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Reliability modelling for wear out failure period of a single unit system

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Abstract

The present paper deals with two time-shifted density models for wear out failure period of a single unit system. The study, considered the time-shifted Gamma and Normal distributions. Wear out failures occur as a result of deterioration processes or mechanical wear and its probability of occurrence increases with time. A failure rate as a function of time deceases in an early failure period and it increases in wear out period. Failure rates for time shifted distributions and expression for mean time to system are also obtained. Finally, the graphically representation of all the measures of reliability are shown. So we can say from the study that in the wear out period the reliability of the system increases and failure rates of the system decreases.

Mathematics Subject Classification: 90B25

Keywords: Wear out period, Single-unit system, Time-shifted, Rayleigh and Gamma density functions

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1 Introduction

Reliability is an important consideration in planning, design and operation of systems. Reliability is a body of ideas, mathematical models and methods directed toward the solution of problems in predicting, estimating, or optimizing the probability of survival, mean life, or more generally, life distribution of components or systems. Other Problems considered in reliability theory are those involving the probability of proper functioning of the system at either a specified or arbitrary time, or the proportional of time the system functioning properly. There are different measures of reliability which help the system to repair and functioning properly.

Govil and Aggarwal proposed a time-shifted Rayleigh density for wear out failures. Bazovsky have evaluated the wear out failures rates for normal and lognormal distributions. Wear out failures occur as a result of deterioration processes or mechanical wear and its probability of occurrence increases with time. Failure rate for time shifted distributions and expressions for mean time to system failure are obtained. For different values of the parameters, the curves for average lifetimes are also drawn.

2 Models

2.1 Model I

Let us consider, the time shifted Gamma wear out failure density given by,

$$f_W(T) = \frac{e^{(-T+2T_W)}(T-T_W)^{\lambda-1}}{\Gamma(\lambda)}; \quad T \ge T_W\,,\ \ \lambda > 0 \tag{1}$$

(1)

where λ is the parameter.

Let $R_W(T)$ be the reliability function at time T in wear out periods (T,∞), then

$$R_W(T) = \int_T^\infty f_W(t)dt = \frac{1}{\Gamma(\lambda)}[\int_0^\infty e^{-(t-T_W)}(t-T_W)^{\lambda-1}dt - \int_0^T e^{-(t-T_W)}(t-T_W)^{\lambda-1}dt]$$

$$R_W(T) = \frac{e^{T_W}(-T_W)^{\lambda-1}}{\Gamma(\lambda)}[\frac{\Gamma(\lambda)}{(1)^\lambda} - \frac{(T_W)^{\lambda-1}e^{-T}}{T} - \int_0^T t^{\lambda-1}e^{-t}dt] \ ; \quad T \ge T_W \tag{2}$$

Hence these two equations give failure density and reliability as a function of T. The wear out failure rate $\lambda_W(T)$ as a function of T is giving by,

$$\lambda_W(T) = \frac{f_W(T)}{R_W(T)} = \frac{\Gamma(\lambda)^2}{e^{(-T+2T_W)}(T-T_W)^{\lambda-1}} \times \frac{1}{A} \tag{3}$$

where

$$A = \frac{\Gamma(\lambda)}{(1)^{\lambda}} - \frac{(T_W)^{\lambda-1} e^{-T}}{T} - \int_0^T t^{\lambda-1} e^{-t} dt \,; \quad T \geq T_W$$

This predicts a linearly increasing failure rate of the component after its useful life period. If we make a prior assumption that a component has survived its useful life period. The mean time to failure ($\mathrm{MTTF_w}$) due to wear out period will be,

$$\mathrm{MTTF_W} = \int_0^T T \cdot f_W(T) dT = \frac{(T_W)^{\lambda-1} e^{T_W}}{\Gamma(\lambda)} [\int_0^{\infty} e^{-T} T^{\lambda} dT - \frac{\Gamma(\lambda+1)}{(1)^{\lambda+1}}] \tag{4}$$

Now, we determine the reliability for an operating time (t) given T, i.e. $R_W(T,t)$,

$$R_W(T,t) = \frac{\int_{T+t}^{\infty} f_W(T) dT}{\int_T^{\infty} f_W(T) dT}$$

where

$$\int_{T+t}^{\infty} f_W(T) dT = \int_0^{\infty} \frac{(T-T_W)^{\lambda-1}}{\Gamma(\lambda)} e^{-(T-T_W)} dT - \int_0^{T+t} \frac{(T-T_W)^{\lambda-1}}{\Gamma(\lambda)} e^{-(T-T_W)} dT$$

$$\int_T^{\infty} f_W(T) dT = \int_0^{\infty} \frac{(T-T_W)^{\lambda-1}}{\Gamma(\lambda)} e^{-(T-T_W)} dT - \int_0^{T+t} \frac{(T-T_W)^{\lambda-1}}{\Gamma(\lambda)} e^{-(T-T_W)} dT$$

$$R_W(T_W,t) = \frac{[\Gamma(\lambda)/(1)^{\lambda}] - \int_0^{T+t} T^{\lambda-1} e^{-T} dT}{[\Gamma(\lambda)/(1)^{\lambda}] - \int_0^{T} T^{\lambda-1} e^{-T} dT} \tag{5}$$

At the wear out failure period T_W,

$$R_W(T_W,t) = \frac{\int_0^{T+t} (T_W)^{\lambda-1} e^{-T_W} dT_W - [\Gamma(\lambda)/(1)^{\lambda}]}{\int_0^{T} (T_W)^{\lambda-1} e^{-T_W} dT_W - [\Gamma(\lambda)/(1)^{\lambda}]}$$

The following Table illustrates the calculation of the reliability, failure rate and $\mathrm{MTTF_W}$ from above equations. Here, we consider some arbitrary values of T_W, T, t, and λ and use Gauss – Lauguarre method is used to evaluate the values $R_W(T)$, $\lambda_W(T)$, $\mathrm{MTTF_W}$ and $R_W(T_W,t)$.

2.2 Model II

Let us consider, the time shifted normal wear out failure density given by,

$$f_W(T) = \frac{1}{\sigma\sqrt{2\pi}}\exp[\frac{-(T-T_W-\mu)^2}{2\sigma^2}]\,;\ \ \sigma > 0\,;\ \ -\infty < \mu < +\infty\,;\ \ T \ge T_W \tag{6}$$

Where μ, σ are the parameters.

Let $R_W(T)$ be the reliability function at time T in wear out period (T, ∞), then

$$R_W(T) = \int_T^{\infty} f_W(t)dt = \int_T^{\infty} \frac{1}{\sigma\sqrt{2\pi}}\exp[\frac{-(t-T_W-\mu)^2}{2\sigma^2}]dt$$

$$= \frac{\sigma}{(t-T_W-\mu)}\exp[\frac{-(T-T_W-\mu)^2}{2\sigma^2}] \tag{7}$$

when $T \ge T_W$.

Hence from these two equations we have the failure density and reliability as a function of T. The wear out failure rate $\lambda_W(T)$ as a function of T is given by,

$$\lambda_W(T) = \frac{f_W(T)}{R_W(T)} = \frac{T-T_W-\mu}{\sigma^2\sqrt{2\pi}} \tag{8}$$

This predicts a linearly fluctuations failure rates of the component after its useful life period. If we make a prior assumption that a component has survived its useful life period, the mean time to failure ($\mathrm{MTTF_w}$) due to wear out period will be,

$$\mathrm{MTTF_W} = \int_0^{\infty} T \cdot f_W(T)dT = \frac{T\sigma}{\sqrt{2\pi}(T-T_W-\mu)}\exp[\frac{T_W+\mu}{2\sigma^2}] \tag{9}$$

Now we can determine the reliability for an operating time (t) given T, i.e. $R_W(T,t)$,

$$R_W(T,t) = \frac{\int_{T+t}^{\infty} f_W(T)dT}{\int_T^{\infty} f_W(T)dT}$$

where

$$\int_{T+t}^{\infty} f_W(T)dT = \frac{\sigma}{\sqrt{2\pi}(T-T_W-\mu)}\exp[\frac{-(T-T_W+t-\mu)^2}{2\sigma^2}]$$

$$\int_{Tt}^{\infty} f_W(T)dT = \frac{\sigma}{\sqrt{2\pi}(T-T_W-\mu)}\exp[\frac{-(T-T_W-\mu)^2}{2\sigma^2}]$$

$$R_W(T,t) = \exp[\frac{t}{2\sigma^2}(t+2T_W-2T+2\mu)]$$

At the wear out failure period T_W,

$$R_W(T,t) = \exp[\frac{t}{2\sigma^2}(t+2\mu)] \tag{10}$$

3 Main Results

3.1 For Model I

Table 1: Reliability, Failure rates and Mean time to failure ($MTTF_w$) at the wear out period

$T = t = 10000$ hrs; $\lambda = 10$					
T_W (wear out periods)	2000	3000	5000	10000	14000
$R_W(T)$ (Reliability in wear out periods)	0.8168	0.5431	0.4321	0.1216	0.1133
$\lambda_W(T)$ (failure rates)	1.3162	4.0218	6.3167	7.0026	9.3123
$R_W(T_W, t)$	0.7116	0.5444	0.3226	0.2161	0.1243
$MTTF_w$	2131.2	2932.6	4561.6	9428.8	12363.2

3.2 For Model II

The following table illustrates the calculation of the reliability, failure rate and $MTTF_w$. Here, we considered some arbitrary assumed values of T_W, T, t, μ and σ. Values of $R_W(T)$, $\lambda_W(T)$, ($MTTF_w$) and $R_W(T_W, t)$ are evaluated from the above equations.

Table 2: Reliability, Failure rates and Mean time to failure ($MTTF_w$) and $R_W(T_W, t)$ at the wear out period

For Normal Distribution				
$T_W \downarrow$	$\mu = 5.0$; $T = 4.0$ days ; $t = 2.0$; $\sigma = 5.0$			
	$R_W(T)$	$\lambda_W(T)$	$MTTF_w$	$R_W(T_W, t)$
0.5	0.7187	4.5016	0.4621	0.8697
1.0	0.6316	3.2316	0.9312	0.8022
1.5	0.5444	2.3134	1.1216	0.7124
2.0	0.4321	2.0133	1.8326	0.7012
2.5	0.4116	1.1666	2.1200	0.6148
3.0	0.3200	1.0016	2.8217	0.4325
3.5	0.2162	1.3421	3.2112	0.3216
4.0	0.1189	1.2622	3.9316	0.2803
4.5	0.1011	1.0821	3.1216	0.2015

4 Graphical Representations

4.1 For Model I

Curves plotted between the different values of T_W and $R_W(T)$, $\lambda_W(T)$, ($MTTF_w$) and $R_W(T_W, t)$, as given below,

Figure 1: Reliability in wear out period

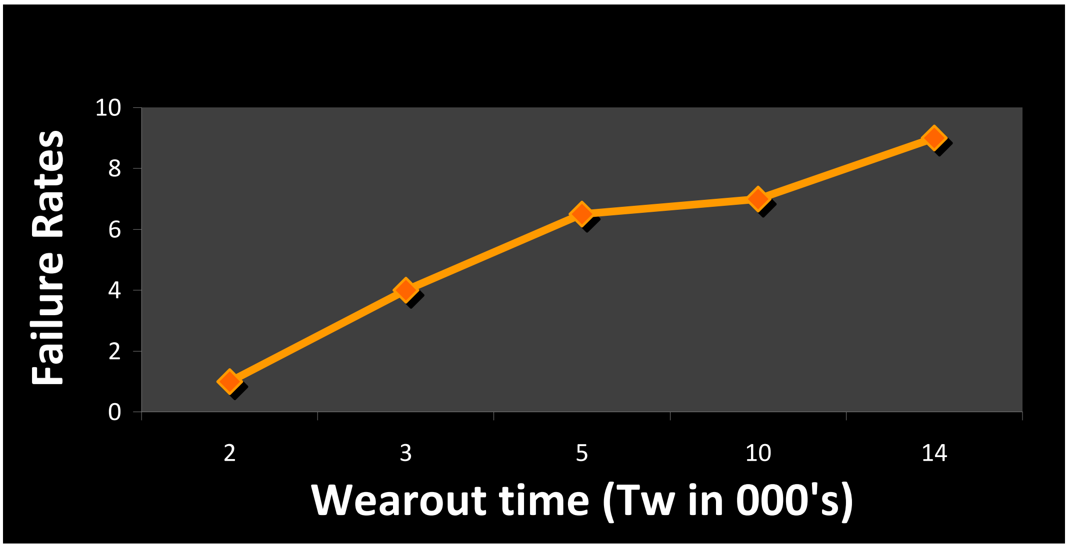

Figure 2: Failure rates in wear out period

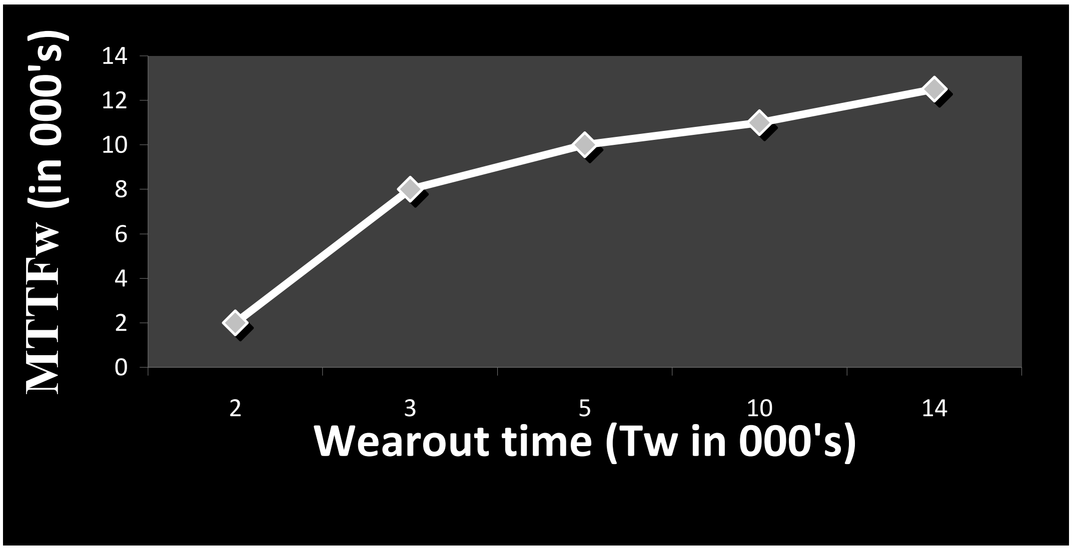

Figure 3: Mean time to failure in wear out period

4.2 For Model II

Curves plotted between the different values of wear out periods and the failure rates in the wear out periods.

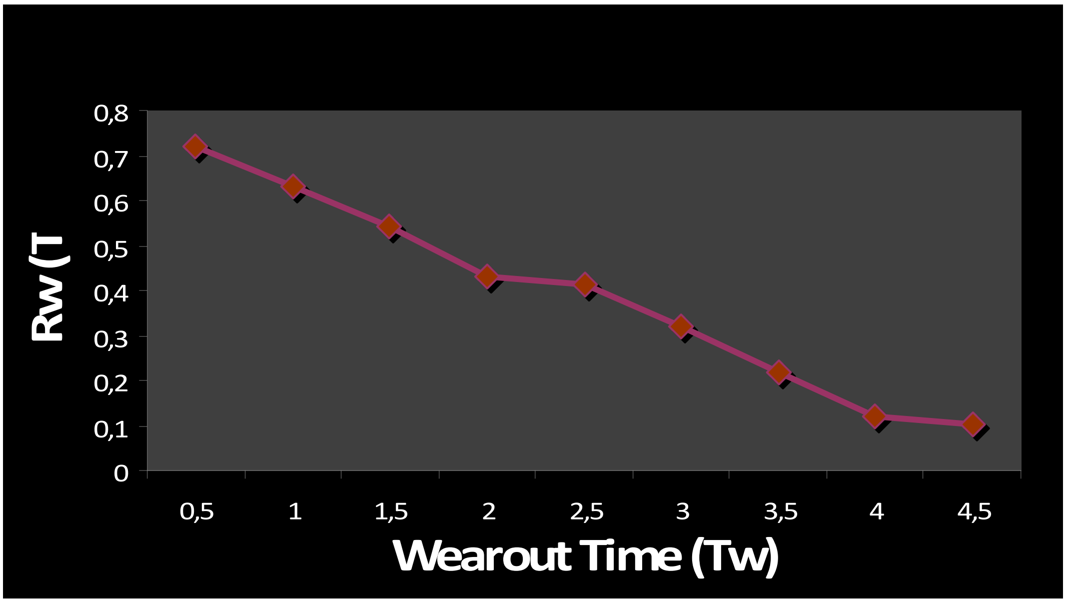

Figure 4: Reliability in wear out period

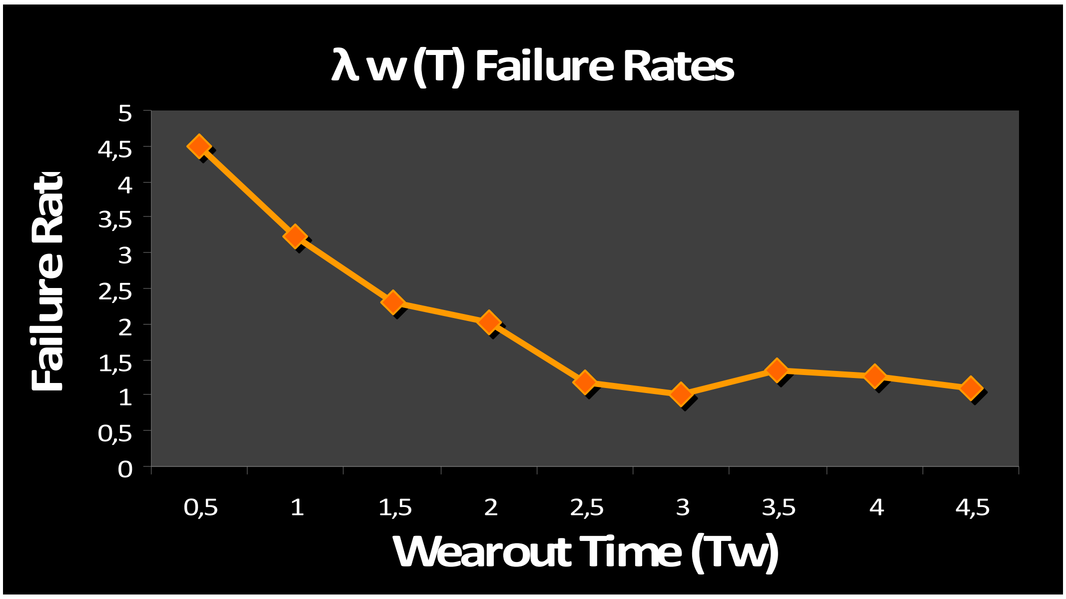

Figure 5: Failure rates in wear out period

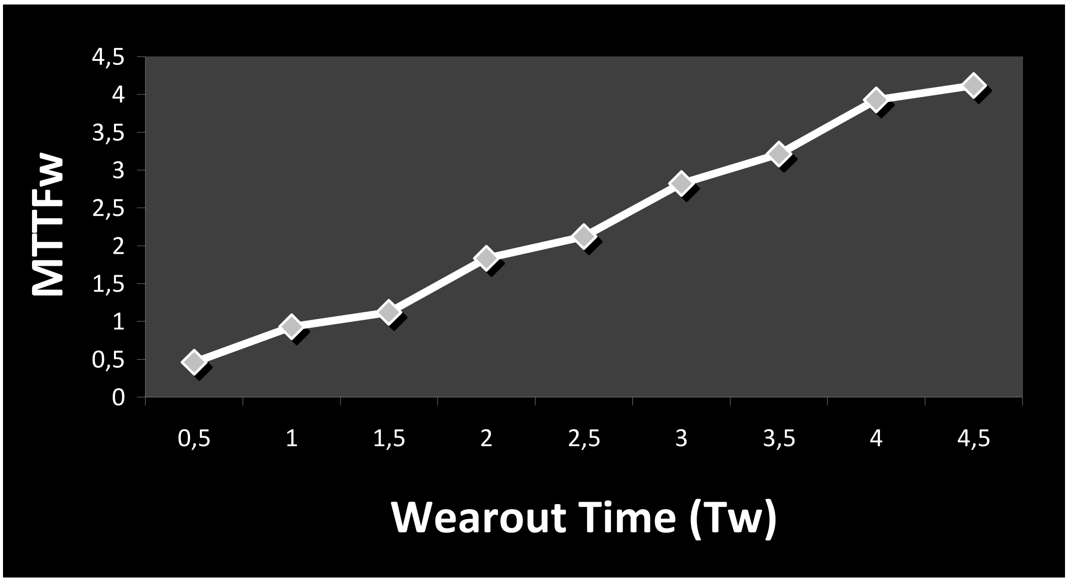

Figure 6: Mean time to failure in wear out period

5 Conclusions

For **Model I:** From the figures, we perceive that,

1) We get the maximum reliability 0.8168 at wear out period 2000 hrs and minimum reliability 0.1133 at wear out period 14000hrs. As the wear out period increase reliability in the wear out period also decreases.
2) We get the maximum failure rate 9.3123 at wear out period 14000 hrs and minimum failure rate 1.3126 at wear out period 2000 hrs. As the wear out time an increase failure rates also decreases.
3) We get the maximum mean time to failure 0.7116 at wear out period 2000 hrs and minimum mean time to failure 0.1243 at wear out period 14000 hrs. As the wear out time increases the mean time to failure decreases.

For **Model II** : From the above charts, it is perceived that as the wear out period increases the reliability in wear out period and failure rates decreases, mean time to failure in the wear out period increases and the reliability between wear out period and operating period decreases.

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