Fuzzy logic and fuzzy set algebra for management of Forex market assets

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Abstract

This paper considers application of fuzzy logic elements for aggregation of technical and fundamental analysis results on the Forex market. It is proposed to use self-similar implications of fuzzy discourse.

Mathematics Subject Classification: 91B26
Keywords: Forex market; fuzzy sets; self-similarity principle; technical analysis; fundamental analysis

1 Introduction

Financial market rarely follows any crisp logic. Fundamental analysis attempts to analyze macroeconomic factors and to forecast market trends on the basis of political, economical and social models, as well as on the basis of politics...
of states, estimations of prospects of various technologies, demographic factors, etc.

The basis of technical analysis contains assumption that macroeconomic factors manifest themselves in change of volumes of trade and of financial assets, and analysis of history of that changes allows to infer about presence (absence) of some or other trends, and to issue recommendations for investors.

Econometrics knows many models used for automation of the exchange trade (advisors) and visualization of the recommendations (indicators). These are models of sliding average MA(q), autoregression AR(p), mixed models, such as ARMA(p,q) [1,2], Elliott waves, Bolinger bands, stochastic models, etc.

It is difficult (or impossible) to find one decision rule, which would operate efficiently in various market situations. Very frequently all recommendations issued on the basis of fundamental and technical analysis are formulated on qualitative level, and, moreover, availability of large number of indicators and advisors requires to develop algorithms for reconciliation of various solutions.

This paper presents system, which allows, on the basis of the fuzzy-set theory and fuzzy logic, to reconcile recommendations obtained on the basis of arbitrary number of technical indicators (5 linguistic terms), and of the estimations resulting from fundamental analysis (3 linguistic terms).

We use embedded system of the decision-making rules, while results of fuzzy logic derivation for two variables are input variable for subsequent logic derivations. With this, the decision-making rules are being preserved. To estimate measures of the logically fuzzy «AND» and «OR», we use “S” and “T” norms, respectively [3]. The algorithm is implemented in the MQL-5 medium and is tested in various time frames for the currency pair “EURUSD” for period 2013-2014.
2 Model

As input parameters of the model, let us use data of n technical indicators \( x_n \), and results of forecasts made on the basis of fundamental analysis \( y_n \). Let us norm the technical analysis data in such a manner that \(-1 \leq x_n \leq 1\):

\[
x_n \rightarrow (x_n - 0.5(\max(x_n) - \min(x_n)))/(0.5(\max(x_n) + \min(x_n)),
\]

where \( \max(x_n), \min(x_n) \) are maximum and minimum values of variables \( x_n \) within the interval \( N \gg 1 \).

Let us define fuzzy set \( A_n = \{x_n, \mu(x_n), x_n \in X_n\} \), where \( \mu(x_n) \) is measure of membership \( 0 \leq \mu(x_n) \leq 1 \). Let us break down the interval into \( T \) linguistic terms, while function of membership therein we define as triangle and trapezoidal fuzzy numbers [3]. Relevant measures look as follows:

\[
\mu(x_n) = \begin{cases}
0, & x_n < a \\
1 + \frac{b-x_n}{a-b}, & a \leq x_n < b \\
1, & x_n = b \\
1 - \frac{x_n-b}{b-c}, & b \leq x_n < c \\
0, & x_n \geq c
\end{cases},
\]

\[
\mu(x_n) = \begin{cases}
0, & x_n < a_1 \\
1 + \frac{b_1-x_n}{a-b_1}, & a_1 \leq x_n < b_1 \\
1, & b_1 \leq x_n < b_2 \\
1 - \frac{x_n-b_2}{b_2-c}, & b_2 \leq x_n < c_1 \\
0, & x_n \geq c_1
\end{cases}
\]

Subsequently, for fuzzy derivations made on the grounds of technical analysis we will use system of five terms, which will be denoted as «0», «1», «2», «3», «4» (Figure.1).

We assume the fuzzy set of the fundamental analysis results as consisting of three linguistic terms: NN - «probable negative tendency – drop of currency exchange rate», NP - «indefinite tendency», PP - «probable positive tendency of
currency exchange rate. Measure inside each term we assume to be constant and equal to zero.

Figure 1: Terms and membership functions for fuzzy sets of technical analysis; left part of the graph is shown: terms «0», «1», «2».

Essential mention should be made, that if depth of forecast of the technical analysis is comparable with result of multiplication of the time frame size by the length of the smoothing interval (for instance – in the MACD method it is length of averaging of the «quick» sliding average), then depth of the fundamental forecast is comparable with the interval of obtaining of the information, on the basis whereof this forecast is built (for instance – statistics of unemployment in USA). For any time frame the second value is immensely larger than the first one, and this fact should be allowed for in the course of testing and application of the proposed algorithm.

If \( x \) and \( y \) are fuzzy sets, then we can introduce operation of intersection \( \cap \) and union \( \cup \) of these sets according to [3], as
\[
\mu(x \cap y) = \min(\mu(x), \mu(y)) \quad (T\text{-norm; } \mu(x \cup y) = \max(\mu(x), \mu(y)) \quad (S\text{-norm}.
\]

Apart from this, it is necessary to introduce logic implication, which interprets fuzzy logic derivation «IF … THEN» or «FOLLOWS» according to [4]:
\[
x \rightarrow y: \mu(x \rightarrow y) = \min(\mu(x), \mu(y))
\]
Using these definitions, we can interpret fuzzy knowledge base, which allows to make decisions on the basis of fuzzy sets, which correspond to the results of both technical and fundamental analysis.

Let \( x_n^m \) - be fuzzy set belonging to term \( n \) for technical indicator \( m \). Fuzzy knowledge base is given in Table 1.

| \( x_n^m \) | \( x_n^{m+1} \) |
|---|---|---|---|---|---|
| 0 | 1 | 1 | 2 | 2 |
| 1 | 1 | 1 | 2 | 2 |
| 2 | 1 | 2 | 3 | 3 |
| 3 | 2 | 2 | 3 | 3 |
| 4 | 2 | 2 | 3 | 3 |

In accordance with the definitions, measure of the fuzzy logic derivation on the grounds of this base is:

\[
x_n^m \cap x_k^{m+1} \rightarrow Z_{nk}^{m,m+1}, \quad \mu(Z_{nk}^{m,m+1}) = \min(\mu(x_n^m), \mu(x_k^{m+1}))
\]

Since number of terms of the fuzzy logic derivation is equal to number of terms of the input variables, let us use the self-similarity principle, i.e. adjoining (aggregation) of the following fuzzy conclusions will be made on the grounds of the same technical analysis knowledge base:

\[
\mu(Z_{nk}^{m,m+1,m+3,m+3}) = \min(\mu(Z_{nk}^{m,m+1}), \mu(Z_{ik}^{m+2,m+3})).
\]

The fundamental analysis results will be aggregated with the technical analysis by means of the fuzzy base given in form of Table 2.
Table 2: Fuzzy knowledge base for aggregation of the technical and fundamental analyses

<table>
<thead>
<tr>
<th></th>
<th>yₙᵐ</th>
<th>xₙᵐ</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>NN</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>NP</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>PP</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

And, finally, we will make fuzzy logic implications inside various methods of fundamental analysis by means of a matrix similar to that in Table 1, but having dimensions 3*3.

Now, on the basis of fuzzy logic derivation unifying technical and fundamental analyses, we can formulate the control action: in our case it is making of decision on entrance to – exit from «short» or «long» position and lot volume.

If fuzzy logic conclusion belongs to term «2», then there is no opening of any positions. If fuzzy logic conclusion belongs to terms «0» «1», then we enter «short» position with lot volume for term «0» and 

\[ l = \varepsilon L \mu(Z) \]

for term «1», if conclusion belongs to terms «3», «4», then we enter «long» position with volumes

\[ l = L \mu(Z) \text{ и } 1 = \varepsilon L \mu(Z) \], respectively.

In these expressions \( \mu(z) \)- is fuzzy derivation measure built on embedded aggregations of fundamental and technical analyses , \( \varepsilon \) - normalizing factor, usually \( \varepsilon = 0.5 \).

3 Testing of the model

The algorithm has been tested on the pair EURUSD for the period 1.01.2013-28.02.2014. At the input we used five technical indicators MACD,
ADX, RSI, MFI, OBV (five fuzzy logic terms 0, 1, 2, 3, 4). One input corresponded to fuzzy logic implication connected with the fundamental analysis (three fuzzy terms NN, NP, PP). The fuzzy knowledge base corresponded to Tables 1 and 2. Maximum admissible number of technical indicators - 24, of fundamental analysis methods - 8.

Volume of trades was 0.5 standard lots, values of quick and slow smoothing for indicator MACD was $p_1=15$, $p_2=30$, and $p_1=15$ for indicators ADX, RSI, MFI, OBV.

When testing at large interval 1.01.2013-28.02.2014, we used term NP of the fundamental analysis (Table 3), while for the short interval 1.01.2014-28.02.2014 the modeling was carried out for all three fussy terms NN, NP, PP (Table 4).

Initial deposit was $10000, leverage - 100. Apart from this, we used filtration based on «heavy tails» [5] with cut-off parameter equal to two root-mean-square deviations.

Exit from «short» and «long» positions was done upon attaining of take profit value equal to 500 points. Variable parameter was value of time frame within the interval of 1-5 min. Results of testing - are number of frames N, probability of profitable trade P, Sharp’s factor Sh, and summary profit $, are given in Tables 3 and 4.

Table 3: Number of frames N, probability of profitable trade P, Sharp’s factor Sh, and profit vs time frame size TF for period 1.01.2013-28.02.2014.

<table>
<thead>
<tr>
<th>TF</th>
<th>N</th>
<th>P</th>
<th>Sp</th>
<th>$</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>46</td>
<td>0.90</td>
<td>0.17</td>
<td>1051.3</td>
</tr>
<tr>
<td>2</td>
<td>55</td>
<td>0.92</td>
<td>0.27</td>
<td>1852.8</td>
</tr>
<tr>
<td>3</td>
<td>62</td>
<td>0.91</td>
<td>0.13</td>
<td>1263.0</td>
</tr>
<tr>
<td>4</td>
<td>54</td>
<td>0.93</td>
<td>0.23</td>
<td>1647.6</td>
</tr>
<tr>
<td>5</td>
<td>70</td>
<td>0.94</td>
<td>0.43</td>
<td>3364.3</td>
</tr>
</tbody>
</table>
Table 4: Number of frames N, probability of profitable trade P, Sharp’s factor Sh, and profit vs time frame size TF at various terms of fundamental analysis for period 1.01.2014-28.02.2014.

<table>
<thead>
<tr>
<th>TF</th>
<th>N</th>
<th>P</th>
<th>Sp</th>
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<tbody>
<tr>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NN</td>
<td>1</td>
<td>7</td>
<td>0.85</td>
<td>0.47</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>8</td>
<td>0.87</td>
<td>0.85</td>
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<tr>
<td></td>
<td>3</td>
<td>6</td>
<td>0.83</td>
<td>0.90</td>
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<tr>
<td></td>
<td>4</td>
<td>3</td>
<td>0.67</td>
<td>-0.18</td>
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<tr>
<td></td>
<td>5</td>
<td>7</td>
<td>0.80</td>
<td>0.68</td>
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<tr>
<td>NP</td>
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<td>6</td>
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<td>0.25</td>
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<tr>
<td></td>
<td>2</td>
<td>22</td>
<td>0.95</td>
<td>1.19</td>
</tr>
<tr>
<td></td>
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<td>15</td>
<td>0.93</td>
<td>0.59</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>15</td>
<td>0.94</td>
<td>0.87</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>17</td>
<td>1.00</td>
<td>1.48</td>
</tr>
<tr>
<td>PP</td>
<td>1</td>
<td>6</td>
<td>0.83</td>
<td>0.52</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>3</td>
<td>0.67</td>
<td>-0.44</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>15</td>
<td>0.92</td>
<td>0.73</td>
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<tr>
<td></td>
<td>4</td>
<td>13</td>
<td>0.87</td>
<td>1.13</td>
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<tr>
<td></td>
<td>5</td>
<td>17</td>
<td>0.88</td>
<td>1.19</td>
</tr>
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</table>

For this interval the results of testing appeared to be profitable for all time frames. With this, at the initial stages of trades in January – March the trade was loss-making with relative drawdown 6-8%.

Within short interval 1.01.2014-28.02.2014 we have detected sensitivity of the trade results to terms of fundamental analysis, and have observed two loss-making strategies for terms «NN» (negative forecast for exchange rate of pair EURUSD) and «PP» (positive forecast for exchange rate of pair EURUSD).
4 Discussion of the results

Obtained results testify about possibility to use the algorithm for management of finances on the Forex market. Essential parameter of the model is depth of normalizing $N$ when mapping outcomes of the technical indicators on interval $[-1,1]$.

In the course of testing, we used interval $N=100$, while its increase up to $N=1000$ essentially deteriorates the results. Derivations of the fuzzy analysis become mostly neutral – term «2». This is connected with the fact that maximum and minimum value of technical indicators at large intervals becomes too large by absolute value, and realization’s recurrence rate rapidly drops. Decrease of the normalizing interval to $N=10$ brings about similar negative effects. In this case terms «0», «1» и «3», «4» – entrance to «short» and «long» positions – become almost equally probable, and poorly correlate with real behavior of the currency exchange rate.

Possibility of loss-making trades at short intervals when using the fundamental analysis trends «PP», «NN» (Table 4) and maximal profit at any time frames for term «NP» may testify about the fact that in interval 1.01.2014-28.02.2014 the forecast for the «slow» (fundamental) trends are changed from «PP» to «NN», i. e. forecasting interval appeared to be shorter than the testing interval.

The model includes free constant parameters $a$, $b$, $c$… defining measure at terms «0»... «4» (1). The testing assumed that $a= -0.75$, $b=-0.5$, $c= -0.25$, $a_1=c$, $b_1= -0.125$, etc. In general case these values can be optimized using some or another extreme method, for instance – the least-squares method. However, it is improbable that this will bring about essential improvement of the results, since it requires too long optimization sequence (large volume of the initial data), what will lead to overlapping of the effects in various time scales, and, as a consequence, to trivial results [5, 6]. Solution of this problem is situated at «tails
of the distributions», i.e. on the border between the statistical and the deterministic methods.

Further development of the model is possible in the following directions: firstly, hedging of the risks for making of decision with use of more than two currency pairs with relevant fuzzy knowledge base; secondly, use of more complicated knowledge bases on the grounds, for instance, three fuzzy variables with subsequent self-similar embedding; thirdly, integration of the algorithm into a system, which uses not only standard indicators, but also alternative decision-making systems, such as the «heavy tails» models, and algorithms connected therewith.

Demo version and full package implementing the proposed algorithm are accessible on the developers’ website [7].

References
