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A Comparison of Extreme Value Theory with Heavy-tailed Distributions in Modeling Daily VAR

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Abstract

In this study, the performances of GARCH models with different distribution assumptions in modeling Value-at-Risk are evaluated by the backtesting procedure for three equity indexes. Recent researches indicate that Extreme Value Theory (EVT) is good candidate to model rare extreme events and unpredictable losses. Due to return series have non-normal characteristics, standardized residuals of GARCH are modeled by EVT and leptokurtic distributions. Empirical findings show that EVT based GARCH model is outperformed according to the backtesting results modeling daily VaR for all equity indexes.

JEL classification numbers: C22, C52, G15
Keywords: Value at Risk; Extreme Value Theory; GARCH estimation; Backtesting

1 Introduction

One of the most important challenges in modeling Value-At-Risk (VaR) is the distribution assumption made for financial return series. Most of the VaR models assume that financial return series are normally distributed. Recent researches indicate that normality assumption of return series is not valid in most cases when return series have heavy tails. Modeling VaR with normality assumption gives underestimate VaR forecasts. Therefore without considering extreme losses in financial return series is the main problem of the risk modeling. Therefore, EVT is good candidate modeling the tail of distribution that contains the extreme events. Many studies, especially McNeil and Frey (2000), Gencay et al. (2003), Gilli and Kellezi (2006), Onour (2010) and Singh et al. (2013, have evaluated the performance of EVT measuring the financial risk and also investigated tail behavior of financial returns series. When analyzed the recent studies, it is shown that EVT which is interested in extreme losses or extreme gains, outperforms with respect to other well-known

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models. McNeil and Frey (2000) proposed a GARCH-EVT model which contains two-step estimation procedure. The main aim of proposed model is to forecast the daily-VaR considering the extreme events in the tail of the distribution. Soltane et al. (2012) compared the GARCH-EVT model with GARCH-normal and other well-known models. Chan and Gray (2006) evaluated the performance of EVT approach to forecast daily VaR in electricity market. Karmakar (2013) also estimated the tail-related risk measure using McNeil and Frey's (2000) GARCH- EVT model in Indian stock market.

Due to returns exhibit skewness and excess kurtosis, normality assumption made for return series causes the underestimation or overestimation of the true VaR. Venkataraman (1997), Zangari (1996) used the mixture of normal distributions which is heavy-tailed and able to capture the extreme events. Lee et al. (2008) used the GARCH model under skewed generalized error distribution (GARCH-SGED) to forecast daily VaR and compared the forecast performance of GARCH-SGED with GARCH-normal. Skewed generalized error distribution is able to capture both skewness and kurtosis in financial returns series. Angelidis et al. (2004) used the GARCH model under student-t and generalized error distribution (GED) to forecast daily VaR considering the leptokurtic structure of the returns series. Ergen (2010) compared the forecast performance of GARC-EVT model with heavy-tailed GARCH models. According to these studies, leptokurtic distributions are able to produce better daily VaR forecasts.

In this paper, VaR forecast performance of GARCH-EVT model is compared with the GARCH-normal, GARCH-student-t, GARCH-GED and GARCH-SGED models for ISE-100, Nikkei-225 and S&P-500 stock exchange indexes. This study has two major aims. First one is to show that how distribution assumption made for residuals in GARCH models affects the daily-VaR forecasts and secondly daily-VaR forecasts of ISE-100, Nikkei-225 and S&P-500 stock exchange indexes are compared with various models by backtesting procedure.

The rest of the paper organized as follows: Section II presents the VaR and EVT comprehensively. Section III presents backtesting and GARCH models based on different distribution assumptions. Section IV presents data, descriptive statistics, empirical evidence, and final section presents the conclusion of study.

2 Financial Risk Measurement Based On EVT

VaR can be simply defined as follows:

$$VaR_{\alpha} = F^{-1}(1-\alpha)$$

where F is the distribution function of financial losses, F^{-1} denotes the inverse of F and α is the quantile at which VaR is calculated. EVT is strong method to capture extreme tails of distribution and also tail behavior of loss distribution. Modeling the extreme events, peaks over threshold methodology is used in recent applications. Peaks over Threshold (POT) method focuses on the distribution of exceedances over a threshold. F_u which is the conditional excess distribution can be defined as follows:

$$F_u(y) = P(x-u \le y / x > u), \quad 0 \le y \le x_F - u \tag{1}$$

where X is a random variable, denotes the financial losses, u is a threshold, $y = x-u$

are the excesses, called as extreme losses, $x_F \leq \infty$ is the right endpoint of F which is the distribution function of X. POT deal with the estimation of distribution function F_u which can be written in terms of F,

$$F_u(y) = \frac{\Pr\{x-u \leq y, x>u\}}{\Pr(x>u)} = \frac{F(y+u)-F(u)}{1-F(u)} = \frac{F(x)-F(u)}{1-F(u)} \tag{2}$$

A theorem by Balkema and de Haan (1974) and Pickands (1975) indicates that, for sufficiently high threshold, the excess distribution function F_u, can be approximated by Generalized Pareto Distribution (GPD):

$$F_u(y) \approx G_{\xi,\sigma}(y), \quad \text{u} \to \infty$$

$$G_{\xi,\sigma}(y) = \begin{cases} 1-(1+\xi \dfrac{x-\mu}{\sigma})^{-1/\xi} & , \ \xi \neq 0 \\ 1-e^{-y/\sigma} & , \ \xi = 0 \end{cases} \tag{3}$$

ξ is shape parameter, μ is the location parameter and σ is the scale parameter for GPD. When $\xi > 0$, it takes the form of the ordinary Pareto distribution which is the most suitable for financial return series. When $\xi = 0$, the GPD takes the shape of exponential distribution and it is known as a Pareto II type distribution for $\xi < 0$ (Gencay and Selcuk, 2004).

$$F_u(y) = \frac{F(y+u)-F(u)}{1-F(u)} = \frac{F(x)-F(u)}{1-F(u)} \tag{4}$$

$F(x)$ can be isolated from (4),

$$F(x) = (1-F(u))F_u(y) + F(u) \tag{5}$$

$F_u(y)$ and $F(u)$ are replaced respectively by GPD and $(n-N_u)/n$, n is the total number of observations and N_u is the number of observations above the threshold. $\hat{F}(x)$ can be obtained as follows:

$$\hat{F}(x) = \frac{N_u}{n}(1-(1+\frac{\hat{\xi}}{\hat{\sigma}}(x-u))^{-1/\hat{\xi}} + (1-\frac{N_u}{n}) = 1-\frac{N_u}{n}(1+\frac{\hat{\xi}}{\hat{\sigma}}(x-u))^{-1/\hat{\xi}} \tag{6}$$

We can obtain the VaR_p inverting (6) for a given probability,

$$VaR_p = u + \frac{\hat{\sigma}}{\hat{\xi}}[(\frac{n}{N_u}p)^{-\hat{\xi}} - 1] \tag{7}$$

Determination of threshold is critical importance for the GPD modeling. The most used method is Mean Excess (ME) Plot for determination of threshold. ME Plot can be defined as follows:

$$(u, e_n(u)), \quad x_1^n < u < x_n^n \tag{8}$$

where, $e_n(u)$ is the sample mean excess function,

$$e_n(u) = \frac{\sum_{i=k}^{n}(x_i^n - u)}{n-k+1}, \quad k = \min\{i \ / \ x_i^n \ > \ u\} \tag{9}$$

and $n-k+1$ is the number of observations exceeding the threshold u (Gilli and Kellezi, 2006).

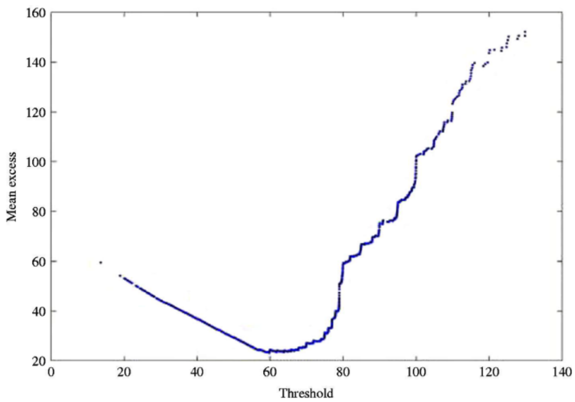

Figure 1: Mean Excess Plot

Figure 1: shows linearity in a region where above the threshold u, the data can be modeling with GPD. Sing et al. (2013) interpreted the linearity as follows:

- Upward linear trend indicates a positive shape parameter (ξ) for the GPD
- Horizontal linear trend indicates a GPD with $\xi \approx 0$
- Linear downward trend can be interpreted as GPD with negative ξ

3 Garch Models In VaR Estimation

Garch-normal model

Let $R_t = \ln(S_t / S_{t-1}) \times 100$ denotes the daily returns of the assets on time t and S_t represents the closed prices of the assets. Engle (1982) introduced the ARCH(q) model and expressed the conditional variance as a linear function of the past q squared residuals. Bollerslev (1986) proposed a generalization of the ARCH model, GARCH(1,1) model with *normal* error distribution can be written as follows:

$$\begin{aligned} R_t &= \mu + e_t \\ e_t &= \varepsilon_t \sigma_t, \ e_t \ i.i.d.N(0,1) \\ \sigma_t^2 &= \omega + \alpha e_{t-1}^2 + \beta \sigma_{t-1}^2 \end{aligned} \tag{10}$$

where respectively, μ and σ_t^2 are the conditional mean and variance. To ensure the stationarity condition and positive variance below equations must be hold.

$\alpha + \beta < 1, \alpha > 0, \beta > 0$ and ω>0

Log-likelihood function of GARCH-normal model under normality assumption can be written as:

$$L(\psi) = -0.5\left(T\ln 2\pi + \sum_{t=1}^{T} \ln \sigma_t^2 + \sum_{t=1}^{T} \varepsilon_t^2 \right) \quad (11)$$

where $\psi = (\mu, \omega, \alpha, \beta)$ is the parameter vector. According to GARCH-N model, one-day-ahead VaR forecast can be calculated as:

$$VaR_{t+1} = \mu + F_\alpha(\varepsilon_t).\hat{\sigma}_t \quad (12)$$

where $F_\alpha(\varepsilon_t)$ is the left quantile of standard normal distribution at α level. Generally, μ is equal to 0 for daily returns of assets.

Garch-student-t model

Engle (1982) assumed the distribution of ε_t as normal. Bollerslev (1986, 1987) proposed the standardized student-t distribution with $\upsilon > 2$ degree of freedom. Student's-t is symmetric distribution and for $\upsilon > 4$, conditional kurtosis greater than 3, which exceeds the normal value. Under this specification, log-likelihood function, for a sample of T observations, can be written as follows:

$$L(\psi) = T\left[\ln\Gamma\left(\frac{\upsilon+1}{2}\right) - \ln\Gamma\left(\frac{\upsilon}{2}\right) - \frac{1}{2}\ln\left[\pi(\upsilon-2)\right] \right] - \frac{1}{2}\sum_{t=1}^{T}\left[\ln\sigma_t^2 + (1+\upsilon)\ln\left(1+\frac{\varepsilon_t^2}{\upsilon-2}\right) \right] \quad (13)$$

where $\Gamma(\upsilon)$ is the gamma function and υ is the thickness parameter of the distribution tails. The one-day-ahead VaR forecast based on SGED distribution can be calculated as follows:

$$VaR_{t+1} = \mu + F_\alpha(\varepsilon_t).\hat{\sigma}_t$$

where $F_\alpha(\varepsilon_t)$ is the left quantile of the student-t distribution at α level.

Garch-GED model

In order to model the excess kurtosis observed asset prices, assumption on ε_t can be relaxed. Nelson (1991) proposed the generalized error distribution GED instead of assuming ε_t is normally distributed. Under this specification, log-likelihood function for GED distributed ε_t :

$$L(\psi) = \sum_{t=1}^{T}\left[\ln\left(\frac{\upsilon}{2}\right) - \frac{1}{2}\left|\frac{\varepsilon_t}{\lambda}\right|^{\upsilon} - (1+\upsilon^{-1})\ln(2) - \ln\Gamma\left(\frac{1}{2}\right) - \frac{1}{2}\ln\left(\sigma_t^2\right) \right] \quad (14)$$

where υ is the tail-thickness parameter and

$$\lambda = \left(\frac{\Gamma\left(\frac{1}{\upsilon}\right)}{2^{\frac{2}{\upsilon}}\Gamma\left(\frac{3}{\upsilon}\right)} \right)^{\frac{1}{2}} \tag{15}$$

where $\Gamma(.)$ is the gamma function. The Gaussian distribution is a special case of GED distribution when $\upsilon = 2$. If $\upsilon < 2$, GED has fatter tails than Gaussian distribution. According to Nelson (1991) specification, log-likelihood function can be written as follows:

$$L(\psi) = \sum_{t=1}^{T}\left[\ln\left(\frac{\upsilon}{\lambda}\right) - \frac{1}{2}\left|\frac{\varepsilon_t}{\lambda}\right|^{\upsilon} - \left(1+\upsilon^{-1}\right)\ln(2) - \ln\Gamma\left(\frac{1}{\upsilon}\right) - \frac{1}{2}\ln(\sigma_t^2) \right] \tag{16}$$

According to GARCH(1,1) model, $\sigma_t^2 = \omega + \alpha e_{t-1}^2 + \beta\sigma_{t-1}^2$ in the above equation. Parameters of the GARCH(1,1) model can be obtained by the numerical maximization procedure. The one-day-ahead VaR forecast based on SGED distribution can be calculated as follows:

$$VaR_{t+1} = \mu + F_\alpha(\varepsilon_t, \upsilon).\hat{\sigma}_t$$

Where $F_\alpha(\varepsilon_t, \upsilon)$ is the left quantile of GED distribution at α level.

Garch-SGED model

Lee et al. (2008) used the SGED distribution which provides a flexible distribution for modeling the empirical distribution of financial data. Probability density function of standardized SGED distribution can be written as follows:

$$f(\varepsilon_t) = C\exp\left(-\frac{|\varepsilon_t + \delta|^{\kappa}}{\left[1 + sign(\varepsilon_t + \delta)\lambda\right]^{\kappa}\theta^{\kappa}} \right) \tag{17}$$

where

$$C = \frac{k}{2\theta}\Gamma\left(\frac{1}{\kappa}\right)^{-1}, \theta = \Gamma\left(\frac{1}{\kappa}\right)^{0.5}\Gamma\left(\frac{3}{\kappa}\right)^{0.5} S(\lambda)^{-1}$$

$$S(\lambda) = \sqrt{1 + 3\lambda^2 - 4A^2\lambda^2}, \delta = \frac{2\lambda A}{S(\lambda)} \tag{18}$$

$$A = \Gamma\left(\frac{2}{\kappa}\right)\Gamma\left(\frac{1}{\kappa}\right)^{-0.5}\Gamma\left(\frac{3}{\kappa}\right)^{-0.5}$$

where κ is the shape parameter with constraint $\kappa > 0$, λ is skewness parameter with $-1 < \lambda < 1$. SGED distribution turns out to be the standart normal distribution when $\kappa = 2$ and $\lambda = 0$. Log-likelihood function of GARCH-SGED model can be written as follows:

$$L(\psi) = -\frac{|R_t - \mu/\sigma_t + \delta|^{\kappa}}{\left[1 + sign\left(R_t - \mu/\sigma_t + \delta\right)\lambda\right]^{\kappa}\theta^{\kappa}} \tag{19}$$

where ψ is the parameter vector. The one-day-ahead VaR forecast based on SGED distribution can be calculated as follows:

$$VaR_{t+1} = \mu + F_\alpha(\varepsilon_t, \kappa, \lambda).\hat{\sigma}_t$$

where $F_\alpha(\varepsilon_t, \kappa, \lambda)$ is the left quantile of SGED distribution at α level.

Garch-EVT model

EVT relies on an assumption of independent and identically distributed (i.i.d.) observations. Generally, it is not true and unrealistic assumption for the financial return series. To overcome this problem, McNeil and Frey (2000) proposed a two-stage approach.

Proposed model can be summarized as follow:

1. First step, GARCH (1,1) model is fitted to the return series by pseudo maximum likelihood estimation (PML) and gives the residuals for step-2 and also 1 day ahead predictions of μ_{t+1} and σ_{t+1}.
2. Second step, EVT-POT method is applied to the residuals of GARCH model. The most important point of this method is selection of threshold u. Using the parameter estimation of EVT-POT method and also predictions of μ_{t+1} and σ_{t+1}, VaR_{t+1} can be calculated easily.

The one-day-ahead VaR forecast based on GARCH-EVT model can be calculated as follows:

$$VaR_{t+1} = \mu + F_\alpha(\varepsilon_t; \xi, \sigma).\hat{\sigma}_t$$

where $F_\alpha(\varepsilon_t; \xi, \sigma)$ is obtained by the POT estimation procedure. To compare the forecasting ability of these models in terms of VaR forecasts, backtesting methodology is used. Kupiec (1995) proposed a LR test for evaluating the model accuracy. The LR test statistic can be written as follows:

$$LR = -2\ln\left[\frac{p^{n_1}(1-p)^{n_0}}{\hat{\pi}^{n_1}(1-\pi)^{n_0}}\right]\chi_1^2 \tag{20}$$

where $\hat{\pi} = n_1/(n_0+n_1)$ is the maximum likelihood estimation of p, n_1 represents the total violations and n_0 represents the total non-violations forecasts. Under the null hypothesis $(H_0 : p = \hat{\pi})$, LR statistics follows a chi-square distribution with one degree of freedom.

4 Empirical Results

The return series contain different sample sizes, 1256 observations for ISE-100 and S&P-500, 1211 observations for Nikkei-225 indexes. Table 1 gives the descriptive statistics of daily log returns for ISE-100, S&P-500 and Nikkei-225 indexes. According to Table 1, for all equity indexes mean returns is closed to 0. Skewness and kurtosis are significantly different from the 0 and 3 for normal distribution and also *JB* test statistics are far greater than the critical value at %5 level and *p-value* is 0. Therefore, log returns of three indexes have the non-normal characteristics, excess kurtosis and fat tails.

Table 1: Descriptive statistics of the daily log returns

	ISE-100	S&P-500	Nikkei-225
Number of observations	1256	1256	1211
Minimum	-0.09013	-0.09469	-0.12111
Maximum	0.12127	0.10957	0.13234
Mean	0.00031	0.00002	-0.00020
Median	0.00085	0.00072	0.00037
Std. Deviation	0.01838	0.01661	0.01813
Skewness	-0.11998	-0.24613	-0.50283
Kurtosis	3.85865	6.94596	7.99124
Jarque-Bera	786.645	2549.15	3409.73
Probability	0	0	0

Table 2. represents the parameter estimates of GARCH(1,1) model. *Normal*, *Students't*, *GED*, SGED and *GPD* are assumed for standardized residuals. To implement GARCH—EVT model, threshold value of GPD is determined with respect to 90th quantile of the standardized residuals. According to Table 2., conditional variance parameters are highly significant and $\omega > 0, \alpha \geq, \beta \geq 0$ and $\alpha + \beta < 1$ conditions are hold for the positive variance and stationarity condition. All parameters satisfy the assumption of GARCH(1,1) model. To evaluate the out of sample performance of these models, rolling window estimation procedure is used. Firstly, to implement the rolling window estimation, window length must be determined. Because of the returns series contains different sample size, window length is differently determined for all equity indexes to evaluate out of sample performance of models with equal forecast period. Window length of equity indexes are respectively 1256 for ISE-100 and S&P-500 and 1211 for Nikkei-225 indexes.

Table 2: Parameter estimates of GARCH(1,1) model for three indexes, assuming four different distributions for the standardized residuals

Parameter	**ISE-100**		**S&P-500**	**Nikkei-225**	
			Normal Distribution		
ω	0.000006	(0.000003)	0.000003 (0.000001)	0.000008	(0.000003)
α	0.112678	(0.020851)	0.111790 (0.015839)	0.135926	(0.022036)
β	0.871191	(0.023936)	0.878648 (0.014850)	0.834648	(0.025347)
LL	3397.101		3708.001	3511.063	
			Student's t Distribution		
ω	0.000005	(0.000002)	0.000002 (0.000001)	0.000007	(0.000003)
α	0.090481	(0.022490)	0.107612 (0.019033)	0.115638	(0.022811)

β	0.896608 (0.025681)	0.888907 (0.016700)	0.856217 (0.026635)
υ	7.923236 (1.678036)	6.024970 (1.171811)	18.019092 (7.823826)
LL	3413.187	3726.328	3514.426
		Generalized Error Distribution	
ω	0.000005 (0.000003)	0.000002 (0.000001)	0.000008 (0.000003)
α	0.103268 (0.023584)	0.109035 (0.019221)	0.126417 (0.023145)
β	0.881287 (0.027161)	0.884197 (0.017566)	0.844314 (0.027000)
υ	1.489908 (0.081856)	1.274209 (0.074468)	1.700551 (0.104832)
LL	3411.697	3726.328	3514.517
		Skewed Generalized Error Distribution	
ω	0.000005 (0.000003)	0.000002 (0.000001)	0.000008 (0.000003)
α	0.098602 (0.022346)	0.106762 (0.018478)	0.124616 (0.022399)
β	0.885828 (0.025862)	0.884948 (0.017236)	0.842935 (0.026793)
λ	0.944994 (0.034729)	0.918362 (0.030316)	0.893770 (0.036767)
κ	1.487202 (0.081611)	1.304730 (0.076731)	1.769865 (0.114814)
LL	3412.857	3739.734	3518.577
		Generalized Pareto Distribution	
ω	0.000006 (0.000004)	0.000003 (0.0000008)	0.000008 (0.0003662)
α	0.1129 (0.03440)	0.1108 (0.01856)	0.1361 (0.03175)
β	0.8709 (0.03901)	0.8796 (0.01552)	0.83845 (0.03296)
ξ	0.04940891	0.1910409	0.06468743
σ	0.5191189	0.7606422	0.55519733
LL	3397.104	3708.001	3511.063
Observations	1256	1256	1256

Standard errors are presented in parentheses.

Out-of-Sample Performance of VaR Models and Backtesting Results

As mentioned above, rolling estimation procedure is used to obtain VaR forecasts at %95 and %99 confidence level for all equity indexes using standart normal distribution, student's t distribution, GED, SGED and generalized parato distribution. According to Table 3., forecast period is the same for all indexes. To evaluate the performance of the models violation based backtesting method is used. According to backstesting results obtained for ISE-100 index, GARCH-EVT model is outperformed with respect to the other model for %95 confidence level. GARCH-EVT and GARCH-SGED model have the same violations (5) for %99 confidence level. *LR-uc* is used to test equality of expected violation and observed violation.

Table 3: Out of sample performance of models according to bactesting results for ISE-100 index

		%95 confidence level		
ISE-100	**Number of Forecasts**	**Expected Violation**	**Observed Violation**	**LR-uc**
GARCH-normal	439	22	27	1.143 (0.285)
GARCH-student's t	439	22	30	2.802 (0.094)
GARCH-GED	439	22	27	1.143 (0.285)
GARCH-SGED	439	22	22	0 (0.991)
GARCH-EVT	439	22	18	0.79506 (0.3726)
		%99 confidence level		
ISE-100	**Number of Forecasts**	**Expected Violation**	**Observed Violation**	**LR-uc**
GARCH-normal	439	4	11	7.089 (0.008)
GARCH-student's t	439	4	6	3.841 (0.464)
GARCH-GED	439	4	6	3.841 (0.464)
GARCH-SGED	439	4	5	0.082 (0.775)
GARCH-EVT	439	4	5	0.082 (0.775)

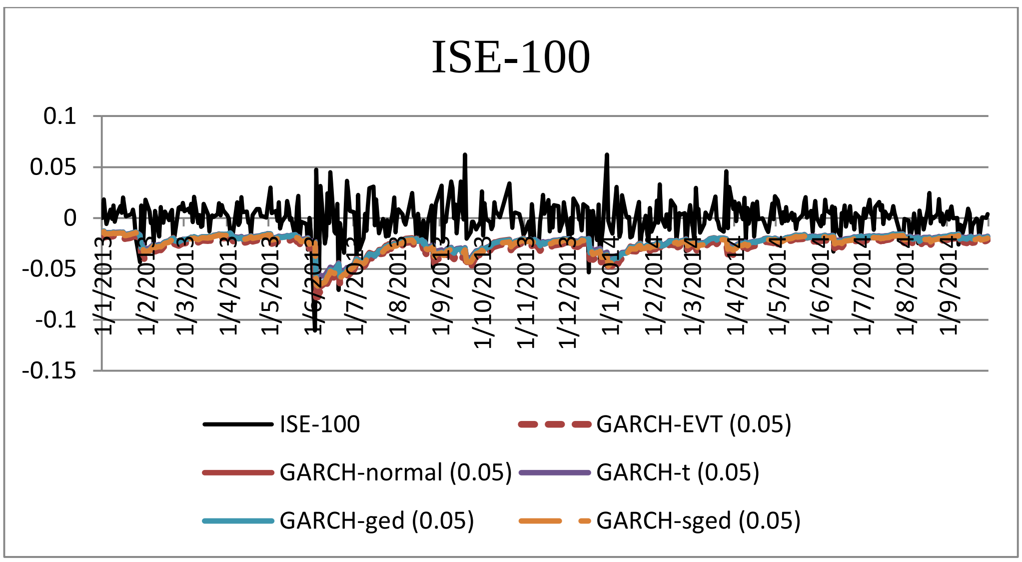

Figure 2: Daily VaR forecasts for ISE-100 index at %95 confidence level

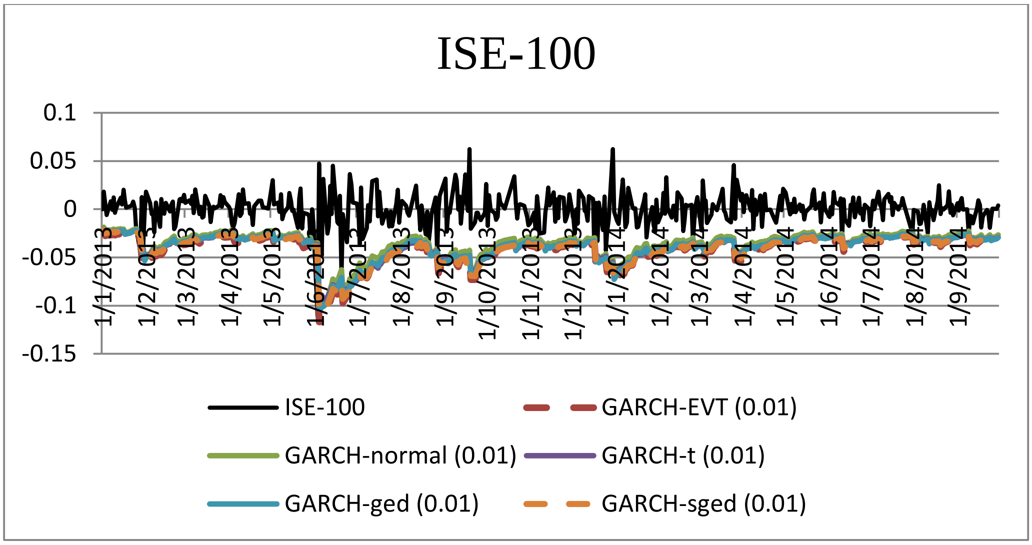

Figure 3: Daily VaR forecasts for ISE-100 index at %99 confidence level

According to backstesting results obtained for S&P-500 index, GARCH-EVT model is outperformed with respect to the other model for %95 confidence level. GARCH-EVT and GARCH-SGED model have the same violations (3) for %99 confidence level.

Table 4: Out of sample performance of models according to bactesting results for S&P-500 index

%95 confidence level				
S&P-500	**Number of Forecasts**	**Expected Violation**	**Observed Violation**	**LR-uc**
GARCH-normal	439	22	22	0 (0.991)
GARCH-student's t	439	22	23	0.052 (0.819)
GARCH-GED	439	22	23	0.052 (0.819)
GARCH-SGED	439	22	21	0.084 (0.834)
GARCH-EVT	439	22	15	2.5936 (0.1073)
%99 confidence level				
S&P-500	**Number of Forecasts**	**Expected Violation**	**Observed Violation**	**LR-uc**
GARCH-normal	439	4	9	3,751 (0.053)
GARCH-student's t	439	4	5	0.082 (0.775)
GARCH-GED	439	4	5	0.082 (0.775)
GARCH-SGED	439	4	3	0.5 (0.479)
GARCH-EVT	439	4	3	0.5 (0.479)

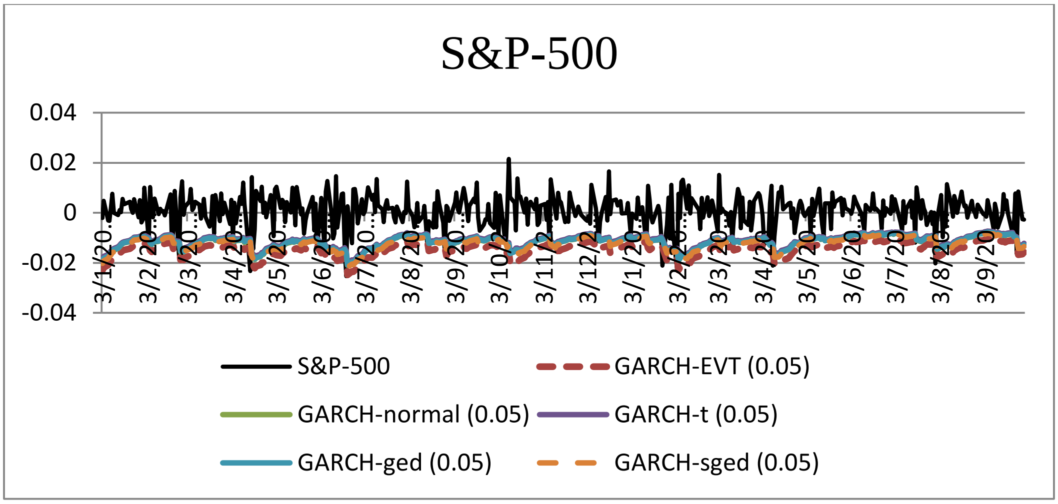

Figure 4: Daily VaR forecasts for S&P-500 index at %95 confidence level

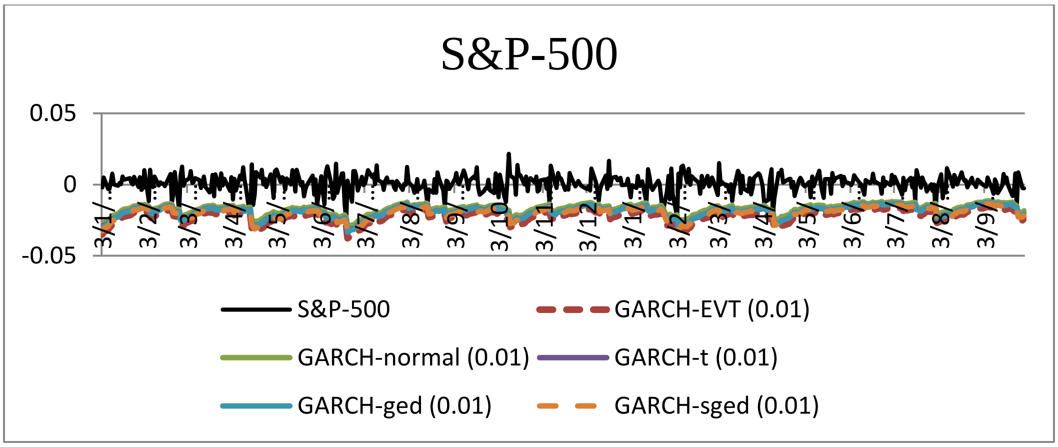

Figure 5: Daily VaR forecasts for S&P-500 index at %99 confidence level

According to backstesting results obtained for Nikkei-225 index, GARCH-EVT model is outperformed with respect to the other model for %95 and %99 confidence level. GARCH-EVT and GARCH-SGED models are outperformed because of the non-normal characteristics (leptokurtic) of the standardized residuals.

Table 4: Out of sample performance of models according to bactesting results for Nikkei-225 index

%95 confidence level				
Nikkei-225	**Number of Forecasts**	**Expected Violation**	**Observed Violation**	**LR-uc**
GARCH-normal	439	22	23	0.052 (0.819)
GARCH-student's t	439	22	23	0.052 (0.819)
GARCH-GED	439	22	23	0.052 (0.819)
GARCH-SGED	439	22	22	0 (0.991)
GARCH-EVT	439	22	20	0.1877 (0.6648)
%99 confidence level				
Nikkei-225	**Number of Forecasts**	**Expected Violation**	**Observed Violation**	**LR-uc**
GARCH-normal	439	4	9	3.751 (0.053)
GARCH-student's t	439	4	6	0.535 (0.464)
GARCH-GED	439	4	5	0.082 (0.775)
GARCH-SGED	439	4	4	0.036 (0.849)
GARCH-EVT	439	4	3	0.5 (0.479)

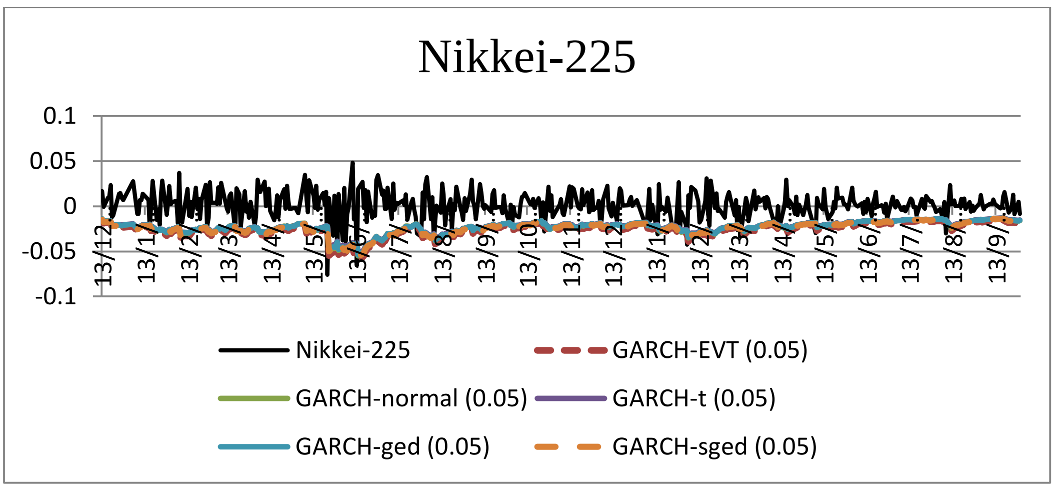

Figure 6: Daily VaR forecasts for Nikkei-225 index at %95 confidence level

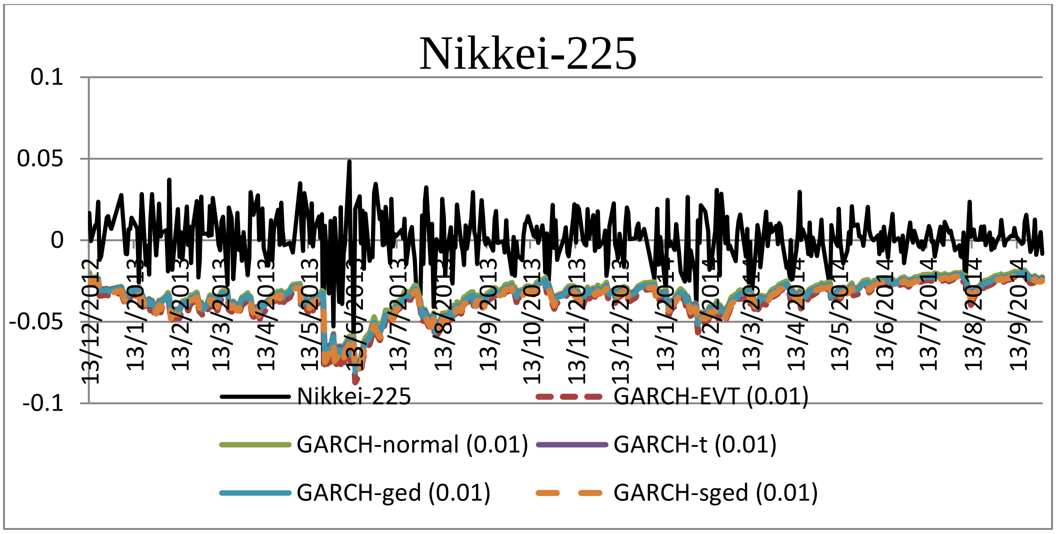

Figure 7: Daily VaR forecasts for Nikkei-225 index at %99 confidence level

5 Conclusion

ISE-100 index is badly affected by the reason of the recent political issues. When analyzed the returns series of ISE-100 index, downside price movements are draw attention. Therefore, financial risk management is become so important for the market regulators and investors.

By the reason of the financial instability, big and unpredictable losses, EVT might be good candidate to model tail behavior of return series measuring the VaR. For this purpose, GARCH-normal, GARCH-student's t, GARCH-GED, GARCH-SGED and GARCH-EVT models are compared by the backtesting results for three equity indexes. This study shows that GARCH-SGED and especially GARCH-EVT model are outperformed when the returns series have non-normal characteristics, excess kurtosis and skewness. GARCH-EVT model take into account the leptokurtosis of the standardized residuals. Other models are insufficient to measure true VaR and cannot respond quickly to changing volatility.

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