Measuring and Modelling the Market Liquidity of Stocks: Methods and Issues

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Abstract

The liquidity of an asset in modern financial markets is a key and, yet, elusive concept. A market is often said to be liquid when the prevailing structure of transactions provides a prompt and secure link between the demand and supply of assets, thus delivering low costs of transaction. Providing a rigorous and empirically relevant definition of market liquidity has, however, provided to be a difficult task. This paper provides a critical review of the frameworks currently available for modelling and estimating the market liquidity of stocks. We discuss definitions of market liquidity that stress the role of the bid-ask spread and the estimation of its components arising from alternative sources of market friction. In this case, measures of liquidity based on intra-daily data are relevant for capturing the core features of a market, and for their ability to account for the arrival of new information.

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1   Introduction

The scope of this paper is to present a guided tour around various measures
for liquidity of stocks proposed in the literature on asset pricing and market
microstructure. Given the large number of liquidity measures and methodolo-
gies employed both by practitioners and academic researchers, we review the
role of each liquidity measure by discussing the logic behind their construction,
and how they are related. Our paper focuses on the stock market. We should
stress, though, that the key concepts and methods can be extended to other
assets traded in markets with an organization of exchanges similar to those for
stocks.

Liquidity is often pointed at as a key concept in financial markets. It is a
very elusive concept though. In general terms, it denotes a desirable function
that should reflect a well organized financial market. A market is often said
to be liquid when the prevailing structure of transactions provides a prompt
and secure link between the demand and supply of assets, thus delivering low
transaction costs. Providing a rigorous definition of market liquidity that is
relevant for practival applications is, however, a cumbersome task.

Differently from a widely-quoted survey of Baker [8], we consider definitions
of market liquidity that emphasize the role of the bid-ask spread and the
estimation of its components. The difference between bid and ask quotes for
an asset provides a liquidity measure applicable to a dealer market, rather
than a broker market. Despite this, it is possible to compute approximations
that mimic the difference between bid and ask quotes even in broker markets.
Hence, the role of intra-daily measures of liquidity can capture the core features
of a market, such as the arrival of new information to the trading parties.

Laying out an operational definition of liquidity involves the specification
of two additional concepts. These are the the transaction time - i.e. the speed
of executing transactions - and the pure transaction costs - i.e. the price paid
by investors for the liquidity services. The time of transaction is related to the
demand pressure generated by the market. This takes the form of a request for

a quick execution of the orders. At the same time, an order request involves the opportunity for the investor to buy or sell a stock at the prevailing price, or at a price close to the one prevailing in the market.

These intuitive considerations provide the ground for a relevant concept of liquidity. In other words, an asset is liquid if it can be quickly exchanged at a minimal cost. A similar definition can be applied also to a market as a whole. In this sense, a market is liquid if it is possible to buy and sell assets at a minimal cost without too much delay from order placement.

Another important aspect concerns the extent to which asset prices are affected by the trading activity. In a liquid market, changes in prices should not be determined by transaction costs. In other words, block size transactions should have a minimal impact on prices. Prices usually change both in anticipation and in response to order flows. Hence, it becomes crucial to understand the extent to which the amount of transactions or order size can determine large price swings. In a 'thin' market, prices are highly responsive to trade size. In a liquid or 'deep' market, prices can be affected by order flows only to a minor extent.

While there are different concepts of asset liquidity, available measures of liquidity focus on alternative aspects of the measurement problem. Some indices concentrate on the role of volume size. They are based on volume information reflect the price impact of transactions. When properly aggregated, they provide also synthetic measures of the liquidity present in an entire market. Other liquidity measures are related to the execution-cost aspect of liquidity. These types of indices evaluate the the cost paid to the market-maker - i.e. either dealer or specialist - for matching the demand and the supply. These analyses are generally based on the bid-ask spread and its variations. In fact, when a dealer or a specialist revises the bid-ask quotes, a careful study of bid-ask spread components can reveal information on the sources of liquidity.

The literature identifies three main component of the bid-ask spread. These arise from order processing, adverse information and inventory costs. A high level of competition between intermediaries allows for a reduction of the order processing component, and improves the liquidity condition of the market. The informational component of the bid-ask spread sheds light on the degree of efficiency due to the presence of hidden information or insider trading.

The content of the paper is organized as follows. Section 2 discusses the

various concepts of liquidity with a view on their implications for asset pricing. Section 3 focuses on liquidity measures obtained from information on traded volumes. Section 4 considers the indices computed from bid and ask prices. Section 5 focuses on the role of transaction costs as a source of asset illiquidity. Section 6 proposes some concluding remarks.

2   Why is market liquidity important?

The relevance of liquidity arises from its connection with the institutional organization of a market. According to Baker [9], we can identify three main aspects i the relation between market orders and prices, which determine key properties of a market:

1. Depth: a market is deep when there are orders both above and below the trading price of an asset.

2. Breadth: a market is broad when there is a large volume of buying and selling orders. The spread is large when the order flow is scarce.

3. Resiliency: a market is resilient if there are many orders in response to price changes. There is a lack of resiliency when the order flow does not adjust quickly in response to price swings.

All these aspects play a crucial role in the evaluation of the structure of a financial market. In fact, the availability of liquidity has important consequences both on the prices of assets, and on the degree of competition between market actors.

How do these aspects of market organization relate to measure of asset liquidity? According to John Maynard Keynes, an asset is liquid if

> "it is more certainly realizable at short notice without loss."

This quotation highlights two aspects. The first one has to do with the riskiness of the realization of an asset value. The second one considers the presence of a marketplace where negotiations can take place without adverse price oscillations. Subsequent contributions have pointed out the role of speed and the costs associated to market exchanges. Massimb and Phelps (1994) focus on the importance of immediacy. Liquidity consists in the

"market ability to provide immediate execution for an incoming market order (often called "immediacy") and the ability to execute small market orders without large changes in the market price(often called "market depth" of "resiliency")."

The key point of the concept of liquidity is the possibility to exchange an asset in the market without dramatic changes in the prevailing market price. Sensible empirical implementations of this idea are hard to construct because the 'true' degree of liquidity is unobservable. This would be well represented by the difference between the observed transaction price and the price that would occur in complete absence of transaction costs.

3   Volume-based liquidity measures

In this section we present the liquidity indices proposed in the early stages of the market microstructure literature. Their emphasis is on the relationship between prices and quantities, as they evaluate the degree of price impact of a transaction of a specific size.

3.1   Trading volume

A rough measure of liquidity is represented by the traded volume. Some researchers consider trading volume as an inappropriate liquidity index, though. The reason lies in the issue of double counting. A transaction on the buy side can be also recorded as transaction on the sell side. Hence, a more suitable measure is provided by the ratio between trading volume and market capitalization.

For a stock, the traded volume is one of the key determinant for the whole pricing structure. For example, Blume, Easley and O'Hara [11] argue that volume traded generates information that cannot be extracted from alternative statistics.

3.2   The conventional liquidity ratio

The liquidity ratio, also called 'conventional liquidity ratio', is probably one of the liquidity measures most frequently applied in the empirical analysis. It provides an index for how much traded volume is necessary to induce a one-percent price change. The analytical expression for asset i is:

$$LR_{it} = \frac{\sum\limits_{t=1}^{T} P_{it} V_{it}}{\sum\limits_{t=1}^{T} |PC_{it}|} \tag{1}$$

where P_{it} denotes the price of asset i on day t, V_{it} is the volume traded, and $|PC_{it}|$ is the absolute percentage price change over a fixed time interval, given by $PC_{it} = P_{it} - P_{it-1}$.

The liquidity ratio is usually computed for a number of assets, and is aggregated over a pool with similar characteristics. The time interval (T, t) adopted to compute the index is typically chosen arbitrarily. However, it is often calculated over a monthly time scale. This way, the numerator denotes the total volume of the traded assets over the previous four weeks. Instead, the numerator is the absolute value of the daily percentage price changes of the stock over the last four weeks. The higher the ratio LR_{it} is, the higher the liquidity of asset i will be. This means that large volumes of trades have little influence on prices. Obviously, this conceptual framework focuses more on the price aspect than on the issues of time or execution costs typically present in a market.

3.3   The index of Martin [38]

Martin [38] proposes a liquidity index where a stationary distribution of price changes is assumed to hold through the entire transaction time. The analytical expression for the index takes the form:

$$MLI_t = \sum_{i=1}^{N} \frac{(P_{it} - P_{it-1})^2}{V_{it}} \tag{2}$$

where P_{it} is the closing price and V_{it} denotes the traded volume. The reader should notice that this index is computed over the total number of asset for the market. MLI_t is considered a suitable index for the market as a whole, while the liquidity ratio is best suited for a single asset.

A higher value for MLI_t implies less liquidity because of the influence of price dispersion. The higher the ratio, the higher the price dispersion relative to the traded volume, and the lower is the liquidity of the market.

Given its properties, Martin's (1975) liquidity index produces meaningful results if computed on a daily basis. To obtain sensible outcomes for longer time horizons, one needs to compute a weighted average of indices derived for shorter time intervals.

3.4   The liquidity ratio of Hui and Huebel [34]

Hui and Huebel [34] introduce an additional measure of liquidity for a single asset. The mathematical expression of the index is:

$$LR_{HH} = \frac{\left(P^i_{\max} - P^i_{\min}\right) / P^i_{\min}}{V^i / \left(S \cdot \overline{P}\right)} \tag{3}$$

where $P^i_{\max}$ is the highest daily price over a 5-day period, $P^i_{\min}$ is the lowest daily price over the same horizon, V^i is the total volume of assets traded over a 5-day period, S is the total number of assets outstanding and $\overline{P}$ denotes the average closing price. A higher value for the index LR_{HH} implies lower liquidity.

Inspecting equation (3) reveals that the logic behind the construction of this index is not very different from that underlying MLI_t. In fact, the denominator of (3) represents the traded volume adjusted for market capitalization. The numerator indicates the largest percentage price change over a 5-day horizon.

The ratio proposed by Hui and Huebel [34] suffers at least of two shortcomings. First, the time period consisting of 5 days is arguably too long for the index to detect market anomalies, given the fact that asset prices can quickly adjust to liquidity problems. The second critical point is related to the choice of variables. For instance, if we focus on stocks quoted in a dealer market, such as the NASDAQ, high-quality price data may not be readily available. In this case, it is possible to replace $P_{\max}$ and $P_{\min}$ with the bid-ask spread. However, this represents a problematic approach because the bid-ask spread quotes are often less volatile than prices. Hence, the use of bid-ask quotes may bias downward the analysis of liquidity.

3.5   The turnover ratio

The turnover ratio TR_t^i for a stock i at time t is defined as follows:

$$TR_t^i = \frac{Sh_t^i}{NSh_t^i} \qquad (4)$$

where Sh_t^i is the number of units traded, and NSh_t^i is the total number of asset units outstanding. This index is computed for a single time period, which could be a day or a month. Often it is averaged out over a given period:

$$TR_T^i = \frac{1}{N_T} \sum_{t=1}^{N_T} TR_t^i \qquad (5)$$

with a number of sub-periods N_T.

The indices outlined earlier can be included into parametric models for asset prices. For instance, we can consider a regression of asset returns to test for the statistical significance of various liquidity measures.

Datar, Naik and Radcliffe [15] use the turnover rate as a proxy for liquidity in a test for hte role of liquidity that can be easily applied. From Amihud and Mandelson [4], we know that, in equilibrium, liquidity is correlated with trading frequency. Therefore, by directly observing the turnover rate, it is possible to back out trading frequency as a proxy for liquidity. Datar, Naik and Radcliffe [15] perform the following regression in cross sectional data:

$$R_t^i = k_0 + k_1 TR_t^i + k_2 b_t^i + k_3 lsize_{t-1}^i + k_4 \beta_t^i + e_t$$

where R_t^i is the return of stock i at month t, TR_t^i is the turnover ratio at month t, b_t^i is the book to market ratio expressed as the natural logarithm of book value to market value for each individual firm, $lsize_t^i$ is the natural logarithm of total market capitalization of firm i at the end of the previous month. The results suggest that stock returns are a decreasing function of the turnover rates. After controlling for b_t^i, $lsize_{t-1}^i$, and β_t^i.

3.6   The market-adjusted liquidity index

Hui and Heubel [34] propose a measure for liquidity that takes into account the systematic sources for market risk. The construction of this market index

involves two steps. In the first step, a market model for the asset return is estimated to control for the effects of average market conditions on price changes. For stock prices, this stage typically consists in estimating the following model:

$$R_{it} = \alpha + \beta R_{mt} + \varepsilon_{it} \tag{6}$$

where R_{it} is the daily return on the i-th stock, R_{mt} is the daily market return on the aggregate stock market index, α is a constant, β measures the systematic risk, and ε_{it} denotes a measure of idiosyncratic risk. The motivation for using this model relies on the idea that part of the stock's specific risk reflects the liquidity in the market. Thus, more liquid stocks display smaller random price fluctuations, and tend to perform as the market model would suggest. In other words, larger price dispersion is a characteristic of stocks with low liquidity, and that deviate from the market model.

The second step in the construction of the index consists in the definition of a model for idiosyncratic risk. This can be formalized as:

$$\varepsilon_{it}^2 = \phi_0 + \phi_1 \Delta V_{it} + \eta_{it} \tag{7}$$

where ε_{it}^2 are the squared residuals from equation (6), ΔV_{it} is the daily percentage change in currency volume traded, η_{it} is an i.i.d. residual with zero mean and constant variance.

The market-adjusted liquidity ratio is identified as the coefficient ϕ_1 in equation (7). A small value of ϕ_1 indicates that prices change little in response to variations in volume. Thus, this measure takes into account the price effect arising from changes in liquidity conditions, which are mimicked by the change in trading volume. A liquid stock is characterized by a low exposure to liquidity risk which is, in turn, measured by a low ϕ_1.

This liquidity measure provides sensisble results on the assumption that asset prices behave according to the market model. If deviations from the market model are due to swings in volume, there is an identification problem. Despite this issue, the market-adjusted liquidity index provides for a simple way to test for liquidity effects. This is widely applied both to dealer and auction markets.

3.7 An explicit illiquidity measure

The role of traded volume is central to the liquidity measures proposed in

the recent years. A relevant index of illiquidity is introduced by Amihud [1]:

$$ILLIQ_T^i = \frac{1}{D_T} \sum_{t=1}^{D_T} \frac{|R_{t,T}^i|}{V_{t,T}^i} \tag{8}$$

where D_T is the available data length, $R_{t,T}^i$ is the return on day t of year T, and $V_{t,T}^i$ is the daily volume. The day-t impact on the price of one currency unit of volume traded is given by the ratio $\frac{|R_{t,T}^i|}{V_{t,T}^i}$. The illiquidity measure (8) is the average of the daily impacts over a given sample period.

The illiquidity measure of Amihud [1] is similar to the liquidity ratio. The latter provides an understanding of the link between volume and price changes. The illiquidity index delivers only a rough measure of the price impact. Differently from the bid-ask spread, the main advantage of this index relies on the wide availability of data for its computaiton, especially for those markets that do not report reliable spread measures.

Amihud [1] has introduced the illiquidity index to investigate the influence of market conditions on stock returns. His framework introduces a cross-sectional test by selecting a sample of stocks quoted on the NYSE. The testing model takes the form

$$\begin{aligned} R_{m,T}^i = & \lambda_0 + \lambda_1 ILLIQMA_{m,T-1}^i + \lambda_2 ATR_{m,T-1}^i + \lambda_3 v_{m,T-1}^i + \lambda_4 p_{m,T-1}^i + \\ & + \lambda_5 c_{m,T-1}^i + \lambda_6 dy_{T-1} + \lambda_7 R_{100}^i + \lambda_8 R_{T-1}^i + \lambda_9 \sigma R_{T-1}^i \\ & + \lambda_{10} \beta_{T-1}^i + u_t^i \end{aligned} \tag{9}$$

The stock return $R_{m,T}^i$ in month m for year T is regressed over several variables, including a constant λ_0, the mean adjusted illiquidity measure at the end of year T-1, $ILLIQMA_{m,T-1}^i$, the mean adjusted turnover ratio $ATR_{m,T-1}^i$, the log of traded volume, $v_{m,T-1}^i$, the log of stock price $p_{m,T-1}^i$, the log of capitalization $c_{m,T-1}^i$, the dividend yield dy_{T-1}, computed as the sum of the annual cash dividends divided by the end-of-year price. Moreover, R_{100}^i and R_{T-1}^i are the cumulative stock returns over the last 100 days and the entire year, respectively, σR_{T-1}^i is the standard deviation of the stock daily return during year T-1, and β_{T-1}^i is the beta of stock i computed for portfolios of stocks of homogeneous size.

We can also consider a mean-adjusted illiquidity measure:

$$ILLIQMA_{m,T}^i = \frac{ILLIQ_{m,T}^i}{AILLIQ_{m,T}}$$

where $AILLIQ_{m,T}$ is the average illiquidity for the stocks included in the regression model. In general, the annual average illiquidity across stocks is defined as:

$$AILLIQ_T = \frac{1}{N_T} \sum_{i=1}^{N_T} ILLIQ_T^i$$

where N_T is the number of stocks in year T. The variable $ATR_{m,T}^i$ is computed in the same way. This transformation allows to take into account the time-series variability in the estimated coefficients, which can arise from high volatility associated to illiquidity.

The empirical results of Amihud [1] show that the illiquidity measure is statistically significant for NYSE stocks during the period 1964-1997. The coefficient on the illiquidity measure has a positive sign. Stock turnover, instead, delivers an estimated negative coefficient. Moreover, if stock returns are computed in excess of the Treasury bill rate, the model results show that the compensation for expected market illiquidity is still sizeable.

3.8   General comments on volume-based measures

We can point out at least three issues arising from the use of liquidity indices based on volume.

First of all, these measures fail to distinguish between transitory and persistent price effects of changes in traded volume. A transitory effect can often be explained as a temporary lack of liquidity in the market, or arise from the pure transaction cost component. A permanent price effect is a price change due to the presence of information asymmetries between traders. This is related to changes in the fundamental value of assets anticipated by part of the market because of inside information.

The distinction between transitory and permanent price effects can also be thought of as a problem similar to the decomposition of a time series into a stationary and a random-walk component, as studied by Beveridge and Nelson [10]. The dichotomy between permanent and non-permanent effects can be identified from pricing errors. These consist in the difference between the 'efficient' unobserved price and the actual transaction price.

A pricing error can be decomposed into an information-related component and an uncorrelated term. The latter arises from price discreteness, temporary

liquidity effects and inventory control. Information-based pricing errors are related to adverse selection. The presence of traders with superior information about assets, and a lagged adjustment of the market to new information.

As discussed by Hasbrouck and Schwarts [29], this decomposition can be obtained only by studying the components of the bid-ask spread. French and Roll [16] suggest that the role of information is crucial in determining the volatility of returns. Price volatility can be the result of informational asymmetry, rather than a consequence of lack of liquidity. These are aspects that cannot be accounted for by volume-based indices.

A second problem with volume indices is that they do no show how a sudden order arrival affects prices. This is the so called 'order-induced effect'. In other words, volume indices take into account only past links between changes in prices and volumes. The reason is that these measures are not based on theoretical models of dealer/specialist behavior.

An additional issue is discussed by Marsh and Rock (1986). They argue that conventional liquidity indices tend to overestimate the impact of price changes on large transaction deals. They also underestimate the effect of price changes on small transactions. This issue arises from the lack of proportionality between prices and volume that characterizes all the volume-based liquidity measures.

Despite these shortcomings, measures based on volume can be employed fruitfully to model liquidity for agency markets rather than for dealer markets. In fact, the problem, especially for the volume indices computed on a daily basis, is that they do not take into account the effect of large block trades, which are instead very common in dealer markets. On the other hand, these indices represent a useful starting point for a more thorough analysis.

4   Price-variability indices

In this category we can include the measures that infer asset or market liquidity directly from price behavior. We consider the Marsh and Rock [37] liquidity ratio and the variance ratio. A second group of indices infers the liquidity condition by using mere statistical techniques.

4.1   The of liquidity ratio Marsh and Rock [37]

Differently from the liquidity measures considered so far, Marsh and Rock [37] assume that price changes are independent from trade size, except for large traded blocks. This is based on the argument that standard liquidity ratios are strongly affected by trade size. The expression for this index is given by:

$$LR_{MR}^i = \frac{1}{M^i} \sum_{m=1}^{M^i} \left| \frac{P_m^i - P_{m-1}^i}{P_{m-1}^i} \right| \cdot 100 \tag{10}$$

where M^i is the total number of transactions for asset i. The expression after the summation term denotes the absolute value of percentage price change over two subsequent periods. Intuitively, the index (10) considers the relation between the percentage price change and the absolute number of transactions. Thus, this index shifts the attention from the aggregate market to the microstructure effects, which are represented by the number of transactions. In fact, differently from the volume-based indices - where traded volumes drive the scaling effect -, the scaling variable is determined by the number of transactions here. This reflects the idea that the liquidity of an asset is better represented by the price effects of transactions, rather by the impact on volumes.

To provide a better explanation, let us consider a market with two stocks. Stock A is traded in large blocks once a day. Stock B is exchanged for the same total volume, but for transactions of smaller size. Common wisdom would suggest that asset B is more liquid than asset A. Unfortunately, even if price changes for both A and B were similar across markets, we would not be able to reach this conclusion by looking at volume-based liquidity measures. This example helps to clarify the value generated by the liquidity index of Marsh and Rock [37].

The main issue with this type of index is determined by the arbitrariness involved in its formulation. In particular, the length of the period over which the index can be computed is not explicitly specified. Owing to its underlying properties, it is reasonable to adopt the March and Rock (1986) ratio for short horizons. Differently from alternative indices though, this measure is suitable for both dealer and auction markets.

4.2   The variance ratio

The variance ratio, also called market efficiency coefficient (MEC), measures the impact of execution costs on price volatility over short horizons. The idea behind the construction of this index can be summarized as follows. With high execution costs, asset markets are characterized by price volatility in excess of the theoretical volatility of equilibrium prices. Therefore, a more liquid market implies a smaller variance of transaction prices around the equilibrium price. The reason is that the difference between actual and equilibrium price in a liquid market is smaller than what one should observe in an illiquid market.

Denote by $var\left(R_T^i\right)$ the long-term variance, and by $var\left(Z_T^i\right)$ the short-term variance of asset return i. T is the number of subperiods into which longer periods of time can be divided. The variance ratio VR^i can be defined as follows:

$$VR^i = \frac{var\left(R_T^i\right)}{T \cdot var\left(Z_T^i\right)} \tag{11}$$

This metric compares the long-term variance with the short-term variance. When $VR^i < 1$, it suggests that the market is illiquid. In other words, the short-term retur is higher than the long-term return. If we assume that the markets are in equilibrium in the long run, this implies a large discrepancy between the short and long-term equilibrium return. Of course, when the two returns coincide, the liquidity index is equal to one.

The variance ratio is often used to test for market efficiency. This is done by measuring the deviation of an asset price from the random-walk hypothesis. Let the asset price P_t follow a random-walk process:

$$P_t = P_{t-1} + \eta_t \tag{12}$$

where η_t is a homoskedastic disturbance uncorrelated over time, i.e. $E\left(\eta_t\right) = 0$, $Var\left(\eta_t\right) = \sigma_\eta^2$, $E\left(\eta_t\eta_\tau\right) = 0$ for all $\tau \neq t$. Under the random walk hypothesis, from (12) we obtain $\Delta P_t = \eta_t$. To construct the variance ratio, we can show that:

$$var\left(P_t - P_{t-2}\right) = var\left(P_t - P_{t-1}\right) + var\left(P_{t-1} - P_{t-2}\right) = 2\sigma_\eta^2$$

$$var\left(P_t - P_{t-T}\right) = T\sigma_\eta^2$$

Therefore, under the random walk hypothesis, $\Delta P_t = R_T$, which delivers the variance ratio:

$$VR_T = \frac{var\left(P_t - P_{t-T}\right)}{T\sigma_\eta^2} = 1$$

For $VR_T = 1$, there are no deviations from the random walk hypothesis.

Now let us consider the implications of the deviation from the random walk hypothesis. We concentrate on the following case:

$$var\left(R_t + R_{t-1}\right) = 2Var\left(R_t\right) + 2Cov\left(R_t R_{t-1}\right)$$

The variance ratio can be constructed from:

$$
\begin{aligned}
VR\left(2\right) &= \frac{2Var\left(R_t\right) + 2Cov\left(R_t R_{t-1}\right)}{2Var\left(R_t\right)} = 1 + 2\left[\frac{Cov\left(R_t R_{t-1}\right)}{2Var\left(R_t\right)}\right] = \\
&= 1 + 2\rho\left(1\right)
\end{aligned}
$$

where $\rho\left(1\right)$ is a proxy for the correlation coefficient, whose expression is given by:

$$\rho\left(1\right) = \frac{Cov\left(R_t R_{t-1}\right)}{2Var\left(R_t\right)}$$

If we generalize this argument, we obtain the following expression for the variance ratio:

$$VR\left(T\right) = \frac{Var\left(R_t\right)}{TVar\left(R_T\right)} = 1 + 2\sum_{s=1}^{T-1}\left(1 - \frac{s}{T}\right)\rho\left(s\right) \qquad (13)$$

With serially-uncorrelated asset returns, i.e. if $\rho\left(s\right) = 0$, for $s > 1$, the variance ratio is equal to 1. With autocorrelation of order 1,

$$R_t = \phi R_{t-1} + \varepsilon_t$$

and $E\left(\varepsilon_t\right) = 0$, $Var\left(\varepsilon_t\right) = \sigma_\varepsilon^2$, the expression for the variance ratio becomes:

$$VR\left(T\right) = 1 + 2\sum_{s=1}^{T-1}\left(1 - \frac{s}{T}\right)\phi^k$$

The variance ratio can be computed over arbitrary time intervals. For example, Hasbrouck and Schwarts [29] calculate it over three distinct time intervals. They consider the ratio of two-day to half-hour variance, the ratio of one-day to one-hour variance, and the ratio of two-day to one-day return variance. The logic behind this analysis lies in the different informational

content of short-term and long-term variance. In fact, a sequence of short-term transactions tends to affect the market price in a way more marked than a set of transactions measured over a longer period.

From these considerations, it is reasonable to expect a value for the variance ratio larger than unity in the presence of sequential information arrival, market-maker intervention and other factors implying undershooting of price level. It is clear, however, that this index cannot account for all the causes of liquidity costs.

The variance ratio displays two additional shortcomings. The first one is related to its sensitiveness to the time interval chosen for its calculation. A second drawback concerns the fact that it is related to a notion of equilibrium price that is unobservable. The variance ratio is, however, measured from actual transaction prices. This implies that it takes into account the trading activity occurred both inside or outside the limits of the bid-ask spread.

4.3   Estimation methods based on vector autoregressions

Vector autoregressive (VAR) models are commonly used in macroeconomics to identify the effects of various shocks on the structure of the economy. There are two types of applications of the VAR methodology for liquidity measurement. Hasbrouck (1988, 1991 and 1993) considers the deviations of actual transaction prices from unobservable equilibrium prices. This modelling approach starts with the decomposition of transaction prices into a random walk and a stationary component, along the same lines proposed in macroeconomics by Phillips and Solo [44]. The random walk component identifies the efficient price. The stationary component pins down the difference between the efficient price and the actual transaction price, also called pricing error. The dispersion of the pricing error is a natural measure of market quality.

The methodology proposed by Hasbrouck [30] can be used to study a second relevant issue. This is the extent to which fluctuations in a given market arise from swings in another market. This issue has opened the door to the use of cointegration analysis in empirical market microstructure.

4.3.1 Cointegration, market microstructure and asset liquidity

Financial markets are characterized by 'price multiplicity'. In particular, different investors can provide different valuations and attach different prices to the same asset. Also, different market venues can be availbale for the same asset. Following Hasbrouck [30], we formulate a statistical model for the joint behavior of two prices for the same asset linked by a no-arbitrage or equilibrium relationship. The two prices incorporate a single long-term component that takes the form of a cointegrating relation.

To provide a simple example, let us consider a security that trades in two different markets. The price on market 1 is denoted as P_{1t}, while the price on market 2 is given by P_{2t}. Suppose that the two prices are driven by the same efficient price as:

$$p_t = \begin{bmatrix} p_{1t} \\ p_{2t} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} V_t + \begin{bmatrix} S_{1t} \\ S_{2t} \end{bmatrix}$$

where V_t is the (unobservable) efficient price, and S_{1t} and S_{2t} are the pricing errors associated to each security. The unobservable efficient price follows a random walk:

$$V_t = V_{t-1} + u_t$$

The component of the pricing error vector can be viewed as originating from bid-ask bounce, price discreteness or inventory effects. In other words, we require that the characteristics of the pricing errors do not generate a permanent effects on prices. This idea can be formalized by assuming that S_{1t} and S_{2t} evolve according to a zero mean-covariance stationary process. From these assumptions, we can obtain a moving-average representation for the return Δp_t :

$$\Delta p_t = \varepsilon_t + \psi_1 \varepsilon_{t-1} + \psi_2 \varepsilon_{t-2} + ...$$

where $\varepsilon_t = [\varepsilon_{1t}, \varepsilon_{2t}]$ consists of innovations reflecting information in the two separate markets. The sum $\Psi(1) = I + \psi_1 + \psi_2 + ...$, with I as a 2×2 identity matrix, reflects the impact of an initial disturbance on the long-term component. The random-walk variance is:

$$\sigma_u^2 = \Psi \Omega \Psi^{'}$$

where $\Omega = Var(\varepsilon)$.

The simplest approach for achieving identification in the impat of innovations considers the random-walk variance contribution from both markets:

$$\sigma_u^2 = \left[\begin{array}{cc} \Psi_1 & \Psi_2 \end{array} \right] \left[\begin{array}{cc} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{array} \right] \left[\begin{array}{c} \Psi_1 \\ \Psi_2 \end{array} \right]$$

If this covariance matrix is diagonal, then we can identify the model to deliver a clean decomposition of the random walk variance between the two markets. However, if the covariance matrix is not diagonal, then the covariation between the two prices cannot be easily attributed to either market. Hasbrouck [28] introduces a bound for the information shares coming from each market through an orthogonalization of the covariance matrix.

The following question is whether there are alternative restrictions that deliver a better identification. A fruitful approach has been introduced by Harris, McInish and Wood [26] who assume a generic stochastic process in place for V_t. This can be denoted as f_t and assumed to be integrated of order 1, although not necessarily a random walk. Thus, to price in multiple markets, Harris, McInish and Wood [26] specify a process for V_t such that $V_t = A_{p_t}$, where $A = [a_1, a_2]$ is subject to a normalization $a_1 + a_2 = 1$.

The approach of McInish and Wood [39] suffers from several shortcomings. For instance, it is unclear why one should not consider stochastic processes where past prices reveal additional information. In general, we can modify this simple model to take into account the different effects of revealing both public and private information. Hasbrouck [30] follows this avenue to to study alternative mechanisms of attribution of price discovery in multiple markets. In this case, the price component of interest is not forced to be a random walk. The application of permanent-transitory decompositions to microstructure data characterizes non-martingale pricing factors, which are inefficient proxies for optimally-formed and updated expectations.

An additional extension to multi-market analysis uses price data with different sampling frequencies. Hasbrouck [30] shows that the usual method of collecting data from different markets in which trades occur simultaneously can provide misleading inferences on price discovery. In this case, information on price leadership may not be accounted for.

5 Measures based on transaction costs

Among the transaction costs measures, the bid-ask spread and its variants are the indicators of market liquidity that are used most commonly. The reason is that they convey insight on information sharing in the market. The intuition behind the use of the bid-ask spread lies in the fact that market prices depend the side of the market that initiates the trade. Buyer-initiated trades are concluded at the ask price, while seller-initiated trade are concluded at the bid price. The difference between the best (lowest) ask price and the best (highest) bid price defines the bid-ask spread.

5.1 The bid-ask spread

In general, the bid-ask spread is a measure of transaction costs in dealer markets like the NASDAQ. A market bid is the highest price at which a dealer is willing to buy a stock, and at which an investor intends to sell. A market ask is the lowest price at which the dealer is willing to sell the stock. We should stress that the expression 'highest price' stands for 'the best market offer'. Since the dealer posts both the bid and ask quotes, the spread between these quantities can be interpreted as the price that the market pays for the liquidity services offered by the dealer.

Huang and Stoll [32] notice that specialists often operate as dealers. This is due to the institutional characteristics of specialists. Typically the specialist disseminates a quote in the market. Market orders are then worked out against limit orders previously placed on the quote posted by the specialist. The disseminated quote is set exactly as the bid-ask spread on the dealer market.

Let A_t denote the ask price, B_t the bid price, and S_t the spread at time t. Formally, the quoted absolute bid-ask spread is defined as:

$$S_t = A_t - B_t \tag{14}$$

Frequently the literature reports also a measure of half the spread, given by $S_t/2$, and the midpoint quote, given by the best bid and ask quotes in effect for a transaction at time t. From (14), we can see that more liquid markets generate lower quoted spreads. This highlights the existence of a negative relationship between the spread and asset prices, as explicitly discussed by Amihud and Mendelson [7].

From this simple measure, it is possible to construct additional indices that are often used to model market liquidity. One of these consists in the percentage term. Given the quote midpoint as $M_t = (A_t + B_t)/2$, a measure of the percentage spread pS_t is given by:

$$pS_t = \frac{A_t - B_t}{M_t} \qquad (15)$$

The spread itself represents a measure of transaction costs, rather than a liquidity index in strict sense. However, high transaction costs represent a source of a low liquidity.

Cohen et al. [14] characterize the distinction between the dealer spread and the market spread. The dealer spread is the simple bid-ask spread defined in (14). The market spread, instead, is the difference between the highest bid and the lowest ask across dealers quoting the same stock at the same time. According to Hamilton [24], a market spread can be lower than a dealer spread. In fact, the cost of immediacy to investors is represented by the size of the market spread. The state of competition and the order processing costs are rather related to the absolute magnitude of the spread.

An additional issue characterizing the simple spread analysis has to do with the fact that it cannot capture the impact of large block-size transactions on market prices. In fact, the spread measure implicitly assumes that trades occur only at the posted quotes. In this case it is difficult to establish if the transactions occurred for a given price are formed inside or outside the quoted spread. Moreover, given the ability of the market spread to drift away from the one consistent with the perfect market hypothesis, the size of the spread can reflect three main microstructural phenomena. These consist of a pure execution cost, an inventory cost position of the dealer, and an information component cost. In order to detect these various components, the literature has proposed many empirical tests and theoretical models. We review these frameworks in the following sections.

The study of the spread in absolute terms represents only a preliminary stage towards a deeper analysis of transaction costs and information asymmetry. In fact, the decomposition of the spread components can allow to disentangle the most important effects arising from trading activities.

5.2   A measure of implied spread

The measure of Roll [47] is one of the most famous liquidity indices proposed in the microstructure literature. Roll's idea consists in using a model to infer the realized spread (the effective spread) that is reflected from the time series properties of observed market prices or returns.

The main drawback of this approach is that it does not offer any insights on the components of the spread. The reason is that this framework is based on the assumption of homogeneous information across traders. Therefore, the adverse selection component is missing. The magnitude of the spread reflects only the so-called order processing costs, which are thought to have transitory effects, in contrast to information effects, that have permanent effects.

Let P_t denote the observed transaction price for a stock oscillating between bid and ask quotes. We assume that this reproduces the negative serial covariance observed in actual price changes, documented by Fama and French [17]. The equilibrium price V_t follows a pure random-walk process with drift:

$$V_t = \overline{V} + V_{t-1} + \varepsilon_t \tag{16}$$

where ε_t is the unobservable innovation in the true value of the asset between transaction $t-1$ and t. This is an i.i.d. term with zero mean and constant variance σ_ε^2. The observed price can be described as follows:

$$P_t = V_t + \frac{S}{2}Q_t \tag{17}$$

where S denote the quoted absolute spread, assumed to be constant over time. Q_t is an indicator function that takes values -1 or 1 with equal probabilities depending on the fact that the t-th transaction may occur at the bid or at the ask.[4] Thus, the change in transaction prices is given by:

$$\Delta P_t = \overline{V} + \frac{S}{2}\Delta Q_t + \varepsilon_t \tag{18}$$

To obtain a reduced form, we need two additional assumptions:

- the market is informationly efficient, i.e. $cov(\varepsilon_t, \varepsilon_{t-1}) = 0$,

- buy and sell orders have equal probability, i.e. $cov(\Delta P_t, \Delta P_{t-1}) = -1$.

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[4]In particular, $Q_t = -1$, for seller inititiated transaction, $Q_t = +1$, for transaction initiated by a buyer at the ask quote.

The probability distribution of trade direction can be represented in the following form:

		$Q_{t-1} = -1$		$Q_{t-1} = +1$	
		Trade at bid		Trade at ask	
Trade Sequence $\rightarrow$		$B_{t-1}B_t$	$B_{t-1}A_t$	$A_{t-1}A_t$	$A_{t-1}B_t$
$\downarrow$		0	2	0	-2
B_tB_{t+1}	0	1/4	0		1/4
B_tA_{t+1}	2	1/4	0	0	1/4
A_tA_{t+1}	0	0	1/4	1/4	0
A_tB_{t+1}	-2	0	1/4	1/4	0

Thus, since buy or sell transactions are equally likely, the joint distribution can be characterized as:

		ΔQ_t		
		2	0	-2
ΔQ_{t+1}	2	0	0	1/8
	0	1/8	1/4	1/8
	-2	1/8	1/8	0

It is not difficult to verify that the autocovariance of trades is given by:

$$Cov\left(\Delta Q_t, \Delta Q_{t+1}\right) = -4 \cdot \frac{1}{8} - 4 \cdot \frac{1}{8} = -1$$

Therefore, the autocovariance function of price variations is:

$$Cov\left(\Delta P_t, \Delta P_{t-1}\right) = Cov\left(\frac{S}{2}\Delta Q_t, \frac{S}{2}\Delta Q_{t-1}\right) = \frac{S^2}{4}Cov\left(\Delta Q_t, \Delta Q_{t-1}\right)$$

from which we obtain:

$$Cov\left(\Delta P_t, \Delta P_{t-1}\right) = -\frac{S^2}{4} \tag{19}$$

Equation (19) provides the measure of spread defined by Roll (1984). This is obtained by estimating the autocovariance of transaction prices and solving for S. The estimator for the serial covariance is:

$$\widehat{Cov} = \frac{1}{n}\sum_{t=1}^{n}\Delta P_t \Delta P_{t-1} - \overline{\Delta P}^2 \tag{20}$$

where $\overline{\Delta P}^2$ is the sample mean of $\{\Delta P\}$. It is possible to show that the population distribution of $\widehat{Cov}$ is asymptotically normal (see Harris [25]). Moreover, the serial covariance estimator has a downward bias in small samples of data with low frequency. In particular, the bias is large for data with frequency higher than daily.

The implications from (19)-(20) are that the more negative the price auto-correlation is, the higher the illiquidity of a given stock will be. As discussed by Lo and MacKinlay [35], there is a relation between the variance ratio and Roll's (1984) measure of liquidity. This link arises from the dependence of the variance ratio on the autocorrelation of daily returns. With returns that exhibit a negative autocorrelation, the measure of Roll [47] generates higher illiquidity, and a variance ratio lower than one.

The main shortcoming of this measure consists in its inability to capture asymmetric information effects. As stressed by Huang and Stoll [33], short-term returns can be affected by a multiplicity of additional factors. The point is that the measure of Roll [47] can be safely applied only under the assumption that the quotes do not change in response to trades. This condition would hold only if there were no informed traders in the market, and the quotes did not adjust to compensate for changes in inventory positions.

According to Huang and Stoll [32], in the case of the NYSE, Roll's measure is much lower than the effective half-spread, while for NASDAQ it is 'virtually' identical to the effective spread. This implies that specialist dealers adjust their quotes in response to the trades because of information effects. At the same time, NASDAQ dealers do not adjust their quotes, thus supporting the assumption of a minimal role for asymmetric information in this market.

5.3   The role of asymmetric information

Glosten [19] is the first contribution that models the role of information asymmetries in market microstructure. It introduces the distinction between the effects arising from order processing and those from adverse information. As previously remarked, the first type is transitory, while the second is permanent. On the other hand, the adverse-information component produces non-transitory impacts because it affects the equilibrium value of the security. This can take place when market-makers engage in trades with investors

who possess superior information. Thus, an order placed by a trader can be correlated with the true value of the asset.

The model of Glosten [19] includes two basic equations:

$$V_t = \overline{V} + V_{t-1} + (1 - \gamma)\frac{S}{2} + \varepsilon_t \qquad (21)$$

$$P_t = V_t + \gamma\frac{S}{2}Q_t \qquad (22)$$

where γ is the fraction of the quoted spread due to order processing costs, and $(1 - \gamma)$ is the share arising from adverse information.[5] Note that ε_t reflects the effect arising from the arrival of public information. Thus, the true price V_t fully reflects all the information available to the public immediately after a transaction, and the information revealed by a single transaction through the sign of the variable Q_t.

We should stress that the model of Roll [47] is nested by this specification, and can be obtained from $\gamma = 1$. It is not difficult to prove that the autocovariance of the price change is equal to:

$$Cov\left(\Delta P_t, \Delta P_{t-1}\right) = -\gamma\frac{S^2}{4} \qquad (23)$$

5.4   The relation between inventory and adverse-information effects

Inventory and information effects are key determinants of liquidity conditions. With information effects, prices move against the dealer after a trade. They fall after a dealer purchase, and rise after a dealer sale. This is often denoted as a 'price reversal', and consists in a situation where a dealer trades against informed agents. In this case, a market-maker can incur in significant losses.

The idea of price reversal arises from the observation that the realized spread is often different from the quoted spread. In Stoll [50], the quoted spread S is taken as constant and depends only on the transaction size, which is constant as well. In practice, the model of Roll [48] assumes that transactions

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[5]The notation for the other variables is the same as the one outlined in the previous sections.

occur either only at the bid or ask quotes. If inventory-holding costs are included into the model, the dealer will have the incentives to change the spread to either induce or inhibit additional trading movements. In fact, after a dealer purchase (a market sale), bid and ask quotes drop in order to induce dealer sales and disincentivate additional dealer purchases. At the same time, bid and ask quotes increase after a dealer sale (a market purchase) to inhibit additional dealer sales.

This type of spread revision operates in the same way both in the case of inventory control and adverse information. However, the reasons for spread revisions are different. With asymmetric information, a buyer (seller) initiated transaction conveys informations on a higher (lower) expected price of the asset. This is due to the expectation by market participants that active traders possess superior information.

Summing up, different reasons for a spread revision can produce similar observed effects. The inventory effect pushes the dealer towards a quote revision in order to avoid a process of trade that would even out his inventory position. With asymmetric information there is the need for the dealer to protect himself from adverse trading directions generated by better informed counterparties.

5.5   The model of Stoll [50]

The model of Stoll [50] presents a way to jointly estimate the three key components of the spread, namely the shares due to order processing, inventory and adverse information. This framework accounts for the fact that order flows arising from different motives need not occur with the same probability. Let θ denote the probability of a price reversal, i.e. the unconditional probability of a trade change: $\theta = \Pr\{Q_t = Q_{t-1}\}$. The size of a price change conditional on a reversal is given by $(1 - \lambda)\,S$. In other words:

$$(1 - \lambda)\,S = \Delta P_t \mid \{Q_t \neq Q_{t-1}\}$$

where

$$\{Q_t \neq Q_{t-1}\} = \left\{ \begin{array}{l} P_{t-1} = B_{t-1}, P_t = A_t, \text{ or} \\ \quad P_{t-1} = A_{t-1}, P_t = B_t \end{array} \right.$$

In the framework of Stoll [50], the price change ΔP_t arises from the fact that the initial trade is at the bid or at the ask. Thus, for transactions starting at the ask price, we have:

$$\Delta P_t = \begin{cases} (B_t - A_{t-1}) = (1 - \lambda)\,S, \text{ with probability } \theta \\ (A_t - A_{t-1}) = -\lambda S, \text{ with probability } (1 - \theta) \end{cases}$$

and for transactions starting at the bid price:

$$\Delta P_t = \begin{cases} (A_t - B_{t-1}) = (1 - \lambda)\,S, \text{ with probability } \theta \\ (B_t - B_{t-1}) = -\lambda S, \text{ with probability } (1 - \theta) \end{cases}$$

Therefore, the expected price change conditional on an initial transaction at the ask is given by:

$$E\left\{\Delta P_t \mid P_{t-1} = A_{t-1}\right\} = -\left(\theta - \lambda\right) S \qquad (24)$$

while the expected price change conditional on an initial transaction at the bid is:

$$E\left\{\Delta P_t \mid P_{t-1} = B_{t-1}\right\} = \left(\theta - \lambda\right) S \qquad (25)$$

The realized spread is the dealer's gain after two transactions, consisting of a purchase and a sale. In particular, it denotes the difference between the expected price change after a dealer purchase and the expected price change after a dealer sale. Given the effective spread s, we have:

$$s = 2\left(\theta - \lambda\right) S \qquad (26)$$

We should note that the realized spread is the remuneration for the services provided by a market-maker, including all the components discussed earlier. The fraction of spread given by (26) includes both the order processing and the inventory component. The adverse information term consists in the fraction of the spread not earned by the market-maker, and is equal to $[1 - 2\left(\theta - \lambda\right) S]$.

To provide an empirical implementation of this approach, we can distinguish between two cases. These occur when we observe directly the trade direction, and when trade data are not available. If market data are available, we can directly estimate a version of equations (24)-(25) as:

$$\left(s_\tau^i \mid B_\tau^i\right) = \left[\left(P_{t+\tau}^i - P_t^i\right) \mid P_t^i = B_\tau^i\right] \qquad (27)$$

for trades at the bid and:

$$\left(s_\tau^i \mid A_\tau^i\right) = \left[\left(P_{t+\tau}^i - P_t^i\right) \mid P_t^i = A_\tau^i\right] \tag{28}$$

for trades at the ask, where τ indicates the time length after which a subsequent price is observed. The choice of the time horizon adopted in the estimation is crucial. If the time frame is too short, the subsequent price may fail to reflect a reversal, and may depict only another trade in the same direction. However, if the time horizon is too long, we might obtain results affected by excessive volatility due to frequent consecutive price changes.

Huang and Stoll [32] use four alternative time horizons, namely between five and ten minutes after the initial trade at t, with the first trade occurring at least five minutes after the initial trade, with the first trade between 30 and 35 minutes after the initial trade, and with the first trade occurring at least 30 minutes after the initial trade. The findings of Huang and Stoll [32] suggest that dealers in the NASDAQ face a lower realized spread than those of the NYSE.

When trade data are not available, we need to resort to information from the statistical patterns characterizing an asset price. It can be shown that the covariance of price changes is given by:

$$Cov\left(\Delta P_t, \Delta P_{t+1}\right) = S^2 \left[\lambda^2 \left(1 - 2\theta\right) - \theta^2 \left(1 - 2\lambda\right)\right]$$

In order to detect inventory costs, Stoll [50] presents also the autocovariance of changes in quotations:

$$Cov\left(\Delta Q_t, \Delta Q_{t+1}\right) = S^2 \lambda^2 \left(1 - 2\theta\right) \qquad Q = A, B$$

Under the assumption of constant quoted spread, this covariance can be computed either from changes in the bid or the ask quotes, so that $Cov\left(\Delta B_t, \Delta B_{t+1}\right) = S^2 \lambda^2 \left(1 - 2\theta\right)$, or $Cov\left(\Delta A_t, \Delta A_{t+1}\right) = S^2 \lambda^2 \left(1 - 2\theta\right)$. The expressions for the covariance delivered by different versions of the model are collected in table 5.5.

The theory of bid-ask spread considered thus far is based on several assumptions that can be challenged. George, Kaul and Nimaledran [18] show that the available estimators of spread components are typically biased and inefficient. This is due to two important facts. The first one is that stock returns are positively autocorrelated. Moreover, transaction returns display a large

Table 1: Covariance in the model of Stoll [50]

Spread Determinant	$Cov\left(\Delta P_t, \Delta P_{t+1}\right)$	$Cov\left(\Delta Q_t, \Delta Q_{t+1}\right)$
Order Processing: $\theta = 1/2$, $\lambda = 0$	$-\frac{1}{4}S^2$	0
Adverse Information: $\theta = 1/2$, $\lambda = 0.5$	0	0
Inventory Costs: $\theta > 1/2$, $\lambda = 0.5$	$\left(-\frac{1}{4}S^2, 0\right)$	$\left(-\frac{1}{4}S^2, 0\right)$

unexpected return component. George, Kaul and Nimaledran [18] introduce time-varying expected returns by assuming $\overline{V}_t \neq \overline{V}$, where the autocovariance of quote changes are positive.

Other approaches for the estimation of the spread components include the so-called 'trading indicators'. The models proposed in this context, such as Glosten and Harris [20], Glosten [19] and Madhavan, Richardson and Roomans [36] do not contain any assumption about the arrival of orders. Only the actual direction of trades affects the parameter estimation.

Given their structure, these models require a very careful specification of the type of market under study. In this respect, they are not general enough. Depending on whether there is a quote or an order-driven market, we can observe a different behavior of the transaction price that is related to the order size. Therefore, for small transactions, the model of Glosten and Harris [20] underestimates the adverse selection component, and overestimates the order processing component. The opposite holds for transactions of large blocks.

Within the class of models of trade indicators, the framework proposed by Madhavan, Richardson and Roomans [36] disentangles the effects from adverse information and inventory changes. The price mechanism proposed displays an asymmetric information component of the spread related to innovations in the order flow. Madhavan, Richardson and Roomans [36] construct a trade-indicator model that considers also trades inside the quotes. The framework is general enough to capture the information advantages linked to unexpected trading movements.

5.5.1   The model of Huang and Stoll [33]

Huang and Stoll [33] introduce a three-part decomposition of the spread.

The model is based on the equilibrium price:

$$V_t = V_{t-1} + \eta \frac{S}{2} Q_{t-1} + \varepsilon_t \tag{29}$$

In equation (29), the term η denotes the percentage of the half spread due to adverse selection, and ε_t is a public information shock serially uncorrelated over time. Q_t is a trade indicator variable, which is equal to 1 if the transaction is buyer-initiated, and is equal to -1 if the transaction is started at the bid.

The last trade conveys information relevant to determine the true value of the stock price. Given a midpoint quote $M_t = (A_t + B_t)/2$, Huang and Stoll [31] assume the following relationship with the unobserved price:

$$M_t = V_t + \delta \frac{S}{2} \sum_{i=1}^{t-1} Q_i \tag{30}$$

where δ measures the inventory effect, and $\sum_{i=1}^{t-1} Q_i$ is the cumulative inventory from market opening until $t-1$. In particular, Q_1 is the initial inventory of the day. Combining (29) and (30), we obtain the change in the midpoint quote:

$$\Delta M_t = (\delta + \eta) \frac{S}{2} Q_{t-1} + \varepsilon_t \tag{31}$$

The traded price P_t is:

$$P_t = M_t + \frac{S}{2} Q_t + u_t \tag{32}$$

In this model, S denotes the traded spread. This is different from the quoted (posted) spread S_t because it reflects also trades inside the quotes but outside the midpoint. From (29) and (32), we obtain:

$$\Delta P_t = \frac{S}{2} \Delta Q_t + (\delta + \eta) \frac{S}{2} Q_{t-1} + \zeta_t \tag{33}$$

where $\zeta_t = \Delta u_t + \varepsilon_t$. Equation (33) reflects only a two-way decomposition of the spread. The order processing cost is defined as $1 - \delta - \eta$. However, by estimating equation (33) alone, it is not possible to draw any conclusion on either the relative importance of the adverse-information component, or the inventory effect. Only a simultaneous three-way decomposition of the spread can fully uncover all these effects jointly. For this purpose, we need to add to the model an additional equation specifying the probability of trade direction.

Huang and Stoll [33] introduce the assumption of serial correlation in trade flows:

$$E(Q_{t-1} \mid Q_{t-2}) = (1 - 2\theta) Q_{t-2} \tag{34}$$

with θ as the probability of change in trade direction (i.e. the probability that a trade at the bid at time $t-1$, is followed by a trade at the ask at time t). For $\theta \neq 0.5$, the change in the true price is given by:

$$\Delta V_t = \eta \frac{S}{2} Q_{t-1} - \eta \frac{S}{2} (1 - 2\theta) Q_{t-2} + \varepsilon_t \tag{35}$$

Equation (35) has three components. The first one, represented by $\eta \frac{S}{2} Q_{t-1}$, displays the information conveyed by the last trade. The second part, given by $\eta \frac{S}{2} (1 - 2\theta) Q_{t-2}$, introduces the additional persistence in information that is not accounted for by the surprise term ε_t. If $\theta = 1/2$ equation (35) collapses into (29). From (34) and (35), the reader should note that changes in the true value of the asset are unpredictable until the release of public information contained in ε_t shows up, so that $E(\Delta V_t \mid V_{t-1}, Q_{t-2}) = 0$. We can combine (35) and (30) to obtain the change in midpoint quote:

$$\Delta M_t = (\delta + \eta) \frac{S}{2} Q_{t-1} - \eta \frac{S}{2} (1 - 2\theta) Q_{t-2} + \varepsilon_t \tag{36}$$

Equation (36) indicates that the inventory effect can be detected only after the trades are executed, and the quotes are revised. This allows to distinguish between the adverse-information component and the inventory component.

Taking the expectation of equation (35) conditional on the information from observing both M_{t-1} and before Q_{t-1} and M_t, we get:

$$E(\Delta M_t \mid M_{t-1}, Q_{t-2}) = \delta \frac{S}{2} (1 - 2\theta) Q_{t-2} \tag{37}$$

This equation shows that the expected change in the midpoint depends only on δ, the inventory cost component. The inventory-quote response to a trade is given by $\delta \frac{S}{2}$. However, from (36), the change in the midpoint quote due to the inventory effect is much smaller. To get the three-way spread decomposition, we can combine (32) and (36) to deliver:

$$\Delta P_t = \frac{S}{2} Q_t + (\delta + \eta - 1) \frac{S}{2} Q_{t-1} - \eta \frac{S}{2} (1 - 2\theta) Q_{t-2} + \zeta_t \tag{38}$$

Thus, by estimating simultaneously (34)-(38), we can identify all the three spread components, namely δ, η and $1 - \delta - \eta$, together with the probability of a trade reversal θ. The reader should note that the effective spread S is estimated. If this is replaced with the posted spread S_t, then model consists of equation (34):

$$\Delta M_t = (\delta + \eta) \frac{S_{t-1}}{2} Q_{t-1} - \eta \frac{S_{t-2}}{2} (1 - 2\theta) Q_{t-2} + \varepsilon_t \tag{39}$$

and the parameter space is reduced.

Huang and Stoll [33] test the model on trades and quotes for large-capitalization actively-traded stocks in 1992. Their result suggest that the average order processing of the traded spread is 61.8%, the average adverse-information component is 9.6%, and the average inventory cost component is 28.7%. Another interesting piece of evidence consists in the fact that the adverse information component of the spread is smaller for large trades. This is due to the fact that large trades usually tend to be negotiated outside the market, so that the price fully reflects the information given by the last trade.

5.5.2 Empirical issues

An integrated approach to the analysis of the spread has been proposed by Huang and Stoll [31]. They consider a two-equation framework where the determinants of quotes and transaction prices are included to test for the relevance of competing microstructure theories.

To shed light on the framework, let us consider the logarithm M_t of the midpoint quote. The price (in logs) can be expressed as follows:

$$P_t = M_t + W_t \tag{40}$$

where W_t is the deviation of the log of observed transaction price P_t from the log-midpoint quote. This suggests that trades can occur also inside the quotes. Thus, the effective spread is always less than the quoted spread. In equation (40), public dealer purchases (sales) result in $W_t > 0$ (< 0).

We take the first difference of equation (40) to get:

$$P_t - P_{t-1} = M_t - M_{t-1} + W_t - W_{t-1} \tag{41}$$

Let us define the return from official quotes $R_t^p = P_t - P_{t-1}$, with $R_t^m = M_t - M_{t-1}$. In order to take the model to the data, we can specify the quote setting behavior from the midpoint change R_t^m. In doing so, Huang and Stoll [31] identify a fourth microstructure effect that is not captured by previous models. The induced order-arrival effect captures the idea that the probability of a public purchase changes through time after a dealer-price adjustment. This is determined by the ability of the market-maker to induce changes in order

arrivals, and to cover for the entire cost of processing the orders. The induced order-arrival effect can be written as:

$$\mathrm{Prob}\left[W_t > 0 \mid (V_t - M_t) > 0\right] > 0.5 \qquad (42)$$

Equation (42) shows that the divergence between the unobservable price V_t and the midpoint quote depends on the inventory holdings of the supplier.

The subsequent step of Huang and Stoll [31] specifies the pattern of quote returns R_t^m:

$$R_t^m = E\left[R_t^V \mid \Omega_{t-1}\right] + g\left(\Delta I_{t-1}\right) + \varepsilon_t \qquad (43)$$

where $E\left[R_t^V \mid \Omega_{t-1}\right]$ is the expected value of the consensus return, i.e. the return earned on the expected price changes of the true price (expressed in logs): $R_t^V = V_t - V_{t-1}$. In equation (43), Ω_{t-1} denotes the information set available at time t, while $g\left(\Delta I_{t-1}\right)$ is the inventory change of the quote return. To study the role of information effects, the expected component in equation (43) can be conditioned on a subset of variables reflecting the availability of public information:

$$E\left[R_t^V \mid \Omega_{t-1}\right] = f\left(W_{t-1}, R_{t-1}^F\right) \qquad (44)$$

In Huang and Stoll [31], the term R_{t-1}^F denotes the change in logarithm of the S&P500 futures price. The presence of the term W_{t-1} reflects the adjustment of the market-maker to public information revealed through trading. If private information was the main source of the bid-ask spread, the quote's midpoint would be adjusted by W_{t-1} because the previous price deviation is the expected value of the private information conveyed by trade.

The general specification of the model for the return on quote revision is:

$$R_t^m = \eta_0 + \eta_1 R_{t-1}^m + \eta_2 R_{t-1}^F + \eta_3 W_{t-1} + \eta_4 H_{t-1} + \eta_5 L_{t-1}^A + \eta_6 L_{t-1}^B + \eta_7 Z_{t-1} + \varepsilon_t \quad (45)$$

The inclusion of R_{t-1}^F is motivated by the fact that trading in stock index futures is cheaper than trading in stocks. Thus, the diffusion of news can be detected more easily through movements of index futures. Huang and Stoll [31] include W_{t-1} to account for the information from previous period's trading. Huang and Stoll [31] also consider the cumulative volume traded on the single asset, given by H_{t-1}. In order to detect the inventory effect, equation (45)

includes two trade indicator variables constructed as follows:

$$L_{t-1}^A \begin{cases} = 1 & if\ W_{t-1} > 0 \quad and \quad Vol_{t-1} > 10,000 \\ = 0 & \text{otherwise} \end{cases}$$

$$L_{t-1}^B \begin{cases} = 1 & if\ W_{t-1} < 0 \quad and \quad Vol_{t-1} > 10,000 \\ = 0 & \text{otherwise} \end{cases}$$

where Vol_{t-1} indicates the share volume traded at time $t-1$ for the asset. The expected impact on the quote revision is positive for L_{t-1}^A and negative for L_{t-1}^B. Another crucial variable in equation (45) is represented by R_{t-1}^m. This allows to take into account non-instantaneous quote revisions, as well as the negative serial correlation in quote returns.

Finally, quote returns are affected by the difference between the logarithm of the quoted volume at the ask (depth at the ask) and the logarithm of the quoted volume at the bid (depth at the bid), which is denoted as Z_{t-1}. The presence of inventory effects would imply a positive impact on R_t^m. In fact, if a dealer has a large inventory position, he has an incentive to reduce quotes and to raise depth at the ask to encourage transactions with the purpose of mitigating the inventory position.

In equation (45) there is also a signalling effect captured by the sign of η_7. If $\eta_7 < 0$, we have a negative impact on quote changes. This means that a large depth at the ask at time $t-1$ signals the presence of sellers in the limit order book, inducing market participants to revise quotes downward.

To close the model, Huang and Stoll [31] make an assumption about the stochastic process for W_t:

$$W_t = \rho W_{t-1} + \xi_t \tag{46}$$

where ξ_t denotes the order arrival shock. For $\rho = 0$, the probability of a purchase or a sale is independent from the sequence of trades. If the activity of dealer pricing creates an inventory effect, then $\rho < 0$. By combining equations (42) and (46), we obtain:

$$R_t^p = R_t^m + (\rho - 1) W_{t-1} + u_t \tag{47}$$

Plugging equation (45) into (47) delivers the expression for observed returns:

$$R_t^p = \eta_0 + \eta_1 R_{t-1}^m + \eta_2 R_{t-1}^F + \eta_3^p W_{t-1} + \eta_4 H_{t-1} + \eta_5 L_{t-1}^A + \eta_6 L_{t-1}^B + \eta_7 Z_{t-1} + u_t \tag{48}$$

where $\eta_3^p = \eta_3 + \rho - 1$, and $u_t = \varepsilon_t + \xi_t$. From this transformation, we see that the coefficient η_3^p can be decomposed into three components:

(i) the asymmetric information effect, given by η_3, which is expected to be positive and represents the information conveyed by the last trade;

(ii) the induced order arrival effect, given by ρ;

(iii) the bid-ask bounce effect. In the absence of information effect, the third component is equal to -1, and represents the tendency of price returns to be autocorrelated.

In empirical applications, the model of equations (45) and (48) can be estimated jointly by the Generalized Method of Moments on intraday data. An example of this application is represented by Huang and Stoll [31], who consider the 20 most actively traded stocks in the NYSE.

The specification described in (48) includes the most important ingredients of the microstructure theory. By setting $\eta_3 = 0$ and $\eta_3^p = -1$, we can test the order processing theory of the bid-ask spread. With $\eta_3 = 1$, $\eta_3^p = 0$, we have the adverse information theory of the spread, where quotes are adjusted in order to reflect the last trade Z_{t-1}. Additionally, the inventory holding cost theory can be obtained by setting $1 > \eta_3 > 0$, $0 > \eta_3^p > -1$ and $\eta_4 > 0$. In this case, the direction of change in quotes follows the last trade. Moreover, quote returns are adjusted by an amount that depends on the inventory change. The induced order-arrival effect theory arises from $\rho < 0$, $\eta_1 < 0$ and $\eta_7 > 0$.

The key implication of this model is that changes in the midpoint quotes affect the order arrivals, leading to serial correlation in W_t. An important factor is captured by changes in market depth. If depth is used to encourage order arrival, then we would expect a positive η_7. Alternatively, depth can be a signal or act as a barrier, leading to a negative sign in η_7. The effect of large trades in the adverse information theory is captured in the form of a sign pattern as $\eta_5 > 0$ and $\eta_6 < 0$.

Summing up, the two equations (45) and (48) produce a full set of testable implications and cross-equation restrictions. Among the various theories of market microstructure that can be tested, we can also include the efficient market hypothesis of index futures. If an asset market is efficient, the predominant prices fully reflect the information contained in the index futures prices. In this case, we would expect $\eta_2 = 0$.

6   Concluding remarks

Our discussion suggests that the market structure of exchanges determines the liquidity of a stock. In other words, market liquidity depends on the organization of trading activities, the presence of alternative intermediaries, the degree of competition between them, and how the transactions are executed - e.g. whether through electronically orders, limit orders, or on an open outcry trading floor. The key point is that the organization of a market depends also on the regulatatory environment that surround the trading activity. In this section, we conclude the paper by discussing how market regulation can affect the structure of exchanges to improvide the liquidity conditions.

There are two main types of factors that determine the organization of a market. The first aspect concerns the types of market actors and the role of infrastructure. These factors include the structure of the demand side - e.g., whether it is driven by institutional or retail investors -, the cost of processing transactions, the scope for automation and the cost for bearing risk. The market structure is also determined by aspects of industrial organization, such as the degree of competition among traders, and the concentration or fragmentation of trading activities.

We can argue that there is a tension between the need for fragmentation and the desire of concentrating the exchanges within a unique market. The demand for fragmentation is, in general, driven by the regulators. Market fragmentation stimulates the creation of competing markets for the same stock, and allows the selection of the most appropriate trading venue in terms of lowest transaction costs and highest liquidity. Exchange concentration, on the other hand, responds to the need for transparency in price formation. It also generates economies of scales for the concentration of transactions in a unique market.

Opposite forms of fragmentation or concentration lead to distortions in market performance. As pointed out by Stoll (1992), fragmentation implies "*the inability of an order on a market to trade with an order on another market*". Advocates of market fragmentation aim for higher competition across markets, which translates into lower transaction costs for the investment services. On the other hand, advocates of market consolidation stress the dangerous effects on liquidity induced by an excessive fragmentation. These negative

effects can be reduced by an adequate technological automation for order and information processing across markets.

The quality of a market can be judged also according to its ability to provide the right amount of liquidity at a given point in time. Therefore, the presence of specialists, dealers or market-makers plays a crucial role in enhancing liquidity and securing a continuous price offering. In the experience of North-American securities markets, limit-order rules play an important role in increasing liquidity and narrowing the spread on markets managed by specialists. These operational rules establish which order is picked up first. Therefore, from the perspective of market structure, an adequate level of development of technological services, coupled with transparent trading rules, can assure the provision and quality of liquidity services.

A final remark is in order about the degree of competition among market-makers. The concentration of market-makers is obviously determined by the operational rules present in a market. These procedures often concern the minimum trade size required to access the market. In order to enhance competition across market-makers, it is imperative to create rules for trading that do not restrict market access only to a specific group of traders - e.g., investors who trade large block transactions - and that may prevent collusion between market-makers. After creating proper rules for trading and connecting links between markets with different characteristics, the way towards higher competition between traders is in place. And so are the premises for improvements in the liquidity conditions of a market.

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