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A Monte Carlo simulation study for Kolmogorov-Smirnov two-sample test under the precondition of heterogeneity : upon the changes on the probabilities of statistical power and type I error rates with respect to skewness measure

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Abstract

Skewness and kurtosis are adopted by many statisticians as the contraventions of parametric statistics. Therefore, using nonparametric tests would give more proper results for skewed and kurtic series. Many observations also suggest that skewness provokes the loss of power for statistical tests. This paper aims to investigate the impact of skewness on statistical power. For this purpose, the paper takes hold of nine different distributions on Fleishman's power function when skewness measures are 1,75, 1,50, 1,25, 1,00, 0,75, 0,50, 0,25, 0,00, -0,25 and kurtosis measure is 3,75, simultaneously. The investigation concentrates on Kolmogorov-Smirnov two-sample test and considers the significance level (α) as 0,05. This paper runs totally 32 representative sample size simulation alternatives, involving four small and equal; twelve small and different; four large and equal; and twelve large and different sample sizes. The Monte Carlo simulation study takes standard deviation ratios as 2, 3 and 4 under the precondition of heterogeneity. According to the results of equal sample sizes, no significant change are observed on the possibility of Type I error for Kolmogorov-Smirnov

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tests, when the skewness measures decrease from 1,75 to -0,25. For both small and large small sizes, the power of the corresponding test decreases when the coefficient of skewness decreases.

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1 Introduction

Parametric tests have pre-conditions of normal distribution and homogeneity of variance. Normal distribution is a kind of symmetric distributions being optimized mathematically and it offers a fine mathematical curve of frequency distributions for behavioral sciences [1]. Homogeneity variance is an assumption that populations from which samples have been drawn have equal or similar variances [2]. Skewness is one of the two occasions that any sample distribution principally diverges from normality and for many researchers it causes the loss of statistical power [3]. Nonparametric tests are very often more advantageous than parametric tests when one or both of these pre-conditions are not satisfied.

Skewed and kurtic data sets are practically experienced more than normal distributed data. Surely, researchers seek to know whether statistical tests being used have sufficient power. Statistical power of a test varies depending on sample size, skewness, kurtosis, standard deviation and mean ratios. Thereof, researchers are expected to make decisions on which parametric and nonparametric tests they will use with respect to these concerning indicators.

This paper investigates how skewness effects statistical power of nonparametric tests and handles Kolmogorov-Smirnov two-sample test (KS-2). This paper concentrates on heterogeneity of variance pre-condition and utilizes from skewed and kurtic data simultaneously, hence it may differ from recent studies that evaluate the statistical power of nonparamatric tests.

2 Kolmogorov-Smirnov Two-Sample Test

There are several tests available to determine if a sample comes from a normally distributed populations, that include the Kolmogorov-Smirnov test, Anderson-Darling test, Cramer-von Mises Test, Shapiro-Wilk test and Shapiro-Francia test [4]. The Kolmogorov-Smirnov test is a method for comparing the distributions of two independent groups that has virtually disappeared from applied research and introductory statistics books for the social sciences [5]. The Kolmogorov-Smirnov test is important in nonparametric stastistical inference, while the limiting distribution of the Kolmogorov-Smirnov statistic under the null hypothesis is well known and has been derived by several different methods. The limiting distribution of this statistic under the alternative hypothesis is unknown so that it is not possible, in general, to compute the power of the Kolmogoro-Smirnov test [6].

The classical one-dimensional Kolmogorov-Smirnov test is a non-parametric statistic for comparing two empirical distributions which defines the largest absolute difference between the two cumulative distribution functions as a measure of disagreement and is used to test the null hypothesis $F = G$ against the alternative $F > G$ where F and G are distribution functions. If the random variables X and Y correspond to F and G, respectively, then the one-sided alternative is that Y is stochastically greater than X [7]; [8]; [9]. The Kolmogorov-Smirnov test is distribution free in the sense that if H_0 is true, the significance level does not depend on F and G [5].

The generalization of the classical Kolmogorov-Smirnov test is appropriate to analyse random samples defined in two or three dimensions [10]. The Kolmogorov-Smirnov test for two independent samples was developed by Smirnov (1939). When a non-directional and two tailed alternative hypothesis is evaluated, Kolmogorov-Smirnov test for two independent samples is sensitive to any kind of distributional difference (i.e., a difference with respect to location/central tendency, dispersion/variability, skewness, and kurtosis). When a directional and one-tailed alternative hypothesis is evaluated, the test evaluates the relative magnitude of the scores in the two distributions [11]; [12]; [13]; [14]. The Kolmogorov-Smirnov test has at least two major advantages over the chi- square test:

- It can be used with small sample sizes, where the validity of the chi- square test would be questionable.
- Often it appears to be a more powerful test than the chi-square test for any sample size [15]; [16]; [17].

The Kolmogorov-Smirnov test statistic D_n , is defined by $D_n = \sup\left|F(x) - F_n(x)\right|$, where n is the sample size, $F(x)$ is a hypothesized cumulative distribution function with fixed parameters, and $F_n(x)$ is a step-function that increases by $1/n$ at each data value [18]. By the Glivenko-Cantelli Theorem, D_n converges to 0 almost surely under H_0 [19]. Computations of the test statistics for the Kolmogorov-Smirnov test for two independent samples involve the comparison of two cumulative frequency distributions. Whereas the Kolmogorov-Smirnov goodness-of-fit test for a single sample compares the cumulative frequency distribution of a single sample with a hypothesized theoretical or empirical cumulative frequency distribution, the Kolmogorov-Smirnov test for two independent samples compares the cumulative frequency distributions of two independent samples. If, in fact, the two samples are derived from the same population, the two cumulative frequency distributions would be expected to be identical or reasonably similar to one another [14]. The order statistics corresponding to two random samples of size *m* and *n* from continuous populations F_X and F_Y, are

$$X_{(1)}, X_{(2)}, \ldots, X_{(m)} \quad \text{and} \quad Y_{(1)}, Y_{(2)}, \ldots, Y_{(n)}$$

Their respective empirical distribution functions, denoted by $S_m(x)$ and $S_n(x)$, are defined as:

$$S_m(x) = \begin{cases} 0 & \text{if } \; x < X_{(1)} \\ k/m & \text{if } \; X_{(k)} \le x < X_{(k+1)} \text{ for } k = 1,2,\ldots,m-1 \\ 1 & \text{if } \; x \ge X_{(m)} \end{cases}$$

$$S_n(x) = \begin{cases} 0 & \text{if } \; x < Y_{(1)} \\ k/n & \text{if } \; Y_{(k)} \le x < Y_{(k+1)} \text{ for } k = 1,2,\ldots,n-1 \\ 1 & \text{if } \; x \ge Y_{(n)} \end{cases}$$

In a combined ordered arrangement of the *m*+*n* sample observations, $S_m(x)$ and $S_n(x)$ are the respective proportions of *X* and *Y* observations which do not exceed the specific value of *x*. If the null hypothesis $H_0 : F_Y(x) = F_X(x)$ for all *x* is true, the population distributions are identical and we have two samples from the sample populations. The empirical distribution functions for the *X* and *Y* samples are reasonable estimates their respective population cumulative distribution function. Therefore, allowing for sampling variation, there should be reasonable agreement between the two empirical distributions if needed H_0 is true; otherwise the data suggest that H_0 is not true and therefore should be rejected [20].

3 Some Preliminaries

The power of a statistical test is the probability of rejecting the null hypothesis when it is false and it depends on three factors such as the α level of the experimenter, sample size and the effect size [21]. The significance criterion represents the standard of proof that the phenomenon exists, or the risk of mistakenly reject the null hypothesis. A critical effect size is briefly is a measure of how strong the theory is minimally important for the society and it is population-specific as well as measurement-specific [22]. The relationship between the effect size and power of a statistical test can be explained as follows. The larger the effect size, when other factors are equal, the greater the power of a test [23]. Moreover, because the sensible estimation of population parameters increases with sample size, this greater sensibility will be reflected in greater statistical power to detect effects, where association is non-linear and a law of diminishing returns revisits [24].

A number of algorithms are developed for calculating the exact powers and level of significance of statistical tests when the true cumulative distribution function $F(x)$ is continuous [25]. In a priori power analysis, sample size N is computed as a function of the required power level (1-β), so the significance level and the population effect size to be detected with this power level. In contrast, post hoc power analyses compute (1-β) as a function of significance level, the population effect size parameter, and sample size(s) of the study. Compromise power analyses, compute both significance and the power level as functions of the effect size, and the error probability ratio, $q = \beta / \alpha$; while sensitivity analyses compute critical population effect size as a function of α, β and sample size. Finally criterion analyses compute α, as a function of 1-β, the effect size and the given sample size [26].

Type I error is frequently assigned by many researchers as the fourth factor effecting statistical power of a test. A Type I error occurs when a true null hypothesis is rejected and the likelihood of committing a Type I error is specified by alpha level being employed in evaluation process. The researcher should employ the lower values of alpha level to eliminate this error [14]. Meanly, Type I error refers to the probability of rejecting the null hypothesis when it is true [21]. The choice of a particular risk level for making a Type I error is dependent on the cost of making a Type I error [27]. Monte Carlo procedures enable to estimate the Type I comparisonwise and experimentwise error rates for multiple comparison procedures used for pairwise comparisons between means [28].

Measures of skewness and kurtosis are often used to describe shape characteristic of a distribution, in tests of normality and in studies of robustness to normal theory [29]. A skewed distribution is neither symmetric nor normal because the data values trail off more sharply on one side than on the other. One of the fundamental problems with skewness in data is many of the most common statistical methods require at least an approximately normal distribution and when these methods are employed on skewed data, the outcomes may well be misleading or just plain wrong. Even when the answers are basically correct, there is often some efficiency lost and the analysis has not made the best use of all of the information in the data set. One solution to this problem is to use transformation to make a skewed distribution more symmetric [30].

The skewness of a random variable X is often measured by third central moment (γ_1) or β_1 as the formulas below,

$$\gamma_1 = \frac{\mu_3}{\sigma_3} \quad \text{or} \quad \sqrt{\beta_1} = \frac{\mu_3}{\mu_2^{3/2}}$$

where μ and σ are the mean and standard deviation respectively. For the Pareto distribution with density $f(x) = \theta / x^{\theta+1}$, for $x \geq 1$ and $\theta > 0$, the non-central moments are found to be $\mu_k' = \theta / (\theta - k)$; therefore, this skewness coefficients becomes arbitrarily large as $\varepsilon \to 0^+$ for $\theta = 3 + \varepsilon$ [31]; [32].

Darlington (1970) [33] argued that kurtosis is best described as a measure of unimodality versus bimodality rather than as a measure of peakedness versus flatness, where the lower kurtosis means the greater bimodality. The fourth standardized moment of a normal distributed population, or its random variable *X*, is given by;

$$\beta_2 = \frac{E(X-\mu)^4}{\left[E(X-\mu)^2\right]^2} = \frac{E(X-\mu)^4}{\sigma^4}$$

where *E* is denoted by the expected value operator. This fourth moment (β_2) traditionally measures kurtosis and equals to 3 for the normal distribution. Unimodal distributions which have higher peaks in the center of the distribution $(\beta_2 > 3)$ and are often described as leptokurtic [34]. Conventional but conservative alpha levels are used to evaluate significance of skewness with small to moderate samples, while for large samples, the shape of the distribution gives the researcher an opinion on skewness and kurtosis [35].

4 Monte Carlo Simulation Study

Monte Carlo simulation establishes to make empirical assessment of a statistic in random samples being obtained from a pseudo-population that consists of resemble samples [36]. At this point, the Monte Carlo principle may be introduced as a pioonering technique in terms of finding approximate solutions to mathematical or physical problems and by using computer-based simulation programs and random sample procedures. The idea behind the Monte Carlo methodology is the law of large numbers and the initial Monte Carlo principle performs,

- to make statistical estimation based on weak mathematical theory,
- to test the null hypothesis when a wide variety of possible cases are available,
- to evaluate the robustness of parametric outcomes when contradictory assumptions are available,
- to appreciate the quality of estimation methods
- to compare the characteristics of two or more estimators [36]; [37].

Additionally, Monte Carlo simulation is also employed in such circumstances that the concerning assumptions are violated or theoretical sample distributions are not presented [38].

This paper utilizes from Monte Carlo simulation and for this purpose runs SAS 9.00 computer package program. RANNOR procedure in SAS, generates random numbers from a standard normal distribution involved in Fleishman's power function with a population mean of zero, and a standard deviation of one. Fleishman's power function uses the following formula,

$$\mathrm{Y} = \mathrm{a} + [(\mathrm{dX} + \mathrm{c})\mathrm{X} + \mathrm{b}]\mathrm{X}$$

where, X denotes a random variable with a mean of zero and a standard deviation of one, and it is generated by RANNOR, as mentioned above. Besides, Y denotes a distribution depending on constant terms; a, b, c, and d are coefficients identified with respect to different values of standard deviation, skewness and kurtosis, where coefficient a is constant. Coefficients b, c and d are introduced by Fleishman, while coefficient a and c are always opposite signed.

This paper concentrates on the distributions with highest coefficient of kurtosis (that is 3,75) in Fleishman's power function to interpret the effect of skewness on statistical power of Kolmogorov-Smirnov two-sample test when the kurtosis value is 'constant'. Fleishman's power function comprises 9 distributions with constant kurtosis value of 3,75, but different skewness values of 1,75, 1,50, 1,25, 1,00, 0,75, 0,50, 0,25, 0,00 and -0,25. The paper deals with totally 32 representative sample size simulation combinations, involving four small and equal; twelve small and different; four large and equal; and twelve large and different sample sizes. Table 1 summarizes these combinations. The paper regards the standard deviation ratios as $\sigma_1 : \sigma_2 = 2$, $\sigma_1 : \sigma_2 = 3$ and $\sigma_1 : \sigma_2 = 4$ in $\alpha = 0{,}05$ significance level, in this way 1152 (9x32x4) syntaxes are written and 30.000 repetitions are performed for every syntaxes.

5 Simulation Results

The simulation results show similarities among 9 distributions of the study, such that in all distributions and combinations, when kurtosis value is constant, decreasing the coefficients of skewness does not affect the probability of Type I error rates for KS-2 test, where all these rates are less than $\alpha = 0{,}05$ significance level. Again, for all distributions, the results indicate that statistical power of KS-2 test increases, when sample sizes increase; therefore standard deviation ratios have favorably effects as well. The most remarkable outcome of the simulation study is the decrease of the statistical power for KS-2 test, with respect to a decrease on the coefficient of skewness (γ_1). In that case, for all sample sizes and standard deviation ratios, the most significant loss of power is recognized when the coefficient of skewness is decreased from $\gamma_1 = 1{,}75$ to $\gamma_1 = 1{,}50$. In addition, statistical power of KS-2 test increases when both the coefficient of skewness and standard deviation ratios increase. The most significant increase on statistical power of KS-2 test is recognized when standard deviation ratio increases from 2 to 3. For both large and small sample sizes; Table 3, Table 4, Table 5 and Table 6 represent the observed values of statistical power of KS-2 test with respect to a decrease on the coefficient of skewness.

Table 1: Sample Size Combinations Being Used in Monte Carlo Simulation

Sample Size	**Sample Size Combinations**
Small and Equal	(5, 5), (10, 10), (15, 15), (20, 20)
Small and Different	(5, 10), (5, 15), (5, 20), (10, 5), (10, 15), (10, 20), (15, 5), (15, 10), (15, 20), (20, 5), (20, 10), (20, 15)
Large and Equal	(25, 25), (50, 50), (75, 75), (100, 100)
Large and Different	(100, 25), (100, 50), (100, 75)

Table 2: Fleishman's Power Function

Skewness (γ_1)	**Kurtosis** (γ_2)	**a**	**b**	**c**	**d**
1.75	3.75	-0.3994966745	0.9296605248	0.3994966745	-0.0364669928
1.50	3.75	-0.2210276210	0.8658862035	0.2210276210	0.0272206992
1.25	3.75	-0,1606425556	0,8188815613	0,1606425556	0,0491651717
1.00	3.75	-0,1194238366	0,7894207442	0,1194238366	0,0615396192
0.75	3.75	-0,0856305956	0,7699520206	0,0856305956	0,0693485545
0.50	3.75	-0,0555244412	0,7573998478	0,0555244412	0,0742591514
0.25	3.75	-0,0273411959	0,7503153411	0,0273411959	0,0769928241
0.00	3.75	0.00	0,7480208079	0.00	0,0778727161
-0.25	3.75	0,0273411959	0,7503153411	-0,0273411959	0,0769928241

Source: C.H. Lee; A Monte Carlo Study of Two Nonparametric Statistics With Comparisons of Type I Error Rates and Power, Unpublished PhD. Dissertation, Oklahoma State University, 2007, p.173-174. [39]

Table 3: The Probabilities of Type I Error Rates of KS-2 Test for Small and Large Sample Sizes

γ_2	n_1	n_2	The Probabilities of Type I Error Rates of KS-2 Test for Small Sample Sizes								
			$\gamma_{1=1,75}$	$\gamma_{1=1,50}$	$\gamma_{1=1,25}$	$\gamma_{1=1,00}$	$\gamma_{1=0,75}$	$\gamma_{1=0,50}$	$\gamma_{1=0,25}$	$\gamma_{1=0,00}$	$\gamma_{1=-0,25}$
	5	5	0,007	0,007	0,009	0,009	0,009	0,008	0,008	0,008	0,008
	5	10	0,020	0,018	0,020	0,020	0,019	0,020	0,018	0,017	0,020
	5	15	0,023	0,023	0,024	0,023	0,022	0,024	0,025	0,023	0,024
	5	20	0,023	0,023	0,026	0,026	0,024	0,026	0,024	0,027	0,027
	10	5	0,020	0,020	0,020	0,020	0,019	0,018	0,019	0,020	0,018
	10	10	0,012	0,012	0,011	0,012	0,013	0,012	0,012	0,011	0,012
	10	15	0,030	0,029	0,031	0,029	0,030	0,031	0,029	0,031	0,029
3,75	10	20	0,030	0,030	0,027	0,030	0,028	0,027	0,030	0,028	0,030
	15	5	0,024	0,024	0,023	0,023	0,023	0,023	0,024	0,023	0,025
	15	10	0,028	0,027	0,028	0,030	0,031	0,026	0,029	0,030	0,029
	15	15	0,027	0,025	0,028	0,026	0,026	0,027	0,026	0,026	0,027
	15	20	0,034	0,034	0,037	0,034	0,036	0,036	0,036	0,034	0,033
	20	5	0,025	0,026	0,025	0,026	0,026	0,026	0,024	0,024	0,025
	20	10	0,031	0,030	0,028	0,027	0,029	0,028	0,027	0,027	0,032
	20	15	0,034	0,036	0,035	0,034	0,038	0,035	0,036	0,034	0,035
	20	20	0,033	0,034	0,034	0,032	0,031	0,033	0,037	0,033	0,034
γ_2	n_1	n_2	The Probabilities of Type I Error Rates of KS-2 Test for Large Sample Sizes								
			$\gamma_{1=1,75}$	$\gamma_{1=1,50}$	$\gamma_{1=1,25}$	$\gamma_{1=1,00}$	$\gamma_{1=0,75}$	$\gamma_{1=0,50}$	$\gamma_{1=0,25}$	$\gamma_{1=0,00}$	$\gamma_{1=-0,25}$
	25	25	0,033	0,034	0,035	0,039	0,038	0,037	0,034	0,037	0,035
	25	50	0,038	0,039	0,040	0,038	0,039	0,039	0,037	0,037	0,039
	25	75	0,038	0,039	0,036	0,038	0,038	0,037	0,038	0,037	0,039
	25	100	0,037	0,036	0,036	0,035	0,038	0,038	0,035	0,036	0,037
	50	25	0,038	0,036	0,041	0,037	0,038	0,038	0,039	0,038	0,037
	50	50	0,037	0,042	0,040	0,037	0,040	0,041	0,040	0,040	0,040
	50	75	0,039	0,037	0,036	0,039	0,038	0,038	0,036	0,038	0,037
3,75	50	100	0,041	0,039	0,043	0,042	0,039	0,040	0,040	0,040	0,040
	75	25	0,038	0,037	0,036	0,038	0,039	0,038	0,038	0,037	0,038
	75	50	0,039	0,037	0,037	0,038	0,038	0,037	0,038	0,036	0,036
	75	75	0,044	0,041	0,044	0,044	0,043	0,040	0,040	0,042	0,042
	75	100	0,039	0,039	0,039	0,039	0,042	0,040	0,038	0,041	0,039
	100	25	0,037	0,036	0,036	0,036	0,036	0,038	0,039	0,037	0,037
	100	50	0,040	0,041	0,038	0,038	0,041	0,039	0,041	0,043	0,041
	100	75	0,042	0,041	0,041	0,041	0,039	0,040	0,038	0,040	0,039
	100	100	0,037	0,035	0,036	0,038	0,037	0,036	0,034	0,035	0,034

Table 4: The Statistial Power of KS-2 Test for Small and Large Sample Sizes when $\sigma_1 : \sigma_2 = 2$

γ_2	n_1	n_2	The Statistical Power Values of KS-2 Test for Small Sample Sizes								
			$\gamma_{1=1,75}$	$\gamma_{1=1,50}$	$\gamma_{1=1,25}$	$\gamma_{1=1,00}$	$\gamma_{1=0,75}$	$\gamma_{1=0,50}$	$\gamma_{1=0,25}$	$\gamma_{1=0,00}$	$\gamma_{1=-0,25}$
3,75	5	5	0,029	0,013	0,014	0,012	0,012	0,012	0,010	0,010	0,010
	5	10	0,157	0,076	0,059	0,051	0,050	0,047	0,045	0,044	0,046
	5	15	0,162	0,094	0,077	0,069	0,067	0,062	0,061	0,063	0,059
	5	20	0,173	0,102	0,087	0,075	0,074	0,073	0,071	0,070	0,072
	10	5	0,049	0,026	0,019	0,018	0,017	0,017	0,018	0,016	0,017
	10	10	0,124	0,043	0,032	0,030	0,026	0,024	0,024	0,024	0,023
	10	15	0,296	0,119	0,091	0,079	0,075	0,067	0,068	0,067	0,066
	10	20	0,306	0,139	0,112	0,093	0,091	0,084	0,081	0,080	0,081
	15	5	0,051	0,024	0,020	0,019	0,018	0,017	0,016	0,018	0,018
	15	10	0,221	0,082	0,065	0,056	0,052	0,048	0,046	0,046	0,046
	15	15	0,382	0,127	0,090	0,080	0,071	0,062	0,065	0,062	0,063
	15	20	0,579	0,225	0,151	0,126	0,114	0,107	0,105	0,101	0,101
	20	5	0,047	0,023	0,020	0,017	0,017	0,016	0,015	0,017	0,016
	20	10	0,294	0,089	0,063	0,056	0,049	0,046	0,046	0,043	0,044
	20	15	0,449	0,155	0,111	0,091	0,081	0,077	0,073	0,075	0,074
	20	20	0,625	0,231	0,155	0,126	0,111	0,105	0,097	0,095	0,101
γ_2	n_1	n_2	The Statistical Power Values of KS-2 Test for Large Sample Sizes								
			$\gamma_{1=1,75}$	$\gamma_{1=1,50}$	$\gamma_{1=1,25}$	$\gamma_{1=1,00}$	$\gamma_{1=0,75}$	$\gamma_{1=0,50}$	$\gamma_{1=0,25}$	$\gamma_{1=0,00}$	$\gamma_{1=-0,25}$
3,75	25	25	0,785	0,318	0,206	0,169	0,146	0,134	0,122	0,130	0,125
	25	50	0,894	0,477	0,329	0,271	0,236	0,218	0,206	0,198	0,204
	25	75	0,952	0,549	0,381	0,314	0,277	0,253	0,238	0,239	0,243
	25	100	0,954	0,584	0,404	0,336	0,296	0,275	0,267	0,254	0,260
	50	25	0,967	0,456	0,274	0,217	0,182	0,160	0,150	0,147	0,157
	50	50	0,997	0,735	0,501	0,398	0,348	0,310	0,291	0,289	0,292
	50	75	0,999	0,821	0,600	0,494	0,434	0,392	0,373	0,366	0,370
	50	100	1,000	0,877	0,680	0,579	0,509	0,465	0,445	0,438	0,443
	75	25	0,994	0,541	0,310	0,232	0,193	0,173	0,159	0,157	0,161
	75	50	1,000	0,849	0,597	0,463	0,403	0,363	0,340	0,331	0,336
	75	75	1,000	0,946	0,758	0,648	0,564	0,526	0,497	0,485	0,493
	75	100	1,000	0,967	0,824	0,712	0,646	0,595	0,566	0,552	0,562
	100	25	0,999	0,582	0,323	0,242	0,200	0,175	0,159	0,157	0,163
	100	50	1,000	0,920	0,686	0,551	0,471	0,421	0,391	0,388	0,389
	100	75	1,000	0,976	0,831	0,713	0,635	0,584	0,554	0,541	0,553
	100	100	1,000	0,990	0,887	0,784	0,714	0,662	0,633	0,626	0,633

Table 5: The Statistial Power of KS-2 Test for Small and Large Sample Sizes When $\sigma_1 : \sigma_2 = 3$

γ_2	n_1	n_2	The Statistical Power Values of KS-2 Test for Small Sample Sizes								
			$\gamma_{1=1,75}$	$\gamma_{1=1,50}$	$\gamma_{1=1,25}$	$\gamma_{1=1,00}$	$\gamma_{1=0,75}$	$\gamma_{1=0,50}$	$\gamma_{1=0,25}$	$\gamma_{1=0,00}$	$\gamma_{1=-0,25}$
3,75	5	5	0,046	0,026	0,022	0,018	0,017	0,017	0,015	0,015	0,014
	5	10	0,234	0,137	0,107	0,094	0,086	0,080	0,079	0,080	0,079
	5	15	0,243	0,162	0,136	0,119	0,113	0,106	0,101	0,103	0,105
	5	20	0,254	0,176	0,150	0,133	0,126	0,117	0,116	0,114	0,120
	10	5	0,090	0,044	0,032	0,029	0,024	0,021	0,024	0,023	0,024
	10	10	0,228	0,105	0,074	0,057	0,053	0,046	0,042	0,043	0,042
	10	15	0,458	0,257	0,185	0,157	0,142	0,133	0,122	0,127	0,125
	10	20	0,470	0,283	0,223	0,190	0,173	0,161	0,152	0,155	0,156
	15	5	0,103	0,044	0,032	0,027	0,025	0,024	0,023	0,019	0,019
	15	10	0,397	0,192	0,132	0,111	0,092	0,085	0,084	0,085	0,084
	15	15	0,593	0,313	0,221	0,179	0,158	0,139	0,131	0,130	0,131
	15	20	0,782	0,491	0,359	0,297	0,256	0,235	0,229	0,221	0,225
	20	5	0,107	0,043	0,030	0,022	0,021	0,020	0,019	0,016	0,019
	20	10	0,514	0,236	0,157	0,128	0,101	0,095	0,089	0,084	0,087
	20	15	0,692	0,371	0,261	0,217	0,185	0,171	0,159	0,163	0,164
	20	20	0,837	0,521	0,376	0,308	0,270	0,242	0,229	0,224	0,230
γ_2	n_1	n_2	The Statistical Power Values of KS-2 Test for Large Sample Sizes								
			$\gamma_{1=1,75}$	$\gamma_{1=1,50}$	$\gamma_{1=1,25}$	$\gamma_{1=1,00}$	$\gamma_{1=0,75}$	$\gamma_{1=0,50}$	$\gamma_{1=0,25}$	$\gamma_{1=0,00}$	$\gamma_{1=-0,25}$
3,75	25	25	0,944	0,679	0,513	0,411	0,375	0,341	0,320	0,314	0,317
	25	50	0,981	0,847	0,706	0,627	0,565	0,527	0,509	0,501	0,512
	25	75	0,996	0,905	0,779	0,697	0,637	0,606	0,586	0,582	0,579
	25	100	0,996	0,918	0,807	0,734	0,679	0,643	0,623	0,611	0,621
	50	25	0,998	0,897	0,708	0,600	0,509	0,471	0,444	0,429	0,442
	50	50	1,000	0,987	0,931	0,866	0,808	0,767	0,742	0,736	0,746
	50	75	1,000	0,996	0,967	0,929	0,891	0,864	0,844	0,840	0,843
	50	100	1,000	0,998	0,986	0,961	0,937	0,919	0,903	0,901	0,902
	75	25	1,000	0,962	0,800	0,670	0,587	0,520	0,496	0,486	0,493
	75	50	1,000	0,999	0,976	0,935	0,893	0,860	0,837	0,836	0,843
	75	75	1,000	1,000	0,997	0,987	0,975	0,962	0,951	0,948	0,954
	75	100	1,000	1,000	0,999	0,993	0,986	0,981	0,977	0,975	0,976
	100	25	1,000	0,981	0,842	0,713	0,623	0,559	0,525	0,511	0,522
	100	50	1,000	1,000	0,993	0,973	0,945	0,922	0,907	0,904	0,908
	100	75	1,000	1,000	1,000	0,996	0,989	0,983	0,977	0,974	0,978
	100	100	1,000	1,000	1,000	0,999	0,997	0,994	0,993	0,991	0,991

Table 6: The Statistial Power of KS-2 Test for Small and Large Sample Sizes When $\sigma_1 : \sigma_2 = 4$

γ_2	n_1	n_2	The Statistical Power of KS-2 Test for Small Sample Sizes								
			$\gamma_{1=1,75}$	$\gamma_{1=1,50}$	$\gamma_{1=1,25}$	$\gamma_{1=1,00}$	$\gamma_{1=0,75}$	$\gamma_{1=0,50}$	$\gamma_{1=0,25}$	$\gamma_{1=0,00}$	$\gamma_{1=-0,25}$
3,75	5	5	0,059	0,037	0,026	0,026	0,022	0,022	0,020	0,018	0,019
	5	10	0,281	0,194	0,155	0,130	0,124	0,116	0,108	0,109	0,108
	5	15	0,295	0,210	0,182	0,165	0,151	0,143	0,139	0,139	0,140
	5	20	0,301	0,225	0,197	0,178	0,164	0,158	0,156	0,153	0,156
	10	5	0,116	0,058	0,045	0,038	0,033	0,028	0,028	0,028	0,026
	10	10	0,290	0,155	0,111	0,089	0,075	0,069	0,067	0,064	0,067
	10	15	0,540	0,349	0,267	0,233	0,206	0,191	0,190	0,179	0,185
	10	20	0,549	0,386	0,312	0,268	0,256	0,236	0,230	0,225	0,221
	15	5	0,147	0,068	0,048	0,035	0,032	0,028	0,026	0,024	0,025
	15	10	0,481	0,278	0,205	0,168	0,143	0,131	0,122	0,121	0,124
	15	15	0,686	0,447	0,334	0,271	0,246	0,221	0,211	0,206	0,211
	15	20	0,865	0,661	0,521	0,452	0,409	0,372	0,355	0,354	0,360
	20	5	0,162	0,063	0,043	0,033	0,029	0,025	0,023	0,024	0,023
	20	10	0,626	0,355	0,246	0,197	0,165	0,147	0,136	0,136	0,135
	20	15	0,787	0,534	0,405	0,332	0,290	0,268	0,253	0,250	0,261
	20	20	0,905	0,695	0,555	0,476	0,428	0,391	0,367	0,364	0,368
γ_2	n_1	n_2	The Statistical Power of KS-2 Test for Large Sample Sizes								
			$\gamma_{1=1,75}$	$\gamma_{1=1,50}$	$\gamma_{1=1,25}$	$\gamma_{1=1,00}$	$\gamma_{1=0,75}$	$\gamma_{1=0,50}$	$\gamma_{1=0,25}$	$\gamma_{1=0,00}$	$\gamma_{1=-0,25}$
3,75	25	25	0,977	0,851	0,719	0,632	0,578	0,540	0,510	0,503	0,514
	25	50	0,996	0,953	0,888	0,834	0,788	0,764	0,745	0,736	0,746
	25	75	0,999	0,980	0,932	0,890	0,855	0,826	0,811	0,811	0,812
	25	100	0,999	0,984	0,948	0,913	0,883	0,858	0,845	0,841	0,850
	50	25	1,000	0,982	0,918	0,843	0,786	0,742	0,709	0,701	0,711
	50	50	1,000	1,000	0,993	0,980	0,968	0,954	0,943	0,940	0,944
	50	75	1,000	1,000	0,998	0,995	0,989	0,983	0,980	0,977	0,980
	50	100	1,000	1,000	1,000	0,998	0,996	0,995	0,992	0,991	0,993
	75	25	1,000	0,997	0,966	0,912	0,856	0,810	0,783	0,776	0,785
	75	50	1,000	1,000	0,999	0,996	0,992	0,986	0,983	0,983	0,983
	75	75	1,000	1,000	1,000	1,000	0,999	0,998	0,998	0,998	0,998
	75	100	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	0,999
	100	25	1,000	1,000	0,980	0,938	0,891	0,854	0,821	0,812	0,824
	100	50	1,000	1,000	1,000	0,999	0,998	0,996	0,994	0,995	0,995
	100	75	1,000	1,000	1,000	1,000	1,000	1,000	1,000	0,999	1,000
	100	100	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000

6 Conclusion and Discussion

As the Monte Carlo simulation study results suggest in this paper, when the kurtosis value is constant with its highest value of 3,75, the statistical power of KS-2 test is not significantly effected by a decrease of the coefficient of skewness. For small sample sizes, the probabilities of Type I error for KS-2 test vary between the values of (0,007) and (0,038). The study observes the smallest value of this probability when the coefficient of skewness decreases from $\gamma_1 = 1{,}75$ to $\gamma_1 = 1{,}50$ for (5, 5) sample size, while it obtains the highest value of Type I probability when the coefficient of skewness decreases from $\gamma_2 = 3{,}75$ to $\gamma_1 = 0{,}75$ for (20, 15) sample size. Similarly, for large sample sizes, the probabilities of Type I error for KS-2 test vary between the values of (0,033) and (0,044). The smallest Type I error probability is observed when the coefficient of skewness is $\gamma_1 = 1{,}75$ for (25, 25) sample size, meanwhile the highest value of the probability is recognized when the coefficients of skewness are $\gamma_1 = 1{,}75$, $\gamma_1 = 1{,}25$ and $\gamma_1 = 1{,}00$ for (75, 75) sample size. The probabilities of Type I error for KS-2 test are found less than α = 0,05 significance level for both small and large sample sizes.

The results of the simulation study in all distributions state that when the sample size increases, the statistical power of KS-2 test also increases. For small sample pairs, when the first sample size is smaller than the second sample size, the observed statistical power of KS-2 test is more than the corresponding power when the second sample size is smaller than the first one. For instance, the observed statistical power of KS-2 test for the sample size of (5, 10) is more than (10, 5) sample size in all distributions and standard deviations. For large sample pairs, just the opposite circumstance is observed, i.e. the observed statistical power for (50, 25) sample size is more than (25, 50) sample size. Moreover, in all distributions and sample sizes, the standard deviation has favorable effect on the statistical power and the most significant statistical power increase is observed when the standard deviation ratio increases from $\sigma_1 : \sigma_2 = 2$ to $\sigma_1 : \sigma_2 = 3$. The results also demonstrate that when the coefficient of skewness decreases, the statistical power of KS-2 test also decreases, except for one observation. The statistical power increases when the coefficient of skewness decreases from $\gamma_1 = 0{,}00$ to $\gamma_1 = -0{,}25$.

In the light of all these simulation results, one may suggest that the researchers concentrating on a non-parametric two-sample test, may observe higher statistical power if they choose their samples from the distribution with the coefficient of skewness, $\gamma_1 = 1{,}75$, and when the coefficient of kurtosis is constant ($\gamma_2 = 3{,}75$) simultaneously. Additionally, the researchers may also observe higher values of statistical power, when $\sigma_1 : \sigma_2$ ratio increases. For small sample sizes, the researchers may prefer the larger values of the first sample size and for large sample sizes; they may prefer the smaller values of the first sample size in order to observe higher statistical power values, numerically.

References

[1] R. J. Shavelson, *Statistical reasoning for the behavioral* sciences, Third edition, Allynand Bacon, Inc., Boston, 1996.

[2] W. P. Vogt, *Dictionary of Statistics and Methodology: A Nontechnical Guide for the Social Sciences,* Third edition, Sage Publications, California, 2005.

[3] J. L. Rasmussen, Parametric vs nonparametric tests on non-normal and transformed data. Unpublished PhD Dissertation, Department of psychology of the graduate school of Tulane University, New Orleans, 1983.

[4] Z. Drezner, O. Turel and D. Zerom, A modified Kolmogorov-Smirnov test for normality, *Communications in Statistics-Simulation and Computation*, **39**(4), (2010), 693-704.

[5] R.R. Wilcox, Some practical reasons for reconsidering the Kolmogorov-Smirnov Test, *British Journal of Mathematical and Statistical Psychology*, **50**(1), (1997), 9-20.

[6] J. Capon, On the asymptotic efficiency of the Kolmogorov-Smirnov test, *Journal of the American Statistical Association*, **60**(311), (1965), 843-853.

[7] N. Smirnov, On the estimation of the discrepancy between empirical curves of distribution for two independent samples, *Bulletin Mathematique de l'Universite de Moscou*, **2**, (1939), 3-14.

[8] M. H. Gail and S. B. Green, Critical values for the one-sided two-sample Kolmogorov-Smirnov statistic, *Journal of the American Statistical Association*, **71**(355), (1976), 757-760.

[9] R.H.C. Lopes, I. Reid and P.R. Hobson, The two-dimensional Kolmogorov-Smirnov test, *Proceedings of XI International Workshop on Advanced Computing and Analysis Techniques Physics Research*, April 23-27 2007, Amsterdam, Netherlands.

[10] G. Fasano and A. Franceschini, A multidimensional version of the Kolmogorov-Smirnov test, *Monthly Notices of the Royal Astronomical Society*, **225**(1), (1987), 155-170.

[11] W. Daniel, *Applied nonparametric statistics*, PWS-Kent Publishing Company, Boston, 1990.

[12] L.A. Marascuilo and M. McSweeney, *Nonparametric and distribution-free methods for the social sciences*, Brooks/Cole Publications Co., California, 1977.

[13] S. Siegel and N.J. Castellan, *Nonparametric statistics for the behavioral sciences*, 2nd edition, Mc-Graw-Hill, New York, 1988.

[14] D. J. Sheskin, *Hanbook of parametric and nonparametric statistical procedures*, 3rd edition, Chapman & Hall/CRC, New York, 2004.

[15] F.N. David and N.L. Johnson, The probability integral transformation when parameters are estimated from the sample, *Biometrika*, **35**(1/2), (1948), 182-190.

[16] F.J. Massey, The Kolmogorov-Smirnov test for goodness of fit, *Journal of the American Statistical Association*, **46**(253), (1951), 68-78.

[17] H.W. Lilliefors, On the Kolmogorov-Smirnov test for normality with mean and variance unknown, *Journal of the American Statistical Association*, **62**(318), (1967), 399-402.
[18] J.H. Drew, A.G. Glen and L.M. Leemis, Computing the cumulative distribution function of the Kolmogorov-Smirnov statistic, *Computational Statistics & Data Analysis*, **34**(1), (2000), 1-15.
[19] Z. Zhang and C. Wang, Investigating the effect of dependence on the performance of Kolmogorov-Smirnov test, Course Project, Michigan State University, December 2010.
[20] J.D. Gibbons and S. Chakraborti, *Nonparametric Statistical Inference*, Fourth edition, Marcel Decker Inc., New York, 2003.
[21] J.P. Stevens, *Applied multivariate statistics for the social sciences*, Fifth edition, Routledge, New York, 2009.
[22] H.C. Kraemer and S. Thiemann, *How many subjects? Statistical power analysis in research,* Sage Publications Inc., California, 1987.
[23] J. Cohen, *Statistical power analysis for the behavioral sciences*, Second edition, Lawrence Erlbaum Associates Inc., New Jersey, 1988.
[24] A. Davey and J. Savla, *Statistical power Analysis with missing Data: A structural equation modelling approach*, Routledge, New York, 2010.
[25] L.J. Gleser, Exact power of goodness-of-fit tests of Kolmogorov Type for discontinuous distributions, *Journal of the American Statistical Association*, **80**(392), (1985), 954-958.
[26] F. Faul, E. Erdfelder, A. Lang and A. Buchner, G*Power 3: A flexible statistical power analysis program for the social, behavioral, and biomedical sciences, *Behavioral Research Methods*, **39**(2), (2007), 175-191.
[27] M. L. Berenson, D. M. Levine and T. C. Krehbiel (2003). *Basic business statistics: Concepts and applications*, Ninth edition, Prentice Hall Inc., New Jersey, 2003.
[28] C. S. Bernhardson, 375: Type I error rates when multiple comparison procedures follow a significant F test of ANOVA, *Biometrics*, **31**(1), (1975), 229-232.
[29] D.N. Joanes and C.A. Gill, Comparing measures of sample skewness and Kurtosis, *Journal of the Royal Statistical Society: Series D (The Statistician)*, **47**(1), (1998), 183-189.
[30] A.F. Siegel, *Practical business statistics*, Sixth edition, Academic Press, Boston, 2012.
[31] R. A. Groeneveld and G. Meeden, Measuring skewness and kurtosis, *Journal of the Royal Statistical Society: Series D (The Statistician),* **33**(4), (1984), 391-399.
[32] J. A. Doornik and H. Hansen, An omnibus test for univariate and multivariate normality, *Oxford Bulletin of Economics and Statistics*, **70**(1), (2008), 927-939.
[33] R. B. Darlington, Is kurtosis really "Peakedness"?, *The American Statistician*, **24**(2), (1970), 19-22.

[34] R. B. D'Agostino, A. Belanger and R. B. D'Agostino Jr., A suggestion for using powerful and information tests of normality, *The American Statistician*, **44**(4), (1990), 316-321.
[35] B. G. Tabachnick and L. S. Fidell, *Using multivariate statistics*, Pearson Education Inc., Boston, 1996.
[36] C. Z. Mooney, *Monte Carlo simulation*. Sage Publications Inc., California, 1997.
[37] J. Jiang, *Large sample techniques for statistics*. Springer Science + Business Media, LLC., New York, 2010.
[38] X. Fan, À. Felsövályi**,** S. A. Sivo and S. C. Keenan, *SAS for Monte Carlo studies: A guide for quantitative researchers*. SAS Publishing, Ary, NC, 2003.
[39] C.H. Lee, A Monte Carlo study of two nonparametric statistics with comparisons of Type I error rates and power. Unpublished PhD Dissertation, Faculty of the Graduate College of the Oklahoma State University, Stillwater, 2007.