*Journal of Finance and Investment Analysis, vol. 2, no.3, 2013, 55-67*
*ISSN: 2241-0998 (print version), 2241-0996(online)*
Scienpress Ltd, 2013

# A Mean-reverting Model of the Short-term Interbank Interest Rate: the Moroccan Case

**Abdelhak Zoglat[1], Elhadj Ezzahid[2], Elhaouari Bouziani[3] and Younés Mouatassim[4]**

## Abstract

Modeling short-run interest rate is of primary importance for all participants in financial markets. In this paper, we attempt to model short-run interbank interest rate in Morocco by the well-known Ornstein-Uhlenbeck model (OU) called also the mean-reverting model. The results show that the speed of reversion is high and that the mean-reverting level is below the key rate and that the estimated volatility of the interbank daily interest rate *r*, when this variable is assumed to be governed by an OU model, is below the historical volatility observed during the studied period.

**JEL classification**: C13, C15
**Keywords**: Mean-reverting model, Short-term Interbank money market, Ornstein-Uhlenbeck process, Morocco.

## 1 Introduction

Interest rate is an important signal in any market economy. It informs about the cost of funds for borrowing agents and the rewards of saving for lending agents. Interest rate is also a mechanism for the transmission of monetary policy (Gray and Talbot, 2006). The problem with interest rate is the existence of many interest rates in the economy.

[1]A. Zoglat, Faculty of sciences, Mohammed V-Agdal University, Rabat, Morocco.
[2]E. Ezzahid, Faculty of Law and Economics, Mohammed V-Agdal University, Rabat, Morocco.
e-mail :ezzahidelhadj@yahoo.fr
[3]E. Bouziani, Ministry of Finance and Economy, Rabat, Morocco.
[4]Y. Mouatassim, Zurich Insurance, Casablanca, Morocco

Article Info: *Received :* April 1, 2012. *Revised :* May 7, 2013.
*Published online* : August 15, 2013

The lending rate $r_l$, which banks charge to their borrowers[5], is different from the deposit rate $r_d$, which the same banks pay to the depositors. The rate $r_{cbl}$, which the central bank charges for the refinancing of banks, is determined by some factors while the interest rate for the financing of the public treasury $r_T$ is determined accordingly to other factors. Finally, the interest rate $r$ that banks charge to each other in their daily transactions for central bank money is commanded by another set of variables. This rate, also called the interbank overnight rate, is of primary importance.

The interbank daily interest rate[6] $r$ is determined in the short-term interbank money market which is a narrower segment of the market of central bank's money. In the interbank market, banks borrow and lend from and to each other for short periods (Gruson, 2005). Naturally, banks will not accept to borrow from each other if the charged interest rate $r$ is greater than what they may pay for getting the same money from the central bank. For this reason, $r$ is less than $r_{cbl}$.

We can think of lending, in interbank money market, as risk-free because the loans are for very short periods (from one day to one week). Another reason is the fact that institutions that operate in this market are in continuous and complex relationships and each operator knows sufficiently the other operators.

Modelling the short-term interest rate is, for at least two reasons, of prime importance[7]. First, the short-term interest rate is charged to risk-free or almost risk-free borrowing agents. Thus, this variable serves as a benchmark for the determination of other interest rates by adding each asset's risk premium. So the prices of many assets are determined as a function of the short-term risk-free interest rate. Second, hedging strategies and options pricing are performed accordingly to the dynamical behaviour of short-term interest rate.

In this paper, we will explore the empirical properties of short-term interbank interest rate in Morocco. We will use a one factor model that is sufficiently general and intuitively appealing. It is the well-known Vasicek model (Vasicek, 1977). This model assumes that $r$ fluctuates around a long-run level and consequently to each deviation from it, a mechanism pulls $r$ back with a certain speed.

The remaining of this paper is structured as follows. In the second section, we discuss the most used models to capture the dynamics of short-term interest rate. The third section contains the presentation of the Ornstein-Uhlenbeck (OU) model. The fourth section is dedicated to the presentation of some empirical facts about the Moroccan interbank money market. In the fifth section, we calibrate an OU model to our data and discuss the results. The sixth section serves to conclude.

---

[5]We must remark that for each client, a bank charges a different rate depending upon his level of risk, his history, his relationship with the bank. Thus, there are as many rates as banks clients.

[6]This rate is also called overnight rate or tom'next rate. The European Central Bank publishes for each open day the EONIA (European over-night index average) which is an average rate weighted by the volume of exchanged funds between banks.

[7]The "short rate is one of the key observables that drive overall term structure dynamics in classical interest rate models" (Pai and Pederson, 1999, p. 388)

## 2 Models of Short-run Interest Rate

Interest rates are important financial variables. They affect directly or indirectly other variables. For this reason, several models have been designed to explore their dynamics especially in the case of the short-term interest rate $r_t$ (Merton, 1973; Vasicek, 1977; Chan et al., 1992; Broze et al. 1996; Chung and Hung, 2000; Varma, 1997). The available models are special cases of a general model which takes into account two main features. First, there is an equilibrium value μ toward which the current short-term interest rate $r_t$ reverts. Second, the volatility of short-run interest rate increases with $r_t$; that is the volatility of $r_t$ per unit of time is proportional to its level. This second feature is concordant with a well-documented stylised fact concerning the dynamics of $r_t$. To take these two features into account, the following stochastic differential equation is used to simulate the path of short-term interest rate.

$$dr_t = \lambda(\mu - r_t)dt + \sigma r_t^{\gamma} dW_t \tag{1}$$

In equation (1), $dr_t$ and $dt$ are respectively the differential of short-term interest rate and the differential of time. The quantities λ, μ and σ are constant parameters. We denote by $W_t$ the standard Brownian motion and thus $dW_t = \varepsilon_t \sqrt{dt}$, where the $\varepsilon_t$'s are standard normally and independently distributed random variables. The parameter λ is the speed of reversion of $r_t$ to its long-run equilibrium value μ. The variance of $r_t$ per unit of time is $\sigma r^{\gamma}$. Remark that the instantaneous volatility of short-term interest rate $\sigma r^{\gamma}$ is proportional to its level $r_t$.
If λ =1, this implies that there is full adjustment of the interest rate to its normal level μ after any deviation observed in the last period. If λ =0, then no mean reversion is observed at all. The most used models for the description of the dynamics of the short-term interest rate are varieties of the model described in equation (1) (Varma, 1997; Chan et al. 1992). Table 1 summarizes some features of four widely used models.

Table 1: Four different models describing the dynamics of short interest rate

| The model | Hypotheses | Mathematical model |
|---|---|---|
| Geometric Brownian motion model | The volatility of interest rate is fully proportional to $r_t$, that is γ=1.<br>α=0. | $dr_t = \beta r_t dt + \sigma r_t dW_t$ |
| Merton model (1973) | The instantaneous volatility of short-run interest rate is constant.<br>There is no mean-reverting behaviour of $r_t$. | $dr_t = \alpha + \sigma dW_t$ |
| Vasicek model (1977) | The level of interest rate does not impact its volatility γ=0.<br>There is a mean-reversion level of $r_t$. | $dr_t = (\alpha + \beta r_t) dt + \sigma dW_t$ |
| Brennan and Shwartz model (1979) | The volatility of interest rate is proportional to $r_t$ that is γ=1. | $dr_t = (\alpha + \beta r_t) dt + \sigma r_t dW_t$ |

We will use, in this paper, the Ornstein-Uhlenbeck model called also the Vasicek model to describe the dynamics of interbank daily interest rate in the case of the Moroccan interbank money market. In the following section, we present this model.

## 3 The Ornstein-Uhlenbeck Process

There is a solid empirical fact behind our choice of the mean-reverting model to capture the dynamics of short-run interbank interest rate. Indeed, we do observe that even if $r_t$ oscillates in the short-term it is pulled, up and down, toward a long term equilibrium level μ which is determined by fundamental variables. It is possible to complicate a bit more our analysis and suppose that the equilibrium interest rate is not constant and it shifts to a new value in each regime. This last possibility will not be explored here.
In this paper, we will suppose that the level of interest rate does not impact its volatility. That is, we will suppose that $\gamma=0$ in equation (1). We assume that the short-run interest rate *r* is described by a mean-reverting model or an OU process as in the paper of Vacisek (1977). The stochastic differential equation (SDE) that governs this process is:

$$dr_t = \lambda\,(\mu - r_t)\,dt + \sigma\,dW_t \tag{2}$$

The initial value of *r* is known and is denoted $r_0$. The mean reversion level and the speed of reversion are respectively μ and λ. As in classic models, $W_t$ is a standard Brownian motion so that $dW_t = \varepsilon_t\sqrt{dt}$ , with $\varepsilon_t \sim$ i.i.d N(0,1). It represents the shock to current interest rate $r_t$. We suppose that the volatility of interest rate is independent of its level $r_t$.
Remark first that:

$$d(e^{\lambda t} r_t) = r_t\,\lambda e^{\lambda t} dt + e^{\lambda t} dr_t \tag{3}$$

This implies that:

$$e^{\lambda t}\, dr_t = d(e^{\lambda t} r_t) - r_t\,\lambda e^{\lambda t} dt \tag{4}$$

Let us multiply each side of equation (2) by $e^{\lambda t}$ to get:

$$e^{\lambda t}\, dr_t = e^{\lambda t}\,\lambda\,(\mu - r_t)\,dt + e^{\lambda t}\,\sigma\,dW_t \tag{5}$$

Combining equations (4) and (5), we get:

$$d(e^{\lambda t}\, r_t) = \lambda\,\mu\, e^{\lambda t}\,dt + e^{\lambda t}\,\sigma\,dW_t \tag{6}$$

The solution of this SDE is:

$$e^{\lambda t}\, r_t = r_0 + \int_0^t \lambda e^{\lambda s}\mu\, ds + \int_0^t e^{\lambda s}\sigma dW_s \tag{7}$$

Multiplying each side of the precedent equation by $e^{-\lambda t}$ leads to the equivalent following equation:

$$r_t = r_0 e^{-\lambda t} + \int_0^t \lambda e^{-\lambda(t-s)}\mu\, ds + \sigma\int_0^t e^{-\lambda(t-s)} dW_s \tag{8}$$

This means that at any moment, the interest rate is the sum of three terms. The first is $r_0 e^{-\lambda t}$, the second is the first integral which is equal to $\mu(1-e^{-\lambda t})$, and the third component of $r_t$ is a normally distributed quantity with a zero expected value and a variance $E\left[\left(\int_0^t e^{-\lambda(t-s)}\sigma\, dWs\right)^2\right]$. Using Ito's Isometry (Kopp, 2011) we get:

$$E\left[\left(\int_0^t e^{-\lambda(t-s)}\sigma dW_s\right)^2\right] = \int_0^t \left(e^{-\lambda(t-s)}\sigma\right)^2 ds = \frac{\sigma^2}{2\lambda}(1-e^{-2\lambda t}) \quad (9)$$

Thus, conditional on $r_0$, the short-run interest rate $r_t$ is a normal variable with an expected value at $t$ that depends upon $r_0$, μ, and λ. Indeed, the value of $r_t$ conditional on $r_0$ is as follows:

$$r_t = \mu + (r_0 - \mu)e^{-\lambda t} + \sigma\int_0^t e^{-\lambda(t-s)}\, dW_s \quad (10)$$

This result is very intuitive. Indeed, the value of $r_t$ at any moment is equal to its long run equilibrium value plus the remaining part of the initial deviation of *r* from its equilibrium value μ. The third component of $r_t$ is of a stochastic nature.
The value of $r_t$ conditional on its initial value contains a deterministic part and a stochastic part. We can compute, conditional on $r_0$, the expected value of $r_t$ and its variance. The latter is equal to the variance of the stochastic component of $r_t$. More precisely we have:

$E(r_t|r_0) = \mu + (r_0 - \mu)e^{-\lambda t}$, and $Var(r_t|r_0) = \dfrac{\sigma^2}{2\lambda}(1-e^{-2\lambda t})$

We must observe that if the speed of adjustment of $r_t$ to μ is very small, i.e. close to zero, we will have a volatility of $r_t$ that goes to infinity which is practically equivalent to say that short-term interest rate never returns to its long-term mean.

## 4 The Dynamics of the Short-term Interbank Interest Rate in Morocco

The interbank monetary market is of prime importance in any market of funds. The interbank rate is a leading indicator about the short-term strains exerted in the market of funds. Furthermore, this rate is the operational target of the central bank in conducting its monetary policy. Indeed, the Moroccan central bank attempts to maintain the short-term interbank rate at a level no far from the key rate.
As discussed earlier, it is difficult to decide which of the interest rates to take as the rate representative of the short-run price of money. Especially when we want to have a dataset that is as homogeneous as possible. Our choice will be pragmatic because our paper seeks just to develop an operational model to describe the dynamics of the short-run interest rate in Morocco. The criteria that the chosen series must meet are the availability of sufficient dataset and the possibility to consider the available rate as a rate that represents the cost of borrowing for an agent with practically no risk of default. The third condition is that the chosen rate must really be a rate that is charged for short periods.

The daily interbank rate or the overnight rate fulfils the above three conditions necessary to consider a rate as the price of money for a short-term. Indeed, the overnight interbank rate is the rate charged by the lending banks to the borrowing ones for a short period, i.e. one day (24 hours). One test of the pertinence/validity of the key rate determined by the central bank is the fluctuation of $r$ around the key rate. In a sense, the key rate must be very close or equal to the long run value $\mu$ toward which $r$ is pulled up and down.

The current operational apparatus for conducting monetary policy[8] by the Moroccan central bank includes three important interest rates. The first one is the key rate $r_{kr}$. The Moroccan central bank provides funds to banks seeking liquidity at this rate for one week in auctions organized each Wednesday. This is the rate intended to represent the fundamental cost of money for a risk free agent for a short period and in the absence of inflation. By its very definition the key rate is in a sense an instantaneous rate. The problem of the key rate is that it is not the result of the confrontation of suppliers and demanders of funds but just decided by the central bank according to the balance of risks concerning inflation. We do not need to say that the central bank uses models and experts' knowledge for the determination of the key rate.

The other two rates that constitute the apparatus of monetary policy in Morocco are the central bank overnight loan rate $r_{cbl}$ and the deposit facility rate[9] $r_{cbd}$. These two rates are supposed to constitute the corridor inside which the interbank rate fluctuates. Indeed, if $r$ is above $r_{cbl}$, then banks prefer to borrow from the central bank. Symmetrically, if the return of money lent to a bank is below $r_{cbd}$, then banks prefer to constitute deposits in the central bank.

In Morocco, the global outlook of the short-term interbank interest rate evolved dramatically during the period 2003-2011. Interbank interest rate had fluctuated during the period spanning from 2003 to 2011 around the key rate which was 3.25%[10] (figure 1). The average short-term interbank interest rate was 3.04% during the period 2003-2011. During 2004, 2005 and 2006, the daily interbank rate $r$ was frequently below the key rate. During 2007, 2008 and 2009, there were much more observations clustered near the key rate. This pattern strengthened during 2010 and in 2011.

During 2009, there was a sharp reduction of banks' liquidity. To preserve the banks' situation, the central bank increased the volume of funds provided to banks. As a consequence, both the volume of funds exchanged on the interbank market and the interbank interest rate decreased.

Many variables are highly linked to the short-term interbank rate. The first one is banks' liquidity position (BLP). Available data show that from the first quarter of 2003 through the second quarter of 2007, banks' liquidity position had been positive with an annual average of 5 billions MAD (DEPF, 2010). Since the second quarter of 2007, banks liquidity position began to be increasingly negative (DEPF, 2011). The second variable commanding the short-term interbank rate is the key rate $r_{kr}$ which serves as a target rate. Indeed, the level of the key rate is a level toward which monetary authorities guide the short-term interbank rate by manipulating the instruments at their hand.

---

[8]The decisional process of conducting monetary policy in Morocco involves Bank Al Maghrib's Board. This board is the structure that decides to increase or decrease the key rate.

[9]From 2003 to march 2012, the key rate was 3.25% and the overnight loan rate is 4.25%. The deposit facility rate is 2.25%. The exception is a short period from September 2008 to March 2009.

[10]A temporary increase of the short-term key rate (+0.25%=25 basis points) was decided by the central bank from September 2008 to March 2009.

The third variable affecting *r* is the rate of required reserves ρ that banks must hold at their accounts in the central bank. These variables, beside others, command the volume of funds exchanged by banks, the level of *r*, and its volatility. Thus, we can write $r=f(\text{BLP}, r_{kr}, \rho)$. Table 2 provides summary annual data about the key variables related to the Moroccan interbank money market during the period 2003-2011.

Table 2: Summary data about the Moroccan interbank money market

| | **2003** | **2004** | **2005** | **2006** | **2007** | **2008** | **2009** | **2010** | **2011** |
|---|---|---|---|---|---|---|---|---|---|
| Exchanged Funds ($10^9$ of MAD) | 11.6 | 15.9 | 20.9 | 28.5 | 26.3 | 35.4 | 32.6 | 31.2 | 40.9 |
| Average banks liquidity shortage ($10^9$ of MAD) | 2.75 | 5.0 | 4,19 | 7.70 | -2.4 | -8.2 | -13.2 | -13.5 | -23.4 |
| Average $r_t$ (%) | 3.24 | 2.37 | 2.76 | 2.58 | 3.29 | 3.37 | 3.26 | 3.29 | 3.29 |
| Volatility of $r_t$ (%) | 0.49 | 0.22 | 0.75 | 0.43 | 0.41 | 0.26 | 0.24 | 0.08 | 0.08 |
| Number of days where $r_t$ is below the key rate | 131 | 255 | 201 | 230 | 100 | 51 | 52 | 13 | 17 |
| Reserve requirements (%) | 16.5 | 16.5 | 16.5 | 16.5 | 16.5 | 15 | 12-10-8 | 6 | 6 |

Number of observations: 2247 in 9 years

The following chart presents the evolution of interbank rate during the period spanning from January, 1st 2003 to December, 30th 2011. The plotted data represent the fluctuations of the daily interbank interest rate with the key rate at the centre of a corridor constituted by the central bank lending rate $r_{cbl}$ and its offered rate for deposits $r_{cbd}$.

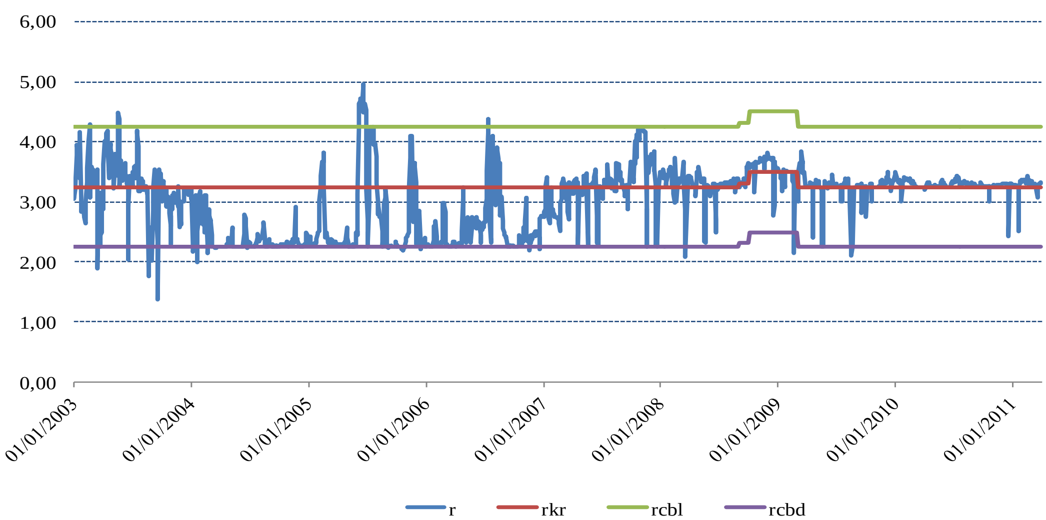


Figure 1: Daily interbank interest rate for the period: 2003- 2011
Source: Bank Al-Maghrib

After this discussion about the evolution of the short-term interest rate in the Moroccan interbank money market, it is time to fit an OU model to our data set.

## 5 Calibration, Results and Discussion

In this paper, the calibration of the OU model to daily interbank interest rate $r_t$ will be performed by the maximum likelihood method (ML method) and the ordinary least squares method (OLS Method)[11]. The use of two different methods to calibrate the OU model to the data aims to gauge the robustness of the estimates.

### 5.1 Calibration of the Model by the ML Method

The use of the MLM is well suited to calibrate an OU process to a specific series because this process is Gaussian. When working with our data we must pass from differential quantities, analytically suitable for continuous time treatment, to first difference equations which are used when only limited number of observations is available. Our $T$+1 observations are the T interbank interest rates observed from $t$=1 to $t$=T and the initial rate $r_0$. These rates are denoted $r_t$ for $t = 0,...,T$. If the interbank daily interest rate is generated by an OU process then each rate is a normal random variable. What we need to estimate is the vector θ of the parameters λ, σ and μ. Using (10), we can write the density of probability of $r_t$ conditional on $r_{t-1}$ as follows:

$$r_t = r_{t-1}e^{-\lambda} + \mu\left(1 - e^{-\lambda}\right) + \sigma\int_{t-1}^{t} e^{-\lambda(t-s)}dW_s \tag{11}$$

Note that, since $\left(W_s\right)_{s\geq 0}$ is the Brownian motion process, the random variables $\varepsilon_t = \int_{t-1}^{t} e^{-\lambda(t-s)}dW_s$, for $t = 1,...,T$, are independent and normally distributed with mean 0 and variance $\frac{1-e^{-2\lambda}}{2\lambda}$. The expected value of $r_t$ and its variance conditional on $r_{t-1}$ are:

$$E\left(r_t \middle| r_{t-1}\right) = \mu + \left(r_{t-1} - \mu\right)e^{-\lambda}, \text{ and } \operatorname{Var}\left(r_t \middle| r_{t-1}\right) = \sigma^2 \frac{1-e^{-2\lambda}}{2\lambda}$$

To simplify the notation, put $\delta^2 = \sigma^2 \frac{1-e^{-2\lambda}}{2\lambda}$. So the conditional probability density of the actual rate $r_t$, given the precedent rate $r_{t-1}$, is given by:

$$f\left(r_t \middle| r_{t-1,\mu,\lambda,\sigma}\right) = \left(2\pi\delta^2\right)^{\frac{1}{2}} \exp\left[-\frac{[r_t - \mu - \left(r_{t-1} - \mu\right)e^{-\lambda}]^2}{2\delta^2}\right]$$

The sample of the T+1 available observations of the daily interbank rate $\left(r_t\right)_{0\leq t\leq T}$ is supposed to be a realisation of an OU process. The likelihood function of this sample may be written as a product of the conditional probability densities of $r_t$ given $r_{t-1}$ for $t = 1,...,T$. More precisely we have that:

[11]For an AR(1), in principle, the same parameters' estimators are obtained by MLM and OLS method when data are independently and normally distributed.

$$f(r,\mu,\lambda,\sigma)=\prod_t^T f\left(r_t \middle| r_{t-1,\mu,\lambda,\sigma}\right)$$
$$=\left(2\pi\delta^2\right)^{-\frac{T}{2}}\exp\left[-\frac{\sum_{t=1}^T[r_t-\mu-(r_{t-1}-\mu)e^{-\lambda}]^2}{2\delta^2}\right]$$

where $r=(r_t, r_{t-1},\ldots,r_1)$.

It is convenient to work with the log-likelihood function. The latter will be denoted by $L(r,\mu,\lambda,\sigma)$, and is given by:

$$L(r,\mu,\lambda,\sigma)=\ln f(r,\mu,\lambda,\sigma)$$
$$=-\frac{T}{2}\ln(2\pi\delta^2)-\frac{1}{2\delta^2}\sum_{t=1}^T[r_t-\mu-(r_{t-1}-\mu)e^{-\lambda}]^2$$

The log-likelihood function, and thus the likelihood function, is maximized for the values $\hat{\mu}$, $\hat{\lambda}$ and $\hat{\sigma}$ obtained as a solution of the system:

$$\frac{\partial L(r,\mu,\lambda,\sigma)}{\partial\hat{\mu}}=\frac{1-e^{-\lambda}}{\delta^2}[\sum_{t=1}^T(r_t-r_{t-1}e^{-\lambda})-T(1-e^{-\lambda})\mu]=0$$
$$\frac{\partial L(r,\mu,\lambda,\sigma)}{\partial\hat{\sigma}}=\frac{\sum_{t=1}^T[r_t-\mu-(r_{t-1}-\mu)e^{-\lambda}]^2-T\delta^2}{\delta^3}\frac{\partial\delta}{\partial\sigma}=0$$
$$\frac{\partial L(r,\mu,\lambda,\sigma)}{\partial\hat{\lambda}}=-T\frac{\partial\delta/\partial\lambda}{\delta}+\frac{\partial\delta/\partial\lambda}{\delta^3}\sum_{t=1}^T[r_t-\mu-(r_{t-1}-\mu)e^{-\lambda}]^2$$
$$-\frac{1}{\delta^2}\sum_{t=1}^T[r_t-\mu-(r_{t-1}-\mu)e^{-\lambda}][(r_{t-1}-\mu)e^{-\lambda}]=0$$

By the second equation, we have:

$$\frac{\partial\delta}{\partial\lambda}\left[-T\frac{1}{\delta}+\frac{1}{\delta^3}\sum_{t=1}^T[r_t-\mu-(r_{t-1}-\mu)e^{-\lambda}]^2\right]=0,$$

and therefore the above system of equations is equivalent to:

$$\begin{cases}\sum_{t=1}^T(r_t-r_{t-1}e^{-\lambda})-T(1-e^{-\lambda})\mu=0\\ \sum_{t=1}^T[r_t-\mu-(r_{t-1}-\mu)e^{-\lambda}]^2-T\delta^2=0\\ \sum_{t=1}^T[r_t-\mu-(r_{t-1}-\mu)e^{-\lambda}][(r_{t-1}-\mu)e^{-\lambda}]=0\end{cases}$$

Using the first equation of this system, the third one leads to:

$$\hat{\lambda} = \ln\left( T\sum_{t=1}^{T} r_{t-1}^2 - \left[\sum_{t=1}^{T} r_{t-1}\right]^2 \right) - \ln\left( T\sum_{t=1}^{T} r_t r_{t-1} - \sum_{t=1}^{T} r_t \sum_{t=1}^{T} r_{t-1} \right)$$

Since $\delta^2 = \sigma^2 \frac{1-e^{-2\lambda}}{2\lambda}$, the solution $(\hat{\mu}, \hat{\lambda}, \hat{\sigma})$ of the system is given by:

$$\begin{cases} \hat{\lambda} = \ln(T\sum_{t=1}^{T} r_{t-1}^2 - [\sum_{t=1}^{T} r_{t-1}]^2) - \ln(T\sum_{t=1}^{T} r_t r_{t-1} - \sum_{t=1}^{T} r_t \sum_{t=1}^{T} r_{t-1}) \\ \hat{\mu} = \frac{1}{1-e^{-\hat{\lambda}}} \frac{1}{T} \sum_{t=1}^{T} (r_t - r_{t-1} e^{-\hat{\lambda}}) \\ \hat{\sigma}^2 = \frac{2\hat{\lambda}}{1-e^{-2\hat{\lambda}}} \frac{1}{T} \sum_{t=1}^{T} [r_t - \hat{\mu} - (r_{t-1} - \hat{\mu}) e^{-\hat{\lambda}}]^2 \end{cases}$$

Using our dataset yields the following estimated values of the parameters of the Ornstein-Uhlenbeck model.

$$\hat{\lambda} = 0.0715$$
$$\hat{\mu} = 0.0304$$
$$\hat{\sigma} = 0.0020$$

The equation giving $r_t$ conditional ob $r_{t-1}$ is:

$$r_t = r_{t-1} e^{-0.0715} + 0.0304(1 - e^{-0.0715}) + 0.0020 \int_{t-1}^{t} e^{-0.0715} dW_s$$

## 5.2 Calibration of the Model by the OLS Method

If we discretize time with a constant increment $\Delta t=1$, and rewrite equation (11)

$$r_t = \mu(1 - e^{-\lambda}) + r_{t-1} e^{-\lambda} + \sigma e^{-\lambda t} \int_{t-1}^{t} e^{\lambda s} dW_s$$

we can remark that the process $(r_t)$ is an AR(1) (Brigo and al, 2007, p. 29). More precisely we have that

$$r_t = c + b r_{t-1} + \delta \varepsilon_t,$$

where the random variable $\varepsilon_t$ is a Gaussian white noise with zero-mean and unitary variance. Comparing this expression of $r_t$ with the previous one, we have:

$$c = \mu(1 - e^{-\lambda}), \quad b = e^{-\lambda}, \text{ and } \quad \delta = \sigma\sqrt{(1 - e^{-2\lambda})/2\lambda}$$

It is clear that we can build estimators of the Vasicek model using those of *b*, *c*, and δ that we can obtain by using OLS method. More precisely, we have that:

$$\hat{\lambda} = -\ln \hat{b}\ ,\ \hat{\mu} = \frac{\hat{c}}{1-\hat{b}}\text{, and } \hat{\sigma} = \frac{\hat{\delta}}{\sqrt{(b^2-1)/2\ln(\hat{b})}}\ , \tag{12}$$

where $\hat{b}$, $\hat{c}$, and $\hat{\delta}$ are the OLS estimators of *b*, *c* and $\delta$.

Before performing the OLS on the equation $r_t = c + b\ r_{t-1} + \delta\ \varepsilon_t$, it is important to test the stationarity of $r_t$ because this property is a symptom/prerequisite of a mean reverting behaviour (Brigo et al., 2007)[12]. To test for the stationarity the ADF test is used. In our case, the ADF statistic has a value equal to -7.72 which is below the critical value -3.96 at 1% level of significance. We therefore accept the stationarity of the daily Moroccan short-term interbank interest rate during the period 2003-2011.

The application of the OLS method to estimate the parameters of the regression of $r_t$ on $r_{t-1}$ yields the following result:

$r_t = 0.0021 + 0.9282\ r_{t-1} + e_t$

The estimated variance of the residuals is $\hat{\delta}^2$=85.16/2268=0.037. Thus, the standard deviation $\hat{\delta}$ of our residual is equal to 0.192. Substituting in the three equations of (12) we recover the OU model parameters:

$$\hat{\lambda} = -\ln \hat{b} = -\ln(0.9289) = 0.0737$$

$$\hat{\mu} = \frac{\hat{c}}{1-\hat{b}} = \frac{0.0021}{1-0.9289.} = 0.0305$$

$$\hat{\sigma} = \frac{\hat{\delta}}{\sqrt{(\hat{b}^2-1)/2\ln(\hat{b})}} = \frac{\sqrt{\frac{0.0083}{2247}}}{\sqrt{(0.9289^2-1)/2\ln(0.9289)}} = 0.0020$$

Finally the equation giving $r_t$ as function of $r_{t-1}$ may be rewritten, in our case by introducing the estimated values of the OU parameters, as follows:

$$r_t = r_{t-1}e^{-0.0737} + 0.0305(1-e^{-0.0737}) + 0.0020\int_{t-1}^{t} e^{-0.0737}dW_s$$

## 5.3 Discussion

In principle, the same parameters estimators are obtained by ML method and OLS method when data are independently and normally distributed. Effectively there are slight differences between the estimates obtained by the two methods. The parameters of an OU process are easily interpreted because this process is intuitively appealing. First, the long-

[12]In a sense, "testing for mean reversion is equivalent to testing for stationarity" (Brigo et al, 2007, p. 25).

run equilibrium level of interest rate may be thought of as the best guess of the cost of funds in the short-run inside a regulated market. Concerning the speed of reversion which is the part of the deviation of $r_t$ from the long-run equilibrium occurred in $t$-1 that is absorbed in period $t$. Faster is the adjustment, shorter is the period necessary to absorb in the next period a deviation of $r_t$ from its long run equilibrium level.
In our case, we have found $\hat{\mu} = 3.05\%$. This implies that a regulated market for short-run funds evaluates the long-run level of the cost of money for a short period to be 3.05%. This rate is below the key rate determined by the central bank, which is 3.25%, by 20 basis points[13]. Concerning the operational efficiency of monetary policy, we can say globally that the central bank was able during the period 2003-20011, to maintain the short-term interbank interest rate around the key rate.
The speed of return of $r_t$ to its long-run level is 0.0737. This implies that if we observe a 1 percent deviation of $r_t$ from its long-run level, then 0.0737 of this deviation is absorbed in the next day. Consequently, 13.4 (1/0.0737) days are, in average, necessary for that market to absorb the previous deviation of $r_t$ from its long-run level. It is important to compare the historical volatility of $r_t$ to its volatility if it is governed by an OU model. Indeed the observed standard deviation of $r_t$ over the period 2003-2011 is 0.0052. This historical volatility is more then twice (2.6) the volatility estimated when we assume that $r$ is governed by an OU model.

## 6 Concluding Remarks

Modelling short-term interest rate is of prime importance because this rate may be used to price interest-rate contingent claims and to hedge interest rate risk. The use of the OU model is justified by the fact that this model is suitable in presence of quantities that fluctuate around an equilibrium value with a bounded variance.
In morocco, the interbank daily interest rate $r$ dances to the key rate tune. The music is monotonous; the key rate remained practically constant over 9 years. It is possible for the central bank to argue that the gravitation of $r$ around the key rate is a validation of its monetary policy. Even if the situation may be a case in favour of the operational efficiency of monetary policy, it is possible that given the conditions of the economy it is more important to explore the possibilities of increasing investment and henceforth growth and employment by reducing the key interest rate.

## References

[1] Bank Al-Maghrib, (2010), Rapport sur la politique monétaire, n° 17
[2] Bank Al-Maghrib, (2010), Rapport sur la politique monétaire, n° 16
[3] Bank Al-Maghrib, (2010), Rapport sur la politique monétaire, n° 14
[4] Bank Al-Maghrib, Monetary policy in Morocco, a paper of the Research and International Relations Department
[5] Berger, E., (1992), Modelling interest rates: fundamental issues, November, 44-47.

[13]In its meeting of March 2012, the Board of Bank Al-Maghrib decided to lower the key rate by 25 basis points. As a consequence, since this date the key rate was set at 3%.

[6] Brennan, M. J. M., and Schwartz, E. S., (1979), "A continuous time approach to the pricing of bonds", Journal of Banking and Finance, **3**, 133-155.
[7] Brière, M., (2005), Formation des taux d'intérêt, anomalies et croyances collectives, Economica, Paris
[8] Brigo, D. N. Dalessandro A., Neugebauer, M., and Triki, F., (2007), A Stochastic processes toolkit for risk management,
[9] Broze, L., Scaillet, O., et Zakoïan, J.-M., (1996), "Estimation des modèles de la structure par terme des taux d'intérêt", Revue économique, n° 3, Mai, 511-519
[10] Chan, K. C., Karolyi, G. A., Longstaff, F. A., Sanders, A. B., (1992), An empirical comparison of alternative models of short-term interest rate, The Journal of Finance, **XLVII**(3), 1209-1227
[11] Chung, C.- F., and Hung, M.-W., (2000), A general model for short-term interest rates, Applied Economics, **32**, 111-121
[12] Direction des études et des prévisions financières, (2011), Fiche relative à l'évolution de la liquidité bancaire et son impact sur le financement de l'économie au cours du premier trimestre 2011, 10 Juin.
[13] Direction des études et des prévisions financières, (2010), Evolution du marché monétaire et obligataire durant l'année 2009.
[14] Gray, S. T., and Talbot, N., (2006), Developing financial markets, Handbooks in central banking, no. 26, Centre for Central Banking Studies, Bank of England
[15] Gruson, P., (2005), Les taux d'intérêt, Dunod, Paris
[16] Kopp E., (2011), From measures to Itô integrals, Cambridge University Press.
[17] Merton, R. C., (1973), "A theory of rational option pricing", Bell Journal of Economics and Management Science, **4**, 141-183
[18] Pai, J. and Pederson, H. W., (1999), Threshold models of the term structure of interest rate, pp. 387-400, Joint day Proceedings Volume of the XXX$^{th}$ International ASTIN Colloquium/9th International AFIR Colloquium, 387-400, Tokyo, Japan
[19] Reimers, M., and Zerbs, M., (1999), A multi-factor statistical model for interest rates, Algo Research Quarterly, **2**(3), September, 53-65
[20] Varma, J. R., (1997), "The stochastic dynamics of the short term interest rate in India", The Indian Journal of Applied Economics, **6**(1), 47-58
[21] Vasicek, O., (1977), "An equilibrium characterization of the term structure", Journal of Financial Economics, **5**, 177-188