Monte Carlo Optimal Distribution of Simulation Dates

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Abstract

Following the 2007-08-credit crisis, investment banks have been focusing on how to increase the accuracy of risk measures. The parameters used for measuring risk have been increasing in relation with the evolution of calculation models. The originality of this work consists in providing, for the first time in literature, a quantitative research on the best method to use for selecting dates when using the popular Monte Carlo simulation. The object of this paper is to find the optimal distribution of simulation dates for measuring counterparty credit risk. We will first analyse a certain counterparty from a real-life portfolio to demonstrate how, when generating the regulatory measures of an Internal Model Method an optimal distribution of observation days is achieved by increasing the number of short- and medium-term time steps. In the second section we will analyse a test portfolio using a flat yield curve to better observe the differences when changing the distribution of simulation dates.

1 Introduction

This work describes how to select a grid of dates on a Monte Carlo simulation, in order to generate the specific measurements required to determine credit and counterparty risk factors. This is a useful assessment because reference grids tend to grow in density together with the performance of the model that generates them. When it comes to banks whose internal model is approved by a national regulator, it is often this regulator demanding an increase in the number of dates in order to better assess risk factors.

The first paragraph introduces all the notions relating to Counterparty Credit Risk [1], [9], [17], [23] that can gradually help us to identify the indicators of capital exposure, specifying the meaning of the internal model and the measures required for the

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management of the corresponding risk, with reference to the EU [22] and to the Bank for International Settlement [2], [3].

The second paragraph introduces the risk measures [6], [7], [8] which we will then analyse in detail in two main application scenarios: an analysis of an existing application in a bank in paragraph 3, and a laboratory application generated with Matlab2015 in paragraph 4.

In the third paragraph, taking a specific internal model for a specific bank as an example, we initially introduce the technical instruments required to obtain a calculation flow of the measurements under consideration. We describe the languages [15], [16] and the flow components [12], [13], [14] used to calculate the measurements. Assuming we have a suite, also necessary for generating these measurements, this paragraph describes the objects involved in this suite and how they are linked together in order to achieve the target. Next we describe the flow used to generate the EPE value, then we clarify the Methodology adopted for calculating the EPE, analysing the results of a calculation of specific risk measures of a specific counterparty. Finally, we explain the selected Methodology of scenario simulation.

The fourth paragraph describes an example of a Matlab test for generating risk measures, showing how these change when various grids are used. We introduce a portfolio of Interest rate swaps [18], [19], [20] defining the 3 selected grids. Then we explain the Methodology of scenario simulation [10], [11]. Finally, we inspect the scenario prices and the Compute exposure by counterparty, analysing each result [4], [5], [21].

The final paragraph summarises the results from the third paragraph of the curves of the counterparty and the analysis of the impact of variable grids from the fourth paragraph. From the third paragraph we can immediately deduce the importance of increasing the density of short-term grids of simulation applied to a real-life portfolio. In the fourth paragraph we analyse the impact that variations in the grid of simulation have on risk measures when increasing the density of short-term time steps first and then long-term ones. We then examine the benefits for both cases in a summarizing table of the portfolio under examination.

2 Preliminary Notes

The originality of this work consists in providing, for the first time in literature, a quantitative research on the best method to use for selecting dates when using the popular Monte Carlo simulation.

The first part of this study aims to demonstrate the benefits of increasing the number of short- and medium-term time steps:

1) Increasing short- and medium-term time steps has a positive influence on the final choice of regulatory measures to adopt.
2) Increasing long-term time steps is less useful because these steps are inaccurate when monitoring risk factor behaviours.

The second part aims to show how:

3) Increasing the density of dates provides more information and measurements can be more accurate if important dates for the portfolio are included.

3 Main Results

3.1 Internal Model Method and Exposure

A strong and resilient banking system is the foundation for sustainable economic growth, as banks are at the heart of the credit intermediation process between savers and investors. Banks also provide critical services to consumers, small and medium-sized enterprises, large corporate firms and governments who rely on them to conduct their daily business, both at a domestic and at an international level.

One of the key lessons learnt from the crisis is the need to strengthen the risk coverage of the capital framework. The failure to capture major on- and off-balance sheet risks, as well as derivative related exposures, was a major destabilizing factor during the crisis.

In addition to raising the quality and level of the capital base, there is a need to ensure that all risk measures are captured in the capital framework. The failure to capture major on- and off-balance sheet risks, as well as derivative related exposures, was a key factor that amplified the crisis.

To determine the default risk capital charge for counterparty credit risk, banks must use the greater portfolio-level capital charge based on the Effective EPE using current market data and the portfolio-level capital charge based on the Effective EPE, using a stress calibration. Counterparty credit risk is the risk that the counterparty to a financial contract will default prior to the expiration of the contract and will not make all the payments required by the contract. Only the contracts negotiated privately between counterparties, over counter derivatives and security financing transactions, are subject to counterparty risk. Exchange-traded derivatives are not affected by counterparty risk, because the exchange guarantees the cash flow promised by the derivative to the counterparty.

Counterparty credit risk, a key driver of the 2007/08-credit crisis, has become one of the main focuses of major global and U.S. regulatory standards. Financial institutions invest large amounts of resources employing Monte Carlo simulations to measure and price their counterparty credit risk.

Basel III makes a number of significant changes to the counterparty credit risk capital regime, including the introduction of a new capital charge designed to capture potential mark to market losses associated with the deterioration in the creditworthiness of a counterparty in relation to OTC derivatives; a number of changes to the capitalization of counterparty credit risk under the internal model method, including the introduction of a stressed effective expected positive exposure capital charge; draft proposals for revising the capitalization of exposures to central counterparties. These proposals are likely to be changed as they may currently discourage acting as a clearing member. Currently all clearing charges are in addition to the pre-existing Basel 3 OTC charges. Given that these proposals are subject to change they have been included in an annex rather than the main body of this document.

The stress calibration should be a single consistent stress calibration for the whole portfolio of counterparties. The greater of the Effective EPE, using current market data and the stress calibration, should not be applied on a counterparty-by-counterparty basis, but to the total portfolio level.

When the Effective EPE model is calibrated using historic market data, the bank must employ current market data to compute current exposures and at least three years of historical data must be used to estimate parameters of the model. Alternatively, market implied data might be used to estimate parameters of the model. In all cases, the data must

be updated quarterly or more frequently if market conditions warrant.

To calculate the Effective EPE using a stress calibration, the bank must also calibrate Effective EPE using three years of data that include a period of stress to the credit default spreads of a bank's counterparties or calibrate Effective EPE using market implied data from a suitable period of stress.

In situations where the bank does not have adequate credit spread data for a counterparty, the bank should map each counterparty to specific credit spread data based on region, internal rating and business type. To assess the adequacy of the stress calibration, the exposure model for all counterparties must use data, either historic or implied, that includes data from the stressed credit period, and must use this data in a manner consistent with the method used for calibrating the Effective EPE model to the current data.

To evaluate the effectiveness of its stress calibration for Effective EPE, the bank must create several benchmark portfolios that are vulnerable to the same major risk factors to which the bank is exposed. The exposure to these benchmark portfolios should be calculated using the current positions at current market prices, stressed volatilities, stressed correlations and other relevant stressed exposure model inputs from the 3-year stress period, the current position at the end of the stress period market prices, stressed volatilities, stressed correlations and other relevant stressed exposure model inputs from the 3-year stress period. Supervisors may adjust the stress calibration if the exposures of these benchmark portfolios deviate substantially.

3.2 Exposures Regulatory Measures used

The regulatory measure, gained from the same distribution of values used for the following measurements, is the Effective EPE; the bank may use this measurement to determine the regulatory requirement for counterparty risk only prior to authorisation by the Bank of Italy.

The measurements with their relative characteristics are Potential Future Exposure, Expected Exposure, Effective Expected Exposure, Expected Positive Exposure, Effective Expected Positive Exposure, Potential Future Exposure 95°, Media Effective Potential Future Exposure 95°.

The daily measure given to the systems used to monitor the credit for OTC derivatives, for the operational calculation of existing credit lines, is the Average of Effective Potential Future Exposure 95°.

The Potential Future Exposure, PFE, shows the evolution in time of the bank's credit exposure in terms of positive MtM for each scenario and for each time step from the date of the analysis to maturity.

Taking into consideration all positions p; of counterparty P, for a given scenario S_i, at time step t_k, if netting is absent the PFE will be defined as:

$$\text{PFE}^{\text{OUT}}(S_i, t_k, P) = \sum_{j=1}^{N} \max(\text{MtM}(S_i, t_k, p_j), 0)$$

If netting is present as:

$$PFE^{NET}(S_i, t_k, P) = \max\left(\sum_{j=1}^{N} MtM(S_i, t_k, p_j), 0\right)$$

Where MtM is the Mark to Market at date t_k.
For each time step, from the date of analysis to that of maturity, the Expected Exposure, EE, is defined as the average of PFE scenarios calculated on Monte Carlo simulations.

$$EE(t_k) = \frac{\sum_{i=1}^{nS} PFE\,(S_i, t_k)}{nS}$$

The Effective EE is a function that never decreases over time because, for each time step, it is the maximum between the EE of the individual time step and that of each preceding moment in time.

$$Eff.\,EE(t_k) = \max(Eff.\,EE\,(t_{k-1}), EE(t_k))$$

The EPE is the average weighted to the time of EEs, Expected Exposure, where the weights are the portion of the entire time interval taken up by the time step of the individual EE. The EPE is calculated on a one-year or less time scale. If all the contracts end during the year, the longest maturity is utilized.

$$EPE = \frac{\sum_{m=1}^{k=\min(1\,year,maturity)} \left(EE(t_m) * (t_m - t_{m-1})\right)}{t_k - t_0}$$

The Effective EPE is the average weighted to the time of Eff. EEs, where the weights are the portion of the entire time interval of the time step of the individual Eff. EE.
When calculating the EAD, where EAD = α x EFF.EPE α > 1, the Effective EPE is multiplied by α that may be 140% or more, at the Bank of Italy's discretion. It is possible to ask for the approval of an internal estimation of α, with, in any case, a floor equivalent to 120%, determined by the relationship between the capital estimated by a joint simulation of market/credit risk factors and the CE of the Effective EPE and the simulation of credit risk factors.

$$Eff.\,EPE = \frac{\sum_{m=1}^{k=\min(1\,year,maturity)} \left(Eff.\,EE(t_m) * (t_m - t_{m-1})\right)}{t_k - t_0}$$

It is possible to define an interval of confidence α associated with the PFE and, in this case, the maximum value of the PFE relative to the α % of the distribution will be considered. In accordance with current legislation we have considered an interval of 95%.
A summary measurement of the PFE 95° is represented by the Add-On, calculated as the relation between the maximum value, peak exposure, of the potential exposure in time, PFE95°, and the notional of the contract.

$$PFE^{\alpha}(t_k) = \inf\{PFE: \Pr(PFE(t_k) > PFE) \le 1 - \alpha\}$$

The Effective PFE 95° average is given by the average weighted by the non-decreasing time of the PFE 95°, Eff. PFE 95°, simulated over the life-span of the netting node, whereby by Eff. PFE 95° is intended to be the maximum between the peak reached over time t and that of each preceding moment in time.

This is the measure used for the measurement of credit lines usage.

$$\text{Eff. PFE}^{\alpha}(t_k) = \max(\text{Eff. PFE}^{\alpha}(t_{k-1}), \text{PFE}^{\alpha}(t_k))$$

$$\text{MeanEff. PFE}^{\alpha} = \frac{\sum_{m=1}^{k=\text{maturity}}(\text{Eff. PFE}^{\alpha}(t_m) * (t_m - t_{m-1}))}{t_k - t_0}$$

3.3 Generation of Exposure Regulatory Measures in a Specific Bank with an Internal Model Method

Here we describe the methodology adopted for calculating the measures of exposure, taking a bank that adopts an internal model as an example. The first step is to give a broad description of the architecture of the computing components used for the final generation of these numbers. Later on we will give first a description of the adopted scenario in relation to the components and then the selected procedure for the calculation of the measures of exposure. Finally, we will analyse the results of the curves obtained for the measures of exposure in relation to a given counterparty.

3.3.1 Calculation tools used

The application component used in this research is the form RiskWatch. This is software developed by IBM to calculate the measurement of risk and is part of the suite of the IBM AlgoSuite. RiskWatch is a calculation engine and financial risk. The RiskWatch application is an enterprise-wide software solution designed to help you measure, monitor, simulate, and restructure financial risk. The standard RiskWatch application provides basic tools for performing these tasks. You can also add a variety of extensions, which provide new functionality and financial models, or integrate RiskWatch with other AlgoSuite components. To access some of the methodologies and functionality, you will need to load extensions into RiskWatch. Extensions are dynamically linked modules that are ready to be loaded into RiskWatch at run time for additional functionality. A Mark-to-Market analysis is the valuation of an institution's overall portfolio from day to day, aggregated by criteria that you determine. To achieve a current value of a portfolio, you must model the fixed elements within the portfolio and the fluctuating risk factors. Once you have defined these elements, you can apply pricing models to the instruments to produce a Mark-to-Market result that you can analyse within the portfolio. You can perform portfolio analysis at various levels using the aggregation capabilities of RiskWatch. Risk managers can look at Mark-to-Market results from various perspectives because aggregation criteria can be defined dynamically. For example, at the aggregate level, you can group by currency, industrial sector, geographic location of the trading desk, or type of business unit.
Risk++ is a powerful C++ class library integrated within the Algorithmics enterprise-wide risk management system. It is capable of significantly extending RiskWatch functionality in a variety of ways. Its dynamic loading capabilities enable you to customize RiskWatch by adding a range of new financial objects to the application at run time. You can add risk

management functionality to RiskWatch by using the library.

The RiskScript language has been designed to help you automate tasks that you perform in RiskWatch on a regular basis. Most RiskWatch functions are available as RiskScript extensions that you can easily run from a script. The RiskScript interface is part of the standard RiskWatch release. You do not require any special files for RiskScript processing. BasicScript is the name of the Basic programming language that is embedded in RiskWatch. The RiskScript language consists of the BasicScript language with RiskWatch extensions, such as functions to access RiskWatch objects, recalculate sessions, and load sessions.

Algo Scenario Engine, ASE, is an advanced scenario generation and management application, which is a primary component of the Mark-to-Future framework. Scenarios created in ASE are based on historical time series data for observable market rates and prices. What we refer to as risk factors. ASE uses risk factors to generate scenarios, which are then used for simulation and stress testing of portfolios.

The MtF Aggregator's, MAG, main role is to calculate portfolio totals from Mark-to-Future, MtF, cubes that hold position, instrument, and portfolio data. In addition, MAG can calculate a number of statistical measures of portfolio and position risks. A MAG request specifies the chain of functions and their parameters. The MtF cube and netting node hierarchy are specified separately. Therefore, the same request can be applied to more than one cube. Each Risk Measure contains one Record Type function, one computation mode function, and one Evaluation function. Simulation functions are optional and can be either an elementary simulation function or a composite function made up of two or more elementary simulation functions.

3.3.2 Flow to generate EPE value

The flow starts from the database that contains the historical data. This data may be updated on a daily, weekly or monthly basis.

The historical data is sent to the Algo Scenario Engine to generate possible scenarios, after identifying the risk parameters used to calculate the final value of risk considered.

The scenarios are as many as the input portfolios, and a simulation is generated for each scenario. With back testing, we can analyse the quality of the previous simulation estimated up to time t, when it arrives at time t, $t = t0 =$ current time, comparing the curve of the past simulation with the real curve described with real data collected up to time t0.

We can stress a simulation; do a stress testing with improving or worsening future conditions, simulating the value of a portfolio by increasing or decreasing a k% of its value. This is a k% stress. Risk Watch, takes input scenarios and generates MtF cubes that are binary files containing the MtM value of financial instruments recalculated on N scenarios, M time step.

A portfolio that generates 1000 scenarios has a match of 1000 cubes in RW. In addition RW generates aggregations of Portfolio Server Hierarchy, PHS, in XML and CSV files that contain, respectively, the structure of the data, XML, and the contents of the data, CSV, ranging input to Algo Hierarchy Server, to subsequently pass the MtF to the Aggregator.

The cubes MtF ranging input to MAG, which in turn generates the risk measure request, specifically, the EPE.

3.3.3 Methodology for the calculation of EPE

This section describes the methodological criteria used for the implementation of the Expected Positive Exposure type internal model for evaluating counterparty risk, both in

terms of regulatory capital requirements and in terms of the operational management of the substitution risk of credit lines.

The methodology adopted uses the Algorithmic calculation engine within the Algo Batch workflow, ASE, RiskWatch and MAG, and it has been implemented in adherence with the requisites of the internal counterparty models of the simulation components of the Potential Future Exposure and of the Expected Positive Exposure.

This approach takes into consideration the distribution of variations in the market value, linked to major fluctuations in the risk factors, of all the contracts with counterparties, and estimates how they will evolve in time, taking into account possible netting agreements.

Here is a synthesis of all the steps taken in this calculation:

Generating set scenarios beginning with a Time series of indicative drivers for each risk factor and a patterns of evolution over time for various risk factors and number of path scenarios and time steps.

Computation of Mark to Future, MtF, in other words the evolution over time of MtM and, therefore, of the max exposure (MtF, 0) of each single instrument within each path scenario for each time step.

Aggregation of exposures according to the correct hierarchy of netting/credit lines.

Computation of the measures adopted for the purpose of management, regulatory conditions and risk analysis.

3.3.4 Results of the calculation

The following figures show the curves of a given counterparty. The images are made up of three curves. The first curve has a simulation grid G0 made up of N dates. The other two curves correspond to grid G1 with the same dates as G0 plus a set of k1 simulation dates and to grid G2 with the same dates as G0 plus a set of k2 simulation dates. The number of elements of set k1 is the same of that of k2. The difference between G1 and G2 is the different dates of k1 and k2 added to the starting grid.

Figure 1. The following image is the MtF of counterparty X with two different number of time steps, N and M, and three different densities of time steps. The source is Intesa Sanpaolo Bank. The following figure is made up of three curves. The first curve is drawn from a simulation grid G0 that is less dense than grids G1 and G2. All three grids are graphed in the figure. We can easily spot the dense section of grids G1 and G2 at the point on the drawing of the curves where we see many peaks. Grid G0, which is less dense, has a straighter line, whereas grids G1 and G2, precisely at the point where they are denser compared to G0, fluctuate more, forming more peaks. The more peaks we have, the more information we can gather about the section concerned. In the final part of the three curves, we see a sudden growth. This growth is linked to the increase in the value of the portfolio at that date.
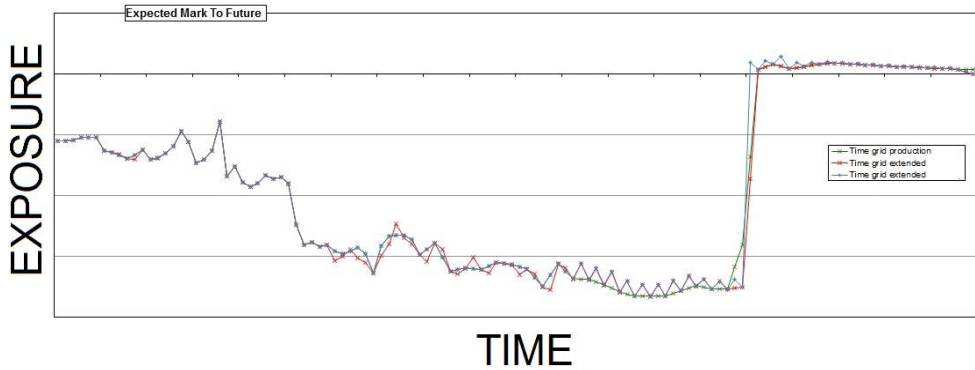
Figure 1: Expected Mark to Future

Figure 2. The next figure shows the curve of the EPE and of the EEPE of counterparty X with two different numbers of time step N and M and three different densities of time steps. The source is Intesa Sanpaolo Bank. In this case also we have three curves, corresponding to grid G0, G1 and G2. The curves G1 and G2 have two different forms of increased dates with respect to grid G0. G1 and G2 have the same number of dates, but the difference is in the area of increased density. In this graph we see that, just like in the previous one, the presence of peaks corresponds to the areas in which the number of dates has been made denser. It is important to notice how, in the long-term area, there is a peak, highlighted by a grid in between the two denser ones, which surpasses the value of the effective EPE. This peak demonstrates a particularly high EPE, which can be explained by the fact that increasing the density of the grid captures an extra information in comparison with the less dense grid, about a certain instrument which, at that date, receives significant variations of exposure, as shown by the peak. These situations generally correspond to deadlines of certain instrument portfolios that had been considered when selecting the risk measures.
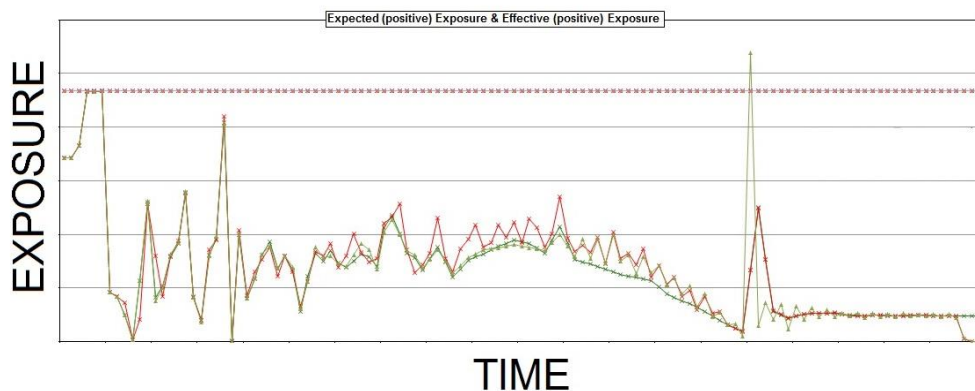

Figure 2: Expected positive Exposure and Effective positive Exposure

Figure 3. The final figure represents the PFE and the EffPFE of counterparty X with two different numbers of time steps N and M and three different densities of time steps. The

source is Intesa Sanpaolo Bank. The curves corresponding to the three different grids used to measure the PFE are extremely interesting mainly because the red curve shows, mostly in the mid-term region, what an important role it plays when determining the effective PFE with this grid. The values captured in this segment, in fact, contribute significantly to identifying the exact value of the effective PFE. The grid used to draw the red curve is, therefore, better equipped than the other two at providing useful information to determine the measurement at hand. Precisely because it gives us more information, the grid corresponding to the red curve is preferred when it comes to selecting a denser grid in comparison with the starting grid. In this case, for this measurement, the grid corresponding to the generation of the red curve gives us a greater contribution.
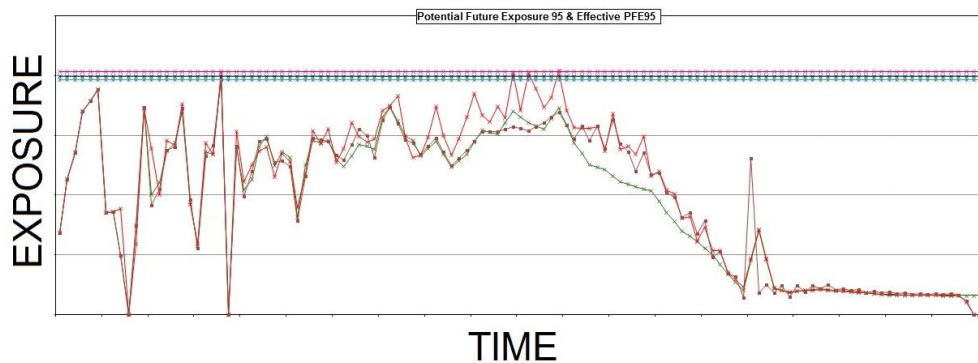

Figure 3: Potential Future Exposure (PFE) 95 and Effective PFE 95

3.3.5 Methodology of scenario simulation

In Monte Carlo multistep scenarios, a stochastic process that defines its distribution for each future time step describes the evolution in time of each risk factor. The estimation of the parameters used is based on the last 3 years in the time series for each risk factor. The models that analytically define such stochastic processes have been chosen according to the typology of the risk factor in consideration, namely:

The Geometric Brownian Motion Model, for equity and commodity, GBM model with null drift. This model is based on a log-normal distribution of the random variable whose temporal evolution is described by the following stochastic differential equation:

$$dx(t) = \sigma x(t)dW(t)$$

Where σ is the historical volatility of the risk factor x (t) and dW(t) is a random walk. The Black-Karasinski Model, for interest rate, fx and credit derivatives, is a mean reverting type model with a log-normal distribution of the underlying variable.

It combines a stochastic process with a log-normal distribution of a Brownian type, with a mean reverting type function, and it allows the creation of scenarios whose value have a limited time-span, thus allowing for the modelling of risk factors, such as interest rates, for example, towards a long-term average. At every step in the simulation, a process which is subject to a mean reversion drifts towards a pre-determined level as well as being subject to random movements.

The process of this variable is described by the following differential stochastic equation:

$$y(t) = \log\big(x(t)\big)$$

$$dy(t) = k\big(a - y(t)\big)dt + \sigma dW(t)$$

Where σ is the historical volatility, a e k are the mean reversion level and the mean reversion speed estimated on a historical basis and dW(t) is a random walk.

Because of the high number of Monte Carlo multistep scenarios that have to be generated, the number of risk factors to be simulated must be kept low in order to maintain the database in a manageable format from both a technical and a performance perspective. The risk factors have, therefore, been linked to drivers whose evolution in time will be functional to the definition of revaluation scenarios.

The choice of drivers has been made according to the importance of risk factors for the bank's portfolio. In other words, according to the importance and/or development of the market. We believe that this simplification does not reduce the importance of the analysis because in this phase an assessment of idiosyncratic risks is less relevant than market risks. The implemented methodology used for an assessment of counterparty risk is therefore based on two main components: an estimation of the value of future exposures for all positions that are subject to counterparty risk through Monte Carlo multistep simulations and a representation of the exposure within the netting hierarchy for every netting node of each counterparty.

The choice of time steps takes into account the structure of the bank's portfolio and the weight that needs to be allocated to exposures in the first year, as these are what are used to calculate the regulatory measures.

This choice tends to reduce to the minimum the possibility of error, roll-off risk, of one-year exposure forecasts, which is coherent with the annual horizon of regulatory measures. From the analysis of exposure profiles, we can also notice how, for the majority of payoffs, the peak is reached in the short or medium term. This is why the choice of time steps has been considered the most appropriate for measuring potential future exposure accurately.

The number of Monte Carlo multistep scenarios currently used is 2.000 for every risk factor.

For certain types of financial instruments the calculation libraries developed internally have to be modified and adapted to enable a correct simulation of the evolution in time of the mark to market. In other words, certain approximations or a different model that is compatible with time evolution will have to be used, taking into account the adjustment requirements or the performance of the simulation.

3.4 Further Investigation on generating the Exposures Regulatory Measures

This case study shows how to compute the credit value for a bank holding a portfolio of vanilla interest rate swaps with several counterparties. In support of the preceding analysis on "increasing the distribution of observation days" for calculating the measures of exposure, this section provides specific data in order to give a broader numerical dimension to this research. The portfolio analysed is a portfolio of vanilla swaps that remains identical for all the tests carried out. The test environment is a Matlab R2015a with a student licence. The test is carried out on three grids with two numbers of observation days, N and M with N<M, and N+k=M. The three grids 0, 1 and 2 have 0 cardinals N and 1 and 2 cardinals M.

The logic of the test is to apply two different increased densities of starting grid 0. The first increased density consists in augmenting the number of dates k distributed in the initial part of starting grid 0. The second increased density consists in augmenting the number of dates k distributed in the final part of starting grid 0.

3.4.1 Interest rate swaps portfolio

The portfolio used for this test is made up of six IRS. Let's take a closer look at the swaps belonging to this portfolio. The following numbers of set id = {1, 2, 3} correspond to counterparties {Alfa Ltd, Beta Ltd, Gamma Ltd}.

The swaps used for generating the risk measurements are as follows:

Table 1: Swap vanilla portfolio

Fixed rate payer: Alfa Ltd; Fixed rate: EUR 20,00 annual Floating rate payer: Strong Financial Corporation Floating rate: 1-year EUR Libor National amount: EUR 500000,00 Maturity: 10 years	Fixed rate payer: Strong Financial Corporation Fixed rate: EUR 40,00 annual Floating rate payer: Alfa Ltd Floating rate: 1-year EUR Libor National amount: EUR 1000000,00 Maturity: 10 years
Fixed rate payer: Beta Ltd; Fixed rate: EUR 19,00 annual Floating rate payer: Strong Financial Corporation Floating rate: 1-year EUR Libor National amount: EUR 1000000,00 Maturity: 20 years	Fixed rate payer: Strong Financial Corporation Fixed rate: EUR 40,00 annual Floating rate payer: Beta Ltd Floating rate: 1-year EUR Libor National amount: EUR 500000,00 Maturity: 20 years
Fixed rate payer: Gamma Ltd; Fixed rate: EUR 5,00 annual Floating rate payer: Strong Financial Corporation Floating rate: 1-year EUR Libor National amount: EUR 12500,00 Maturity: 25 years	Fixed rate payer: Strong Financial Corporation Fixed rate: EUR 10,00 annual Floating rate payer: Gamma Ltd Floating rate: 1-year EUR Libor National amount: EUR 250000,00 Maturity: 25 years

3.4.2 Definition of observation days

In this test, the simulation grids are composed, respectively, of one with 37 dates and two with 49 dates. Grid 0 contains 37 simulation dates, whereas grid 1 contains the same dates as 0 plus an increased density of short- and medium- term dates. Grid 2, on the other hand, contains the same dates as 0, plus an increased density of long-term dates.
This is how grids 0, 1 and 2 are composed:

Table 2: Observation days grids

Grid 0	Grid 1	Grid 2
14-Dec-2007	14-Dec-2007	14-Dec-2007
15-Dec-2007	15-Dec-2007	15-Dec-2007
16-Dec-2007	16-Dec-2007	16-Dec-2007
17-Dec-2007	17-Dec-2007	17-Dec-2007
18-Dec-2007	18-Dec-2007	18-Dec-2007
19-Dec-2007	19-Dec-2007	19-Dec-2007
20-Dec-2007	20-Dec-2007	20-Dec-2007
21-Dec-2007	21-Dec-2007	21-Dec-2007
22-Dec-2007	22-Dec-2007	22-Dec-2007
23-Dec-2007	23-Dec-2007	23-Dec-2007
24-Dec-2007	24-Dec-2007	24-Dec-2007
07-Jan-2008	25-Dec-2007	07-Jan-2008
14-Jan-2008	26-Dec-2007	14-Jan-2008
14-Feb-2008	27-Dec-2007	14-Feb-2008
16-Mar-2008	28-Dec-2007	16-Mar-2008
16-Apr-2008	29-Dec-2007	16-Apr-2008
17-May-2008	30-Dec-2007	17-May-2008
17-Jun-2008	01-Jan-2008	17-Jun-2008
18-Jul-2008	02-Jan-2008	18-Jul-2008
18-Aug-2008	03-Jan-2008	18-Aug-2008
18-Sep-2008	04-Jan-2008	18-Sep-2008
19-Oct-2008	05-Jan-2008	19-Oct-2008
19-Nov-2008	06-Jan-2008	19-Nov-2008
20-Dec-2008	07-Jan-2008	20-Dec-2008
27-Dec-2009	14-Jan-2008	27-Dec-2009
03-Jan-2011	14-Feb-2008	03-Jan-2011
10-Jan-2012	16-Mar-2008	10-Jan-2012
16-Jan-2013	16-Apr-2008	16-Jan-2013
23-Jan-2014	17-May-2008	23-Jan-2014
30-Jan-2015	17-Jun-2008	30-Jan-2015
06-Feb-2016	18-Jul-2008	06-Feb-2016
12-Feb-2017	18-Aug-2008	12-Feb-2017
19-Feb-2018	18-Sep-2008	19-Feb-2018
25-Mar-2023	19-Oct-2008	26-Feb-2019
27-Apr-2028	19-Nov-2008	04-Mar-2020
31-May-2033	20-Dec-2008	11-Mar-2021
04-Jul-2038	27-Dec-2009	18-Mar-2022
	03-Jan-2011	25-Mar-2023
	10-Jan-2012	31-Mar-2024
	16-Jan-2013	07-Apr-2025
	23-Jan-2014	14-Apr-2026
	30-Jan-2015	21-Apr-2027
	06-Feb-2016	27-Apr-2028
	12-Feb-2017	04-May-2029
	19-Feb-2018	11-May-2030
	25-Mar-2023	18-May-2031
	27-Apr-2028	24-May-2032
	31-May-2033	31-May-2033
	04-Jul-2038	04-Jul-2038

Since we are taking vanilla swaps into consideration, we will use the following constant vector as a rate in order to better spot the differences between the three different grids:

Table 3: Zero rate vector

Date/mm	Zero rate
3	0.01
6	0.01
12	0.01
5 X 12	0.01
7 X 12	0.01
10 X 12	0.01
20 X 12	0.01
30 X 12	0.01

These two vectors may obviously be modified according to other curves of the swap rate.
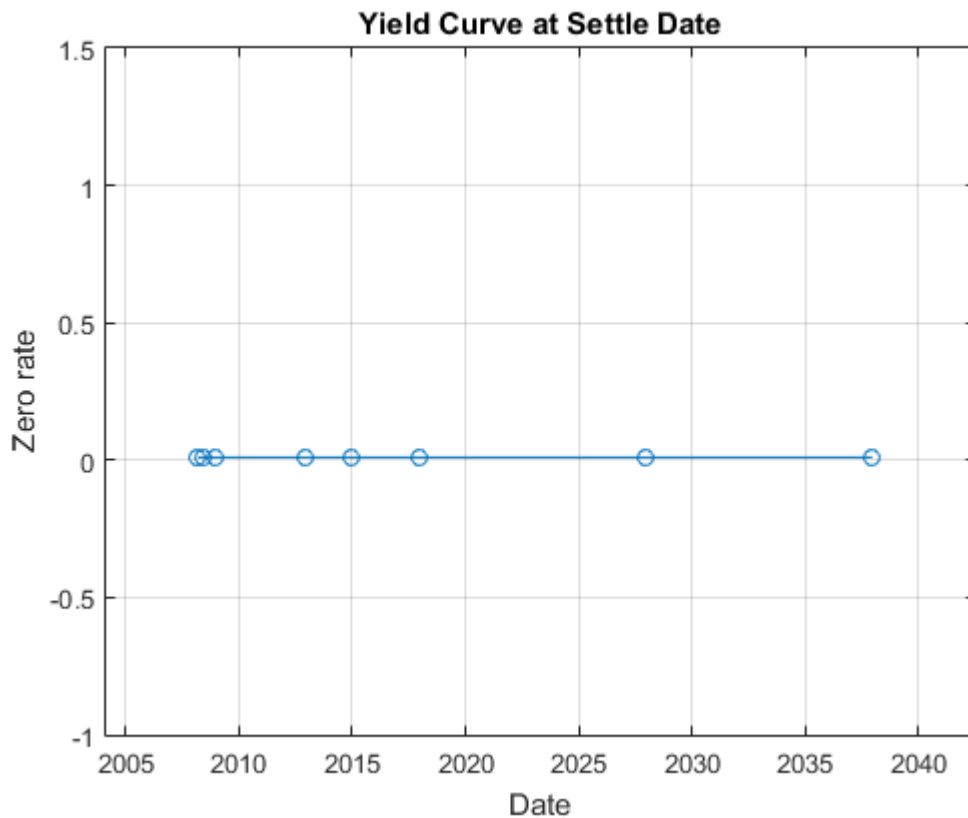

Figure 4: Yield Curve with Zero Rate = 0.01

3.4.3 Methodology of scenario simulation

Having set up the three Monte Carlo simulation grids, we define the total number of scenarios to be generated as N=1000.

The risk factor we will simulate to value our contracts is the zero curve. For this case study we will model the interest rate term structure using the one factor Hull White model. Alpha = 0.2 and Sigma = 0.015;

For each scenario, we simulate the future interest rate curve at each valuation date using the Hull-White one-factor interest rate model. The surface plot of the yield curve evolution for scenario n.5 is the following:
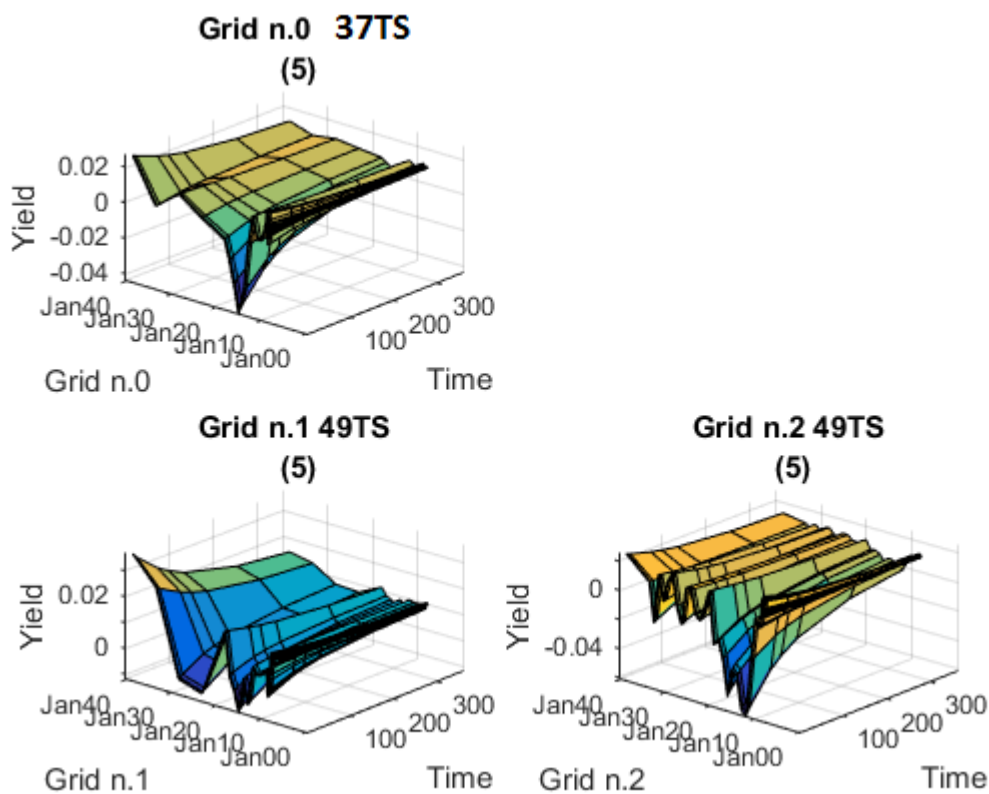

Figure 5: Scenario Yield Curve evolution on grid 0, grid 1 and grid 2

Observing these three surfaces, we see how increasing the density creates more peaks in the applied grids. In quantitative terms, having more peaks means giving a better representation of the projection of the variations of a set of values applied at the beginning of the process. The increase in density is applied in the short-term in grid 1 and in the long-term in grid 2.

3.4.4 Inspect scenario prices

We create a plot of the evolution of all swap prices for a particular scenario. For each scenario the swap portfolio is priced at each future simulation date. Prices are computed using a price approximation function. Since the simulation dates do not correspond to the swaps cash flow dates, we estimate the latest floating rate with the 1-year rate interpolated between the nearest simulated rate curves. The swap prices are then aggregated into a cube of values, which contains all future contract values at each simulation date for each scenario. The resulting cube of contract prices is a 3 dimensional matrix where each row

represents a simulation date, each column a contract, and each page a different simulated scenario.

Figure 6: Swap prices along scenario 5 on grid 0, grid 1 and grid 2

If we compare grid 1 to grid 2, we can see how sampling occurs mainly where the grids are denser. In the first case we have increased the number of short-term dates, in the second case the number of long-term ones. With the increase in density of short-term time steps of grid 1, we can spot a more regular trend towards the beginning of the price curve.

3.4.5 Compute exposure by counterparty

It is possible to plot the total portfolio value for each scenario of our simulation. As each scenario moves forward in time, the value of the contracts will increase or decrease according to changes in the modelled interest rate term structure. As the swaps get closer to maturity, their value will begin to approach zero since the aggregate value of all remaining cash flows will decrease after each cash flow date. The exposure of a particular contract, i, at time t is the maximum of the contract value, Vi, and 0. Several exposure profiles are useful when analysing the potential future exposure of a bank to a counterparty. Here we compute several, non-discounted, exposure profiles per counterparty as well as for the entire portfolio.

If we observe the beam of total exposures, we can see quite clearly that there is a swelling in the central section which corresponds to the curves generated by grid 1, characterised by an

increased density in the central part of the date axis. Another interesting element is the swelling of the beam of exposure generated in grid 2 towards the end of the axis of dates. This is coherent with our expectations. In the areas with an increased density of dates we have a swelling in the curves of the simulation graph. We notice a swelling in the short term in grid 1 and a swelling in the long-term in grid 2. In all three cases, the exposures are at a maximum in the initial section of the three simulation grids.
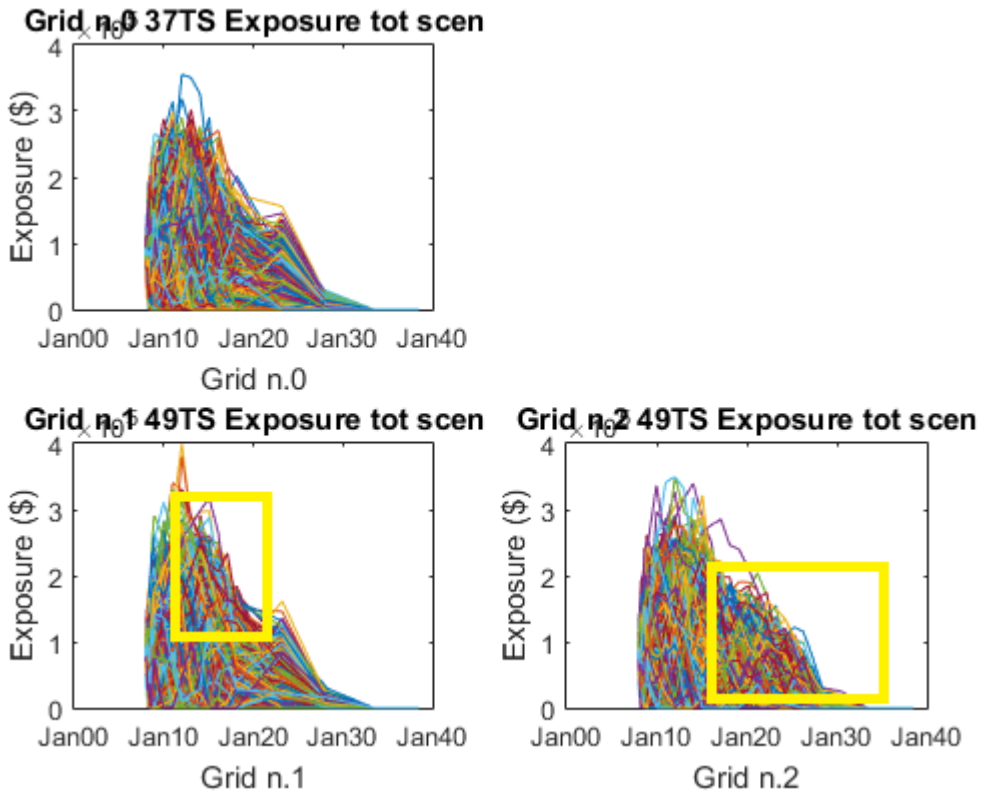

Figure 7: Portfolio exposures for all scenarios on grid 0, grid 1 and grid 2

Let us now examine the exposure of out initial swap portfolio. The blue line represents the PFE (Potential Future Exposure). The red line represents the Maximum Potential Future Exposure. The yellow line represents the Expected Exposure, or, in other words, the mean average of the distribution of exposures at each date. The purple line represents the Expected Positive Exposure, the weighted average over time of the expected exposure. Finally the blue line represents the Effective Expected Positive Exposure, the weighted average of the effective expected exposure.
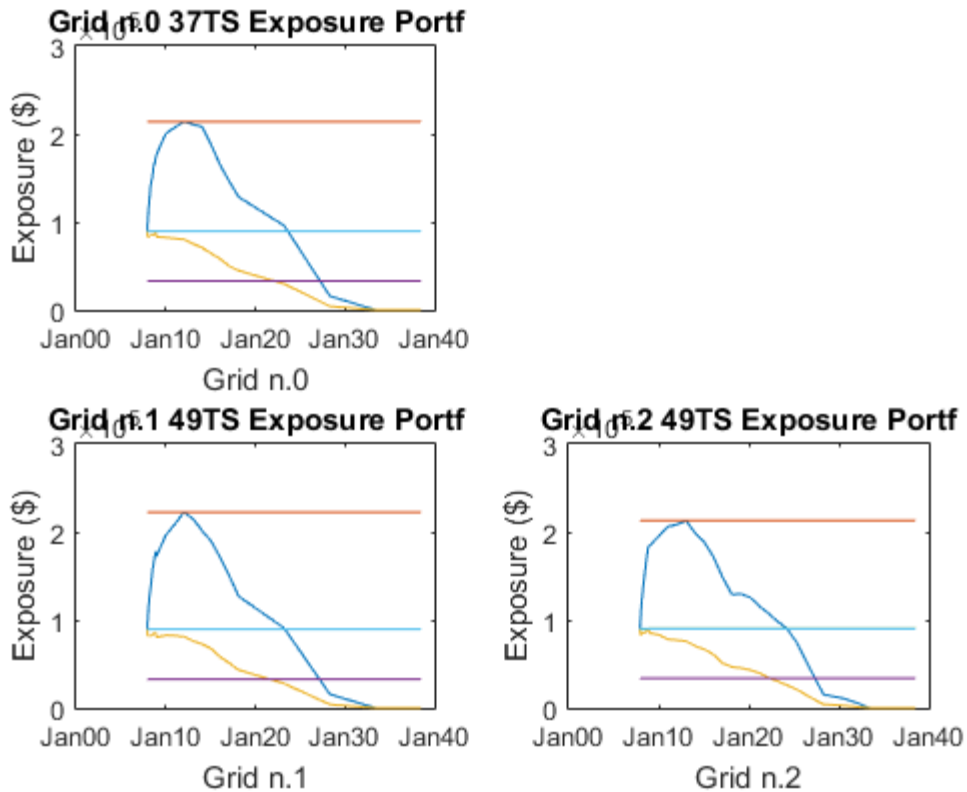
Figure 8: Portfolio exposure profiles on grid 0, grid 1 and grid 2

If we observe the three curves of the EE and of the PFE for the entire portfolio, we notice how when we move from the 37 dates of grid 0 to the 49 of the other two grids, grid 1 looks smoother towards the centre. This means that variations in value are better captured in that section. We would like to add that the section with more peaks corresponds to the section where the grid is denser. On the curve corresponding to grid 2, on the other hand, we notice a smoothening in the final part, which corresponds to an increased density of the grid. The values calculated are as follows:

Table 4: Portfolio exposure profiles on grid 0, grid 1 and grid 2

	Grid 0	Grid 1	Grid 2
EPE	3.306e+04	3.276e+04	3.388e+04
EffEPE	8.953e+04	8.952e+04	9.019e+04
MPFE	2.139e+05	2.224e+05	2.129e+05

We can see the difference in values according to the grids used. The value of the EPE decreases when we use the denser short-term grid. The highest EPE value is obtained by using the denser long-term grid because it better captures the information regarding the maturity of twenty and twenty-five years swaps. The maximum value of Effective EPE is obtained by grid 2, while the maximum value of max PFE is obtained by grid 1.

Now let's analyse the measures of risk on the three grids 0, 1 and 2 with reference to counterparty 1, with a ten-year maturity of a vanilla swap. We can see immediately that the best contribution towards the risk measures is provided by the initial part of curve EE and of PFE. Grid 2 does not capture any extra information because its increased density starts after the maturity of counterparty 1.
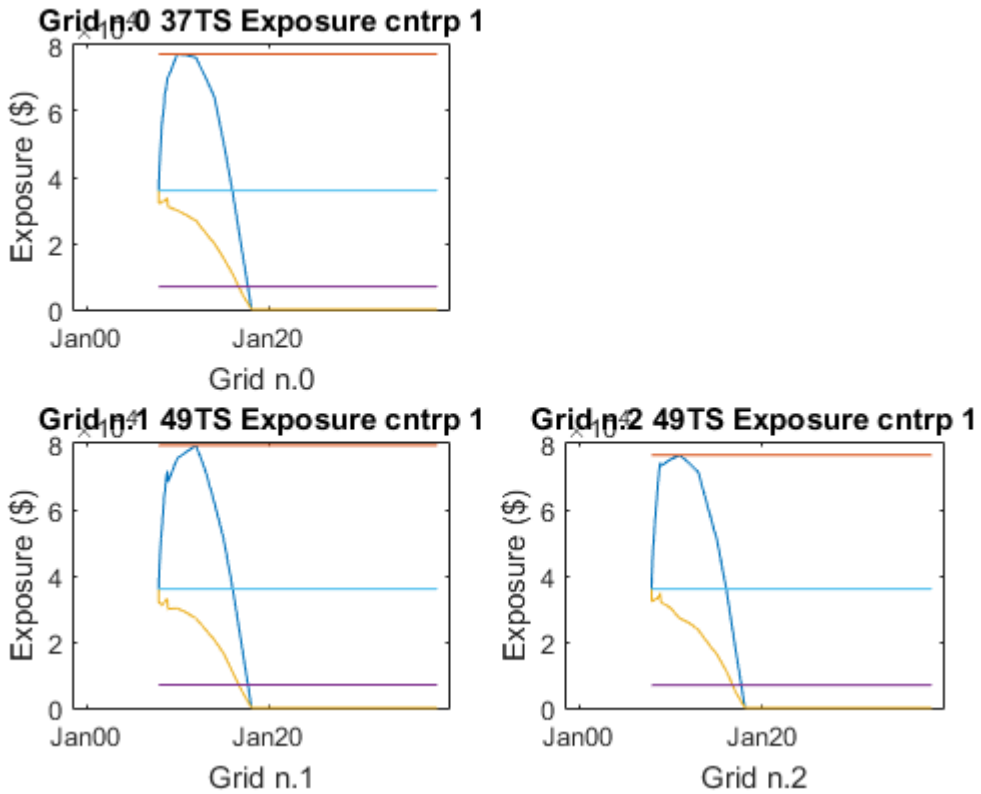

Figure 9: Counterparty Alfa Ltd Exposure Profile on grid 0, grid 1 and grid 2

In these figures we analyse the counterparty made up of two swaps with a ten-year maturity as shown by the curve of the expected exposure on the grid that reaches zero at maturity. The maximum EPE value is obtained by grid 1, whereas the most significant value of the effective EPE is given by grid 0. The most important max PFE is provided by grid 1.

Table 5: Counterparty Alfa Ltd Exposure Profile on grid 0, grid 1 and grid 2

	Grid 0	Grid 1	Grid 2
EPE	6912	6971	6924
EffEPE	3.601e+04	3.597e+04	3.597e+04
MPFE	7.728e+04	7.95e+04	7.649e+04

Now let us examine the risk measures on the three grids 0, 1 and 2 regarding counterparty 2, with a 20-year maturity vanilla swap. Information about this counterparty is better captured by grid 2 with denser long-term time steps, which also capture the maturity of the swap.
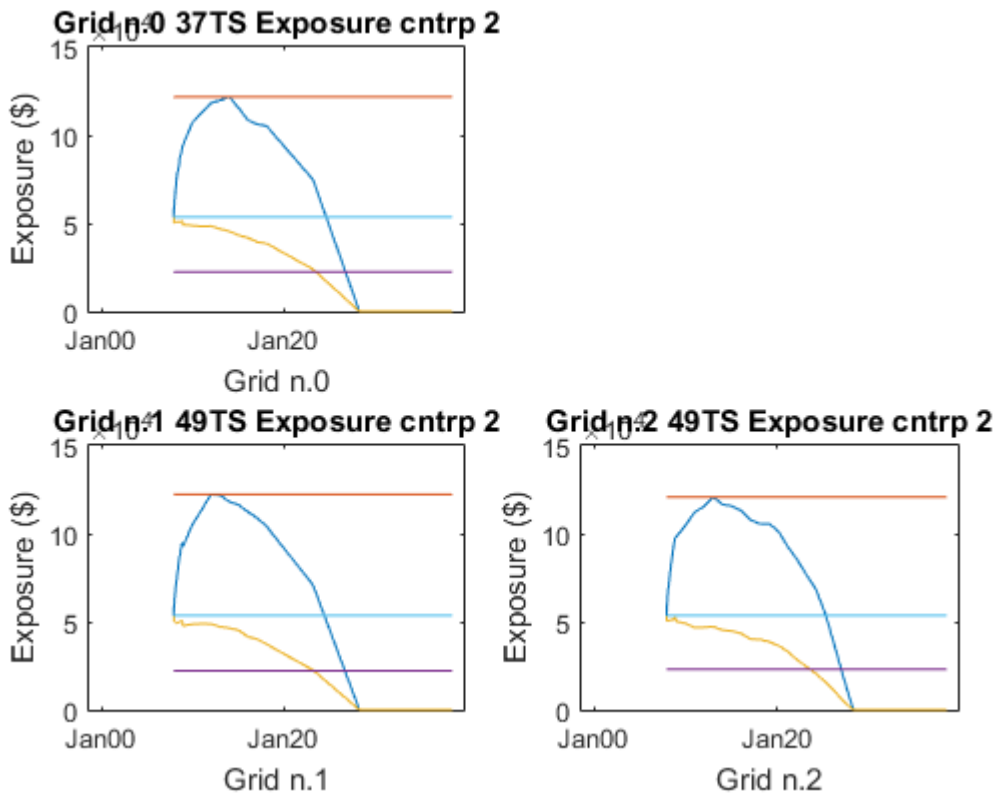
Figure 10: Counterparty Beta Ltd Exposure Profile on grid 0, grid 1 and grid 2

The EE and PFE of grids 0 and 2 are not as smooth as grid 1 which has denser short-term time steps. The EE and PFE of grid 2, on the other hand, are smoother towards the end in correlation with the density increase of grid 2.

The maximum value of EPE is provided by grid 2. The effective EPE is the same for all three grids and the max PFE is shown on grid 1.

Table 6: Counterparty Beta Ltd Exposure Profile on grid 0, grid 1 and grid 2

	Grid 0	Grid 1	Grid 2
EPE	2.242e+04	2.216e+04	2.306e+04
EffEPE	5.355e+04	5.355e+04	5.355e+04
MPFE	1.217e+05	1.223e+05	1.208e+05

Let us now analyse the results of counterparty 3 on grids 0, 1 and 2, when the vanilla swap has a 25-year maturity. Even in this case the long-term density increase captures more information during the maturity period of counterparty 3. The curve of grid 1 has a more regular trend in the short-term, contrary to curves 0 and 2, which show some slight irregularities. The curve of grid 2, also, has a smoother trend in the long-term part.
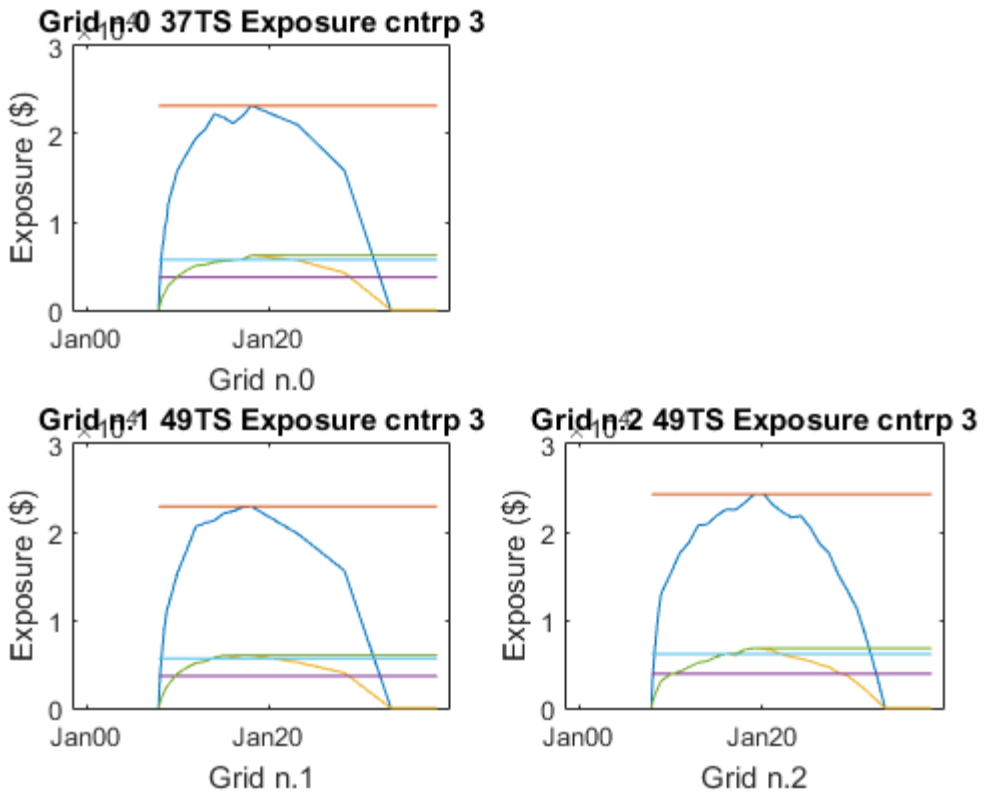
Figure 11: Counterparty Gamma Ltd Exposure Profile on grid 0, grid 1 and grid 2

For this counterparty, grid 2 provides the highest values of EPE, effective EPE and MPFE. Grid 2 gives higher values for all three measurements. Grid 1, on the other hand, gives the lowest EPE, PFE and MPFE. Grid 0 gives values in between grid 2 and 1.

Table 7: Counterparty Gamma Ltd Exposure Profile on grid 0, grid 1 and grid 2

	Grid 0	Grid 1	Grid 2
EPE	3728	3634	3897
EffEPE	5712	5613	6125
MPFE	2.321e+04	2.289e+04	2.4283+04

4 Conclusion

Observing the results of figures 1, 2 and 3, we notice some peaks towards the end of the grid for certain counterparties. These are linked to the long distance between one time step and the next. Although the Mark-to Future may seem to improve, in reality, because of the long distance, the exposure may appear incorrectly because they are mainly collateralized counterparties.

The denser grids allow us to notice more irregular profiles such as, for example, different payment frequencies on the two legs of the swaps. Usually, however, these phenomena have a minor impact.

A denser grid, which also affects the short-term, allows for a better representation of the Effective EPE. In some cases, a denser grid also allows for a better representation in the medium-term.

If we look closely at the curves generated, we can state that a greater number of peaks imply a better representation. The higher the number of intercepted peaks, the better the curve. A variation in the portfolio has an impact on the Effective EPE, therefore if a peak is observed it means that the grid used is of higher quality.

Density is an important aspect in the distribution of time steps; however this density must be dependent on time. The number of time steps must be greater in the short-term, average in the medium-term and lower in the long-term. In the long-term, it is important to make sure that certain risk factors don't produce unwanted results during the later days of observation. To eliminate these unwanted results in the long-term, the grid would need to be denser, but with a decreasing distribution of time steps over time.

To support the observations we have made in the first section, let us now analyse the impact of switching from grid 0, to grids with a higher number of dates, taking into consideration two different ways of increasing the density: the short-term one and the long-term one. From our graphs of the risk measures of the portfolio and of the counterparties, we can deduct that the peaks, which provide a greater contribution towards identifying these measures, are all generated in the short-term. Moreover, we must also highlight the importance of inserting certain dates on the simulation grid, which correspond to the maturity of the instruments of the portfolio.

If we observe the further numerical analysis of grids 0, 1 and 2 on the 6-swap portfolio, we can make some further considerations about the strategy for increasing the density by monitoring the following relations of percentage change:

Table 8: Portfolio exposure profiles, grid0 vs grid1, grid0 vs grid2

	Grid 0 VS $\delta 1$ * grid 1	Grid 0 VS $\delta 2$ * grid 2
EPE portf	$\delta 1 = 0{,}990$	$\delta 2 = 1{,}024$
Eff EPE portf	$\delta 1 = 0{,}999$	$\delta 2 = 1{,}007$
MPFE portf	$\delta 1 = 1{,}039$	$\delta 2 = 0{,}995$

We can observe a positive percentage change of the EPE and of the effective EPE measurement when moving from grid 0 to grid 2, whereas a positive percentage change can be seen only for the max PFE when moving from grid 0 to grid 1.

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