
<td>Total</td>
<td>16</td>
<td>143</td>
<td>17</td>
<td>21</td>
<td>268</td>
<td>38</td>
</tr>
<tr>
<td>Per year</td>
<td>3.2</td>
<td>15.9</td>
<td>3.4</td>
<td>4.2</td>
<td>29.8</td>
<td>7.6</td>
</tr>
</tbody>
</table>

Table 1A shows that there was a malnutrition period in Denmark, 1999 - 2007. The excess number of death in the malnutrition period 1999-2007:

Men \( 143 - 0.9(16+17) = 113.3 \)

Women \( 268 - 0.9(21+38) = 214.9 \)

Total \( 328.2 \)
Table 2A: Calculated number of Deaths from Stroke associated with Malnutrition – related to age groups, 1994-2012.

<table>
<thead>
<tr>
<th>Age group</th>
<th>APC METHOD</th>
<th>EXPANSION METHOD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Men</td>
<td>Women</td>
</tr>
<tr>
<td>55-59</td>
<td>0/14</td>
<td>15/17</td>
</tr>
<tr>
<td>60-64</td>
<td>0/0</td>
<td>16/20</td>
</tr>
<tr>
<td>65-69</td>
<td>4/4</td>
<td>21/24</td>
</tr>
<tr>
<td>70-74</td>
<td>5/5</td>
<td>135/149</td>
</tr>
<tr>
<td>75-79</td>
<td>0/0</td>
<td>148/162</td>
</tr>
<tr>
<td>80-84</td>
<td>28/28</td>
<td>323/343</td>
</tr>
<tr>
<td>85+</td>
<td>61/46</td>
<td>574/473</td>
</tr>
<tr>
<td>Total</td>
<td>98/97</td>
<td>1232/1188</td>
</tr>
</tbody>
</table>

Excess number of deaths in the malnutrition period 1999-2007:

**APC METHOD**

- Men: \((1232 - .9(98 + 194)) = 969.2\)
- Women: \((2038 - .9(165 + 279)) = 1638.4\)

**EXPANSION METHOD**

- Men: \((1188 - .9(97 + 203)) = 918.0\)
- Women: \((2004 - .9(174 + 331)) = 1549.5\)
Appendix 2. The estimated Equations

Models created according to the Expansion Method and supplemented by the cohort effect.

The models are calculated with Weighted Least Squares with age as weight, for seven age groups from 55-59 to 85+, for the period 1994 to 2012.

Men

The Expansion Method

Log(Strm) = \[-2.442 + .122\text{Age}\]
\[-(13.73) (49.92)\]
\[+ [12.050 - .660\text{Age} + .00526\text{Age}^2]TD^2\]
\[1.82 (-3.61) (4.14)\]
\[+ [0.0181] Dmalm\]
\[1.62\]
\[+ [(2.171 - .0252*\text{Age}) TD^2]Dmalm(-1)\]
\[4.40 (-4.34)\]
\[+.256 e(-1)\]
\[2.87\]

\[R^2 = .9984 \quad Adj. R^2 = .9983 \quad Obs = 126 \quad DW = 2.00\]

The Age-Period-Cohort Method

Log(Strm) = \[-3.052 + .127\text{Age}\]
\[-(13.47) (46.92)\]
\[+ [38.783 - 1.274\text{Age} + .008718\text{Age}^2] TD^2\]
\[3.77 (-4.90) (5.27)\]
\[+ [.00268] Dmalm\]
\[2.61\]
\[+ [(1.808 - .0209\text{Age})TD^2] Dmalm(-1)\]
\[3.90 (-3.84)\]
\[+.995 Bm + .200 e(-1)\]
\[5.32 (2.11)\]

\[R^2 = .9986 \quad Adj. R^2 = .9985 \quad Obs = 121 \quad DW = 2.05\]
Women

The Expansion Method

\[
\log(\text{Strw}) = [-4.086 + .141\text{Age}] \\
+ [54.29 - 1.851\text{Age} + .0134\text{Age}^2]TD^2 \\
+ [.119 - .00138\text{Age}]Dmalw \\
+ [(18.204 - .440\text{Age} + .00266\text{Age}^2)TD^2]Dmalw(-1) \\
+ .313 e(-1) \\
\]

\( R^2 = .9984 \quad \text{Adj.} R^2 = .9983 \quad \text{Obs} = 126 \quad \text{DW} = 2.03 \)

The Age-Period-Cohort Method

\[
\log(\text{Strw}) = [-4.893 + .149\text{Age}] \\
+ [82.351 - 2.500\text{Age} + .0171\text{Age}^2]TD^2 \\
+ [.105 - .00121\text{Age}]Dmalw \\
+ [(18.960 - .466\text{Age} + .00286\text{Age}^2)TD^2]Dmalw(-1) \\
+ .686 Bw + .176 e(-1) \\
\]

\( R^2 = .9986 \quad \text{Adj.} R^2 = .9985 \quad \text{Obs} = 121 \quad \text{DW} = 2.01 \)
Appendix 3. Mathematical details

The multicollinearity problem: Equation (4) can be rewritten as:

\[
\log(\text{Str}) = (\beta_1 + \beta_2 \text{Age} + \beta_3 \text{Age}^2) \\
+ (\beta_4 + \beta_5 \text{Age} + \beta_6 \text{Age}^2) \text{TD}^2 \\
+ (\beta_7 + \beta_8 \text{Age} + \beta_9 \text{Age}^2) \text{Dmal} \\
+ (\beta_{10} + \beta_{11} \text{Age} + \beta_{12} \text{Age}^2) \text{TD}^2 \text{Dmal} \\
+ (\beta_{13} + \beta_{14} \text{Age} + \beta_{15} \text{Age}^2) \text{Dmal}(-1) \\
+ (\beta_{16} + \beta_{17} \text{Age} + \beta_{18} \text{Age}^2) \text{TD}^2 \text{Dmal}(-1)
\]

Table 3A: The correlation matrix for variables

For women

<table>
<thead>
<tr>
<th></th>
<th>TD^2</th>
<th>DMALW</th>
<th>DDMALW*TD^2</th>
<th>DMALW(-1)</th>
<th>DDMA(-1)*TD^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>TD^2</td>
<td>1.00000</td>
<td>-0.007517</td>
<td>0.066115</td>
<td>0.058841</td>
<td>0.131514</td>
</tr>
<tr>
<td>DMALW</td>
<td>-0.007517</td>
<td>1.00000</td>
<td>0.984044</td>
<td>0.872504</td>
<td>0.831423</td>
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<tr>
<td>DDMALW*TD^2</td>
<td>0.066115</td>
<td>0.984044</td>
<td>1.000000</td>
<td>0.904967</td>
<td>0.888784</td>
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<tr>
<td>DMALW(-1)</td>
<td>0.058841</td>
<td>0.872504</td>
<td>0.904967</td>
<td>1.000000</td>
<td>0.985235</td>
</tr>
<tr>
<td>DDMA(-1)*TD^2</td>
<td>0.131514</td>
<td>0.831423</td>
<td>0.888784</td>
<td>0.985235</td>
<td>1.000000</td>
</tr>
</tbody>
</table>

For men

<table>
<thead>
<tr>
<th></th>
<th>TD^2</th>
<th>DMALM</th>
<th>DDMALM*TD^2</th>
<th>DMALM(-1)</th>
<th>DDMA(-1)*TD^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>TD^2</td>
<td>1.00000</td>
<td>-0.041946</td>
<td>0.031466</td>
<td>0.018549</td>
<td>0.085354</td>
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<td>DDMALM*TD^2</td>
<td>0.031466</td>
<td>0.980658</td>
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<td>0.563864</td>
<td>0.535537</td>
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<td>0.568936</td>
<td>0.563864</td>
<td>1.000000</td>
<td>0.982936</td>
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<tr>
<td>DDMA(-1)*TD^2</td>
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<td>0.516904</td>
<td>0.535537</td>
<td>0.982936</td>
<td>1.000000</td>
</tr>
</tbody>
</table>

Fat numbers gives the presence of ”harmful” collinearity; see Belsley [4], and Kristensen [14], which means that the partial interpretation of the coefficients to e.g. the variables (TD^2*Dmalm) and Dmalm(-1) are not reliable and may even have wrong signs, which in the worst case could be highly “significant”.
Table 4A. The correlation matrix for the included variables.

For women

<table>
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<tr>
<th></th>
<th>TD^2</th>
<th>DMALW</th>
<th>DDMA(-1)*TD^2</th>
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<td>TD^2</td>
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<td>DMALW</td>
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<td>DDMA(-1)*TD^2</td>
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<td>1.000000</td>
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</table>

For men

<table>
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<tr>
<th></th>
<th>TD^2</th>
<th>DMALM</th>
<th>DDMA(-1)*TD^2</th>
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The Cohort Dummies

A person born in 1927 is 75 years old in 2002, 80 years old in 2007, 85 years old in 2012, and is thus based on 3 dummies.

A person born in 1909 is 85 years old in 1994, and is thus followed by only one dummy (see Table 5A).

Using data from 1983 we can go back to a year of birth for a 85-year-old person: 1983-85=1898.

From 1898 to 1903 there will, however, only be one dummy to indicate the cohort born in 1898. Thus, in this interval there is no residual on the calculated death rate.

When we start our data in 1898 we estimate from 1899. Our coefficient to Coh1909 will thus be based on 3 dummies.

In the estimations with $B$ the birth period was truncated to 1909-1952.
Table 5A: The CohYear Dummy System

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<th>Year</th>
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<th>1914</th>
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