Impact of Calendar Effects
in the Volatility of Vale Shares

Lucas Lúcio Godeiro[1]

Abstract

The paper aims to estimate the impact of calendar effects in volatility of the preferred and ordinary shares of Vale. The data researched were the stocks prices Vale between January 2, 1995 and October 26, 2011. The Stochastic Volatility Model(SV) was the Model and the Kalman Filter was the estimation method used. The results indicate that the privatization and the public offer of the stocks of Vale changed the behavior of volatility of the shares. The calendar effects have effect in volatility. The calendar effects had a greater explanatory power over the ordinary shares.

1   Introduction

In the financial market there is alternation between periods of calm and panic, because it is influenced by several facts, news and economic aspects. In

---

[1] Federal Rural University of Semi-Arid, Brazil.

periods of panic it is assumed that there is an increase in volatility. Morais and Portugal (1999, p.304) "state that the correct prediction of volatility is important not only in outline strategies optimal hedge with derivatives as also captures moments of uncertainty in the market."

The authors estimate the deterministics volatilities from IBOVESPA with GARCH and stochastic volatility models with the Kalman Filter. The same include calendar effects and think only tuesday significant in explaining volatility.

Sobrinho (2001) estimate GARCH Model and Stochastic Volatility Model for Ibovespa in the series between 1994 and 2000. The author emphasizes that "both approaches have fulfilled its purpose well, and GARCH models have slight superiority." The author comes to the conclusion "that the models are complementary, not rivals and that both should be used by risk professionals in the market."

Ziegelmann and Valls Pereira (1997) estimate the stochastic volatility model with temporal deformation. According to the authors "the temporal deformation model is justified by the arrival of new information to the financial market." They compare the model results to discrete time and continuous time and concluded that "the models in discrete time with temporal deformation was more erratic, which did not happen in continuous time models."

After reviewing the literature, it is clear that most research uses a series of Ibovespa. Thus, this research aims to analyze the volatility of Vale shares. The analysis of individual assets and not the index has its importance because the asset can be influenced by volatility in their side of unsystematic risk. The international price of iron ore and one of the factors that may affect the volatility of the shares of Vale. Thus, the aim of the research is to estimate the volatility of ordinary and preferred shares of Vale taking into account the influence of calendar effects.

Besides this introduction and closing remarks, the research will have three more sections. The second will discuss the Stochastic Volatility model. The third will approach about the Kalman filter, the Local Level model and data used. The fourth show the search results.

2   Model

The volatility in the market is used to measure the level of risk present in an asset. The stochastic volatility model is used to extract the conditional variance in a series. First we will set the model without the influence of the weekdays to then be introduced the model calendar effects.

2.1   Stochastic Volatility Model

The definition of the compound return of an active according as Valls Pereira (2011) is given by the following equation:

$$r_t = \Delta ln\left(p_t\right) \tag{1}$$

According to the same author the distribution of returns, conditional on past information, is normal with mean zero and conditional variance given by σ_t^2. The logarithm of this variance is characterized by δ an AR (1) with non-zero average. Thus the model which describes the returns is:

$$r_t = \varepsilon_t \sigma_t \quad \varepsilon_t \sim NI\left(0,1\right) \tag{2}$$

$$ln\sigma_t^2 = \delta + \phi ln\sigma_{t-1}^2 + \sigma_n \eta_t \quad \eta_t \sim NI\left(0,1\right) \tag{3}$$

Making the transformation $ln\left(.\right)$ in (2) we obtain the following equations:

$$lnr_t^2 = ln\sigma_t^2 + ln\varepsilon_t^2 \tag{4}$$

$$ln\sigma_t^2 = \delta + \phi ln_{t-1}^2 + ln\sigma_n \eta_t \tag{5}$$

According as Valls Pereira (2011) the equation (4) connects the observations, that is logarithm of the squared return to a not observed, the logarithm of the conditional variance. Equation (5) shows with the variance evolves in time.

2.2   Stochastic Volatility Model with Calendar Effects

According Valls Pereira (2011) the stochastic volatility model with calendar effects incorporates increased in volatility of given day of the week. The goal is to check if any specific day the volatility is higher than the others. Further according to the author above this effects are incorporated if to model the volatility by the equation below:

$$h_t^* = h_t + \beta' x_t \tag{6}$$

Where x_t is weekly dummies . We will incorporate into the model five dummies, one for each day of the week. The dummy x_{1t} will be 1 in the monday and 0 otherwise. Assuming that $y_t = lnr_t^2, \xi_t^2 = ln\varepsilon_t^2$ and $\sigma_\xi = 1$ we have:

$$y_t = \xi_t exp\left(\frac{h_t^*}{2}\right) \tag{7}$$

$$h_t = \phi h_{t-1} + \sigma_\eta \eta_t \tag{8}$$

Making the transformation $log\left(.\right)^2$ in (7) we have:

$$log\left(y_t^2\right) = h_t + \beta' x_t + ln\left(\xi_t^2\right) \tag{9}$$

$$h_t = \phi h_{t-1} + \sigma_\eta \eta_t \tag{10}$$

According to Valls Pereira (2011) this is the linear state space representation for stochastic volatility model with calendar effects.$\beta' x_t$ are observed deterministic variables that can affect the observations. The function of it is to capture the effects calendar.

3   Methods

This section will present the Kalman filter, the Local Level Model and the variables used in the research. The Kalman filter is a method of estimating the coefficients of the model. The model is used to decompose the series in: level (with fixed variance), cycle and irregular. The cycle will follow an autoregressive model of order 1.

3.1   Kalman Filter

The purpose of this part is to briefly present the Kalman Filter. Hamilton (1994) defines the Kalman Filter as algorithm that updates sequentially the linear projection for a system. Some advantages of the method above are cited by author: the generating autocovariances array for spectral analysis and the estimation of vector coefficients that vary over time.

The filter requires the state-space representation of y_t . Define y_t as vector $(n\,x1)$, which can be described in terms of a vector not observed $(r\,x1)$. The state-space representation is given by:

$$\xi_t = F\xi + \upsilon_{t+1} \tag{11}$$

$$y_t = A^{'}x_t + H\xi_t + w_t \tag{12}$$

$F, A^{'}$ and H are parameter array of dimension (r x r), (n x k) and (n x r), respectively. x_t is a vector of exogenous variables or predetermined. The equation (11) is known as the state equation and (12) is the equation of the observations. υ_t and w_t are vectors of white noises not correlated.

The representation of a univariate process ARMA (p, q) is given by Hamilton (1994). The representation of the state equation is $r \equiv max\,[p, q + 1]$:

$$\begin{aligned} y_t - \mu \;=\; & \phi_1\,(y_{t-1} + \mu) + \phi_2\,(y_{t-2} + \mu) + \cdots + \phi_r\,(y_{t-r} + \mu) + \varepsilon_t \\ & + \theta_1\varepsilon_{t-1} + \theta_2\varepsilon_{t-2} + \theta_{r-1}\varepsilon_{t-r+1} \end{aligned} \tag{13}$$

Assume $\theta_j = 0 \; \forall \, j > p$ and $\phi_j = 0 \; \forall \; j > p$. The general representation for the equation of state is:

$$\xi_{t+1} = \begin{bmatrix} \phi_1 & \phi_2 & \dots & \phi_{r-1} & \phi_r \\ 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \dots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & 0 \end{bmatrix} \xi_t + \begin{bmatrix} \varepsilon_{t+1} \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

The observation equation is given by:

$$y_t = \mu + \begin{bmatrix} 1 & \theta_1 & \theta_2 & \dots & \theta_{r-1} \end{bmatrix} \xi_t \tag{14}$$

To continue the process, it is necessary to define the matrices Q, dimension (R x R) and R of dimension (n x n). $E\left(\upsilon_t\upsilon_t^{'}\right) = Q$ if $t = \tau$ and 0 otherwise. $E\left(w_t w_t^{'}\right) = R$ if $t = \tau$ and 0 otherwise.

The objective of the algorithm is to calculate the forecast for the state vector in t +1 given information up to t (Hamilton, 1994, p.377).

$$\hat{\xi}_{t+1|t} \equiv \hat{E}\left(\xi_{t+1}|\vartheta_t\right) \tag{15}$$

$$\vartheta_t \equiv \left(y_t', y_{t-1}', \ldots, y_1', x_t', x_{t-1}', \ldots, x_1'\right)' \tag{16}$$

The variance-covariance matrix (RxR) of the process is represented by:

$$P_{t+1|t} \equiv E\left[(\xi_{t+1} - \hat{\xi}_{t+1|t})(\xi_{t+1} - \hat{\xi}_{t+1|t})'\right] \tag{17}$$

The forecast equation for $\hat{\xi}_{t+1|t}$ is:

$$\hat{\xi}_{t+1|t} = F.\hat{E}\left(\xi_t|\vartheta_t\right) + \hat{E}\left(\upsilon_{t+1}|\vartheta_t\right) \tag{18}$$

$$\hat{\xi}_{t+1|t} = F.\xi_{t|t} + 0 \tag{19}$$

The forecast equation for covariance-variance matrix is:

$$P_{t+1|t} \equiv E\left[(\xi_{t+1} - \hat{\xi}_{t+1|t})(\xi_{t+1} - \hat{\xi}_{t+1|t})'\right] \tag{20}$$

$$P_{t+1|t} = E\left[(F\xi_t + \upsilon_{t+1} - F\hat{\xi}_{t|t})(F\xi_t + \upsilon_{t+1} - F\hat{\xi}_{t|t})'\right] \tag{21}$$

$$P_{t+1|t} = F.E\left[(\xi_t - \hat{\xi}_{t|t})(\xi_t - \hat{\xi}_{t|t})'\right].F' + E\left[\upsilon_{t+1}\upsilon_{t+1}'\right] \tag{22}$$

$$P_{t+1|t} = FP_{t|t}F' + Q \tag{23}$$

The forecast error is defined by:

$$y_{t+1} - \hat{y}_{t+1|t} = A'x_{t+1} + H'\xi_{t+1} + w_{t+1} - A'x_{t+1} - H'\hat{\xi}_{t+1|t} \tag{24}$$

$$y_{t+1} - \hat{y}_{t+1|t} = H'\left(\xi_{t+1} - \hat{\xi}_{t+1|t}\right) + w_{t+1} \tag{25}$$

The variance of forecast error is:

$$E\left[(y_{t+1} - \hat{y}_{t+1|t})(y_{t+1} - \hat{y}_{t+1|t})'\right]$$
$$= E\left[H'(\xi_{t+1} - \hat{\xi}_{t+1|t})(\xi_{t+1} - \hat{\xi}_{t+1|t})'H\right] + E\left[w_{t+1}w_{t+1}'\right] \tag{26}$$

$$E\left[(y_{t+1} - \hat{y}_{t+1|t})(y_{t+1} - \hat{y}_{t+1|t})'\right] = H'P_{t+1}H + R \tag{27}$$

The gain matrix K of Filter is defined by:

$$K_t \equiv FP_{t|t-1}H\left(H'P_{t|t-1}H + R\right)^{-1} \tag{28}$$

With this update equations are given by:

$$\xi_{t+1} = \xi_{t+1|t} + F\left(\xi_t - \hat{\xi}_{t|t-1}\right) + K_t\left(y_t - A'x_t - H'\hat{\xi}_{t|t-1}\right) + \upsilon_{t+1} \qquad (29)$$

$$P_{t+1} = \left(F - K_t H'\right) P_{t|t-1}\left(F' - HK_t'\right) + K_t RK_t' + Q \qquad (30)$$

The maximum likelihood estimators of the parameters can be obtained using the decomposition of the forecast error:

$$\ell(\psi) = -\frac{NT}{2}ln\left(2\pi\right) - \frac{1}{2}\sum_{t-1}^{T}ln|\left(y_t - \hat{y}_{t|t-1}\right)| \qquad (31)$$

$$-\frac{1}{2}\sum_{t-1}^{T}\left\{E[(y_t - \hat{y}_{t|t-1})(y_t - \hat{y}_{t|t-1})']\right\}'\left[(y_t - \hat{y}_{t|t-1})^{-1}\right]\left\{E[(y_t - \hat{y}_{t|t-1})(y_t - \hat{y}_{t|t-1})']\right\}$$

3.2   Level Local Model

In this research, We will estimate a local level Model for squared return of assets. According to Harvey (1992) Model of Local Level is represented by the equation:

$$y_t = \mu_t + \varepsilon_t \qquad (32)$$

$$\mu_t = \mu_{t-1} + \eta_t \qquad (33)$$

The variables ε_t and η_t are white noise and uncorrelated with each other, and with variance σ_ε^2 and σ_η^2 respectively. Further according to the author above the reduced form for the ARIMA model is a (0,1,1):

$$\Delta y_t = (1 - \theta L)\xi_t \quad -1 \leq \theta \leq 0 \qquad (34)$$

$$\theta = \left(\sqrt{q^2 + 4q} - 2 - q\right) \qquad (35)$$

$$q = \frac{\sigma_n^2}{\sigma_\varepsilon^2} \qquad (36)$$

The model can be rewritten as follows:

$$y_t = \frac{\eta}{\Delta} + \varepsilon_t \qquad (37)$$

$$\hat{\varepsilon}_t = \frac{(1 - F)\sigma_\varepsilon^2}{1 + \theta F\sigma^2}\xi_t \qquad (38)$$

$$\hat{\eta}_t = \frac{1}{1 - \theta F}\frac{\sigma_\eta^2}{\sigma^2}\xi_t \qquad (39)$$

Where $F = L^{-1}$ and the estimators ε_t and η_t depend of future innovations and can be represented by an AR (1) and ARMA (1,1) respectively (Harvey, 1992, P.378).

$$\hat{\eta}_t = \hat{\eta_{t-1}} + q\varepsilon_t \quad 0 \leq q \leq \infty \tag{40}$$

The Level Local Model is a structural model. Harvey (1989) defines a structural models as one that has a direct interpretation and the serie is decomposed in level, trend, seasonality, cycles and irregular components.

3.3   Data

The variables used for the research are the ordinary and preferred shares of company Vale S. A.. The ordinary shares have the ticker on the BOVESPA (VALE3) and VALE5 is the ticker of preferred share. We obtained the closing prices of the assets for the period from January 2, 1995 and October 26, 2011.

From the prices were calculated from the compound return of shares VALE3 and VALE5 through the difference of the log and they were centralized on average. Then, from returns mean centering were calculated the logarithm of squared return. The squared return logarithm will be the variable modeled.

To capture the Calendar Effects, were constructed 5 binary variables that represent the days of the week. Through the significance of these proxies will be checked the influence of the weekday in the asset volatility.

4   Results

The Figure below show the prices and compound return of Vale shares:

A visual inspection of the graphs can identify periods of high volatility, such as the devaluation of the real in 1999 and the sub-prime crisis of 2008. It is noticed also that both actions reached their historic peak in 2008, just before the collapse of Lehman Brothers. In 2010 the Shares came close to achieving the same value of 2008, but the debt crisis in Europe sparked greater risk aversion in the markets and share prices fell again.

The next step was to obtain the log of square return centered on average. Need to center the return on average because there are days when the
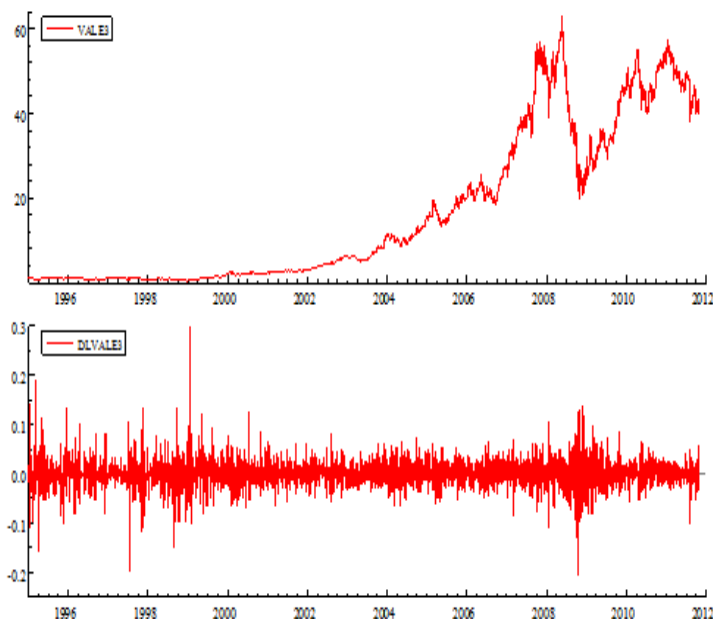
Figure 1: Prices and Compound Returns VALE3

compound return is equal to zero. The graphs of the transformed variables are:

We will estimate the parameters Stochastic Volatility Model using a Local-level model with level of variance fixed, cycles following an autoregressive of order 1 and a irregular component.

It appears that Vale's preferred shares have a higher persistence in volatility, given that the coefficient of the autoregressive part of the model was slightly higher. With respect to variations in volatility influenced by calendar effects, it is observed that the ordinary shares are more influenced. Just Monday was not significant for the VALE3 Shares. In both assets researched there was a reduction in volatility on Friday. The preferred stocks of Vale suffer less influence from the days of the week, because only the Thursday and Friday were significant. The next stage of research will extract the conditional variance of assets surveyed. The conditional variance of assets is shown in the graphs below. The graph below represents the volatility with and without calendar effects:

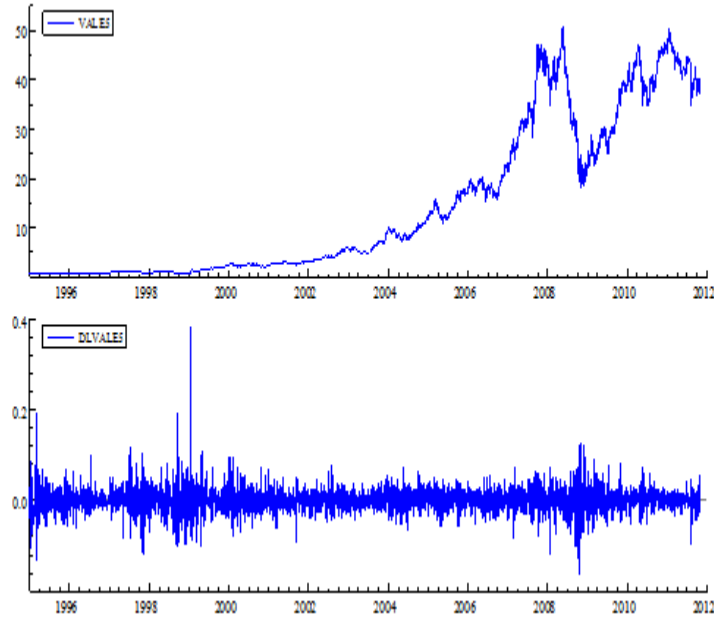One of the findings of the research, analyzing the graphs 6 and 7 is that

Figure 2: Prices and Compound Returns VALE5

Table 1: Estimated Parameters

Variables	VALE3 Parameter	VALE3 Variance	VALE5 Parameter	VALE5 Variance
AR(1)	0.9533*	0.8707	0.9686*	0.7614
DMON	0.1173	0.1204	0.0904	0.1073
DTUE	0.1978***	0.1206	0.0681	0.1074
DWED	0.2142**	0.1206	0.1685	0.1074
DTHU	0.2931**	0.1204	0.1946***	0.1073
DFRI	−0.2932**	0.1188	−0.1946***	0.1073

*,** and *** indicate significance at 1%, 5% and 10% respectively.

the preferred shares of ordinary stock had disparities between the years 1997 and 1998. One explanation may be the fact that Vale was privatized in this period, more precisely in May 1997. Privatization may have diminished the roles ordinary free float on the BOVESPA . The model identifies increases

Figure 3: Logarithm of square return centered in the mean VALE3 and VALE5

volatility in the Asian crisis in 1995, the devaluation of the real in 1999 and real state crisis in 2008. Facts such as the crisis of the 2002 elections did not lead to large increase in volatility in shares of Vale. In the post 2008 crisis we can see a downward trend in volatility in 2009, the year of recovery BOVESPA, but by mid 2010 and the first half of 2011 the risk aversion dominates the market and volatility reverts to bullish .

Another spectrum of analysis of the research will be the time trajectory of the coefficients of the dummies that enter the model as explanatory variables. This analysis is important in order to observe in which period the weekdays influenced more or less the volatility of assets surveyed.

Note that the coefficients of the days Monday and Wednesday from VALE3 exhibit similar behavior. The same follow the behavior of a cycle, which could be described by a cosine function. The parameters of the Tuesday and Thursday were a small influence in volatility over time. The Friday of VALE3 performance was stable throughout time.

With respect to the coefficients of the Monday and Tuesday VALE5 are similar because they lessen its influence over time. They have a strong decay in
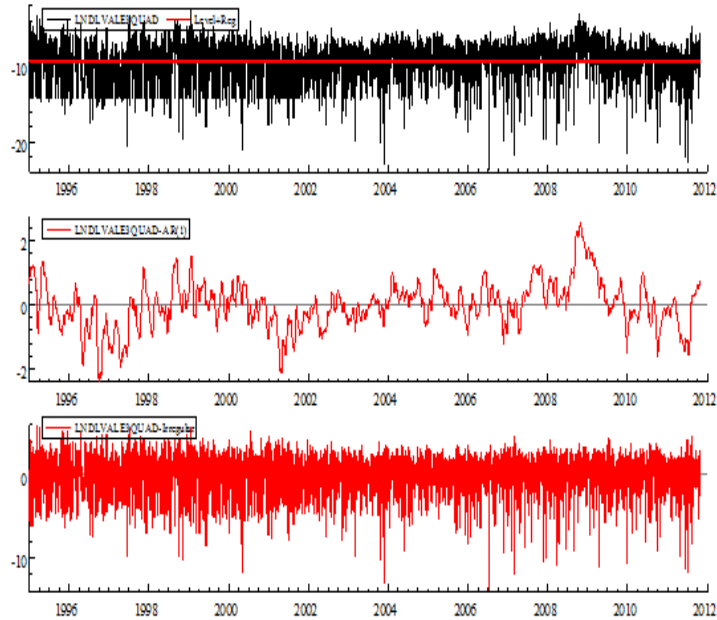
Figure 4: Estimated Model for VALE3

the beginning and then converged to stability. The parameters of Wednesday, Thursday and Friday are stable.

5   Conclusions

The research purposed to estimate the volatility of the shares of Vale since 1995. One important aspect to highlight is that the model was able to predict times of economic crisis, pointing to the increased volatility in these periods. Regarding the effects calendars they had a slightly larger influence on the Ordinary Shares of Vale.

The model captured a significant difference in volatility between the two actions for the period of privatization of Vale. At that time, between the years 1997 and 1998 the preferred share had an increase in volatility. In the crisis of 2008 notes a higher volatility of the ordinary shares of VALE, that fact can be explained because the company sold a greater number of shares in the public offering in July 2008.
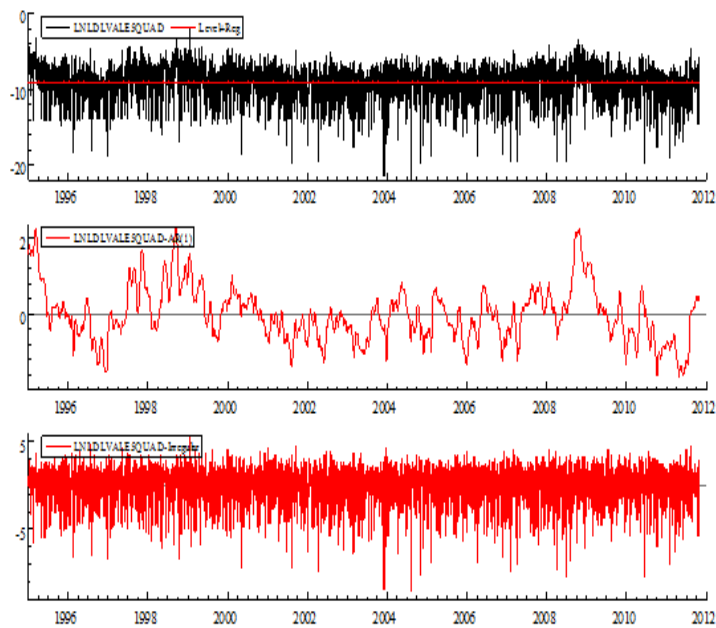
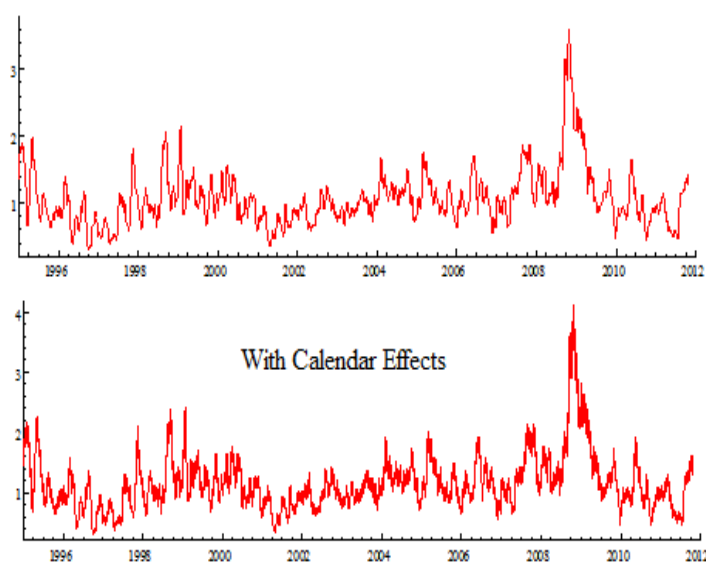Figure 5: Estimated Model for VALE5
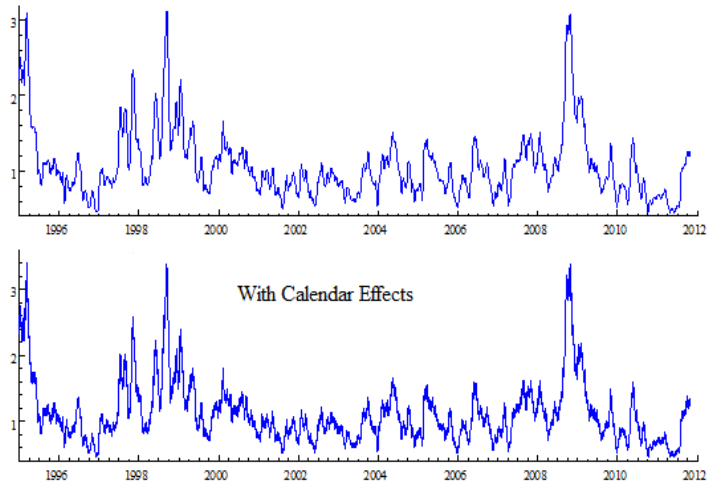

Figure 6: Volatility of VALE3
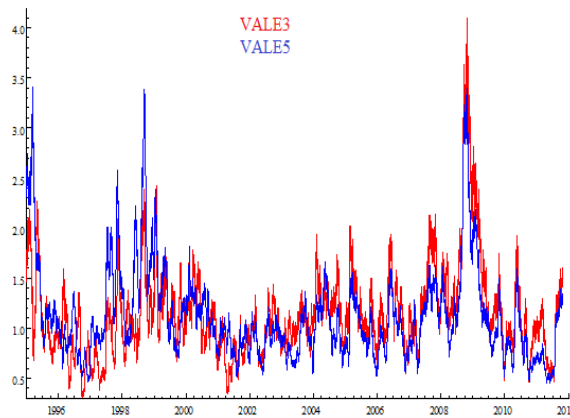
Figure 7: Volatility of VALE5

Figure 8: Volatility of VALE3 and VALE5 with Calendar Effects

The estimation of the volatility of an asset and not the index is important because one of the ways to beat the market is choosing an asset that obtains a better performance than the marke. This is the case of shares of VALE, that for the period covered performed better than the main stock market index in Brazil. Therefore it is suggested that future research work not only with the Ibovespa, but also assets that were best and worst returns the same to thereby make a comparative analysis between the estimated volatilities.
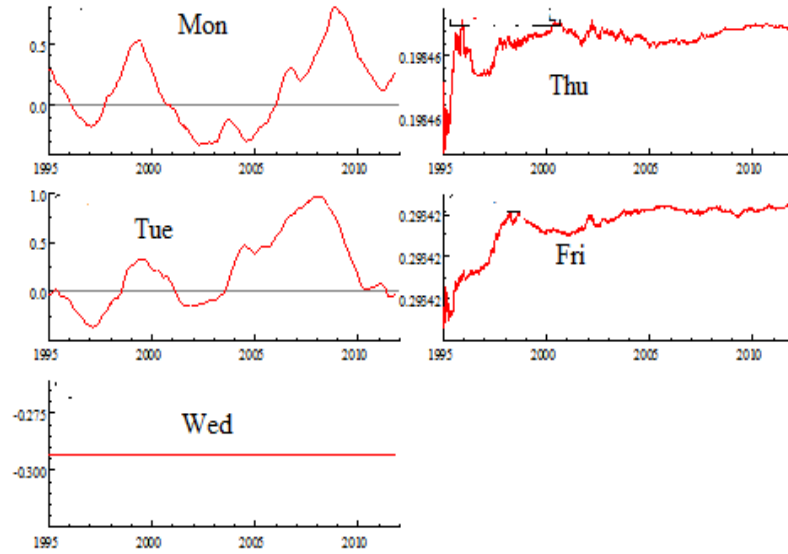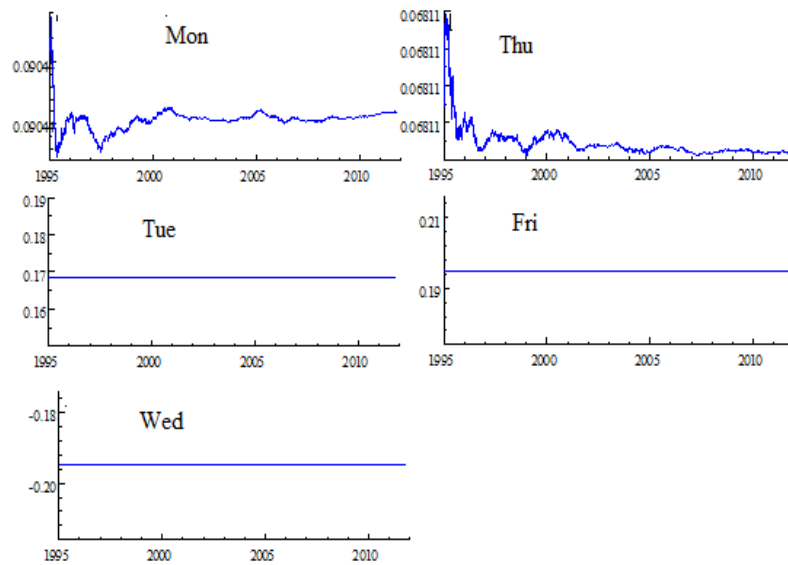
Figure 9: Time-Varying coefficients of VALE3 Dummies

Figure 10: Time-Varying coefficients of VALE5 Dummies

References

[1] J.D. Hamilton, *Times Series Analisys*, Princeton University Press, 1994.

[2] A.C. Harvey, Diagnostic Checking of Unobserved-Components Time Series Models, *Journal of Business & Economic Statistics*, **10**(4), (October, 1992).

[3] A.C. Harvey, *Forecasting Structural Time Series Models and the Kalman Filter*, Cambridge University Press, 1989.

[4] I.A.C Morais, & M.S. Portugal, Modeling and Forecasting Deterministic and Stochastic Volatility for Series of Ibovespa, *Economic Studies*, São Paulo, **29**(3), (1999), 303-341.

[5] N.F.S. Sobrinho, Extraction of volatility of Ibovespa, *Review BM & F*, **144**, (2001).

[6] P.L. Valls Pereira, *Kalman Filter*, Lecture notes, EESP-FGV, São Paulo, 2011.

[7] F.A. Ziegelmann, & P.L. Valls Pereira, Stochastic Volatility Models with temporal deformation: an empirical study for the Bovespa index, *Research and Economic Planning*, **27**(2), (1997), 353-376.