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Stochastic Valuation of Segregated Fund Contracts in an Emerging Market

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Abstract

Stochastic valuation modeling is an important area for financial professionals who deal in products such as equity insurance, especially segregated fund contracts. A stochastic analysis of the guarantee liabilities under any given segregated fund contract requires a credible long-term model of the underlying equity (stock) return process. This paper introduced econometric models far less complex than the Wilkie model for valuing and managing financial risks associated with combined guaranteed minimum maturity benefit and minimum death benefit (GMMB/GMDB) regarding segregated fund contracts in an emerging market (India). Finally, we assess the valuation model via simulation under the GMMB/GMDB for a life age 50 with varying assumptions about the margin offset. The simulation results clearly indicate that, the net present value of outgo is mostly in the negative.

JEL classification numbers: G12, C15, G22
Keywords: Stochastic simulation, Investment guarantees, Guarantee liabilities

1 Introduction

The basic segregated fund contract is a single premium policy, under which most of the premium is invested in one or more mutual funds on the policyholder's behalf. The name "segregated fund" refers to the fact that the premium, after deductions, is invested in a fund separate from the insurer's funds. The management of the segregated funds is often independent of the insurer.

A stochastic analysis of the guarantee liabilities under any given segregated fund contract requires a credible long-term model of the underlying equity (stock) return process.

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However, there are many stochastic models in common use for the equity return process. Actuaries have no general agreement on the form of such a model (see [1]). There are vast numbers of potential stochastic models for equity returns. For instance we have the traditional lognormal stock return model, regime-switching lognormal (RSLN) processes for modeling monthly equity returns popularized by [1] and many more.

A model of equity returns and treasury bond for long-term applications was developed by Wilkie (see [2] and [3]) in relation to the United Kingdom market, and subsequently fitted to data from other markets, including the United States and Canada. The Canadian data (1923-1993) were used for the figures for quantile reserves for segregated fund contracts in [4]. In spite of the usefulness of the Wilkie's model, it has been subjected to vigorous criticisms. For details on these criticisms see [5].

The aim of this paper is twofold. The first is to introduce two different time series econometric processes for modeling long-term equity returns and treasury bonds. The second is to apply a dynamic hedging approach which uses financial engineering technique for finding a replicating portfolio with payoff equivalent to the payoff of the guaranteed liabilities. The remaining of this paper is organized as follows. Section 2 introduces the vector autoregressive (VAR) and co-integrated vector autoregressive (COVAR) processes for modeling the long-term equity returns and treasury bonds respectively. Section 3 illustrates the empirical results of the VAR process using monthly data from the Colombo stock, Bombay stock and Karachi stock exchanges and COVAR process using monthly data from the India money market from August 1997 to June 2009. Developing countries are also known as emerging markets are gradually becoming the propellers of growth around the world. This paper focuses on India because, among emerging markets, it is considered to be the largest alongside China. Quite apart from that, the India unit-linked insurance contracts are also separate account insurance quite similar to the Canadian segregated fund contracts. However, the regulations governing unit linked products are still being developed to follow closely that of Canadian products. The choice of Colombo and Karachi stock indices is to allow accurate estimation of parameters of the long-term equity return model. Extension of the models to incorporate the valuation formulae for the combined guaranteed minimum maturity benefit/guaranteed minimum death benefit (GMMB/GMDB) contract and numerical results are discussed in section 4. Finally, section 5 provides concluding remarks.

It is imperative to mention that, this paper is a follow up to our previous studies of the same markets (see [6]). In that study, the data used were from August 1997 to July 2007, however, it could not consider the effect of the margin offset on the hedge cost (or profitability) and the probability of a loss. This paper differs from [6] in two ways. One, the sample period considered in the present paper is from August 1997 to June 2009. Two, the behavior of the hedge cost (or profitability) and probability of a loss at varying values of the margin offset is investigated.

2 Long-Term Equity Return and Treasury Bond Models

In this section, we provide a brief description of the VAR model for capturing the long term equity returns. Similarly, the COVAR model in capturing the treasury bonds.

2.1 Long-Term Equity Return Model

The long term equity return process follows the VAR model. Prior to modeling the returns, we first transformed it by taking the logarithm transformation as follows:

$$x_t = log(1 + r_t) \tag{1}$$

where x_t is the logarithm of the returns and r_t is the actual returns which is obtained using the following relation:

$$r_t = \frac{S_t}{S_{t-1}} \tag{2}$$

where S_{t-1} and S_t are the equity (stock) price at time $t - 1$and t respectively.
A multivariate time series x_t follows a VAR (p) model if it satisfies

$$x_t = c + \Phi_1 x_{t-1} + \cdots + \Phi_k x_{t-k} + \varepsilon_t \qquad p > 0 \tag{3}$$

where c is a k dimensional vector, Φ is a $k \times k$ matrix and $\{\varepsilon_t\}$ is a sequence of serially uncorrelated random vectors with mean zero and covariance matrix Σ which is positive definite. VAR models in economics were made popular by [7] and VAR of order 1 is obtained by letting $p = 1$ or VAR (1) for short.
We use two widely known methods in time series econometrics to test the suitability of the individual equity returns prior to fitting the VAR model. Basically, these methods check the existence of unit-root in a time series and they are the Augmented Dickey Fuller (ADF) test by [8] and Phillip and Perron (PP) test by [9]. To measure correlation in this paper, the cross correlation analysis is performed and a method proposed by [10] is employed to check the statistical significance of the correlation coefficients at different lags.
The estimation of the parameters of the VAR model can be achieved by the ordinary least squares (OLS) method or the maximum likelihood (ML) method. For the OLS method for the VAR model; see [11] or [12]. Details of the ML estimation method for the VAR model are discussed in [13]. The two methods are asymptotically equivalent under some regularity conditions and the estimates are asymptotically normal. Hence asymptotically valid t-test on coefficients may be constructed in the usual way. The lag length selection process is a procedure employed to accurately re-estimate the VAR model. The process is first to fit a VAR (p) model with orders $p = 0, \ldots, p = p_{max}$ and choose the value of p which minimizes some model selection criteria. In this paper, we used two of the well know selection criteria. They are the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC). For more information on the use of model selection criteria in VAR models (see [12]).

2.2 Treasury Bond Model

The treasury bond model follows the COVAR process. Modeling several unit-root nonstationary time series leads to cointegration. The step by step procedure for cointegration in this paper is similar to what is presented in [14]. To better understand cointegration, we re-express (3) in another form such that c is replaced by: $c_t = c_0 + c_1 t,$ where c_0 and c_1 are k-dimensional constant vectors. If the zeros of the characteristic polynomial

$|\Phi(B)| = \left|I - \Phi_1 \mathrm{B} - \cdots - \Phi_p B^p\right|$ lies outside the unit circle, x_t is stationary $(I(0))$. However, if $|\Phi(1)| = 0,$ then x_t is unit-root nonstationary$(I(1))$.

A Vector Error Correction Model (VECM) for the VAR (p) model x_t is:

$$\Delta x_t = c_t + \Pi x_{t-1} + \Phi^*_1 \Delta x_{t-1} + \cdots + \Phi^*_{t-p+1} \Delta \mathrm{x}_{t-1} + \varepsilon_t \tag{4}$$

We refer to the term Πx_{t-1} as the error-correction term, which is the key component in the study of cointegration. Assume $0 < rank(\Pi) = m < k,$ then x_t is said to be cointegrated with m linearly independent cointegration vectors, and has $k - m$ unit-roots that give $k - m$ common stochastic trend of x_t.
To estimate the COVAR (p) process, the maximum likelihood estimation technique recommended by [14] is employed. The cointegration test involves ML test for testing the rank of Π in (4). In this paper, both cointegration trace test and the likelihood ratio (sequential procedure) test proposed by [15] are used. The critical values of the test statistics of these tests are nonstandard, however are evaluated by way of simulation.

3 Empirical Results

This section provides the empirical results of the processes discussed under section 2. To proceed, we first examine the statistical properties of the stock market indices of Colombo stock exchange (CSE), Bombay stock exchange (BSE) and Karachi stock exchange (KSE). In a similar manner, the statistical properties of the “up to 14 days”, “15-91 days”, “92-182 days” and “183-364 days” yield to maturity (YTM) from the India money market are investigated as well. We now direct our attention to the statistical summaries of the monthly stock returns of CSE, BSE and KSE from August 1997 to June 2009.

3.1 Descriptive Statistics

Table 1 presents the summary statistics for the monthly stock returns of CSE, BSE and KSE. The table shows that, the highest mean return is reported for KSE followed by BSE and CSE. The table further reveals that, BSE and KSE are negatively skewed, however, the CSE is skewed to the right. The three (3) national stock market indices do not only show evidence of positive kurtosis, but also heavy tailed. The normality test based on the Jarque-Bera (J-B) statistics is also shown in table 1. Apart from KSE, the rest showed a probability value greater than the five (5) percent significant level. On the basis of this information, it can be said that KSE is not normal.

Table 1: Statistics of Monthly Stock Market Returns

Stock Index	Mean	Volatility	Skewness	Kurtosis	JB Statistic	P-Value
CSE	0.00312	0.03318	0.10680	3.45200	1.48800	0.47520
BSE	0.00369	0.03542	-0.39780	3.48400	5.16980	0.07540
KSE	0.00389	0.04543	-1.03960	6.38800	94.14280	0.00000

Table 2: Statistics of Monthly Yield to Maturity (YTM)

YTM (Days)	Mean	Volatility	Skewness	Kurtosis	JB Statistic	P-Value
Up to 14	0.06377	0.01731	0.57490	3.39800	8.82150	0.01210
15-91	0.06805	0.01885	0.50240	3.06100	6.03700	0.04890
92-182	0.07124	0.02095	0.90300	4.71200	36.90630	0.00000
183-364	0.07370	0.02077	0.31710	2.27000	5.57500	0.06160

Taking a closer look at India's money market, it is obvious that movements of the treasury bond rates stimulate further interest to investigate the applicability of all the 4 YTM in the valuation of segregated funds in India. Also, summary statistics of the 4 YTM displayed in table 2 indicate that the highest mean YTM is the 183 to 364 days followed by the 92-182 days, 15-91 days and up to 14 days YTM. The largest volatility is exhibited by the 92-182 days, followed by the 183-364 days, then 15-91 days and up to 14 days YTM. Table 2 further reveals that, all the 4 YTM are positively skewed. However, the only YTM which is not heavy tailed is the 183-364 days YTM. Normality checks based on the J-B statistic performed on the YTM, show that, only the 183-364 days YTM do follow the normal distribution when the test is done at the 5 percent significant level.

3.2 Unit-Root Tests and Lag Length Selection

This part of the empirical analysis further examines the time series properties of the stock market return indices from the 3 national stock markets. A similar analysis is performed on the various YTM from India's money market. The unit-root tests used in this paper to examine the time series properties are the ADF test and the PP test. For analytical completeness, however, we repeat the unit-root test under the ADF approach by considering no trend and deterministic trend.

Table 3: ADF and PP Tests for the Three National Stock Markets

ADF Test				PP Test		
Stock Index	**Hypothesis**	**Test Statistic**	**P-Value**	**Stock Index**	**Test Statistic**	**P-Value**
CSE	No Trend	-5.05100	0.00004	-	-	-
BSE	No Trend	-4.28100	0.00071	-	-	-
KSE	No Trend	-4.22600	0.00087	-	-	-
CSE	With Trend	-5.06400	0.00028	CSE	-126.30000	0.00000
BSE	With Trend	-4.28300	0.00447	BSE	-135.10000	0.00000
KSE	With Trend	-4.21200	0.00562	KSE	-136.50000	0.00000

The test results are reported in Table 3. The results indicate that there is no evidence of unit roots in the stock market returns of CBS, BSE and KSE at the five (5) per cent level over the entire sample periods. Therefore the null hypothesis of a unit-root in the stock market returns of CBS, BSE and KSE can be rejected at the 5 per cent significant level in all cases. The YTMs from the Indian money market show evidence of unit-roots at the 5 per cent level over the entire sample periods (Table 4). Therefore the null hypothesis of a unit root in the Indian money market cannot be rejected at the 5 per cent significant level in all cases.

Table 4: ADF and PP Tests for the Treasury Bond Market

ADF Test				PP Test		
YTM (Days)	**Hypothesis**	**Test Statistic**	**P-Value**	**YTM (Days)**	**Test Statistic**	**P-Value**
Up to 14	No Trend	-1.68000	0.43920	-	-	-
15-91	No Trend	-1.69600	0.43130	-	-	-
92-182	No Trend	-2.48000	0.12270	-	-	-
183-364	No Trend	-2.14500	0.22780	-	-	-
Up to 14	With Trend	0.43920	0.40140	Up to 14	-17.90000	0.01660
15-91	With Trend	0.43130	0.65390	15-91	-12.91000	0.06120
92-182	With Trend	0.12270	0.28750	92-182	-10.27000	0.11990
183-364	With Trend	0.22780	0.52760	183-364	-6.17800	0.32650

The next task is to determine the appropriate lag length for fitting and re-estimating both the VAR and the COINT-VAR processes. For the VAR model, both the AIC and BIC criteria are computed with a maximum lag length of 6. The AIC criterion is minimized when p =2 while the BIC criterion is minimized when p =1. For the COINT-VAR model, again priority is given to the BIC criterion where p =1. The test results are reported in Table 5.

Table 5: Appropriate Lag Length Selection Criteria

Model	Equity Process		Bond Process	
	BIC	**AIC**	**BIC**	**AIC**
One	-1503.34	-1538.38	-4088.44	-4146.84
Two	-1484.43	-1545.75	-4051.05	-4156.17
Three	-1450.95	-1538.55	-4034.77	-4186.61
Four	-1411.34	-1525.22	-3973.90	-4172.46
Five	-1370.27	-1510.43	-3926.21	-4171.49
Six	-1342.91	-1509.35	-3886.96	-4178.96

3.3 The VAR (1) Process

To fit the VAR (1) model to the long-term equity return process, there is the need to check whether the individual return series are correlated.
The asymptotically 5 percent critical value of the sample correlation is 0.09 using the method proposed by [10]. It is seen from Table 6 that, significant cross-correlation at the approximate 5 percent level appears at lags one, two and three. However, priority is given to lag one on the grounds of parsimony. An examination of the sample cross-correlation matrices further indicates that, strong linear dependence exists between CSE and BSE and between BSE and KSE at lag 1.

Table 6: Cross Correlation Matrices (CCM)

Lag	One	Two	Three	Four	Five	Six
CSE/BSE	0.1992*	0.0628	-0.1098*	0.0470	0.0100	-0.0649
CSE/KSE	0.0232	0.2562*	0.1041*	-0.0449	-0.0015	-0.0195
BSE/KSE	0.1792*	0.1317*	-0.0183	0.0183	0.0378	0.0713

* means statistically significant at the 5 percent level.

Table 7: Coefficients of the VAR (1) Model

Coefficients	CSE	BSE	KSE
Intercept	0.0026	0.0032	0.0042
Standard Error	0.0028	0.0030	0.0039
Test Statistic	0.9460	1.0846	1.0782
CSE. Lag 1	0.0553	0.0924	-0.0237
Standard Error	0.0881	0.0938	0.1229
Test Statistic	0.6280	0.9847	-0.1928
BSE. Lag 1	0.1821	0.0207	0.0053
Standard Error	0.0839	0.0894	0.1171
Test Statistic	2.1702	0.2317	0.0453
KSE. Lag 1	-0.0306	0.1222	0.0397
Standard Error	0.0636	0.0677	0.0888
Test Statistic	-0.4815	1.8041	0.4477

The re-estimated VAR (1) model is displayed in table 7. The second, third and fourth columns of the table gives the respective estimated coefficients of CSE, BSE and KSE equations.
The estimated matrix equations from the three national stock markets are as follows:

$$\begin{bmatrix} CSE_t \\ BSE_t \\ KSE_t \end{bmatrix} = \begin{bmatrix} 0.0026 \\ 0.0032 \\ 0.0042 \end{bmatrix} + \begin{bmatrix} 0.0553 & 0.1821 & -0.0306 \\ 0.0924 & 0.0207 & 0.1222 \\ -0.0237 & 0.0053 & 0.0397 \end{bmatrix} \begin{bmatrix} CSE_{t-1} \\ BSE_{t-1} \\ KSE_{t-1} \end{bmatrix} + \epsilon_t \quad (5)$$

where $\epsilon_t \sim N(0, \Sigma)$ and $\Sigma = \begin{bmatrix} 0.14852489 & 0.04542068 & 0.03931771 \\ 0.04542068 & 0.16846791 & 0.05991232 \\ 0.03931771 & 0.05991232 & 0.28935903 \end{bmatrix}$.

3.4 Cointegration Test and VECM Representation

Usually the number of linearly independent vectors in a COINT-VAR model test is not unique, so both the trace and the maximum eigen value statistical tests are performed in this sub-section to ascertain the exact number. Table 8 focuses on the tests for cointegration ranks. From the table, the 4 estimated eigen values are less than 1, indicating that the test is stable. Both trace and maximum tests reject H (0), H (1), and H (2) but fail to reject H (3) at both 1 and 5 per cent significance levels. Therefore, there exist 3 linearly independent cointegrating vectors and a common stochastic trend.

Table 8: Cointegration Rank Test

Null Hypothesis	Eigen Value	Trace Test			Maximum Eigen Value Test		
		Statistic	**95% CV**	**99% CV**	**Statistic**	**95% CV**	**99% CV**
H(0)++**	0.5576	208.0230	53.12	60.16	115.8130	28.14	33.24
H(1)++**	0.3271	92.2103	34.91	41.07	56.2537	22.00	26.81
H(2)++**	0.2021	35.9565	19.96	24.60	32.0591	15.67	20.20
H(3)	0.0271	3.8974	9.24	12.97	3.8974	9.24	12.97

Table 9: VECM Coefficient

Item	YTM (Days)			
	Up to 14	15-91	92-182	183-364
Cointegrating Vector	1.0000	-3.0316	1.3445	0.5774
Standard Error	-	0.3659	0.2196	0.2163
Test Statistic	-	-8.2560	6.1223	2.6691
Cointegrating 1	-0.0665	0.1264	-0.1966	-0.1137
Standard Error	0.0539	0.0506	0.0480	0.0414
Test Statistic	-1.2352	2.4960	-4.0980	-2.7433

Note: Intercept = 0.0051 (Standard Error = 0.0047 and Test Statistic = 1.0844)

Now that the number of cointegrating vectors is known, the maximum likelihood estimates of the full VECM can be obtained. A comprehensive result of the computed VECM is shown in Table 9. Since the 4 YTM are cointegrated with a common stochastic trend, then the specified stationary series is given as:

$$w_t \approx x_t - 3.0316y_t + 1.3445z_t + 0.5774m_t \tag{6}$$

where: x = Up to 14 Days YTM, y = 15-91Days YTM, z = 92-182 Days YTM and m = 183-364 Days YTM.
The fitted VECM is given as:

$$\Delta x_t = \begin{bmatrix} -0.0665 \\ 0.1264 \\ -0.1966 \\ -0.1137 \end{bmatrix} [w_{t-1} + 0.0051] + e_t \tag{7}$$

where $e_t \sim N(0, \Sigma)$ and $\Sigma = \begin{bmatrix} 0.012887 & 0.006807 & 0.001986 & 0.001968 \\ 0.006807 & 0.011382 & 0.008082 & 0.006409 \\ 0.001986 & 0.008082 & 0.010228 & 0.006000 \\ 0.001968 & 0.006409 & 0.006000 & 0.007629 \end{bmatrix}$

However, an easy way to obtain simulated values from the VECM representation is to convert it to a VAR model. The simulated values for the 15-91 day YTM are used as the risk-free rate to discount all corresponding future income (margin offset) to their present values in the next section.

4 Valuation Model

This section applies the results of the preceding section and the theory of option pricing (see [16]) in the valuation of segregated fund contracts in India.

4.1 Dynamic Hedging for Separate Account Contract

As an introduction, we provide a review of the valuation formulae for the combined GMMB/GMDB contract as presented in [1]. For a combined GMMB/GMDB contract, the death benefit $(G - F_t)^+$ is paid at the end of month of death, if death occurs in month $t - 1 \rightarrow t$, and the maturity benefit is paid on survival to the end of the contract. Then the total hedge price at t for a GMMB/GMDB contract, conditional on the contract being in force at t, is¨

$$H^c(t) = \sum_{w=t}^{n-1} {}_{w-t|}q_{x,t}^d \, P(t,w) + {}_n p_{x,t}^{\tau} P(t,n) \tag{8}$$

The hedge price at t unconditionally is determined by multiplying (8) by ${}_t P_x^{\tau}$ to give

$$H(t) = \sum_{w=t}^{n-1} {}_{w|}q_{x,t}^d \, P(t,w) + {}_n p_x^{\tau} P(t,n) \tag{9}$$

The hedging error which represents the gap between the change in the stock part and the change in the bond part at discrete time interval is calculated as the difference between the hedge required at t, which include any payout at that time, and the hedge brought forward from $t-1$ to t. The required hedge at t conditional on the contract being in force at t is given as:

$$H^c(t) = Y_t^c + S_t \Psi_t^c \tag{10}$$

where Y_t^c is the bond part, $S_t \Psi_t^c$ is the stock part and $\Psi_t^c = \frac{\delta}{\delta S_t} H^c(t), Y_t^c = H^c(t) - S_t \Psi_t^c$. Similarly, $H(t) = Y_t + S_t \Psi_t$

where $\Psi_t = {}_n p_x^t \Psi_t^c$, $Y_t = {}_{n-t} p_t^\tau Y_t^c$. The hedge portfolio brought forward from $t-1$ to t whether or not the contract remains in force is given:

$$H(t^-) = Y_{t-1} e^{r/12} + S_t \Psi_t^c \tag{11}$$

The hedging error conditional on surviving to $t-1$ is

$$H_t^c = p_{x,t-1}^\tau [H^c(t) - H^c(t^-)] + q_{x,t-1}^d [(G - F_t)^+ - H^c(t^-)] + q_{x,t-1}^l [0 - H^c(t^-)] \tag{12}$$

The hedging error unconditional on surviving to $t-1$ then is

$$HE_t = p_x^\tau \left[p_{x,t-1}^\tau H^c(t) + q_{x,t-1}^d \left((G - F_t)^+) - H^c(t^-) \right) \right] \tag{13}$$

$$= H(t) + {}_{t-1|}q_x^d \left((G - F_t)^+) - H^c(t^-) \right) \tag{14}$$

In this paper, we assume transaction costs are proportional to the absolute change in the value of the stock part of the hedge which is a common practice in finance.
The transactions costs at t unconditional on survival to t are

$$\tau S_t \left| \Psi_t - \Psi_{t-1} \right| \tag{15}$$

where τ is a percentage or proportion of the change in the stock part of the hedge.

4.2 Numerical Investigation for Joint GMMB/GMDB Contract

The contract details are as follows:

i.	Mortality:	See Appendix
ii.	Premium:	$100
iii.	Guarantee:	100% of premium on death or maturity
iv.	Monthly Expense Ratio (MER):	0.25% per month
v.	Margin Offset (MO):	0.02%, 0.04%, 0.06%, 0.10% and 0.12%
vi.	Term:	10 years

The simulation details are as follows:

i.	Number of Simulation:	5000
ii.	Volatility:	20% per year
iii.	Equity Return Model:	VAR (1)
iv.	Treasury Bond Model:	VECM
v.	Transaction Costs:	0.2% of the change in the value of stocks
vi.	Rebalancing:	Monthly

At the end of each month, the outgo is calculated as follows:

- Sum of all mortality payout
- plus transactions costs from rebalancing the hedge
- plus the hedge required in respect of future guarantees
- minus the hedge brought forward from the previous month

The income at the end of each month is calculated as follows:

- Margin offset multiplied by fund value at the end of each month, except the last.
- The present value is calculated using the simulated 15 to 91 days YTM.

At each month end, outgo and income are calculated. Since the present study is simulating a loss random variable (Outgo - Income), negative values indicate that the simulated 15-91 YTM income exceeded outgo. Figures 1 - 6 display the simulated probability density function for the net present value of outgo when the margin offset is set at 0.02%, 0.04%, 0.06%, 0.08%, 0.10% and 0.12% respectively. It is obvious from figures 2 - 6 that the bulk of the distribution falls in the negative part of the graph. This gives a clear indication that in most cases, the margin offset is adequate at meeting all the hedge costs and leave some profit. However, in the case of figure 1, there is a very small part of the distribution in the positive quadrant reflecting an insignificant probability of a loss.

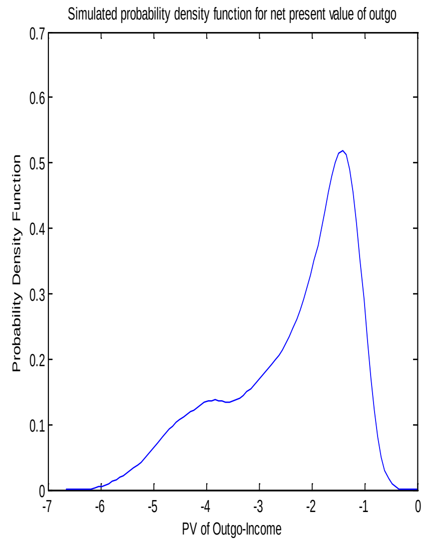

Figure 1: Margin Offset - 0.02%

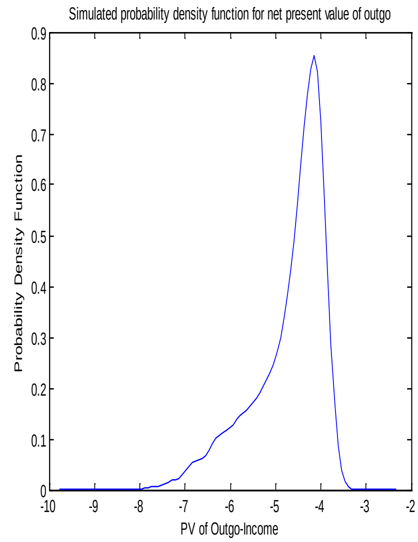

Figure 2: Margin Offset - 0.04%

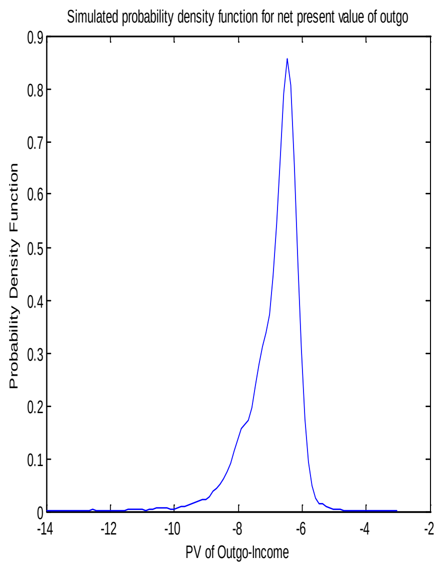

Figure 3: Margin Offset - 0.06%

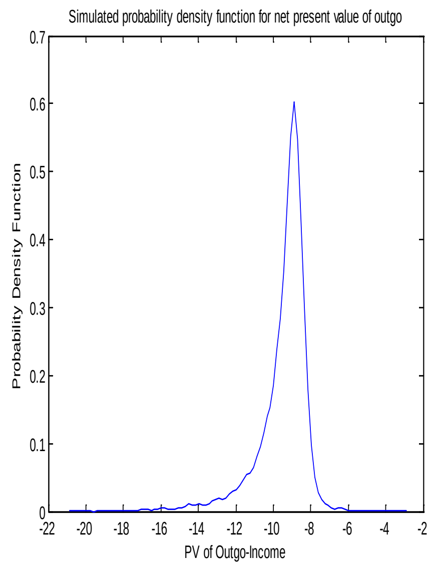

Figure 4: Margin Offset- 0.08%

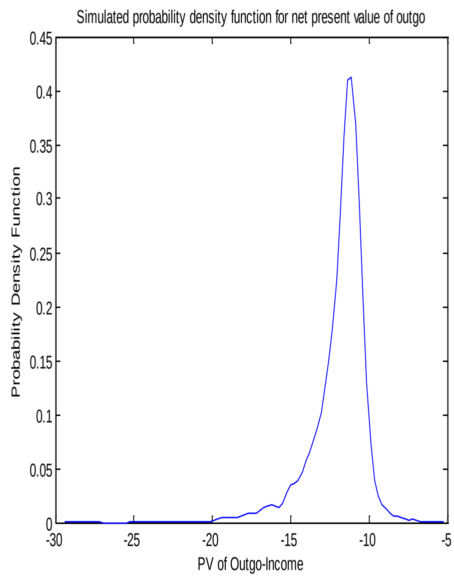

Figure 5: Margin Offset - 0.10%

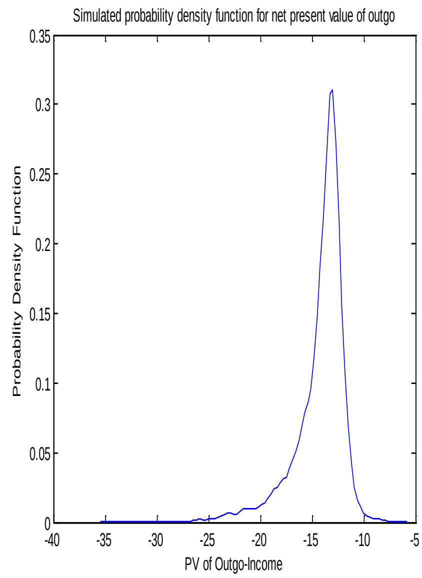

Figure 6: Margin Offset - 0.12%

5 Conclusion

In this paper, we studied the stock markets of Sri Lanka, India and Pakistan by considering the respective return series from August 1997 to June 2009. We also analyzed the treasury bond market of India for the same period. Based on the results, we draw the following conclusions.

First, the stock markets of Sri Lanka, India and Pakistan had no evidence of unit roots, but the returns are correlated. Therefore, the most appropriate model capable of capturing the long-term equity return process for a practical dynamic hedging of segregated fund contracts in India is the VAR (1) process.

Second, the treasury bond market of India did provide evidence of unit-root and a long-run stochastic trend. Consequently, the VECM model is chosen to describe the security bond process in the valuation of segregated fund contracts in India. However, to discount all future income to their present values, the 15 to 91 YTM simulated values are used.

Finally, the valuation results using a life age 50, at a premium of $100 for a contract with combined GMMB/GMDB maturing in 10 years indicate an extremely high probability of a profit than a loss when the margin offset is set above 2%. This is a strong indication that the model has the capability of meeting all the hedge cost and leave some profit.

References

[1] M. Hardy, *Investment Guarantees: Modeling and Risk Management for Equity Linked Life Insurance*, John Wiley and Sons Inc., Hoboken, New Jersey, 2003.

[2] A. D. Wilkie, A Stochastic Investment Model for Actuarial Use, *Transactions of the Faculty of Actuaries*, **39**, (1986), 341-381.

[3] A. D. Wilkie, More on a Stochastic Asset Model for Actuarial Use, *British Actuarial Journal*, **1**, (1995), 341-381.

[4] P. P. Boyle and M. R. Hardy, Reserving for Maturity Guarantees, Ontario, Canada: University of Waterloo, *Institute for Insurance and Pension Research*, (1996), (96-18).

[5] P. P. Huber, A Review of Wilkie’s Stochastic Asset Model, *British Actuarial Journal*, **3**(1), (1997), 181-210.

[6] E. Thompson, and R. Ambagaspitiya, *Valuation of Segregated Funds in India*, Lambert Academic Publishing, Saabrücken, Germany, 2012.

[7] C. A. Sims, Macroeconomics and Reality", *Econometrica*, **48**(1), (1980), 1-48.

[8] D. A. Dickey and W. A. Fuller, Likelihood Ratio Tests for Autoregressive Time Series with a Unit Root, *Econometrica*, **49**(4), (1981), 1057–1072.

[9] P. C. B. Phillip and P. Perron, Testing for Unit Root in Time Series Regression, *Biometrika*, **75**(2), (1988), 335–346.

[10] G. C. Tiao and G. E. P. Box, Modeling Multiple Time Series with Applications, *Journal of the American Statistical Association*, **76**(376), (1981), 802–816.

[11] J. D. Hamilton, *Time Series Analysis*, Princeton University Press, Princeton, New Jersey, 1994.

[12] H. Lütkepohl, *Introduction to Time Series Analysis*, Springer Verlag, Heidelberg, Germany, 1991.

[13] R. Tsay, *Analysis of Financial Time Series*, John Wiley & Sons, New York, 2005.

[14] R. Tsay, *Analysis of Financial Time Series*, John Wiley & Sons, New York, 2010.

[15] S. Johansen, Estimation and Hypothesis Testing of Cointegration Vectors in Gaussian Vector Autoregressive Models, *Econometrica*, **59**(6), (1991), 1551–1580.

[16] F. Black, and M. Scholes, The Pricing of Options and Corporate Liabilities, *Journal of a Political Economy*, **81**(3), (1973), 637-654.

Appendix

Mortality and Survival Probabilities

In this appendix, we give the mortality and survival rates used in the valuation of the segregated funds under the combined GMMB/GMDB contract. At $t = 0$, the life is assumed to be age 50, time t is in months. Independent mortality rates are from the Canadian Institute of Actuaries male annuitants' mortality rates.

t	${}_tp_x^\tau$	${}_{t\|1}q_x^d$	t	${}_tp_x^\tau$	${}_{t\|1}q_x^d$	t	${}_tp_x^\tau$	${}_{t\|1}q_x^d$
0	1	0.00029	21	0.86361	0.0003	42	0.74479	0.00031
1	0.99307	0.00029	22	0.85757	0.0003	43	0.73953	0.00031
2	0.98618	0.00029	23	0.85157	0.0003	44	0.7343	0.00031
3	0.97934	0.00029	24	0.84561	0.0003	45	0.72911	0.00031
4	0.97255	0.00029	25	0.8397	0.0003	46	0.72396	0.00031
5	0.9658	0.00029	26	0.83382	0.0003	47	0.71883	0.00031
6	0.95909	0.00029	27	0.82797	0.0003	48	0.71374	0.00031
7	0.95243	0.00029	28	0.82217	0.0003	49	0.70869	0.00032
8	0.94581	0.00029	29	0.8164	0.0003	50	0.70366	0.00032
9	0.93923	0.00029	30	0.81067	0.00031	51	0.69867	0.00032
10	0.9327	0.00029	31	0.80498	0.00031	52	0.69372	0.00032
11	0.92621	0.00029	32	0.79933	0.00031	53	0.68879	0.00032
12	0.91976	0.00029	33	0.79371	0.00031	54	0.6839	0.00032
13	0.91336	0.00029	34	0.78813	0.00031	55	0.67903	0.00032
14	0.907	0.0003	35	0.78259	0.00031	56	0.6742	0.00032
15	0.90067	0.0003	36	0.77708	0.00031	57	0.66941	0.00032
16	0.89439	0.0003	37	0.77161	0.00031	58	0.66464	0.00032
17	0.88816	0.0003	38	0.76618	0.00031	59	0.6599	0.00032
18	0.88196	0.0003	39	0.76078	0.00031	60	0.6552	0.00032
19	0.8758	0.0003	40	0.75541	0.00031	61	0.65052	0.00032
20	0.86968	0.0003	41	0.75008	0.00031	62	0.64588	0.00032

t	${}_tp_x^{\tau}$	${}_{t\vert 1}q_x^d$	t	${}_tp_x^{\tau}$	${}_{t\vert 1}q_x^d$
63	0.64127	0.00032	101	0.48655	0.00034
64	0.63668	0.00032	102	0.48297	0.00034
65	0.63213	0.00032	103	0.47942	0.00034
66	0.62761	0.00033	104	0.47589	0.00034
67	0.62311	0.00033	105	0.47239	0.00035
68	0.61865	0.00033	106	0.46891	0.00035
69	0.61421	0.00033	107	0.46545	0.00035
70	0.6098	0.00033	108	0.46201	0.00035
71	0.60542	0.00033	109	0.45859	0.00035
72	0.60107	0.00033	110	0.4552	0.00035
73	0.59675	0.00033	111	0.45183	0.00035
74	0.59246	0.00033	112	0.44848	0.00035
75	0.5882	0.00033	113	0.44515	0.00035
76	0.58396	0.00033	114	0.44185	0.00035
77	0.57975	0.00033	115	0.43857	0.00035
78	0.57557	0.00033	116	0.4353	0.00035
79	0.57141	0.00033	117	0.43206	0.00035
80	0.56728	0.00033	118	0.42884	0.00035
81	0.56318	0.00033	119	0.42564	0.00035
82	0.55911	0.00033	120	0.42247	0.00035
83	0.55506	0.00033	121	0.41931	0.00035
84	0.55104	0.00033	122	0.41617	0.00035
85	0.54704	0.00034	123	0.41306	0.00035
86	0.54307	0.00034			
87	0.53913	0.00034			
88	0.53521	0.00034			
89	0.53132	0.00034			
90	0.52745	0.00034			
91	0.52361	0.00034			
92	0.5198	0.00034			
93	0.516	0.00034			
94	0.51224	0.00034			
95	0.5085	0.00034			
96	0.50478	0.00034			
97	0.50108	0.00034			
98	0.49742	0.00034			
99	0.49377	0.00034			
100	0.49015	0.00034			