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Single Factor Interest Rate Models in Inflation Targeting Economies of Emerging Asia

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Abstract

Based on two single factor interest rate models of Vasicek and Cox Ingersoll Ross, the divergence in speed of mean reversion and the accompanying instantaneous volatility effect in inflation targeting economies of Emerging Asia are evident. A higher degree of undershoot and overshoot risk of inflation target, for economies such as Indonesia and the Philippines has been identified. The effectiveness of monetary policy erodes as it departs from the objective of central banks and financial regulators. In an environment of weak mean reversion of interest rates that is subject to external shocks, distortion in financial market related interest rate could be severe for Indonesia and the Philippines.

Mathematics Subject Classification: C18, C58
Keywords: Vasicek and Cox Ingersoll Ross Interest Rate Models, Speed of Interest Rate Mean Reversion, Instantaneous Volatility, Inflation Targeting, Emerging Asia

1 Introduction

The difference between price level and inflation rate targeting is best illustrated by considering how central banks respond when the inflation target is missed (see Ito and Hayashi, 2004)[3]. In a price level targeting framework, central banks and financial

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[3]Ito. T and Hayashi. T (2004), ‘Inflation Targeting in Asia’. HKIMR Occasional Paper No.1, March. Key findings on inflation target economies of South Korea, Indonesia, Thailand and the Philippines indicate central bank independence and the design of target range and horizon would promote more credible framework. The findings also show that introduction of inflation targeting contributed to economic recovery in these four Asian countries, partly by stabilizing inflation expectations and increasing the effectiveness of monetary policy. The authors note a numerical target, instead of a vague concept of price stability makes the goal of monetary policy clear and it

regulators set a target of *k* per-cent growth in the level of prices each year. If the target undershoots in a period, central banks and financial regulators would need to engineer an increase in prices above the *k* per-cent target in the following year to make up the gap and achieve the target. Even though price level targeting offers a purer form of long-run price stability, it involves fine-tuning of the inflation rate in an attempt to compensate for past policy mistakes.

By contrast, in an inflation targeting framework, central banks and financial regulators need not necessarily respond to the undershoot in the inflation target but instead seek only to try to meet the inflation target in the following period. Inflation targeting consists of two approaches, a framework for making policy choices and a strategy for communicating the context and rationale of the policy choices to the broader public[4].

In the first approach of inflation targeting, policy framework of inflation targeting involves constrained discretion that attempts to achieve a balance between the inflexibility of strict policy rules and the potential lack of discipline and structure that is inherent in the policy maker's discretion. In conditions of constrained discretion, the central bank is free to do its best to stabilize output and unemployment when short run shocks occur in the economy. In conditions where information imperfection of the economy and effects of policy exist, policy discretion is applied. Conducting policy discretion within a constrained discretion framework involves the central bank having to maintain a strong commitment of keeping inflation firmly under control since monetary policy influences inflation with a lag. In keeping inflation under control, it would require the central bank to anticipate future movements in inflation and move in a pre-emptive fashion. The policy discretion within a constrained discretion is essentially a forward looking policy approach of monetary policy.

In the second approach of inflation targeting, communications strategy involves the central bank having regular communication with political authorities, financial markets and the general public. Aspects of communication that have been particularly emphasized by inflation targeting central banks are the public announcement of policy objectives, open discussion of the central bank's policy framework and public release of the central bank's forecast or evaluation of the economy. Effective communication policies are considered as a useful way to make the financial market a partner in the policy making process. By explaining the central bank's overall approach, clarification of its plans, objectives and providing assessment of economy, the central bank would be able to manage and stabilize financial market expectations.

Economies such as South Korea, Indonesia, Thailand and the Philippines adopt inflation targeting to manage financial market expectations. Taguchi and Kato (2011) find that by having a well-functioning inflation targeting framework, it is consistent with enhanced monetary autonomy under a floating exchange rate regime[5]. Viewed from an

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is easier to hold the central bank accountable for its decisions.

[4]Bernanke S.B (2003), Remarks made at the Annual Washington Policy Conference of the National Associations of Business Economists, Washington D.C, March.

[5]Taguchi, H and Kato, C. (2011), 'Assessing the performance of inflation targeting in East Asian economies'. *Asia-Pacific Economic Literature*, 25(1), pp 93-102. The study compares the money-inflation relationship under different monetary regimes, the inflation targeting regime with floating exchange rate in the post-1997/98 crisis period and pegged exchange rate regime without inflation targeting in the pre-1997/98 crisis period. The methodology used is co-integration testing

accountability perspective, inflation targeting framework is set based on consultation between the central bank and the government. In the case of South Korea, Bank of Korea (BoK) officials have to appear periodically at parliamentary hearings to face questions about the conduct and success or failure of monetary policy. In Thailand and the Philippines, the Bank of Thailand (BoT) and Bank Sentral Ng Philippines (BSP) officials need to write an open letter to the government explaining their actions when the inflation target is missed (see Table 1).

This research evaluates two significant components in inflation targeting economies of Emerging Asia, the speed of interest rate mean reversion and effects of instantaneous volatility. A slow speed of mean reversion of interest rates will subject inflation targeting an objective that is difficult to be achieved with persistence in either overshooting or undershooting the inflation target. In tandem with slow speed of mean reversion of interest rates, a rise in instantaneous volatility will further exacerbate the overshoot and undershoot of inflation target.

2 Preliminary Notes

Definition 2.1 *The modelling of interest rate markets of inflation targeting economies in Emerging Asia uses two single factor mean reversion interest rate models, which is the Vasicek and Cox Ingersoll Ross model*.

While advanced interest rate models[6] have been applied for derivatives pricing and gauging impact of monetary policy in financial markets, the appropriateness of using the single factor mean reversion interest rate model in this study suits with the assumption that inflation targeting Emerging Asia economies interest rate markets are at a developing

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for identifying the existence of a long-run relationship between money and inflation and estimating an error-correction model for examining the price adjustment speed against money supply.

[6]See James and Webber (2000) who list models as being the following types:

- The traditional one, two and multifactor equilibrium models known as affine term structure models. These include Gaussian affine models such as Vasicek and Hull-White and Steeley, where the model describes a process with constant volatility and models that have square root volatility such as Cox - Ingersoll - Ross. These models use constant parameters including a constant volatility, the actual parameters are calculated from actual data and implied volatilities which are obtained from exchange traded option contracts.
- Whole yield curve models include the Heath - Jarrow –Morton model.
- Interest Rate Market models include the Jamshidian model.
- Consol models include the Brennan and Schwartz.model.
- No arbitrage models fit precisely with the observed term structure of the yield curve, therefore the observed bond yields are in fact equal to the bond yields calculated by the model. The arbitrage free model is intended to be consistent with the currently observed zero coupon yield curve and the short rate drift rate being dependent on time because the future average path taken by the short rate is determined by the shape of the initial yield curve.

Some of these models include,

- The Ho –Lee model.
- The Hull-White model.
- The Black-Derman-Toy model.
- The Black – Karasinki model.

stage. The initial step is by identifying the period of study which is from 2nd June 2008 to 30th September 2011[7], consisting three years and three months. The policy interest rates for inflation targeting Emerging Asia economies are identified at the beginning of the study period[8] while the underlying instrument that is used in this analysis is the 3-month by 3-month forward starting interest rate swaps. The sensitivity of this financial instrument in measuring monetary policy changes, the short duration of its tenor and features that incorporate financial market expectations are factors that impart valuable information to agents in the financial market place. Forward starting swaps are different when compared to interbank offered rates which have a daily fixing feature that is averaged by central banks and financial regulators. Interbank offered interest rates do not reflect financial market expectations accurately. In the case of forward starting swaps, the interest rate equilibrium is determined by agents in the interest rate market, thus giving it a financial market expectations flavour that is sensitive to changes in monetary policy expectation.

The second step involves estimating the mean of short term interest rates that is used as the underlying instrument in both the single factor mean reversion interest rate models, identifying the speed of mean reversion and the instantaneous volatility. The next step in modelling interest rate markets of inflation targeting economies in Emerging Asia is to calibrate the single factor mean reversion interest rate models for the estimation of coefficients in the model.

The Vasicek model is represented as:

$$d\acute{r}_t = \alpha(\beta - \acute{r}_t)d_t + \sigma dw_{t,} \qquad (1)$$

Where α is speed of mean reversion, β is the long term mean of 3-month by 3-month forward starting swaps, σdw_t is the instantaneous volatility with a Weiner process, $\acute{r}$ is the spot interest rate of 3-month by 3-month forward starting swaps and $\alpha(\beta - \acute{r}_t)d_t$ is the drift term.

The Cox Ingersoll Ross model is represented as:

$$d\acute{r}_t = \alpha(\beta - \acute{r}_t)d_t + \sigma\sqrt{\acute{r}_t}dw_{t,} \qquad (2)$$

The coefficients in the Cox Ingersoll Ross model are the same as with the Vasicek model with the exception of the instantaneous volatility which is $\sigma\sqrt{\acute{r}_t}dw_t$ .In calibrating the single factor mean reversion interest rate models for estimation of coefficients, this exercise uses a stochastic differential equation in the framework of Ornstein-Uhlenbeck process. The process involves computing the following equation

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[7]The period of study is from 2nd June 2008 to 30th September 2011. The time period takes into account of the GFC of 2008 which at its peak witnessed the collapse of investment bank Lehman Brothers on 15th September 2008 when it filed for Chapter 11 bankruptcy.

[8]Initial policy rate as of 2nd June 2008 –South Korea Overnight call rate of 5.0%, Indonesia BI Reference rate of 8.25%, Thailand BOT policy rate of 3.25%, Philippines BSP overnight repo rate of 7.0%.

$$r_{i+1} = r_i e^{-\alpha\delta} + \beta\left(1 - e^{-\alpha\delta}\right) + \sigma\sqrt{\frac{1-e^{-2\alpha\delta}}{2\alpha}} \tag{3}$$

Where equation (3) shows that the interest rates in the period ahead r_{i+1} is determined by actual interest rates at period r_i with respect to a mathematical constant of *e* approximately equal to 2.71828. This is adjusted to the speed of mean reversion parameter and the fixed time step of δ. The fixed time step is measured as a single trading day divided by the 252 trading days in a year which is 0.00397. The mean of the 3-month by 3-month forward starting swap, β is adjusted by the difference against the mathematical constant of 2.71828 that is powered to the mean reversion coefficient and the time step. The stochastic differential equation takes into account of instantaneous volatility in the framework of the mathematical constant, the speed of mean reversion and the fixed time step. The relationship between r_i and r_{i+1} is linear with a normal random error of ϵ allowing for the estimation of the following equation

$$r_{i+1} = \acute{b} + \acute{a} r_i + \epsilon \tag{4}$$

From equation (4), interest rates in the period ahead r_{i+1} is determined by actual interest rates at period r_i, the constant $\acute{b}$ and a normal random error of ϵ .Equation (4) is estimated using a linear square method and the random error of ϵ is tested for stability using the Breusch-Godfrey serial correlation LM test.

The residual from equations (4) is tested for heteroscedasticity to identify if estimated variance of the residuals is dependent on the values of the independent variables using the Breusch-Pagan-Godfrey test. Once it has been identified that there is no evidence of serial correlation and the equation specification is stable, the coefficients from this equation are calibrated into the Ornstein-Uhlenbeck process, where:

$\acute{\alpha} = e^{-\alpha\delta}$, $\acute{b} = \beta(1 - e^{-\alpha\delta})$ and standard deviation of $\epsilon = \sigma\sqrt{\frac{1-e^{-2\alpha\delta}}{2\alpha}}$

Rewriting this gives,

$$\alpha = \frac{-ln\acute{a}}{\delta}, \tag{5}$$

$$\beta = \frac{\acute{b}}{1-\acute{a}} \tag{6}$$

$$\text{and } \sigma = std.dev\,(\epsilon)\sqrt{\frac{-2ln\acute{a}}{\delta(1-\acute{a}^2)}} \tag{7}$$

The final step in modelling of interest rate markets of inflation targeting economies in Emerging Asia is calibrating the Ornstein-Uhlenbeck process by using the coefficients that were estimated from equation (4) to find the values of α, β and σ for both the single factor mean reversion interest rate models of Vasicek and Cox Ingersoll Ross. This is obtained from rewriting

$$\alpha = \frac{-ln\acute{a}}{\delta}, \beta = \frac{\acute{b}}{1-\acute{a}} \text{ and, } \sigma = std.dev\,(\epsilon)\sqrt{\frac{-2ln\acute{a}}{\delta(1-\acute{a}^2)}}.$$

3 Main Results

3.1 Serial Correlation and Heteroscedasticity Test

Estimated $\acute{\alpha}$ coefficient of equation (4) reflects financial market related interest rates r_i influences interest rate behavior in a forward manner r_{i+1}, which is consistent with the behaviour of agents in the financial market taking into account of current behaviour of interest rates in shaping their forward expectation of interest rates.

Initial estimates of equation (4) shows strong evidence of serial correlation with the error term being correlated and the presence of heteroskedasticity, where the error term has a different variance. In both instances the ordinary least squares assumption were violated, rendering equation (4) as not correctly specified and unstable. Omission of relevant explanatory variables may have been a factor, however in modelling imperfections in interest rate markets, it is evident that the behaviour of interest rate markets are subject to amplitude of randomness gaining admission into the interest rate market, and these randomness is notable during the period of study which include the GFC of 2008.

Rectifying serial correlation presence was done by taking into account of lagged periods of the residuals in the Breusch – Godfrey test. The lagged periods differed for each interest rate markets in inflation targeting economies of Emerging Asia. In the case of eliminating heteroskedasticity, the Breusch – Pagan – Godfrey test increased the number of regressors against a second moment residual.

Following the rectification of serial correlation and heteroskedasticty, the stability of the equation improved based on chi-square (x^2) values for the Breusch-Godfrey serial correlation LM test show no evidence of serial correlation in the residuals, indicating non rejection of the null hypothesis of no serial correlation, and variance estimates of the residuals showing no sign of heteroscedasticity implying variance of the residual as constant (see Table 2).

3.2 Mean Reversion of Interest Rates in Inflation Targeting Economies of Emerging Asia

Estimates of the α coefficient (see Table 3) indicate the speed of mean reversion in single factor interest rate models for Indonesia and the Philippines as weak. Weak mean reversion of interest rates towards the long term mean β suggests high probability of agents in financial markets failing to interpret monetary policy signalling efficiently and financial market related interest rate unable to achieve equilibrium. In such cases the effectiveness of monetary policy erodes as it departs from the objective of central banks and financial regulators. In an environment of weak mean reversion of interest rates that is subject to external shocks, distortion in financial market related interest rate could be severe. Measures undertaken by central banks and financial regulators to correct this distortion using monetary policy could instead destabilize financial markets.

3.3 Instantaneous Volatility

The instantaneous volatility component in Vasicek and Cox Ingersoll Ross model measures the instant by instant amplitude of randomness gaining admission into interest rate markets. Though elevation of instantaneous volatility was observed in both interest rate models, the elevation was more in the Cox Ingersoll Ross model compared to the Vasicek model in all four inflation targeting economies of Emerging Asia interest rates markets (see Table 4).Increased randomness penetrating the interest rate markets of Indonesia and the Philippines is due to the weak monetary policy signalling effect which dilutes information flow from central banks to agents in the financial market, compared to South Korea and Thailand (see Figure 1 for Implied model of Vasicek and Cox Ingersoll interest rates for inflation targeting economies of Emerging Asia).

4 Figures and Tables

Table 1: Accountability Framework of Inflation Targeting Economies in Emerging Asia

Inflation Targeting countries	**Target set by**	**Target Horizon**	**Open Letter**	**Parliament Hearings**
S.Korea	Government and Central Bank	3 years	No	Yes
Indonesia	Government and Central Bank	Medium term	No	No
Thailand	Government and Central Bank	8 quarters	Yes	No
The Philippines	Government and Central Bank	2012-2014	Yes	No

Source: Hammond (2012), HSBC Asian Central Bank Guide 2012

Table 2: Estimated Coefficients of $\acute{\alpha}$, $\acute{b}$ and ε

	$\acute{\alpha}^{*}$	$\acute{b}$	ε^{**}	p-values ($\chi^2(1)$)	p-values ($\chi^2(2)$)
South Korea	1.0009	-0.0012	0.0490	0.1855	0.0020
Indonesia	0.9303	0.5647	0.8363	0.0591	0.0801
Thailand	0.9962	0.0098	0.1000	0.0776	0.0524
The Philippines	0.9544	0.1876	0.3731	0.0844	0.1561

Source: Authors calculation.

Notes : $\acute{\alpha}$, $\acute{b}$ and ε are based on equation (4) where $r_{i+1} = \acute{\alpha} r_i + \acute{b} + \varepsilon$. , where $\acute{\alpha} = e^{-\alpha\delta}$, $\acute{b} = \beta(1 - e^{-\alpha\delta})$ and

$\varepsilon = \sigma\sqrt{\frac{1-e^{-2\alpha\delta}}{2\alpha}}$ *Significant at 1% and 5% *t*- stat critical values.

** To estimate ε , the standard error of regression is used.

p-values ($\chi^2(1)$) for Breusch-Godfrey serial correlation LM test. The χ^2 estimates for Breusch-Godfrey serial correlation LM test are significant at critical values of 5%.

p-values ($\chi^2(2)$) for Breusch-Pagan-Godfrey heteroscedasticity test. The χ^2 estimates for Breusch-Pagan-Godfrey heteroscedasticity test are significant at critical values of 5%.

Table 3: Estimated Coefficients of Vasicek and Cox Ingersoll Ross Interest Rate Models after calibrated into the Ornstein-Uhlenbeck process

	α	β	σdw_t	$\sigma\sqrt{\acute{r}_t}dw_t$
S.Korea	2.1482	3.7732	2.2073	9.8715
Indonesia	18.1790	8.1106	13.8026	48.0524
Thailand	0.9446	2.6259	1.5810	8.7698
The Philippines	11.7393	4.1204	6.0901	23.0184

Source: Authors calculation.

Notes: Estimated coefficients of equation (1) and (2). Vasicek model, where $d\acute{r}_t = \alpha(\beta - \acute{r}_t)d_t + \sigma dw_t$, and

CIR where $d\acute{r}_t = \alpha(\beta - \acute{r}_t)d_t + \sigma\sqrt{\acute{r}_t}dw_t$.

The coefficients estimated above are in % terms, where $\alpha = \frac{-ln\acute{a}}{\delta}$, $\beta = \frac{b}{1-\acute{a}}$ and $\sigma = std.dev\ (\varepsilon)\ \sqrt{\frac{-2ln\acute{a}}{\delta(1-\acute{a}^2)}}$.

The coefficients of α and β are the same for both models with the difference being the instantaneous volatility. The speed of mean reversion and instantaneous volatility are coefficients that provide inference on behaviour of agents in the interest rate market.

Table 4: Instantaneous Volatility in Vasicek and Cox Ingersoll Ross Model

	σdw_t	$\sigma\sqrt{\acute{r}_t}dw_t$	Relative intensity of instantaneous volatility between Vasicek and CIR model
S.Korea	2.2073	9.8715	7.6642
Indonesia	13.8026	48.0524	34.2498
Thailand	1.5810	8.7698	7.1888
The Philippines	6.0901	23.0184	16.9283

Source: Authors calculation.

Notes: The parameters estimated above are in % terms. Relative intensity is measured as the difference between $\sigma\sqrt{\acute{r}_t}dw_t$ and σdw_t

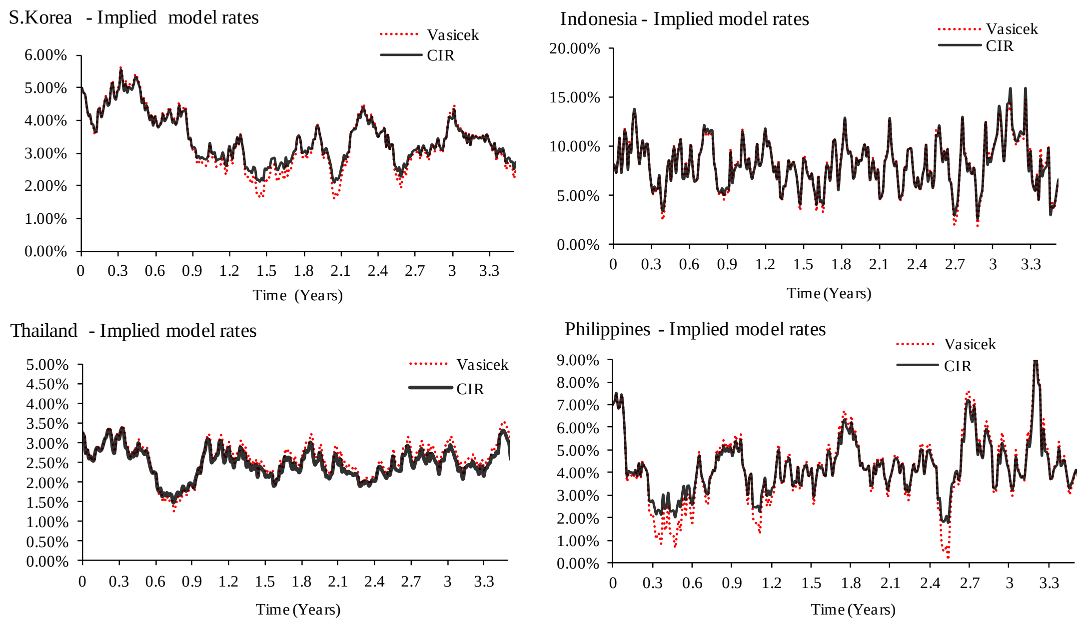

Figure 1: Implied Vasicek and Cox Ingersoll Ross Interest Rates in Inflation Targeting Economies of Emerging Asia

Source: Authors computation.

5 Conclusion

Based on two single factor interest rate models of Vasicek and Cox Ingersoll Ross, the divergence in speed of mean reversion and the accompanying instantaneous volatility effect in inflation targeting economies of Emerging Asia are obvious. A higher degree of undershoot and overshoot risk of inflation target for economies such as Indonesia and Philippines has been identified. The mechanism of monetary policy transmission in both these economies is still at an infancy stage compared to South Korea and Thailand. While there are external factors that may have contributed to the slow speed of mean reversion of interest rates in Indonesia and Philippines, the internal factors based on a simple domestic underlying instrument such as forward starting swaps suggest monetary policy transmission in both Indonesia and Philippines will need further fine turning, consistent with respective inflation targets.

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