The dynamic relationship between stock index and exchange rate: Evidence for Tunis

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Abstract

The analysis of time varying correlation between stock index and exchange rates in the context of international investments was been well researched in the literature in last few years. In this paper, we study the interdependence of stock index and exchange rate for the Tunis during the global financial crisis. Hence, we approve a DCC-FIAPARCH model to study the dynamic conditional correlation, throughout the period spanning from January 1, 2006 until January 1, 2017. The empirical results recommend asymmetric responses in correlations between the stock index and exchange rate from Tunisia. Moreover, the results indicate an increase of exchange rates and stock index correlations through the crisis periods, telling the different currencies vulnerability. Finally, we find some significant decreases in the estimated dynamic correlations, indicating the existence of a “currency contagion effect” during turmoil periods.

JEL classification numbers: G32, G33, C22, C53, G15.
Keywords: volatility, DCC-FIAPARCH, Global financial crisis, exchange rates, stock index and currency contagion.

1 Introduction

The modeling of the volatility of financial returns has become a subject that is the subject of the most research in the field of financial econometrics (Sadorsky (2012)). The importance of volatility modeling comes in large part from the fact that it agrees to survey the uncertainty of the performance of a given asset such as an exchange rate or a stock market index, for example. Likewise, the fact that the index of these assets are very variable can not be neglected, especially when we invest our money. Nowadays, any investor is aware of the fluctuations that introduce an element of risk into his portfolio. This is why investors want to choose the degree of 'exposure' to risk consistent with their level of tolerance to the latter. Thus, knowledge of the volatility of financial returns plays a key role in the valuation and hedging of any investment, especially when it is risky. Figlewski (1997) considers volatility a synonym for risk, he thinks that high volatility means a disruption of the market. It can be measured by using the variance of index returns that reflect the loss or gain over a given

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period of time. Tsay (2005) indicates that a highly volatile market implies that index fluctuate wildly. Thus, the modeling of the dynamics of volatility makes it possible to take into account its properties, such as the grouping of volatility, long memory and asymmetry effect.

The present study provides a robust analysis of dynamic linkages between exchange rate and stock index from Tunisia that goes beyond a simple analysis of correlation breakdowns. The time-varying DCC are captured from a multivariate FIAPARCH-DCC model which takes into account long memory behavior, market information speed. The rest of the paper is organized as follows. First part presents the data and the econometric methodology. Second part displays and discusses the empirical findings and their interpretation, while final part provides our conclusion.

2 Review of empirical work

Over the past decade, a growing number of studies have emerged in an attempt to model asymmetric long-volatility volatility in exchange rates and stock index such as the work of Abdalla and Murinde (1997), Ajayi, Friedman and Mehdian (1998), Granger, Huang and Yang (2000), Smyth and Nandha (2003) Phylaktis and Ravazzolo (2005), Kodongo and Ojah (2012). Aydin (2002), for example, examined the performance behavior of the Istanbul stock market index using the ARCH model (1,1) and the author found that there volatility are clusters negative asymmetry, large flattening, and autocorrelation in time series data. Baillie and Bollerslev (2000) have pointed out that the integrated fractional GARCH model, noted by FIEGARCH, can adequately describe the volatility of DEM / USD. Degiannakis (2004) used the FIAPARCH model to examine the characteristics of conditional volatility in three European stock markets. The author has documented that ARCH processes generate more accurate forecasts of stock index volatility. Akgün and Sayyan (2005) examined the asymmetric response of Istanbul stock market yields using asymmetric conditional heteroskedasticity models (EGARCH, GJR, APARCH, FIEGARCH, FIAPARCH) for the period 04 January 2000 through 25 May 2005. Their results show that the APARCH and FIAPARCH models provide more accurate volatility forecasts. Kang and Yoon (2006) used the FIEGARCH model to capture the effect of asymmetry and long-term memory in the volatility of stock market returns from South Korea, Japan, Hong Kong, and Singapore. Their study indicated the presence of long memory and the asymmetry effect in the sample that was used. Kang et al. (2009) also examine the volatility predictive power of GARCH models. They show the superiority of FIGARCH in modeling and predicting the persistence of volatility. Kasman and Torun (2009) investigate the long-term memory of stock market returns in eight central and eastern European countries. Their results provide evidence of the presence of long memory for the markets considered. More recently, Kang, Cheong, and Yoon (2010) test whether long memory is present in Chinese stock market returns, and these authors have found that the FIGARCH model (1, d, 1) has a higher return than the GARCH and IGARCH models. Conrad, Karanasos, and Zeng (2011) estimate the FIAPARCH model uni-variate and multivariate versions for stock market returns in eight countries, and consider that this model is applicable because it takes into account both long memory and asymmetry. Chkili, Aloui, and Nguyen (2011) use a Markov-Switching EGARCH model to analyze the dynamic relationship between exchange rates and stock market returns in four emerging countries, namely Singapore, Hong Kong, Mexico, and Malaysia, for normal and turbulent periods. The predictive superiority of FIAPARCH over other GARCH models is supported by
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the work of Dimitriou and Kenourgios (2013). Riadh El Abed and Samir Maktouf (2016) suggest that the FIAPARCH model is the best specification for modeling the conditional volatility. Li and Giles (2015) examined exchange rate volatility for developed countries such as the United States and Japan and six Asian countries such as China, India, Malaysia, Indonesia, the Philippines and Thailand using the BEKK-MGARCH model. Bentes (2016) adopts a fractionally integrated GARCH model (FIGARCH) to examine the impact of global crises using daily data for the period 1985-2009. His study finds evidence of a long memory in the conditional variance throughout the sampling period, but mixed evidence in the subsample analysis. Ogega Haggai Owidi and Freshia Mugo-Waweru (2016) examine the behavior of the volatility of Kenyan stock market performance by the FIEGARCH model (1, d, 1). These authors have found that the FIEGARCH model has the capacity to capture the asymmetry effect by taking into account the characteristics of the long memory of volatility.

3 Econometric Methodology

3.1 FIAPARCH (p, d, q) Model

The AR (1) process is one of the most common models for describing a time series \( r_t \) of stock returns or exchange rates. Its formulation is given as:

\[
(1 - \xi L)r_t = c + \varepsilon_t, \quad t \in \mathbb{N}
\]  

(1)

\(|\xi| < 1 \) and \(|c| \in [0, +\infty)\) represents the average value of returns and \( \varepsilon_t \) is innovation, that is, what distinguishes the daily yield from its average value by incorporating the latest news. This innovation can be modeled by a FIAPARCH model which is built from the APARCH model, initially introduced by Ding, Granger and Engle (1993) and then taken up by Tse (1998), in order to take into account the persistence of the shocks in the variance conditional.

\[
\varepsilon_t = \sigma_t z_t
\]

\[\sigma_t\] represents the conditional variance and \( z_t \) is an independent and identically distributed random variable (i.i.d.) that follows a reduced normal centered distribution. Tse (1998) uses a FIAPARCH (1, d, 1) model to examine the conditional heteroscedasticity of the yen-dollar exchange rate. Its formulation is given as:

\[
\sigma_t^\delta = \frac{w}{1 - \beta(L)} + \frac{1 - (1 - \phi(L)(1 - L)^d)}{1 - \beta(L)} (|\varepsilon_t| - \gamma \varepsilon_t)^\delta
\]

(3)

\((1 - L)^d\): The operator of financial differentiation in terms of hyper-geometry (Conrad et al., 2011), \( \delta \): The power term parameter (a Box-Cox transformation) that takes positive (finite) value Tse (1998) and \( \gamma \): Leverage coefficient.

The advantage of our analysis over the above studies is the FIAPARCH specification of conditional variance, while existing studies use the simple GARCH model.

3.2 DCC - FIAPARCH (p, d, q) Model

In the following, we present the multivariate FIAPARCH (M-FIAPARCH) process taking into account the conditional dynamic correlation (DCC) (Dimitriou (2013)) advanced by Engle (2002). This approach generalizes the FIAPARCH conditional constant correlation (CCC) of Conrad (2011) multivariate model. In addition, the FIAPARCH model increases the flexibility of the conditional variance specification by allowing for an asymmetric response of
volatility to positive and negative shocks and to the long-term dependence of volatility. At the same time, this model allows the data to determine the power of returns for which the predictable pattern in the volatility model is highest (Conrad, Karanasos, and Zeng, 2011). The DCC multivariate model of Engle (2002) and Tse and Tsui (2002) has two steps to estimate the \( H_t \) of the conditional covariance matrix.

First, it corresponds to using a model of FIAPARCH \((1, d, 1)\) a variable in order to obtain the estimates of \( \sqrt{h_{it}}\).

The daily returns are supposed to be generated by a process AR (1) several variables of the type:

\[
Z(L)r_t = \mu_0 + \varepsilon_t
\]

\[
\mu_0 = [\mu_{0,i}]_{i=1, \ldots, n} : \text{le vecteur-colonne -dimensional } N \text{ des constantes; } |\mu_{0,i}| \in [0, \infty[ ;
\]

\[
Z(L) = \text{diag}(\psi(L)) : A \text{ diagonal matrix } N \times ; \psi(L) = [1 - \psi_i L]_{i=1, \ldots, n}; |\psi_i| < 1
\]

\[r_t = [r_{i,t}]_{i=1, \ldots, N} : \text{The vector } N\text{-dimensional column of returns}; \varepsilon_t = [\varepsilon_{i,t}]_{i=1, \ldots, N} : \text{The vector } N\text{-dimensional column of residuals.}
\]

\[
\varepsilon_t = z_t \odot h_t^{\frac{1}{2}}
\]

\(\odot\): the product Hadamard; \(\wedge\): the elementwise exponentiation.

\(h_t = [h_{i,t}]_{i=1, \ldots, N}^t\) is \(\Sigma_{t-1}\) measurable and the stochastic vector \(z_t = [z_{i,t}]_{i=1, \ldots, N}\) independent and identically distributed with mean zero and positive definite covariance matrix

\(\rho_t = [\rho_{i,t}]_{i=1, \ldots, N}\) with \(\rho_{i,i} = 1\) for \(i=j\). Note that \(E(\varepsilon_t / \mathcal{F}_{t-1}) = 0\) et

\[
\varepsilon_t = z_t \odot h_t^{\frac{1}{2}}
\]

The FIAPARCH multivarié \((1, d, 1)\) is given by:

\[
B(L)\left(h_t^{\delta/2} - \omega\right) = [B(L) - \Delta(L)\Phi(L)][I_N + \Gamma_t]|\varepsilon_t|^{\delta} 
\]

\(|\varepsilon_t|\) is the vector \(\varepsilon_t\) with elements stripped of negative values.

\[
B(L) = \text{diag}(\beta(L)) \text{ with } \beta(L) = [1 - \beta_i L]_{i=1, \ldots, N}\text{ et } |\beta_i| < 1
\]

\[
\Phi(L) = \text{diag}(\phi(L)) \text{ with } \phi(L) = [1 - \phi_i L]_{i=1, \ldots, N}\text{ et } |\phi_i| < 1
\]

\[
\omega = [1 - \omega_i]_{i=1, \ldots, N} \text{ with } \omega_i \in [0, \infty[ \text{ et } \Delta(L) = \text{diag}(d(L)) \text{ with }
\]

\[
d(L) = [(1 - L)^{d_i}]_{i=1, \ldots, N}\forall 0 \leq d_i \leq 1.
\]

Finally, \(\Gamma_t = \text{diag}(\gamma \odot s_t)\) avec \(\gamma = [1 - \gamma_i]_{i=1, \ldots, N}\) et \(s_t = [s_{it}]_{i=1, \ldots, N}\) ou \(s_{it} = 1\) si \(\varepsilon_{it} < 0\) et 0 if not

In the second step, we estimate the conditional correlation using transformed return-of-inventory residues, which are estimated by their standard deviations from the first step. The multivariate conditional variance is specified as follows:

\[
H_t = D_t R_t D_t
\]

\(D_t = \text{diag}(h_{11t}^{1/2}, \ldots, h_{Nt}^{1/2})\) denotes the conditional variance derived from the univariate model.
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AR (1) - FIAPARCH (1, d, 1) and \( R_t = (1 - \theta_1 - \theta_2)R + \theta_1\psi_{t-1} + \theta_2 R_{t-1} \) is the conditional correlation matrix. Engle (2002) derives a different form from the DCC model. The evolution of the correlation in DCC is given by:

\[
Q_t = (1 - \alpha - \beta)Q + \alpha z_{t-1} + \beta Q_{t-1}
\]

(8)

\( Q = (q_{ij}) \) is the covariance matrix \( N \times N \) time-variable of \( \text{de } z_t \), where \( \bar{Q} = E[z_t z_t'] \) denotes the unconditional variance matrix \( n \times n \) of \( z_t \), while \( \alpha \) and \( \beta \) are satisfactory non-negative parameters \( (\alpha + \beta) < 1 \) since \( Q_t \) does not usually have units on the diagonal. The conditional correlation matrix \( R_t \) is derived by the \( Q_t \) scale as follows:

\[
R_t = \left( \text{diag}(Q_t) \right)^{-1/2} \left( \text{diag}(Q_t) \right)^{-1/2}
\]

(9)

\( \theta_1 \) and \( \theta_2 \) are the satisfactory non-negative parameters \( (\theta_1 + \theta_2) < 1 \) et \( R = \{ \rho_{ij} \} \) is a symmetric positive matrix of defined parameters \( N \times N \) with \( \rho_{ij} = 1 \) and \( \psi_{t-1} \) is the correlation matrix \( N \times N \) de \( \varepsilon_t \).

4 Data and Preliminary Analyses

We use data from stock market indices and daily exchange rates, not only to obtain a sufficient number of observations to examine the recent global financial crises, but also to avoid the inefficiency that could arise if the samples studied are small in order to consider long memory in the measures of volatility. Weekly data to analyze the links between stock index and exchange rates do not take into account long memory efficiently, while monthly data may be insufficient to track short-term developments. We consider from January 1, 2006 to January 1, 2017. The sample consists of 8760 observations. All data are from the Board of Governors of the Federal Reserve System and are available at the following links: (https://www.federalreserve.gov/) and (https://www.finance.yahoo.fr).

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std. Devi</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>JB</th>
<th>Q^2(10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>R (TUNINDEX)</td>
<td>0.030311</td>
<td>0.4716</td>
<td>-0.5665</td>
<td>21.3672</td>
<td>56693.95</td>
<td>996.372***</td>
</tr>
<tr>
<td>R (TND/USD)</td>
<td>-0.013432</td>
<td>0.37515</td>
<td>-0.20603</td>
<td>9.192877</td>
<td>6449.145</td>
<td>171.232***</td>
</tr>
</tbody>
</table>

In table 1 the Skewness coefficient is negative for all series of returns, this indicates an asymmetric distribution tail on the left. The results of the Jarque-Bera test indicate the rejection of the normality assumption for all series, suggesting non-linear behavior. Thus, this non-linear behavior will allow us to opt for a long memory process. The Ljung-Box test for correlating series of 10 offsets allowed us to reject the null hypothesis of autocorrelation. This equates that the conditional volatility of the returns of the series contains long memory.
Figures 1 and 2 illustrate the evolution of stock index and exchange rate over time. These figures show that all stock index and exchange rate quaked since 2008 and display volatility clustering, revealing the heteroscedasticity existence. This characteristic cares the use of GARCH family models to investigate stock returns dynamics.

5 Empirical Results

5.1. Crisis period specifications
Recent crises have some unique characteristics, such as length, reach and origins of crisis. Many studies use key economic and financial events to determine crisis duration and beginnings (Forbes and Rigobon, 2002; Chiang et al., 2007). Nevertheless, other studies follow a statistical approach using the Markov regime change processes to identify endogenously the crisis period (Boyer et al., 2006; Rodriguez, 2007). We should note that economic and statistical approaches are at least partly arbitrary. Some studies help to avoid discretion in defining the crisis period by using discretion in choosing the econometric model to estimate the location of the crisis period over time. Baur (2012) used key financial and economic events, he estimated excessive volatility to identify the crisis period, and he studied the transmission of the global financial crisis from the financial sector to the real economy.

In this study, we specify the duration of global financial crisis and their phases according to economic and statistical approaches. We follow a statistical approach based on a Markov-
dynamic regression model (MS-DR), which takes into account the endogenous structural breaks and thus allows determining the beginning and the end of each phase of the crises.

Figure 3: Regime classification of stock index “conditional volatilities”

Figure 4: Regime classification of exchange “conditional volatilities”

Figure 3 and Figure 4 shows Regime 0, in light blue, corresponds to periods of stable and low volatility. Regime 1, in grey, denotes periods of rising and persistent volatility returns. The red columns indicate the smoothed regime probabilities, while the grey shaded spaces are the regimes of excess volatilities according to MS-DR model.

Engle and Ng (1993) proposed a set of asymmetric volatility tests named by the sign and size bias tests. These tests were used to determine whether an asymmetric model is needed for a given series or whether the symmetric GARCH model is verified.

<table>
<thead>
<tr>
<th>Table 2: Tests for Sign and Size Bias</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td><strong>Sign bais test</strong></td>
</tr>
<tr>
<td>Coeff  P-value  Coeff  P-value</td>
</tr>
<tr>
<td>Sign bais</td>
</tr>
<tr>
<td>Negative sign bias</td>
</tr>
<tr>
<td>Positive sign bias</td>
</tr>
</tbody>
</table>
The results in table 2 show that symmetric GARCH model residuals for TND/USD and TUNINDEX returns do not suffer from sign biases and negative size biases. On the other hand, these present a positive size bias. The joint effect at significant values of 1% for all these variables, which demonstrates a rejection of the null hypothesis of non-asymmetries. The overall results would therefore suggest a motivation for estimating an asymmetric volatility model for these variables.

According to test results Geweke and Porter-Hudak (1983) the null hypothesis of no long memory is rejected at the level of 10% for absolute returns and square returns. Evidence of the presence of long-term memory in absolute returns and square returns of stock index and for absolute returns and square returns of exchange rates is consistent with several earlier studies such as Kasman and Torun (2009), Kang, Cheong and Yoon (2010) for stock markets, and Abed and Maktouf (2016) for exchange rate and stock index.

Table 3: Long Memory Tests: GPH test-d Estimates

<table>
<thead>
<tr>
<th></th>
<th>Squared returns</th>
<th>Absolute returns</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$m = T^{0.5}$</td>
<td>$m = T^{0.6}$</td>
</tr>
<tr>
<td>TND/USD</td>
<td>0.5001*</td>
<td>0.5405*</td>
</tr>
<tr>
<td></td>
<td>0.825**</td>
<td>0.5801*</td>
</tr>
<tr>
<td>TUNINDEX</td>
<td>0.6929**</td>
<td>0.5082*</td>
</tr>
</tbody>
</table>

Notes: ***, ** and * désignent respectivement une signification statistique à 1%, 5% et 10%.

Table 4: FIAPARCH (1, d, 1)

<table>
<thead>
<tr>
<th></th>
<th>TND/USD</th>
<th>TUNINDEX</th>
</tr>
</thead>
<tbody>
<tr>
<td>ESTIMATION</td>
<td>Coeff</td>
<td>t-prob</td>
</tr>
<tr>
<td>CST(M)</td>
<td>0.0019</td>
<td>0.0000</td>
</tr>
<tr>
<td>AR(1)</td>
<td>0.0044</td>
<td>0.0000</td>
</tr>
<tr>
<td>CST(V)</td>
<td>0.0151</td>
<td>0.4689</td>
</tr>
<tr>
<td>d</td>
<td>0.5531</td>
<td>0.0000</td>
</tr>
<tr>
<td>ARCH(Phi1)</td>
<td>0.2912</td>
<td>0.0000</td>
</tr>
<tr>
<td>GARCH(Beta1)</td>
<td>0.7406</td>
<td>0.0000</td>
</tr>
<tr>
<td>APARCH(Gamma1)</td>
<td>-0.3840</td>
<td>0.0000</td>
</tr>
<tr>
<td>APARCH(Delta)</td>
<td>1.6827</td>
<td>0.0000</td>
</tr>
<tr>
<td>Df</td>
<td>8.8146***</td>
<td>0.0000</td>
</tr>
</tbody>
</table>
The ARCH and GARCH parameters (Phi1 and Beta1) are statistically significant and non-negative for all the returns of the series of exchange and stock index which justifies the relevance of the specification FIAPARCH (1, d, 1). The t-student degree of freedom parameter (df) is very significant for all exchange rate and stock index returns. This result confirms our preliminary analysis and, subsequently, the choice of t-student as an appropriate distribution. In addition, for all exchange rate stock index returns, Term (γ) leverage estimates are statistically significant, indicating an asymmetric response of volatilities to positive and negative shocks. Estimates of the power term (δ) are very significant for all exchange rate returns and the stock index.

Conrad, Karanasos and Zeng (2011) show that when the series is very likely to follow a non-normal error distribution, the superiority of a squared term (δ = 2) is lost and other power transformations can be more appropriate. In addition, all currencies display a significant fractional (d) parameter, which indicates a high degree of persistence behavior. This implies that the impact of negative shocks and their persistence on the conditional volatility of exchange rate and stock index returns.

From table 5, we can see that ARCH and GARCH coefficient estimates capture the dependence of shocks and persistence of volatility in the conditional variance equations. These coefficients are statistically significant at the risk thresholds of 1%, 5% and 10%. The size of the ARCH (q) model is very small, it is an indicator that the conditional volatility does not change very quickly under the impressions but, it tends to fluctuate gradually over time. In addition, the number of model coefficient delays is very narrow and the change in conditional volatility is symmetrical. Thus, the parameter of degrees of freedom T-Student (v) is significant, which favors the choice of this distribution. We accept the null hypothesis of serial correlation on standardized residuals and therefore an absence of the lack of static precision.

<table>
<thead>
<tr>
<th>Table 5: DCC- FIAPARCH (1, d, 1)</th>
<th>TND/USD - TUNINDEX</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel B:</td>
<td>Coeff</td>
</tr>
<tr>
<td>rho</td>
<td>0.0838</td>
</tr>
<tr>
<td>alpha</td>
<td>0.0063</td>
</tr>
<tr>
<td>beta</td>
<td>0.9915</td>
</tr>
<tr>
<td>df</td>
<td>8.7677</td>
</tr>
<tr>
<td>Panel B:</td>
<td>t-prob</td>
</tr>
<tr>
<td>rho</td>
<td>0.0493</td>
</tr>
<tr>
<td>alpha</td>
<td>0.0404</td>
</tr>
<tr>
<td>beta</td>
<td>0.0000</td>
</tr>
<tr>
<td>df</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Hosking(20) 147.378 [0.0001]  
Hosking²(20) 546.957 [0.0000]  
Li - McLeod(20) 147.379 [0.0000]  
Li – McLeod²(20) 546.162 [0.0003]
Figure 5: The DCC behavior over time

Figure 5 illustrates the evolution of the estimated dynamic conditional correlations between foreign exchange rate and stock index. Compared to the pre-crisis period, the estimated DCCs show a decline during the post-crisis period. Such evidence is in contrast with the findings of previous research on foreign exchange and stock index, which show increases in correlations during periods of financial turmoil (Dimitriou et al., 2013).

Nevertheless, the different path of the estimated DCCs displays fluctuations for all pairs of exchange rate and stock index across the phases of the global financial crises, suggesting that the assumption of constant correlation is not appropriate. The above findings motivate a more extensive analysis of DCCs, in order to capture contagion dynamics during different phases of crises.

5.2. The DCC behavior during different crisis periods

We next provide further results on the contagion effects during different phases of the two crises. Using various dummy variables allows us to identify which of the sub-periods exhibit contagion effects the exchange rate and stock index of the Tunisia. We create dummies, which are equal to unity for the corresponding phase of crisis and zero otherwise, to the following mean equation in order to describe the behavior of DCCs over time:

\[
\rho_{ij,t} = c_0 + \sum_{p=1}^{P} \psi_p \rho_{ij,t-p} + \sum_{k=1}^{\lambda} \beta_k \text{dummy}_k \eta_{ij,t}
\]  

(10)

where \(c_0\) is a constant term, \(\rho_{ij,t}\) is the pairwise conditional correlation between the exchange rate and stock index of the Tunisia, such that i corresponds to TND while j corresponds to TUNINDEX, and \(k = 1… \lambda\) are the number of dummy variables corresponding to different periods of the two crises, which are identified based on an economic and a statistical approach.

Furthermore, the conditional variance equation is assumed to follow an asymmetric GARCH (1,1) specification of Glosten, Jagannathan, and Runkle (1993) including the dummy variables identified by the two approaches:

\[
h_{ij,t} = \alpha_0 + \alpha_1 h_{ij,t-1} + \sum_{k=1}^{\lambda} \zeta_k \text{dummy}_k \eta_{ij,t} + \eta_{ij,t-1}^2 + \alpha_2 \eta_{ij,t-1}^2 I(\eta_{ij,t-1} < 0)
\]

(11)

As the model implies, estimated dummy coefficients significance indicates structural changes in mean or/and variance shifts of the correlation coefficients due to external shocks during the different periods of the two crises. According to Dimitriou and Kenourgios (2013), a positive and statistically significant dummy coefficient in the mean equation indicates that the correlation during a specific phase of the crisis is significantly different from that of the previous phase, supporting the presence of spillover effects among currencies. Furthermore, a positive and statistically significant dummy coefficient in the variance equation indicates a higher volatility of the correlation coefficients. This suggests that the stability of the correlation is less reliable, causing some doubts on using the estimated correlation coefficient as a guide for portfolio decisions.
Table 6: Tests of changes in dynamic correlations during the phases of Crisis

<table>
<thead>
<tr>
<th>Mean Eq</th>
<th>Coeff</th>
<th>signif</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_0$</td>
<td>0.0074</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\psi_1$</td>
<td>0.8739</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.0036</td>
<td>0.0612</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>0.0088</td>
<td>0.7213</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>0.0062</td>
<td>0.0356</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variance Eq.</th>
<th>-</th>
<th>-</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_0$</td>
<td>0.0004</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>0.1017</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\nu_1$</td>
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<td>$\zeta_4$</td>
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<td>LB(20)</td>
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<td>0.2621</td>
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<tr>
<td>LB*(20)</td>
<td>11.6054</td>
<td>0.7661</td>
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$LB(20)$ and $LB^*(20)$ denote the Ljung-Box tests of serial correlation on both standardized and squared standardized residuals.***, **, and * represent statistical significance at 1%, 5%, and 10% levels, respectively.

Table 6 shows the estimations of the mean and variance equations, setting a dummy variable for each phase of the crisis according to the economic approach. The constant terms $c_0$ and the autoregressive term ($\psi_1$) are both statistically significant for all DCCs, with the latter taking values close to unity, indicating a strong persistence in the conditional correlations among the examined currencies. For the mean equation, dummy coefficient $\beta_1$ for the phase 1 of the global financial crisis is positive and significantly. This evidence suggests that the DCCs between TND/USD and TUNINDEX were amplified during phase 1, supporting the existence of a difference in the vulnerability of the currencies. At the phase 2 of the GFC, the dummy coefficient $\beta_2$ is positive and not statistically significant for the exchange rate and stock index, supporting a decrease in DCCs. This suggests that the relationship between exchange rate and stock index actually decreased during this phase. We could define this finding as a “currency contagion effect”. During the phase 3 of macroeconomic deterioration, positive and statistically significant dummy coefficient $\beta_3$ exist for only the pair of currencies, implying an increase of DCCs. Finally, the estimations of the variance were reported in Table 6. The dummy coefficients $\zeta_{k,t}$ where $k = 1, 2, 3, 4$ for TND/USD and TUNINDEX are positive and statistically significant across several crisis phases. This finding means that the volatility of correlation coefficients is increased, implying that the stability of the correlations is less reliable for the implementation of investment strategies.
6 Conclusion

Whereas time fluctuating correlations of stock market returns and foreign exchange rate have seen large research, reasonably little attention was given to the dynamics of correlations within a market. In this paper, we evaluate the dynamic conditional correlation between the TND exchange rate expressed in (USD) and stock index and TUNINDEX by means of the Dynamic Conditional Correlation (DCC) model. First, we assessed the univariate FIAPARCH model on exchange rate and stock index in the asymmetry effect existence. Second, we used the DCC-FIAPARCH model to examine and analyze contagion risk between them. Our empirical results point out that foreign exchange market and Tunisia stock market exhibit asymmetry and no asymmetry in the conditional variances. For that reason, the results point to the importance of applying a suitably flexible modeling framework to truthfully estimate the interaction between exchange market and stock market co-movements. The conditional correlation surrounded by the TND/USD and Tunisia stock index displays higher dependency when it was driven by negative innovations to variations than it is by positive innovations. In addition, the stock index correlations turn out to be more volatile throughout the global financial crisis. The empirical analysis of the configuration of the time-varying correlation coefficients, during the main crisis periods, provides evidence in approval of contagion effects due to steering behavior in Tunisia stock markets and exchange rate. Our empirical results seem to be essential to researchers and practitioners and principally to active investors and portfolio managers who comprise in their portfolios equities from the Tunisia stock market. Actually, the high correlation coefficients, during crises periods, involve that the advantage from international diversification, by holding a portfolio involving of diverse stocks from the contagious stock markets, drop. The findings lead to essential implications from investors’ and policy makers’ perception. They are of great consequence for financial choices of international investors on managing their risk disclosures to exchange rate and stock index fluctuations and on taking advantages of prospective diversification opportunities that may rise due to dropped dependence among the exchange rates and stock index. The growth of exchange rates and stock index linkages throughout crisis periods shows the different currencies vulnerability and implies a decline of portfolio diversification benefits, meanwhile holding a portfolio with various currencies is less subject to systematic risk. Additionally, this correlations’ behavior considered as confirmation of non-cooperative monetary policies nearby the world and highlight the need for some form of policy organization among central banks. As a final point, the different patterns of dynamic linkages between Tunisia stock index and exchange rate might influence intercontinental trade flows and the accomplishments of multinational corporations, as they create ambiguity with concern to exports and imports.

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References

The dynamic relationship between stock index and exchange rate: Evidence for Tunis


