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Environmental Regulation Modeled as a Public Input Proportional to Capital Investment

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Abstract

Public inputs, when referred to as unpaid factors, are included in production specifications characterized by constant returns to scale in all inputs. In equilibrium, the public input rent generated by this configuration must be allocated among firms based on some measure of productive activity. Herein, decentralized environmental regulation is modeled as a public input of the unpaid type. Environmental rent is then rationed to private firms in proportion to their capital investment. Results reveal that source based capital taxation will capture these environmental rents, however, will not yield efficiency in the devolved interjurisdictional framework. The capital tax must be complemented by benefit taxation in order to provide efficient local public expenditures and socially optimal levels of environmental quality.

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1. Introduction

Public inputs, such as transportation networks and general public infrastructure, provide enhancing potential for production of most private goods. In the relevant public finance literature, production function frameworks where output is a function of private inputs (e.g., labor (L) and capital (K)) and public inputs (e.g., roads and public safety) appear as convention. Several public input specifications are common and differ by specific characteristics such as rivalry, congestion, firm aggregation, scale economies and price exclusion (Feehan 1989). Generally two, more broad, specifications that focus on scale economies are prevalent. Pure or *factor-augmenting* structures exhibit constant returns to scale in private inputs only. Proportional scaling occurs between output and private inputs without need for scaling up public inputs. Secondly, public inputs referred to as *unpaid factors* are included in configurations characterized by constant returns to scale in all inputs of production. In this case, an increase in private inputs requires the same proportional increase in public inputs.

In their originating work that focuses on efficiency of decentralized environmental regulation, Oates and Schwab (1988) treat polluting waste emissions (E) as a non-purchased (unpaid) input to production. They go on to assume that local authorities set a physical quantity of a concentration of pollutants allowed within the jurisdiction. In effect, the local authority determines the sum total of polluting waste emissions for the entire community. Modeling environmental policy this way allows for emissions to be captured in a single variable that enters both production and utility functions. In production, allowed emissions enhance output − regarding utility, emissions are viewed as a public bad.

Oates and Schwab (1988) argue that the command and control determined emissions aggregate must be allocated among firms based on some measure of productive activity. They suggest that a firm's allowed emissions are proportional to its labor force. Modeled as constant returns to scale, production is defined, using Euler's theorem,

$$Q = F(L, K, E) = F_L L + F_K K + F_E E \,, \tag{1}$$

where subscripts denote positive marginal product partial derivatives. Dividing through by the labor input yields,

$$\frac{F}{L} = F_L + F_K \frac{K}{L} + F_E \frac{E}{L} \,, \tag{2}$$

where the far right term represents additional output stemming from allowed emissions per labor input. Oates and Schwab (1988) allow this emissions rent to be captured by jurisdictional residents through their income-consumption constraint,[2]

[2] Equivalent to equation (4) p. 338, Oates and Schwab (1988).

$$c = \frac{F}{L} - r\frac{K}{L} + y\,, \tag{3}$$

where r is the net-of-tax return to capital and y represents any exogenous income.
In this paper, mobile capital replaces labor as the measure of productive activity. Specifically, allowed emissions are proportional to firms' employment of capital. This proposed rationing rule is not trivial, in a somewhat related treatment Oates and Schwab (1991) regard 'unpaid' public input provision as a direct in-kind subsidy to capital investment. By doing so they demonstrate that efficient public input provision is achieved by taxing public input rents away via a source-based capital tax. Will similar efficiency outcomes carry over to the 'unpaid' emissions input specification proposed herein?
In the next section, the augmented model is fully developed and optimal conditions are derived. Section 3 lays out two key propositions. Propositions reveal that source based capital taxation alone will not yield efficiency in the augmented model. The capital tax must be complemented by benefit taxation in order to provide efficient local public expenditures and socially optimal levels of environmental quality. Section 4 concludes and provides future research avenues.

2. The Augmented Model

The economy is made up of a large number of symmetric jurisdictions. Competitive polluting firms in each jurisdiction produce a numeraire output. There are two primary, private factors of production − an immobile factor, L, and mobile capital, K. A jurisdiction's fixed supply of the immobile factor is owned entirely by the jurisdiction's residents. Capital is perfectly mobile both within and across jurisdictions, but fixed in supply ($\overline{K}$) in the broader economy. New capital formation is not considered as the model concentrates on location choices of the existing capital stock. Capital moves between jurisdictions until the net-of-tax return to capital, r, is equalized throughout. Competitive firms and 'small' jurisdictions view r as parametric. Jurisdictions are 'small' enough in the sense that their policy dealings have no influence on price, yet large enough that firms' pollution externalities are purely localized.[3]
In addition to the primary factors of production, pollution emissions, E, enter production as an 'unpaid factor' (Oates and Schwab 1988). Local governments use a command and control strategy when setting environmental standards. Authorities specify the aggregate level (E) of allowed pollution concentration within the jurisdiction. As described in the introduction, including pollution emissions as a factor of production is akin to the treatment of public goods as inputs enhancing the production of private goods (Feehan 1989; Oates and Schwab 1991).

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[3] The latter externality fits Oates (2002) 'Benchmark Case 2: Local Public Goods".

Constant-returns-to-scale *jurisdictional* production is denoted as $F(L, K, E)$. All marginal products are positive and diminish. Negative definiteness of the function also requires,

$$F_{LL}F_{KK} - F_{KL}^2 > 0\,, \tag{4}$$

and,

$$2F_{KL}F_{KE}F_{LE} - F_{LL}F_{KE}^2 - F_{KK}F_{LE}^2 + F_{EE}\left(F_{LL}F_{KK} - F_{KL}^2\right) < 0\,. \tag{5}$$

Subscripts as before denote partial derivatives. The first term in equation (5) is ambiguous in sign yet would support the inequality constraint if negative. Of the three cross-partials in the term, assuming that the fixed factor and emissions are technical substitutes appears reasonable ($F_{LE} < 0$). Capital-fixed factor and capital-emissions are presumed technical complements. For functions continuous in the partials, Young's theorem defines $F_{LK} = F_{KL}$, $F_{KE} = F_{EK}$, etc.
Repeating equation (1), production follows,

$$F(L, K, E) = F_L L + F_K K + F_E E\,, \tag{6}$$

where differentiating equation (6) with respect to $\theta = \{L, K, E\}$ yields,

$$LF_{L\theta} + KF_{K\theta} + EF_{E\theta} = 0\,. \tag{7}$$

As discussed above, treating emissions as an unpaid factor to production generates rents, $F_E E$. Somewhat related to Oates and Schwab (1991), these rents are rationed to firms in proportion to their capital investment.[4] This rationing framework implies that in addition to the marginal product of capital, investment creates an additional return,

$$R = \frac{F_E E}{K}\,, \tag{8}$$

where the profit-maximizing condition for capital investment becomes,

$$r = F_K + R - t\,. \tag{9}$$

The variable t denotes a sourced-based unit tax on capital. The following derivatives of equation (8) are noteworthy,

[4] See Kunce and Shogren (2005) for a model that rations these rents directly to perfectly mobile firms.

$$R_K = \frac{\partial R}{\partial K} = \frac{KF_{KE}E - F_E E}{K^2}, \tag{10}$$

$$R_E = \frac{F_E + F_{EE}E}{K}. \tag{11}$$

When a jurisdiction's capital-emissions complementarity is sufficiently strong, the comparison expressed in equation (10) could be positive, yet a negative relationship seems more natural and accepted. The numerator of equation (11) reflects how total emission rents change with changes in allowed emission levels, *E*. Intuitively, a positive relationship is reasonable, pollution rents increase when a jurisdiction relaxes regulatory standards (higher *E*).

Expenditures on public goods are financed by taxing capital. Public goods may be interpreted as publicly provided private goods or Samuelsonian public goods where each unit produced is consumed jointly by all residents of a jurisdiction (Wilson 1986). The public budget constraint becomes,

$$G = tK. \tag{12}$$

Each symmetric jurisdiction consists of residents identical in preferences and ownership share of the fixed factor. Jurisdictional residents' income consists of returns to the fixed factor and any *exogenous* income, *y*, that includes any jurisdictional returns from capital ownership. When using equations (6), (8) and (9) jurisdictional income-consumption is equal to,

$$X = F(L, K, E) - (r + t)K + y. \tag{13}$$

Residents of a jurisdiction receive utility from consumption and local public goods, but suffer disutility from the level of allowed pollution emissions. Jurisdictional utility takes the form, $U(X,G,E)$, where U_X and $U_G > 0$, but $U_E < 0$. Higher *E* corresponds to poorer environmental quality where *E* represents a pure public bad. In keeping with the Arrow-Debreu (Wilson 1999) separation assumption for general equilibrium constructs, residents have two distinct roles in the model. First, as consumers, they seek to maximize utility over a bundle of goods and public services. Second, supplying fixed factor inputs to production and in return receiving income for consumption. More of the mobile factor enhances local production and can provide residents with higher incomes hence more consumption. However, in order to attract the mobile factor, the jurisdiction lowers taxes (effecting the provision of *G*) and/or relaxes environmental regulations (lowering utility directly) thus setting up a characteristic economic tradeoff.

Benchmark social efficiency requires the maximization of the jurisdictional residents' utility subject to (i) utility in all other jurisdictions is equalized to a fixed level, (ii) aggregate production and consumption clear, and (iii) the mobile capital stock is allocated entirely among jurisdictions (clears). The resulting social optimum conditions from the standard model are well known (see Oates and Schwab 1988; Wilson 1999) therefore derivation discussion here is keep to a minimum. Social efficiency becomes,

$$MRS(G,X) = \frac{U_G}{U_X} = 1 \quad \forall \text{ jurisdictions}, \tag{14}$$

$$MRS(E,X) = \frac{-U_E}{U_X} = F_E \quad \forall \text{ jurisdictions}, \tag{15}$$

Equation (14) represents the familiar 'Samuelson condition' for the provision of public goods (Wilson 1986). This optimality condition suggests that the jurisdictional marginal rate of substitution ($MRS(G,X)$) between the public good and consumption equals the marginal cost of providing an incremental increase in the public good. Given equations (12) and (13), the marginal rate of transformation in this context is one for one. Equation (15) shows that jurisdictions should choose a combination of environmental quality and consumption such that the marginal rate of substitution between the two equals the marginal product of emissions (recall that $U_E < 0$). Equation (15) then represents a Samuelson rule for environmental quality, if you are so inclined (Kunce and Shogren 2005).
Jurisdictional authority, acting as a benevolent dictator, maximizes jurisdictional utility subject to constraint equations (9), (12) and (13) forming the Lagrangean,[5]

$$\begin{aligned} \text{Max}_{X,G,E,K,t} \quad & U(X,G,E) + \lambda_1[F_K + R - t - r] + \lambda_2[tK - G] + \\ & \lambda_3[F(L,K,E) - (r+t)K + y - X]. \end{aligned} \tag{16}$$

First-order-conditions become,

$$X: \quad U_X - \lambda_3 = 0, \tag{17}$$

$$G: \quad U_G - \lambda_2 = 0, \tag{18}$$

$$E: \quad U_E + \lambda_1(F_{KE} + R_E) + \lambda_3 F_E = 0, \tag{19}$$

$$K: \quad \lambda_1(F_{KK} + R_K) + t\lambda_2 + \lambda_3(F_K - r - t) = 0, \tag{20}$$

[5] A reviewer of this paper wanted to see two relevant comparative statics − how mobile capital is effected by changes in t and E. See the appendix to this paper for an alternative derivation using total differentials that adapts the use of these mobile capital comparisons (equations (A6) and (A7)).

$$t: \quad -\lambda_1 + \lambda_2 K - \lambda_3 K = 0, \tag{21}$$

where details for the partial derivatives R_K and R_E are found in equations (10) and (11). Solving equations (17), (18) and (21) for the Lagrange multipliers yields,

$$\lambda_1 = K(\lambda_2 - \lambda_3), \tag{22}$$

$$\lambda_2 = U_G, \tag{23}$$

$$\lambda_3 = U_X. \tag{24}$$

Note that Lagrange multipliers measure the sensitivity of the optimally valued utility function to changes in the constraints. Binding equality constraints require non-zero multipliers. Multiplier λ_1 is associated with the price for (returns to) capital, λ_2 measures the effect of changes in public expenditures, lastly, λ_3 reflects changes in income/consumption. The right-hand-side of equation (22) reflects that local public goods will not be efficiently provided. Recall the social optimum for public goods provision requires $U_G = U_C$ from equation (14). The multiplier, λ_1, is interpreted as the marginal utility of the capital price when residents' utility is maximized. Necessitating an interior solution, equations (19) and (20) reinforce a 'positive' marginal utility of the capital price, hence, public goods will be underprovided ($U_G / U_C > 1$).
Substituting equations (9) and (22) through (24) into equations (19) and (20) facilitates the reduction of the first-order-conditions to two,

$$E: \quad U_E + K(U_G - U_X)(F_{KE} + R_E) + U_X F_E = 0, \tag{25}$$

$$K: \quad tU_G + K(U_G - U_X)(F_{KK} + R_K) - U_X R = 0. \tag{26}$$

Solving equations (25) and (26) simultaneously using equations (10) and (11) yields the optimal conditions of interest, with suitable rearrangement,

$$MRS(E,X) = F_E \cdot MRS(G,X) + (KF_{KE} + EF_{EE})(MRS(G,X) - 1), \tag{27}$$

$$t = R + (KF_{KK} + EF_{KE})\left(\frac{1}{MRS(G,X)} - 1\right), \tag{28}$$

or when using equation (7) first with $\theta = E$, then $\theta = K$,

$$MRS(E,X) = F_E \cdot MRS(G,X) - LF_{LE}(MRS(G,X) - 1), \tag{29}$$

$$t = R - LF_{KL}\left(\frac{1}{MRS(G,X)} - 1\right). \tag{30}$$

3. Propositions

Proposition 1. (a) *A meaningful* (*interior*) *solution requires* (*t*) *and therefore* (*G*) *to be positive. Assuming capital−fixed factor technical complementarity and using the equation (22) result that capital taxation under provides local public goods, the tax rate* (*t*) *is unambiguously positive.* (b) *Assuming that the fixed factor and emissions are technical substitutes, jurisdictions will set environmental standards below the social optimum.*

The standard argument in the literature, originating with Wilson (1986) and Zodrow and Mieszkowski (1986), is that capital tax financing of local public goods leads to distorting competition for mobile capital resulting in the under-provision of local public services. Because jurisdictions finance a unit increase in public goods with an increase in the capital tax, capital will flee in response to the tax increase. Therefore, the marginal cost of a unit increase in the public good includes not only the direct resource cost but also the loss in tax revenues due to capital flight. The loss of local tax revenue is not viewed as a social cost because other jurisdictions realize a fiscal benefit from the inflow of capital. The cost of local public goods is then overestimated by the jurisdiction which will choose an inefficiently low level of public goods (Wilson 1999).
An interior solution to the Lagrangean requires G to be positive, no corner solutions. Imagine a two-good world with G on the horizontal axis. Optimality requires a tangency where the marginal rate of subsitution, $MRS(G,X)$, equals the marginal rate of transformation, $\mathrm{d}X/\mathrm{d}G$. Under-provision of G is an intersection moving to the left of tangency on the horizontal axis. This intersection occurs where $MRS(G,X) > \mathrm{d}X/\mathrm{d}G$, or $MRS(G,X) > 1$ herein. Under-provision of G forces the bracketed portion of equation (30) negative. Assuming $F_{KL} > 0$, the tax rate t is unambiguously positive resulting in G being positive. Given that the jurisdiction chooses a positive capital tax rate and public goods are under-provided because the capital tax alone is not an effective revenue source, what level of environmental quality should be set in order to maximize jurisdictional utility? Equation (29) shows that the marginal rate of substitution between allowed emissions and consumption exceeds the marginal cost F_E when $MRS(G,X) > 1$ and $F_{LE} < 0$. Consequently, jurisdictions will allow pollution emissions beyond the socially efficient level. Since more lax environmental measures lower abatement costs and increase emission rents, there is an incentive for local authorities to lower standards in order to lure mobile capital.

Proposition 2. *If local taxes become 'benefit taxes' − public goods provision is not distorted, environmental standards are set efficiently, and the tax on capital is equal to the emissions rent.*

Oates and Schwab (1991) further assume that jurisdictions have access to 'benefit taxes' that allow more degrees of freedom when setting the tax on capital. Local taxes become benefit taxes and the provision of public inputs and local public services is efficient. The tax on capital equals the value of increased production (rent) from a marginal increase in the public input to production while a tax on workers efficiently provides local public goods. Results herein are somewhat related − removing the public goods distortion leads to efficient environmental quality choices and requires that the rent on emissions is taxed away, $t = R$. However, taxing the emissions rent is not enough to finance efficient levels of local public expenditures. In our case, as long as the capital tax rate is set to capture emissions rent, there must be an additional non-distorting tax in order to provide local public goods. Without such a tax, higher capital taxes would be required, $t > R$, resulting in a deviation from the efficiency forwarded in Proposition 2. In order to illustrate this point, solve equations (29) and (30) simultaneously by eliminating the $MRS(G,X)$ term yielding,

$$MRS(E,X) = \frac{-LF_{KL}(F_E - LF_{LE})}{-LF_{KL} + (t-R)} + LF_{LE}\,. \tag{31}$$

If t does not equal R, efficiency will not be achieved in environmental quality reinforcing the need for an additional non-distorting tax instrument.

4. Concluding Remarks

The examination presented herein reveals that source based capital taxation alone will not yield efficiency in the augmented model. The capital tax must be complemented by a benefit tax in order to provide efficient local public expenditures and socially optimal levels of environmental quality. Efficiency requires that rent-seeking private inputs to production are taxed to remove the rent accruing to them, but taxes must be applied to the fixed factor as well as mobile capital.

Concerning future research, as described above firms treat E as exogenous generating an amount of emissions rent that must clear in equilibrium. Therefore, at a given wage w and return to capital r, firms will hire the two private factors in a cost minimizing manner. Since the production function is linearly homogenous, the capital/labor ratio will be determined solely by the wage/capital-return ratio. The level of E set by local authorities influences the levels of factors employed but not the cost minimizing capital/labor ratio. Consequently, a rationing scheme based on income shares results. The share of total income accruing to labor becomes, $\mu = wL / (wL + rK)$, with $(1 - \mu)$ going to capital. Hence, the contribution to output from allowed emissions, $F_E E$, can be allocated based on these shares. Potential efficiency outcomes from this rationing scheme warrant further exploration.

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Appendix

The alternative derivation relies on the total differential method. First, we need to redefine the consumption constraint, equation (13) in the text. By substituting text equation (12) into equation (13), the jurisdictional residents' total income-consumption can now be defined,

$$X = F(L, K, E) - rK - G + y. \tag{A1}$$

Jurisdictional authorities choose over a policy variable vector $\Omega = \{t, E\}$ that maximizes jurisdictional utility subject to text equation (12) and equation (A1). First order conditions become,

$$\frac{U_G}{U_X}\frac{\partial G}{\partial \Omega} + \frac{U_E}{U_X}\frac{\partial E}{\partial \Omega} + \frac{\partial X}{\partial \Omega} = 0, \tag{A2}$$

where $MRS(E,X) = -U_E/U_X$ and $MRS(G,X) = U_G/U_X$, continuing,

$$\frac{\partial G}{\partial \Omega} = t\frac{\partial K}{\partial \Omega} + K\frac{\partial t}{\partial \Omega}, \tag{A3}$$

noting that L is fixed, and using text equation (9),

$$\frac{\partial X}{\partial \Omega} = (t - R)\frac{\partial K}{\partial \Omega} + F_E\frac{\partial E}{\partial \Omega} - \frac{\partial G}{\partial \Omega}. \tag{A4}$$

Text equation (9) provides the necessary system required to determine K as an implicit function of each policy variable in Ω. This system yields the relevant comparisons needed to complete and interpret the total differential first order conditions. Defining the implicit function,

$$I_1: \;\; F_K + R - t - r = 0, \tag{A5}$$

we can now derive,

$$\frac{\partial K}{\partial t} = \frac{-\partial I_1/\partial t}{\partial I_1/\partial K} = \frac{1}{F_{KK} + R_K}, \tag{A6}$$

$$\frac{\partial K}{\partial E} = \frac{-\partial I_1/\partial E}{\partial I_1/\partial K} = \frac{-(F_{KE} + R_E)}{F_{KK} + R_K}, \tag{A7}$$

where the terms R_K and R_E are defined in text equations (10) and (11). Using the accepted and intuitive assumptions from the text, equations (A6) and (A7) show that capital is deflected by higher taxes and lower levels of E denoting stricter environmental standards.

Substituting equations (A3), (A4), (A6), (A7) and text equations (10) and (11) into equations (A2) yields more complete first order conditions,

$$\Omega = t: \quad \left(\frac{U_G}{U_X} - 1\right)\left(\frac{t}{F_{KK} + \dfrac{KF_{KE}E - F_E E}{K^2}} + K\right) + (t - R)\left(\frac{1}{F_{KK} + \dfrac{KF_{KE}E - F_E E}{K^2}}\right) = 0, \tag{A8}$$

$$\Omega = E: \quad \frac{U_E}{U_X} + F_E + \left(\frac{U_G}{U_X} - 1\right)\left(\frac{-t\left(F_{KE} + \dfrac{1}{K}(F_E + F_{EE}E)\right)}{F_{KK} + \dfrac{KF_{KE}E - F_E E}{K^2}}\right)$$
$$+ (t - R)\left(\frac{-\left(F_{KE} + \dfrac{1}{K}(F_E + F_{EE}E)\right)}{F_{KK} + \dfrac{KF_{KE}E - F_E E}{K^2}}\right) = 0. \tag{A9}$$

Solving equations (A8) and (A9) simultaneously yields the optimal conditions,

$$\frac{-U_E}{U_X} = F_E \frac{U_G}{U_X} + (KF_{KE} + EF_{EE})\left(\frac{U_G}{U_X} - 1\right), \tag{A10}$$

$$t = R + (KF_{KK} + EF_{KE})\left(\frac{1}{U_G/U_X} - 1\right), \tag{A11}$$

which are equivalent to equations (27) and (28) in the text.