Evaluating Process Capability Indices for some Quality Characteristics of a Manufacturing Process

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Abstract

The effectiveness of a manufacturing product produced depends on whether the specified standards by the management / production engineers are met or not. It is possible that a process will be stable but will not meet the specification set by the production engineer or management. In order to determine whether a process meets the specification of the process engineer or management, process capability indices (PCIs) are often used. This study is aimed at evaluating the capability and the percentage of nonconforming unit of a manufacturing process. Data on two quality characteristics were extracted from the records of Evans Medical PLC, Agbara, Nigeria. The process capability indices were used to determine the capability of the process and the percent non-conforming was used to check the amount of the process that does not meet the required standard. The obtained results reflect the amount of the products produced during the period of study that do not conform to specification and the gain of testing for the normality assumption as reflected in the parts per million defects for the process data. The confidence interval derived for the process indices can be used to improve the capability of the process.

Keywords: Process, Stability, Capability Index, Defect Parts Per Million (PPM), Percent Non-Conforming

1   Introduction

The challenge in today's competitive markets is to be on the leading edge of producing high quality products at minimum costs. A process is a unique combination of machines, materials, methods, and people engaged in producing a measurable output (Oakland, 2002). The Process Capability is a measurable property of a process to the specification. The output of this measurement can be used to predict how many parts will be produced out of

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specification.   Process capability analysis together with statistical process control and design of experiments are statistical methods that have been used for decade with purpose to reduce the variability in industrial processes and products (Albing, 2006). Process capability analysis deals with how to assess the capability of a manufacturing process, where information about the process is used to improve the capability. One can determine how well the process will perform relative to product requirement or specifications. Before assessing the capability of a process, it is important that the process is stable and repeatable. To check if a process is stable, statistical process control is usually applied in order to be able to detect and eliminate assignable cause of variation and control charts are usually used in order to determine if the process is in statistical control and reveal systematic patterns in process output (Montgomery, 2008). The most frequent tools used when performing a capability analysis is some kind of process capability index.

A process capability index is a numerical summary that compares the behavior of a product or process characteristic to engineering specification (Kotz and Johnson, 1993). A process capability index is a unit-less measure that quantifies the relationship between the actual performance of the process and its specified requirements. (McCoy, 1991) described process capability indices as the common way to summarize the process performance. The most widely used process capability indices in the industry today analyze the capability of a process under the assumption that the process is stable and that the studied characteristics is normally distributed (Spiring, 1991). Process capability studies are conducted on processes which have all assignable causes eliminated, and insofar as possible, have all common causes reduced to a minimum. By definition, these conditions exist when a process is in statistical control. A large value of the index indicates the current process is capable of producing parts that, in all likelihood, will meet or exceed the customer's requirements. A capability index is convenient because it reduces complex information about the process to a single number. The indices are used to communicate how well the process has performed. For stable or predictable process, it is assumed that these indices also indicate expected future performance. Suppliers may also use capability indices for different characteristics to establish priorities for improvement activities. Similarly, the effect of a process change can be assessed by comparing capability indices calculated before and after the change. In the literature, there are many process capability indices available. These are C_p, C_{pk}, C_{pm} and C_{pmk}. However, these indices operate under the assumption of normality (Lee, 2001). Clements (1989) used a Pearson distribution curve to estimate the non-normal process capability index, Vännman and Albing (2007) proposed family of quantile based process capability indices for situation when the process data is non-normal, Czarski (2008) used what he called the exact method to estimate the quantiles proposed by Vännman and Albing (2007). Thus, process capability indices should be determined based on the distribution of the process data.

In this paper, the capability of the manufacturing processes of Cofta tablet production is considered using the most common process capability indices. The normality assumption was first tested for the quality characteristics under study, thereafter the test for the stability of the process was carried out. If the output of the characteristic is stable, then the capability was then evaluated using the indices of process capability to ascertain the amount of nonconforming product that may reach the factory floor. Furthermore, the confidence intervals for the capability indices were constructed.

2 Process Capability Indices

Process capability indices (PCIs) can be categorized under two cases. These are for situations when the distribution of the process data is normal and non-normal. In this paper, the indices considered are the process capability ratio (C_p) and C_{pk}.

2.1 Process Capability Indices For Normal Process

The C_p Index is a process capability index that indicates the process' potential performance by relating the natural process spread to the specification (tolerance) spread. It is often used during the product design phase and pilot production phase. The index simply makes a direct comparison of the process natural tolerance to the engineering requirements. Assuming the process distribution is normal and the process average is exactly centered between the engineering requirements, a C_P index of 1 would give a "capable process." However, to allow a bit of room for process drift, the generally accepted minimum value for C_P is 1.33. In general, the larger the C_P is the better (Merton, 2003). The C_p index is

$$C_p = \frac{(USL - LSL)}{6\sigma} \tag{1}$$

where, USL is the Upper Specification Limit, LSL is the Lower Specification Limit, and σ is the standard deviation of the characteristic under study. In this paper, the concept of confidence interval proposed by Kotz and Johnson (1993) is incorporated. The 100(1- α) % confidence interval for the C_p index is

$$\hat{C}_p \sqrt{\frac{\chi^2_{\alpha/2,n-1}}{n-1}} \leq C_p \leq \hat{C}_p \sqrt{\frac{\chi^2_{1-\alpha/2,n-1}}{n-1}} \tag{2}$$

where $\hat{C}_p = \dfrac{C_p}{\sqrt{\dfrac{\chi^2_{n-1}}{n-1}}}$. It should be noted that if the sample size is large enough (e.g. n > 100),

then $E\left[\hat{C}_p\right] \approx C_p$.

The C_{pk} index is the measurement of performance of a process considering both the location and the dispersion information about it. It is known as the shorter standardized distance from the center of the process to either USL or LSL (Alsup and Watson, 1993). The C_{pk} index indicates the process actual performance by accounting for a shift in the mean of the process toward either the upper or lower specification limit. It is often used during the pilot production phase and during routine production phase (Kane, 1986). A C_{pk} of at least + 1 is required, and +1.33 is preferred. Note that C_{pk} is closely related to C_P, and that the difference between C_{pk} and C_p represents the potential gain to be had from centering the process. The computation formula is

$$C_{pk} = \min\left[\frac{USL - \bar{X}}{3\sigma}, \ \frac{\bar{X} - LSL}{3\sigma}\right] \tag{3}$$

Lee (2001) presented an alternative formula for computing the C_{pk} index. The formula is

$$C_{pk} = (1-k)\frac{T}{6\sigma} \tag{4}$$

where $k = \dfrac{2*|\bar{X} - \mu|}{T}$, $\bar{X}$ and σ are the mean and standard deviation respectively,

calculated from a random sample of size n, $\mu = \dfrac{USL + LSL}{2}$ and $T = USL - LSL$

In this paper, the Lee (2001) approach was adopted for the calculation of C_{pk}. It should be noted that C_{pk} can be calculated even if only one specification limit exists or if a minimum or maximum is specified. The $C_{pk(max)}$, that is C_{pk} for Upper Specification Limit is determined using:

$$C_{pk(max)} = \frac{(USL - \bar{X})}{3\sigma} \tag{5}$$

Similarly, the C_{pk} for Lower Specification Limit is determined by:

$$C_{pk(min)} = \frac{(\bar{X} - LSL)}{3\sigma} \tag{6}$$

Bissel (1990) constructed a confidence interval for C_{pk} using Taylor's series under the assumption of normal distribution of $\hat{C}_{pk}$. The Bissel confidence interval is defined as

$$\left[\hat{C}_{pk} - Z_{(\alpha/2)}\sqrt{\frac{1}{9n} + \frac{\hat{C}_{pk}^2}{2(n-1)}}, \; \hat{C}_{pk} + Z_{(\alpha/2)}\sqrt{\frac{1}{9n} + \frac{\hat{C}_{pk}^2}{2(n-1)}}\right] \tag{7}$$

where $Z_{\alpha/2}$ is the $(\alpha/2)$ upper quantile of standard normal distribution.

2.2 Process Capability Indices For Non-Normal Process

If the process data is non-normal, calculating C_p and C_{pk} directly based on the raw data may lead to over estimation of the PCIs. One possible way to deal with such problem is to transform the data to normal distribution using data transformation such as Bob-Cox transformation. In some cases, the process data distribution is so far from normal distribution and can not be transformed to normal. Clements(1989) proposed a method based on quantile of the data. Actually these methods are suitable for any process distributions. So it is called the robust method for PCA. Pan and Wu (1997) used the S-plus computer language to simplify the calculation of non-normal process capabilities indices. The Pan and Wu (1997) formula for the common PCIs are presented below:

$$C_p = \frac{USL - LSL}{q_{0.99865} - q_{0.00135}} \tag{8}$$

and

$$C_{pk} = \min\left\{ \frac{USL - q_{0.50}}{q_{0.99865} - q_{0.50}}, \frac{q_{0.50} - LSL}{q_{0.50} - q_{0.00135}} \right\} \tag{9}$$

Peng (2010) used Vännman (1995) unified four basic PCIs to estimate PCIs. The non-normal process capability indices can be obtained using:

$$C_p(u,v) = \frac{d - u|M - m|}{3\sqrt{\left(\frac{q_{0.99865} - q_{0.00135}}{6}\right)^2 + v(M - T)^2}} \tag{10}$$

where q_α is the αth quantile of the process. i.e., $P(X < q_\alpha) = \alpha$, M is the median of the data, T is the target value, d is the half length of the specification interval, and m is the midpoint of the specification interval. That is d = (USL – LSL)/2 and m = (USL + LSL)/2.

From Equation (10), the two capability indices used in this paper can be deduced as follows:

$$C_p(u,v) = \begin{cases} C_p, & \text{if } u = v = 0 \\ C_{pk}, & \text{if } u = 1 \text{ and } v = 0 \end{cases} \tag{11}$$

It should be noted that the Pan and Wu (1997) Equations and Equation (10) are equivalent and will therefore, yield the same indices values.

2.3 Evaluating Parameters

2.3.1 Percentage Non-conforming (PNC)

The process percent nonconforming (PNC) is the long term process yield that can be expected from the process if it is allowed to operate at the capability. For process with two – sided specification limits, the percentage of non-conforming items (PNC) can be calculated as

$PNC = p(X > USL) + p(X < LSL)$

On the assumption of normality of the process, the percentage of non-conforming items can be expressed as

$$PNC = 1 - \Phi\left(\frac{USL - \mu}{\sigma}\right) + \Phi\left(\frac{LSL - \mu}{\sigma}\right) \tag{12}$$

Where $\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution and μ and σ are the process mean and standard deviation, respectively. Pearn and Kotz (2006) noted that if the process is perfectly centered at the specification range($\mu = m$), then the PNC can be expressed as $2\Phi(-3C_p)$ and when $\mu \neq m$, it only provides a lower bound on

PNC. Thus, The process percent nonconforming and process yield for the Cp index is defined as:

$$PPN_{C_p} = [2(1 - \Phi(3C_p))]100 \tag{13}$$

The process percent nonconforming and process yield for the C_{pk} index is defined as:

$$PPN_{Cpk} = [\Phi(-3C_{pk}) + \Phi(-3(2C_p - C_{pk})]100 \tag{14}$$

It should be noted that the percent nonconforming for C_p is twice the percent nonconforming for C_{pk} (Albing, 2006)

2.3.2 Defects parts per million (ppm)

The defect parts per million (ppm) is obtained directly from the percentage non-conforming defined by:

ppm = 10000* PNC

For example, if C_p = 1.00, the corresponding PNC = 0.27 which gives 2700 parts per million.

3   Main Results

Data from a manufacturing company that produces Cofta tablet is used in this paper. Cofta tablet is produced with many quality characteristics among them are weight, thickness, Ammonium chloride content and the hardness of the tablet. Two out of these important characteristics of Cofta tablet were considered in this paper. The two characteristics are the weight and thickness of the tablet. The specification limits for the weight is 950 − 1050 (mg) while the specification limits for the thickness is 3.57 − 3.97 (mm). The Data were collected from the record book of the company for the period of 30 months with the sample size of 10 per month. Thus, there are 300 data points for each of the characteristic under study.

3.1 Weight of Tablet

The specification limits for this variable are USL = 1050 and LSL = 950.   The data was first tested for normality. The histogram of the data is well spread and approximately normal. However, the cdf compare with the normal distribution shows a wide variation with the distribution of the data. This is an indication that the data is non-normal. Further analysis using the Kolmogorov-Smirnov Goodness-of-Fit Test and the Shapiro-Wilk test was conducted. The Kolmogorov-Smirnov Goodness-of-Fit Test gives the following results: KS = 0.143 and p = 0.000. The Shapiro-Wilk test gives W = 0.932 with p = 0.000. Therefore, the weight process data can be adjudged to be non- normal. To test for stability of the process data, the $\bar{X}$ and S control charts was used. From the data $\bar{\bar{X}}$ =995.29 , $\bar{S}$ =11.6461 and the sample standard deviation (S) = 12.9426. For   n =10, A_3 = 0.975, B_4 = 1.716, and B_3 = 0.284 are read off from SQC table (see e.g. Montgomery, 2010).   The plot of the control charts for $\bar{X}$ and S charts shows that no points fall outside the control limits. Therefore, the process of the weight of Cofta tablet is stable.

Since the population standard deviation is unknown, then the samples estimate of the population standard deviation (S) equals 12.9426 was used. Thus using Equation (1), the C_p index is obtained to be:

$$C_p = \frac{1050 - 950}{6(12.9426)} = \frac{100}{77.6556} = 1.29$$

To determine the C_{pk} index, we know that USL = 1050 and LSL = 950, then $\mu = \frac{1050 + 950}{2} = 1000$, $T = 1050 - 950 = 100$ and $k = \frac{|995.29 - 1000|}{50} = 0.0942$.

Substituting these values into Equation (4), we have:

$$C_{pk} = (1 - 0.0942)\frac{100}{6(12.9426)} = 1.17$$

Using Equations (13) and (14) respectively, the percent nonconforming units are 0.011and 0.024 for C_p and C_{pk}. To determine the confidence intervals for the computed indices, Equations (2) and (7) were used respectively for C_p and C_{pk} for a situation when the data is assumed normal. Using the obtained values of Cp = 1.29 and Cpk = 1.17, the confidence intervals are $1.19 \leq C_p \leq 1.39$ for the C_p index and $1.07 \leq C_{pk} \leq 1.27$ for the C_{pk} index

However, the data was confirmed to be non-normal, therefore, the above indices values can be misleading. Thus, using Equations (10) and (11), the non-normal based PCIs for the data were computed. From the data $q_{0.99865}$ = 1025, $q_{0.0135}$ = 953, and $q_{0.50}$ = 998. Substituting these values into Equations (10) and (11) we have

$$C_p = \frac{1050 - 950}{1025 - 953} = 1.39$$

and

$$C_{pk} = \min\left[\frac{1050 - 998}{1025 - 998}, \frac{998 - 950}{998 - 958}\right] = \min[1.93, 1.20] = 1.20$$

Using Equations (13) and (14), the percent non-conforming unit are -0.003 and 0.016 respectively, for C_p and C_{pk} with confidence intervals $1.28 \leq C_p \leq 1.50$ for the C_p index and $1.10 \leq C_{pk} \leq 1.30$ for the C_{pk} index.

3.2 Thickness of Tablet

The specification limits of the thickness quality characteristic are USL = 3.97 and LSL = 3.57. Therefore,

$$\mu = \frac{3.97 + 3.57}{2} = 3.77, \quad T = 3.97 - 3.57 = 0.4 \text{ and } \quad k = \frac{|3.799 - 3.77|}{0.4} = 0.1495.$$

The data was first tested for normality using the Kolmogorov-Smirnov Goodness-of-Fit Test and the Shapiro-Wilk test. The results of the Kolmogorov-Smirnov Goodness-of-Fit Test is KS = 0.1814 with

p = 0.000. Also the Shapiro–Wilk test gives W = 0.90 with p = 0.000. Thus, the thickness characteristic data is adjudged to be non-normal. From the data, $\bar{\bar{X}} = 3.799, \bar{S} = 0.0164,$ and S = 0.1319. The plot of $\bar{X}$ and S control charts indicate that no points fall outside the control limits, and no evidence of systematic change. This is an indication that the process is stable. Hence we proceed to determine the capability of the characteristic process.

The C_p index using Equation (1) is $C_p = \dfrac{3.97 - 3.57}{(6 \times 0.1319)} = 0.5054 \gg 0.51,$ and the C_{pk}

index using Equation (4) is $C_{pk} = (1 - 0.1495)\dfrac{0.4}{6(0.1319)} = 0.4299 \gg 0.43$

The percent nonconforming products using the above computed indices are 12.60 and 13.69 for C_p and C_{pk}, respectively. Also the confidence intervals for the computed indices are $0.47 \leq C_p \leq 0.55$ for the C_p index and $0.38 \leq C_{pk} \leq 0.48$ for the C_{pk} index

Since the data is tested to be non-normal then the above computation can lead to over estimation of the index. Using Equations (10) and (11), the non-normal based PCIs for the data were determined to be

$$C_p = \frac{3.97 - 3.57}{3.98 - 3.57} = 0.98$$

and

$$C_{pk} = \min\left[\frac{3.97 - 3.84}{3.98 - 3.84}, \frac{3.84 - 3.57}{3.84 - 3.57}\right] = \min[0.93, 1.0] = 0.93$$

The percent nonconforming products using the above computed indices are 0.33 and 0.36 for C_p and C_{pk}, respectively. Also the confidence intervals for the computed indices are $0.90 \leq C_p \leq 1.06$ for the C_p index and $0.85 \leq C_{pk} \leq 1.01$ for the C_{pk} index

4 Discussion of Result

The summary of the capability indices, the percent non-conforming products, the part per million that are non-conforming and the length of the intervals for the two characteristics under study are presented in Table 1.

Table 1: Summary of Capability Indices for Weight and Thickness

Characteristic	Normal based PCIs				Non-normal based PCIs			
	Index	PNC	ppm	Limit Interval	Index	PNC	ppm	Limit Interval
Weight	$C_p = 1.29$ $C_{pk} = 1.17$	0.011 0.024	110 240	0.22 0.18	$C_p = 1.39$ $C_{pk} = 1.20$	0.003 0.016	30 160	0.2 0.2
Thickness	$C_p = 0.51$ $C_{pk} = 0.43$	12.60 13.69	126000 136900	0.16 0.16	$C_p = 0.98$ $C_{pk} = 0.93$	0.33 0.36	3300 3600	0.08 0.10

The process capability indices for the weight of Cofta tablet are very close under the two conditions that were used in this study as reflected in Table 1. This may be due to the fact that the distribution of the data is not far from normal distribution. However, the parts per million defects for the non-normal based indices show an appreciable gain over the normal based indices. The results for the thickness shows that the process for the thickness is not capable and not well centered since the C_p and C_{pk} are not equal. Also the non-normal based indices show appreciable gain in the parts per million defects.

The confidence interval for the C_p and C_{pk} were also determined for the purpose of future management of the weight and thickness of the tablet process. However, the obtained Cp and Cpk for the thickness are not capable of producing according to the specifications. Hence, the confidence intervals obtained based on the computed values can not be used for future management.

5   Conclusion

The two quality characteristics of Cofta tablet considered in this study reflect that the process were not capable and not well centered. The appreciable gain obtained by using the non-normal based capability indices is a reflection of the importance of testing the data first for the normality assumption. If the normal based capability indices had been used to study the capability of the process, wrong and invalid inference would have been made and therefore a wrong conclusion would have been drawn and thus faulty signal of higher parts per million defects will be reported. Based on the obtained results, it will be recommended that the company should adjust the specification limits for the weight and more importantly for the thickness of the tablets in order to increase the tolerance band which will lead to the improvement of the precision of the process.

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