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A Time Varying Parameter State-Space Model for Analyzing Money Supply-Economic Growth Nexus

Olushina Olawale Awe[1], Ian Crandell[2], A. Adedayo Adepoju[3] and Scotland Leman[4]

Abstract

In this paper, we propose a time-varying parameter state space model for analyzing predictive nexus of key economic indicators such as money supply and Gross Domestic Product (GDP). Economic indicators are mainly used for measuring economic trends. Policy makers in both advanced and developing nations make use of economic indicators like GDP to predict the direction of aggregate economic activities.
We apply the Kalman filter and Markov chain Monte Carlo algorithm to perform posterior Bayesian inference on state parameters specified from a discount Dynamic Linear Model (DLM), which implicitly describes the relationship between response of GDP and other economic indicators of an economy. In our initial exploratory analysis, we investigate the predictive ability of money supply with respect to economic growth, using the economy of Nigeria as a case study with an additional evidence from South African economy. Further investigations reveal that

[1] Obafemi Awolowo University,Ile-Ife, Nigeria. E-mail:oau.lisa@gmail.com
[2] Department of Statistics, Virginia Tech, USA. E-mail: ian85@vt.edu
[3] Department of Statistics, University of Ibadan, Nigeria. E-mail: pojuday@yahoo.com
[4] Virginia Tech, USA. E-mail:leman@vt.edu

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leading variables like capital expenditure, the exchange rate, and the treasury bill rate are also useful for forecasting the GDP of an economy. We demonstrate that by using these various regressors, there is a substantial improvement in economic forecasting when compared to univariate random walk models.

Mathematics Subject Classification: C11; C32
Keywords: Bayesian inference; Time-Varying Parameters; Money Supply; MCMC; State-Space Model

1 Introduction

In recent years, researchers have been able to reasonably predict monetary aggregates with respect to economic growth [1]. There is a popular notion that stock of money varies with the economic activities [2], therefore, it is important to examine whether the relationship between money supply and economic growth is stable over time or not. Although there are apparent mixed reactions among economists on the effect of money supply and economic growth, their relationship has received increasing attention in recent years. Money supply exerts considerable influence on economic growth in both developed and developing economies, hence it is believed that the low level of supply of monetary aggregates in general and money stock in particular is responsible for the fundamental failure of many African countries to attain tangible economic growth and development [3].

Researchers like [4] and [5] have suggested that economic growth is not possible without an appropriate level of money supply, credit, and appropriate financial conditions. However, [2] have proposed that variation in the quantity of money is the most important determinant of economic growth, meaning that countries that devote their time to regulating the behavior of aggregate money supply rarely experience much variation in their economic activities. Much empirical evidence from the Nigerian economy, for instance, has shown that since the 1980's, some close relationships exist between stock of money and economic growth [6].

Using Nigeria as a case study, we set out to investigate the empirical nexus between GDP, money supply, and some key economic indicators using our proposed discount factor state-space model, with time-varying parameters. Over the years, Nigeria has been controlling her economy through variation in her stock of money and other economic indicators. When an economic indicator is studied in isolation, it is not too complicated to extract the desired information. However, the analysis becomes more useful and accurate when we use more realistic dynamic regression models to study the causal behavior of multiple variables. In our empirical analysis using Nigerian macroeconomic data, a time-varying parameter state space model is estimated, using discount factors to estimate the evolution variance, in order to capture the feedback effects and growth dynamics of the Nigerian economy. It is natural and usual to model economic time series as solutions of difference equations with some stochastic perturbations [7, 8]. The classical multiple linear regression, vector autoregressive model or the error correction model are all candidate models that have been used for analyzing similar macroeconomic variables in the past [9]. However, many of these methods share a common possible shortcoming in that they all assume that the growth rate of the economic variables are linear and stationary. For instance, the classical time series methodology requires at least a preliminary transformation of the data in order to achieve stationarity. On the other hand, a Bayesian state-space modeling framework, such as the one we use in this paper, does not assume a regular pattern and stability of the underlying system. To the best of our knowledge, literature on the application of time varying parameter state- space models to the Nigerian economy, using discount factors to estimate the stochastic evolution variance parameter is quite scanty and insular.

The Economy of Nigeria is one of the fastest growing economies in the world. It was ranked 31st in the world in terms of GDP with an estimated growth rate of about 7.4 percent as at 2013. However, the world bank has listed Nigeria as one of the world's poorest countries despite placing the West African country as the biggest economy in Africa. The figures show that Nigeria, the biggest oil producer in Africa, has climbed to become the 26th largest economy in the world as at year 2014. According to the International Monetary Fund (IMF), countries are supposed to recalculate their GDPs after every five years, but Nigeria's was last computed in 1999 when there was limited growth in most economic sectors. The new calculation, released by Nigeria's National Bureau of Statistics (NBS), now includes previously unaccounted industries like telecoms, information technology, music, on-line sales, airlines, and film production. In spite of this, the world bank's statistics of Nigeria are as follows: 70 percent of the population (115 million people) in Nigeria are living on less than $1.25 per day. 20 percent of the population (40 million people) are living on less than $3 per day. 5 percent (7 million people) are living on less than $10 per day. 4 percent (6 million people) are living on $12-$30 a day, while 1 percent (1.7 million people) are living on over $30 a day. Effective functioning of the economy depends on the availability of reliable, timely, and properly interpreted economic data. This has motivated us to study models that can describe and predict the dynamics of key economic indicators of the Nigerian economy, since there is an apparent disconnect between Nigeria's fast growing GDP rate and her high poverty rate. We believe that such models will be useful for econometricians and policy makers to proffer suggestions as to the way forward.

The rest of this paper is organized as follows: In section 2, we present a brief review of literature on state space models. Section 3 is on the Bayesian econometric model and MCMC methods used in this paper. Section 4 deals with the application of our method to the Nigerian economy. Finally, section 5 gives an extensive discussion and conclusion of the paper.

2 State Space Models

State space models consider a time series as the output of a dynamic system perturbed by random disturbances, hence they represent the role of history differently in a finite-dimensional state. They are suitable for modeling a wide array of data (univariate and multivariate time series) in the presence of non-stationarity, structural changes and irregular patterns (see [10] and [11]). State space models became prominent in time series and econometrics literature in the seventies in works involving Markovian evolution models and dynamic regressions like [12],[13], [14] and [15]. They became established in the eighties and nineties when authors like [16], [17], [18], [19] and [20] applied various dynamic linear models to the modeling of economic time series data. Various other authors extended Hamilton's original model to allow for time-varying, Markov switching or duration dependent state-space models and applied such models to investigate U.S business cycle in the works of researchers such as [21],[22], [23], [24], [25], [26], [27], and [20].

Construction of robust econometric models for prediction and analysis of financial-economic time series is an extremely urgent task [28, 29], hence it is imperative for researchers to consider models that can capture the dynamic movements of economic indicators. Time-varying parameters in linear regression and econometric time series models have become more relevant in recent times. Some of the rationales behind time-varying parameter models are documented in [30]. Even when the underlying parameters are stable, situations arise in which a time-varying coefficient approach will prove to be effective. [31] suggests introducing randomness to time-varying coefficients in order to avoid overparameterization problems which are usually common when working with time-varying coefficient models. However, the randomness in these parameters fits quite well into the Bayesian framework because there is no strict distinction between fixed and random parameters in the Bayesian context. Hence, in estimating econometric models with time-varying parameters, state-space representation of the system is necessary [32, 33, 34, 35].

In the last few decades, there has been an increasing interest in the application of state- space model and its variants in econometrics and time series analysis, partly because of its flexibility and because of the development of modern Monte Carlo methods for dealing with non-Gaussian situations [36]. State space models build on the dependence structure of a Markov chain to define more complex models for the observations. Usually, it is assumed that there is an unobservable Markov chain θ_t called the 'state' and that Y_t is an imprecise measurement of θ_t. In econometric applications,we think of the state θ_t as an auxiliary time series that facilitates the task of specifying the probability distribution of the observable time series Y_t. The states are usually assumed to follow a Markovian transition model.

Formally speaking, a Markovian state - space model can be defined as one which consists of an R^p-valued time series θ_t:$t = 0, 1, 2, ..., T$ and an R^k-valued time series Y_t:$t = 1, 2, ..., T$ which satisfies the following assumptions:

- θ_t is a Markov chain.

- Conditional on θ_t, the Y_t's are independent and Y_t depends on θ_t only.

State space models in which the states are discrete-valued random variables are called Hidden Markov Models [11]. Markovian dependence is the simplest

$$\begin{array}{ccccccccccccc}
\theta_0 & \rightarrow & \theta_1 & \rightarrow & \theta_2 & \rightarrow & \cdots & \theta_{t-1} & \rightarrow & \theta_t & \rightarrow & \theta_{t+1} & \rightarrow \cdots \\
 & & \downarrow & & \downarrow & & & \downarrow & & \downarrow & & \downarrow & \\
 & & Y_1 & & Y_2 & & & Y_{t-1} & & Y_t & & Y_{t+1} &
\end{array}$$

Figure 1: Dependence Structure of State Space Models

form of dependence among the $Y_t's$ in which time has a definite role, hence state space models are useful for modeling time-varying scenarios [32].

3 Model Specification and Methodology

We specify a dynamic linear regression model to assess the relationships between economic growth (proxied by GDP), money supply and some key economic indicators of the Nigerian economy. Our model specification takes the following general form:

$$\begin{aligned}
y_t &= X_t\theta_t + v_t \quad v_t \sim N(0, V), && (1)\\
\theta_t &= G_t\theta_{t-1} + w_t \quad w_t \sim N_p(0, W_t), && (2)\\
\theta_0 &\sim N_p(m_0, C_0).
\end{aligned}$$

Equation (1) is known as the observation equation while equation (2) is the evolution equation. G_t is a known matrix of order $p \times p$ that determines how the observation and state equations evolve in time (see [10]. We assume that all v_t's are independent from the w_t's. Since each parameter at time t only depends on results from time $t-1$, the state parameters are time-varying and constitute a Markov chain.

In our model, the response y_t is the annual GDP of Nigeria from 1960 to 2009. The matrix X consists of economic indicators measured concurrently with the GDP and includes a column of 1's representing a dynamic intercept term. θ_t are time varying regression coefficients which model the relationship between the regressors and the response at each time t. As typical in dynamic regression models, the state- space evolution matrix G_t is constant and equal to the identity matrix.

3.1 Bayesian Estimation of the Model Parameters

Parameters of interest which are to be estimated are the matrix θ, the error variances V and W_t, and the one-step-ahead forecasts f_t. Since normality is assumed, θ and f_t can be estimated using the Kalman filter [37]. V is assumed to be distributed inverse-gamma a priori and is estimated using a Gibbs sampler, while we estimate W_t using discount factors which shall be explained later in this section.

3.2 The Kalman Filter Algorithm

Due to the Markovian structure of the states θ_t, we estimate the model by the method of Kalman filter [37, 19, 38] by computing the predictive and filtering distributions of θ_t inductively starting from $\theta_0 \sim N(m_0, C_0)$. The Kalman filter calculates the mean and variance of the unobserved state θ_t, given the observations. It is a recursive algorithm i.e the current best estimate is updated whenever a new observation is obtained.

Let $\theta_{t-1}|y_{1:t-1} \sim N(m_{t-1}, C_{t-1})$, where $y_{1:t-1}$ denotes all observations up to time $t-1$. The one-step-ahead predictive density $\theta_t|y_{1:t-1}$ is Gaussian with parameters

$$E(\theta_t|y_{1:t-1}) \quad = m_{t-1} \equiv A_t, \tag{3}$$

$$Var(\theta_t|y_{1:t-1}) \quad = C_{t-1} + W_t \equiv R_t. \tag{4}$$

The one-step-ahead predictive density of $Y_t|y_{1:t-1}$ is Gaussian with parameters

$$f_t \quad = E(Y_t|y_{1:t-1}) = X_t A_t, \tag{5}$$

$$Q_t \quad = Var(Y_t|y_{1:t-1}) = X_t R_t X_t' + V. \tag{6}$$

The filtering density of θ_t given $y_{1:t}$ is Gaussian with parameters

$$m_t \quad = E(\theta_t|y_{1:t}) = A_t + R_t X_t' Q_t^{-1} e_t, \tag{7}$$

$$C_t \quad = Var(\theta_t|y_{1:t}) = R_t - R_t X_t' Q_t^{-1} X_t R_t, \tag{8}$$

where $e_t = Y_t - f_t$ is the forecast error.

3.3 Discount Factors for W_t

For the Kalman filter to run, it is necessary to know V and W_t. Estimation of V is done using the Gibbs sampler described below. Here, we propose the use of discount factors to estimate W_t following the description given in [11].

The idea behind discount factors is to represent W_t as a proportion of filtering distribution variance C_t. If C_t is large then there is high uncertainty in moving from θ_{t-1} to θ_t. Since W_t represents this uncertainty, it is natural to model it as proportional to C_t. Thus, we select a discounting parameter δ and set

$$W_t = \frac{1-\delta}{\delta} C_{t-1}. \tag{9}$$

This method has the advantage of giving a natural interpretation to W_t while also allowing it to vary through time to model changes in volatility. All that remains is to estimate δ. We tune δ by examining the predictive performance using mean squared prediction error of the model under a set of different values. This is the approach we adopted in this paper.

3.4 FFBS Algorithm and Gibbs Sampler

In order to estimate V we use Gibbs sampling. This requires us to draw samples from $V|\theta$ as well as from $\theta|V$. The latter draw is performed using the Forward Filtering Backwards Sampling (FFBS) algorithm [39].

We begin by initializing $V^{(0)}$ and running the Kalman filter on the data using these initial values for V. We denote

$$p(\theta_0, ..., \theta_T | D_T) = \prod_{t=0}^{T} p(\theta_t | \theta_{t+1}, ..., \theta_T, D_T)$$

and note that, by the Markov property,

$$p(\theta_t | \theta_{t+1}, ..., \theta_T, D_T) = p(\theta_t | \theta_{t+1}, D_T).$$

Then, our FFBS algorithm proceeds as follows:

1. We sample from $p(\theta_T|D_T)$ using the filtering density above. It can be shown that this distribution is $N(h_t, H_t)$ where:

$$h_t = m_t + C_t G' R_{t+1}^{-1}(\theta_{t+1} - a_{t+1}) \tag{10}$$
$$H_t = C_t - C - tG' R_{t+1}^{-1} G C_t \tag{11}$$

2. This allows us to sample from $p(\theta_{T-1}|\theta_T, D_T)$.

3. We proceed inductively until we have a complete sample from $p(\theta_0, ..., \theta_T|D_T)$. Since we sampled from t = T to t = 0, this is called backwards sampling.

To sample from $V|\theta$ we impose a gamma prior on V^{-1} and derive the posterior hyperparameters. Let $V^{-1} \sim Gamma(a_0, b_0)$, then

$$V^{-1}|\theta \sim Gamma(a_0 + \frac{T}{2}, b_0 + \frac{1}{2}\sum_{t=1}^{T}(y_t - X_t\theta_t)^2)$$

The Gibbs sampler proceeds as thus. First, initialize $V^{(0)} \sim Gamma(a_0, b_0)$. Then, for $i = 1, \dots, M$,

1. Sample $\theta^{(i)}$ using FFBS

2. Sample $V^{-1(i)}|\theta^{(i)} \sim Gamma(a_0 + \frac{T}{2}, b_0 + \frac{1}{2}\sum_{t=1}^{T}(y_t - X_t\theta_t^{(i)})^2)$

We tried multiple values for δ in the GIbbs sampler and saw which one worked best. We then used that value for the final estimate of W_t.

4 Empirical Analyses and Results

In this section, we apply the proposed model and filtering algorithm detailed in Section 3 to a set of Nigerian economic indicators in order to better understand some of the driving factors behind the Nigerian economy. Additionally, we demonstrate how these indicators can be combined to enhance future economic predictions. We proceed by describing and discussing the economic indicators which comprise this study.

4.1 Data

The data used in this research are Nigerian economic indicators sourced from websites of the Central Bank of Nigeria (CBN)(http://www.cenbank.org /economic-indicators) and World Bank (http://data.worldbank.org/country /nigeria). We apply our methods to some key economic data of Nigeria including annual Money Supply (MS), Lending Rate (LR), Gross Domestic Product (GDP), Exchange Rate (ERT), Capital Expenditure (CE), External Debt (ED) and Treasury Bill Rate (TR) for the period between 1960-2009. All economic variables were inflation-adjusted to 1960 values and log transformed prior to analysis.

From the plots of the economic indicators shown in Figure 2 below, it appears that the Nigerian economy was stable during the years immediately after independence and into the oil boom years. Nigerian GDP rose strongly from 2007 probably because of growth in non-oil sectors as a result of increased money stock due to decreased lending rate. Interest rates (lending rates) were fixed and this was highly beneficial to farmers who could borrow and have access to more money for sustainable business activities.

According to experts, the assertion that money supply influences economic growth depends on several other macroeconomic variables [5]. Hence, in order to investigate other possible macroeconmic variables that might have contemporaneous prediction effect on GDP alongside money supply, we include in our model each of the other variables aforementioned in turns.

5 Introduction

5.1 Model Selection and Discussion of Results

Predictive performance of the variables was assessed using one-step-ahead Mean Squared Prediction Error (MSPE). Our Gibbs sampler was run, using the range of values of discount factors $\delta \in \{.01, .02, \ldots, .99\}$ while the δ with the lowest MSPE was chosen in each model in Table 1. Because results are based on Kalman filtering and Gibbs sampling, we assess convergence through visual inspection of trace plots, in conjunction with the Geweke test [40, 33]. In general, we ran chains of length 10,000 with an additional burn-in period

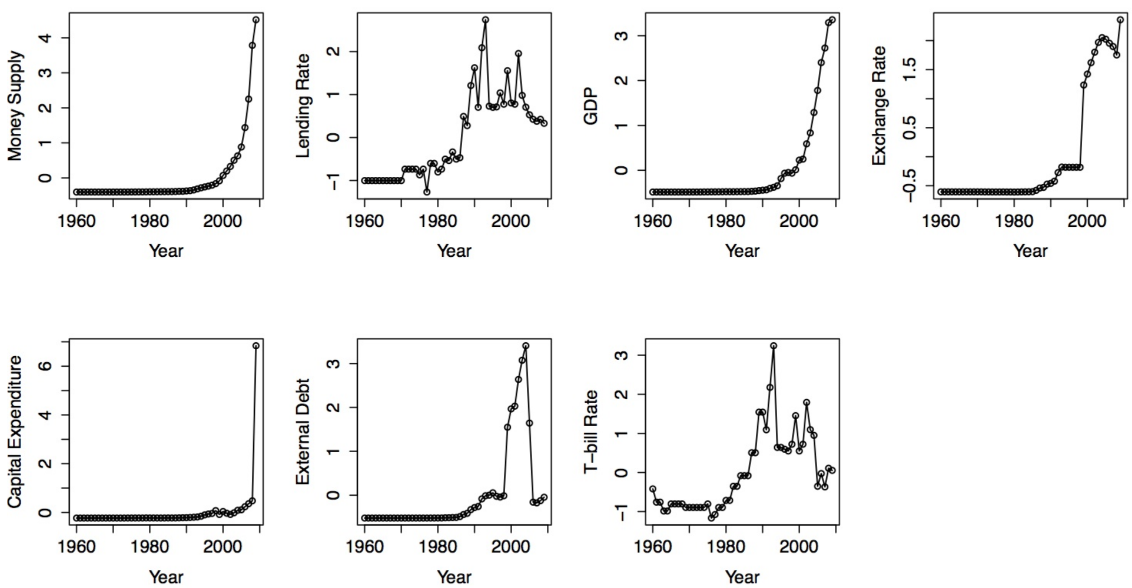

Figure 2: Annual time-series data on Money Supply (MS), Lending Rate (LR), Gross Domestic Product (GDP), Exchange Rate (ERT), Capital Expenditure (CE), External Debt (ED) and Treasury Bill Rate (TR)

of 2,000. We highlight that convergence is rapid for our DLM model, thus additional sampling is not required. Finally, we examined the effective sample size to ensure we had sufficient replications to estimate V. The plots of estimates of the best regressors vs GDP in terms of MSPE are shown in Figures 5 and 6 below. Figures 7 and 8 shows the plots of the tied models.Standard convergence diagnostics from the Geweke test statistics and visual assessment of trace plots did not raise any reason for concern, so we proceeded and used the output of the Gibbs sampler for posterior inference. All of the (absolute) Geweke z statistics are below the 1.96 threshold, indicating a failure to reject the null hypothesis of stationary means in each time series. All effective sample sizes for the two-variable regressors are above 5000. The traces of the simulated variances do not show any particular sign of nonstationary behavior.

Assuming that economic theory provides little or no guidance to the inclusion or exclusion of the variables in the model, we selectively include all the variables in the model in a stepwise fashion to achieve parsimony, while making GDP the response variable as shown in Table 1. Before introducing any regres-

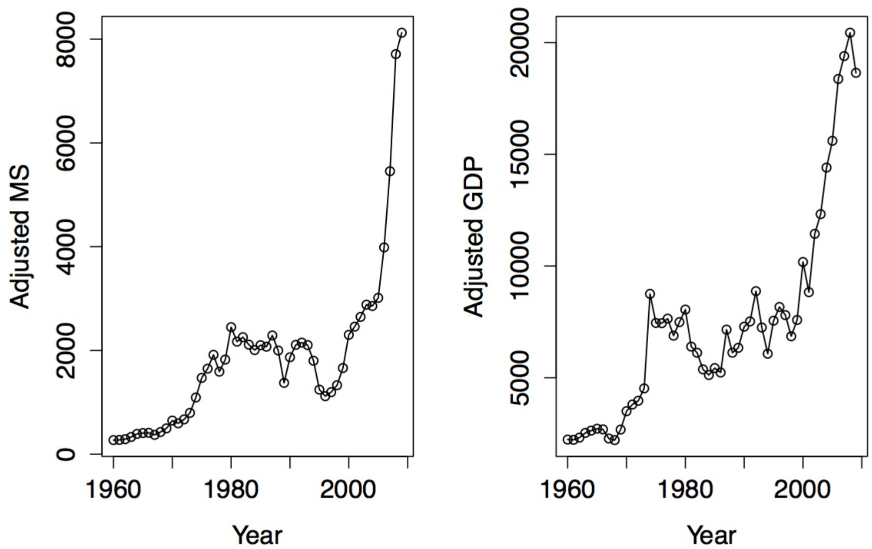

Figure 3: Temporal trends of adjusted GDP and Money Supply

sors, we first model GDP with an Integrated Random Walk Model (IRWM) with no regressors. We discovered that the MSPE of the IRWM is much higher than when we introduce regressors. Further, we examine the predictive effect of each regressor on GDP. The model involving money supply performs better than other variables in terms of predictive performance with respect to economic growth (GDP). Next we examine the contemporaneous predictive effects of five other key economic indicators along with money supply (Table 2). We find that money supply does better in predicting GDP when combined with capital expenditure. This implies that, ceteris paribus, while controlling money supply, expenditure on infrastructural investment and productive activities ought to contribute positively to economic growth. However, economies in transition do spend heavily on physical infrastructure to improve the economic welfare of the people and facilitate the production of goods and services. If government spends money on assets, investment in roads, education, health, agriculture and other areas, these investments will have direct social and economic benefits to the people. Notice that treasury bill rate and exchange rate have the same MSPE value of 1.232 when combined with money supply as

Model	Regressor	MSPE	V	δ
1	None	1.410	0.106	0.93
2	CE	1.264	0.024	0.81
3	ED	1.236	285.087	0.33
4	MS	**1.224**	0.012	0.70
5	TR	1.309	0.024	0.70
6	ERT	1.239	111.536	0.69
7	LR	1.240	0.010	0.28

Table 1: Mean Squared Prediction Error (MSPE), Observation Variance and δ for Various One-Regressor Models.

predictors in models 9 and 12 respectively. This is not unconnected with the apparent fact that both variables have concomitant effects, and play important roles in describing the money supply-economic growth nexus in Nigeria over the years.

Model	Regressors	MSPE	V	δ
8	MS+LR	1.230	4.791	0.81
9	MS+ERT	1.232	0.012	0.81
10	MS + CE	**1.222**	0.010	0.74
11	MS + ED	1.235	0.014	0.80
12	MS + TR	1.232	0.015	0.81

Table 2: Mean Squared Prediction Error (MSPE), Observation Variance and δ for Various Two-Regressor Models.

5.2 Empirical Evidence from South African Economic Data

We find a similar predictive trend in the South African economic data involving key economic variables like Gross Domestic Product (GDP), Money Supply (MS), Exchange Rate (ERT), Government Spending (GS), Lending Rate (LR) and Treasury- bill Rate (TR) data downloaded from the global economy website (www.theglobaleconomy.com). Table 3 shows that the model involving money supply is the one with the best predictive power due to the fact that it has the lowest MSPE. In all cases, GDP is the response variable which serves as a proxy for economic growth.

Model	Regressor	MSPE	V	δ
1	MS	**11.401**	844711.115	0.150
2	ERT	302.231	242.354	0.960
3	GS	111.959	49.125	0.900
4	LR	11.447	1.293	0.350
5	TR	11.456	0.009	0.260

Table 3: Mean Squared Prediction Error (MSPE), Observation Variance and δ for Various One-Regressor Models for South Africa.

Model	Regressors	MSPE	V	δ
1	MS +ERT	11.379	0.378	0.640
2	MS +GS	11.379	2.069	0.590
3	MS +LR	11.408	0.009	0.490
4	MS +TR	11.408	1.441	0.510

Table 4: Mean Squared Prediction Error (MSPE), Observation Variance and δ for Various Two-Regressor Models for South Africa.

As shown in the MSPE column in Table 1, several of our candidate models performed better than predictions based on no regressors (Integrated Random Walk (IRW) model). In our empirical analysis, it is evident that since the 1980's, some relationship exists between stock of money and economic growth (see Figure 3). Notice also the spikes and sudden jumps in the late 1980's and mid 1990's. This may be as a result of the fact that in the 1990's, there was re-capitalization in the banking sector, mergers and acquisition which resulted in increased bank branches, reduced lending rates, growth in the capital market and overall increase in stock of money and money aggregates. The Nigerian capital market experienced a bullish trend when it started the year 2008 with a market capitalization of N10.284 trillion and went on to achieve its highest value ever of 66,371 on March 5, 2008, with a market capitalization of N12.640 trillion. In the period 1960-70, the GDP recorded 3.1 per cent growth annually. During the oil boom era, around 1970-78, Nigerias GDP grew positively

by 6.2 percent annually. However, in the 1980s, GDP had negative growth rates. In the period between 1988-1997 which constitutes the period of Structural Adjustment Program (SAP) and economic liberalization in Nigeria, the GDP responded to economic adjustment policies and grew at a positive rate of 4.0. In the years after independence, industry and manufacturing sectors had positive growth rates due to investment activities in the mining sub-sector, especially petroleum. Gross domestic investment as a percentage of GDP, which was 16.3 percent and 22.8 percent in the periods 1965-73 and 1973-80 respectively, decreased to almost 14 percent in 1980-88 and increased to 18.2 percent in 1991-98.
The contribution of government investment and agriculture to GDP, which was 63 percent in 1960, declined to 34 percent in 1988, due to the neglect of the agricultural sector. As a result of this, by 1975, the economy had become a net importer of basic food items. Notice also the sudden departure in pattern at the tail end (year 2009) of the chart in Figure 6. This depict an outlier as shown in the data of capital expenditure and further suggests that the new government regime in 2009 took drastic actions to spend more money on assets, infrastructure and services to improve the standard of living of the people. Several other works that have studied or found strong support for positive relationship between economic growth and money supply or government expenditure in Nigeria include [41], [42], [7], [43], [44], [6] and [3].

Consequent upon the effect of the collapse of oil price in 1981 and the balance of payment deficit experienced during this period, various measures of stabilization were used such as fiscal monetary policies [3]. The changes in the money supply trend tends to be highly associated with the trends in exchange rate (LR) and treasury bill rate (TR), when combined together as predictors. Structural changes in both series tend to have a contextual and economical interpretation. Central banks often engage in buying or selling of treasury bills or foreign currency in order to stabilize the exchange rate which might have gone up due to excess liquidity in the inter-bank money market. Excess liquidity means that banks have more money than immediately needed. This can prompt increase in the exchange rate. It can also lead to inflation with banks giving out the excess in the form of credit hence leading to too much money in the economy. These two indicators should be noted and regulated by policy makers in order to maintain steady economic growth and stabilize the economy of Nigeria. More so, exchange, interest and inflation rates are fundamental macroeconomic variables that are capable of changing the direction and growth pattern of a country's economic development and stability. According to a recent central bank of Nigeria's report, exchange rate channel sometimes moves in surprising directions by amplifying the effects of policies thereby complicating monetary policy.

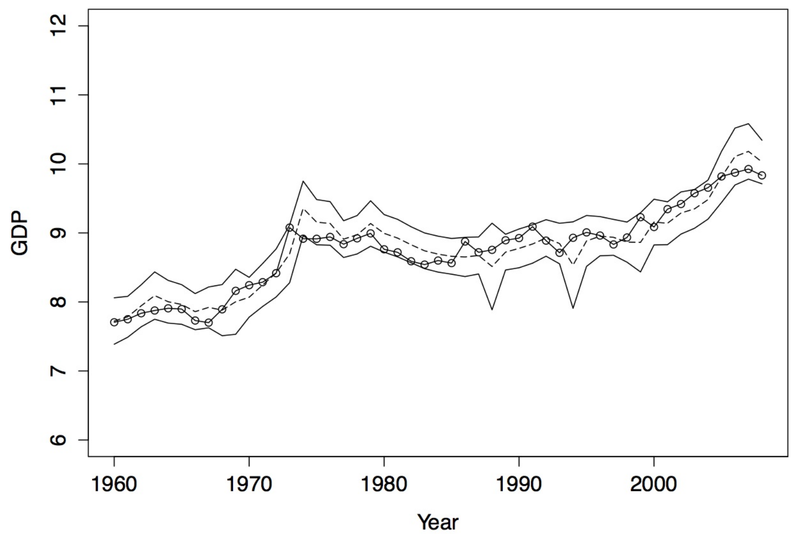

Figure 4: Dynamic Linear Regression of GDP on Money Supply

6 Conclusion and Policy Recommendation

In this paper, we proposed and estimated a time-varying parameter dynamic linear regression model, with application to key economic indicators of the Nigerian economy, using discount factors to estimate the state evolution variance of the model.Economic indicators are standard metrics for measuring how well an economy is performing or thriving, and are of paramount importance for predicting future economic trends. As a simple example, a country's Gross Domestic Product (GDP) is typically perceived as an indicator of economic growth and improvement in the standard of living of its inhabitants.

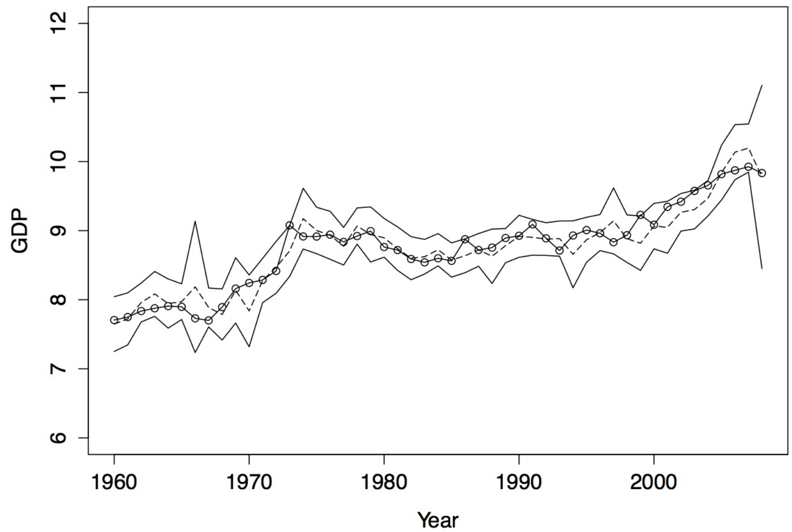

Figure 5: Dynamic Linear Regression of GDP on Money Supply and Capital Expenditure

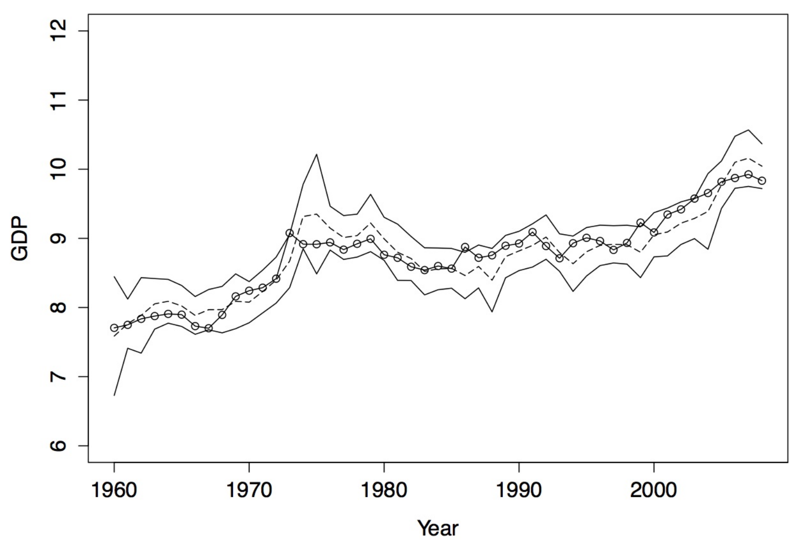

Figure 6: GDP vs Money Supply + TBR

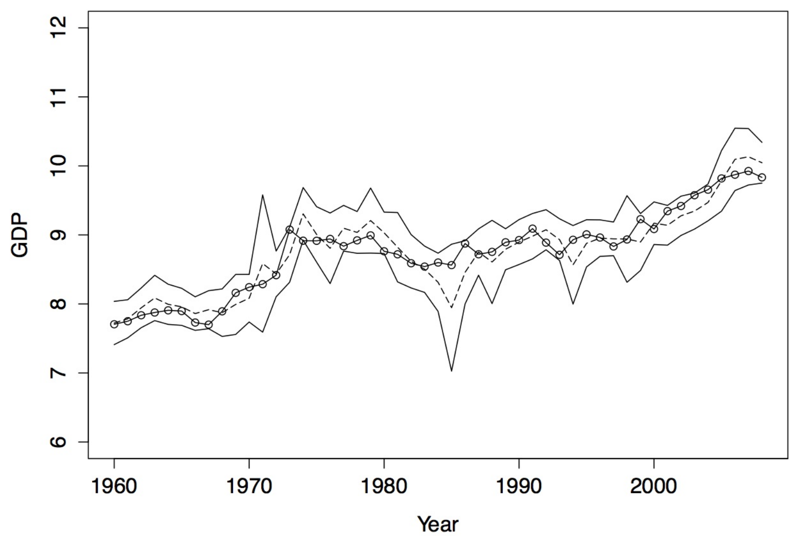

Figure 7: GDP vs Money Supply + Exchange Rate

Policy makers in both advanced and developing nations make use of economic indicators like GDP to predict the direction of aggregate economic activities. However, because varying countries have different economic contributors, each country will have it's own indicators which are useful for measuring and predicting economic growth.

Time-varying parameters were estimated and sampled via Markov chain Monte Carlo procedure, which is stable and fast to converge.The estimated model indicates that our TVP dynamic linear model fits the Nigerian economic data well and is able to detect regime shifts and sudden jumps in the economic series considered. In our model estimation and analysis, we find that the economic indicator that best predicts GDP along with money supply is capital expenditure for the period under consideration. According to some economic analysts, slow implementation of policy reforms has been a key impediment to the growth of the Nigerian Economy. Hence, we recommend that the government should continue to formulate appropriate monetary policies while expending on capital expenditures that will benefit the populace. Formulating a country's monetary policy is extremely important when it comes to promoting sustainable economic growth. More specifically, monetary policy focuses on how a country determines the size and rate of growth of its money supply in order to control inflation within the country.

To increase or decrease the money supply in the economy, the central bank could buy or purchase securities from banks. The funds acquired from security sales could be used as loans to individuals and businesses. Limited access to capital slows down economic growth as investment decreases, while the more money available in the capital market for lending, ceteris paribus, the lower the rates on these loans become, which causes more borrowers to access cheaper capital. This easier access to capital leads to greater investment and will often stimulate the overall economy. Finally, we observe from our empirical analysis that some of the structural changes in the Nigerian economy, as revealed by the Figures in section 4, coincides with the beginning of recession or boom in the economy due to regime shifts. Hence, we suggest that the new central bank regime and economic policy makers in Nigeria should embrace benign policies that will bring down inflation,encourage private sector investment by keeping lending rates low and affordable for businesses that requires finance to thrive in sectors like agriculture and manufacturing so as to ensure equitable distribution of wealth and resources in a dynamically growing economy like Nigeria.

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