Journal of Finance and Investment Analysis, vol.1, no.3, 2012, 21-31
ISSN: 2241-0988 (print version), 2241-0996 (online)
Scienpress Ltd, 2012

Estimation risk modeling in portfolio selection: Implicit approach implementation

Asma Graja Elabed[1] and Amel Baccar[2]

Abstract

This paper contributes to portfolio selection methodology using bayesian theory. A new estimation approach is applied to forecast the mean vector and covariance matrix of returns. The proposed method accounts for estimation errors. We compare the performance of traditional Mean Variance optimization of Markowitz with Michaud's Resampled Efficiency approach in a comprehensive simulation study for bayesian estimator and Implicit estimator. We carried out a numerical optimization procedure to maximize the expected utility using the MCMC samples from the posterior and the predictive distribution.

Keywords: Bayesian theory, Implicit distribution, Resanpled efficiency, Mean variance optimisation, MCMC algorithm

---

[1] University of Gabes, COFFIT – FSEGS, Tunisia, e-mail: asmagraja2002@yahoo.fr
[2] FSEGS, University of Sfax, COFFIT – FSEGS, Tunisia, e-mail: emel_baccar@yahoo.fr

Article Info: *Received* : February 27, 2012. *Revised* : April 4, 2012
Published online : August 31, 2012

1 Introduction

Portfolio selection is one of the most important problem in practical investment management. The first paper in this field goes back at least to the mean variance paradigm of Markowitz (1952), which analytically formalizes the risk-return tradeoff in selecting optimal portfolios. Even when the mean variance is a static one period model, it has widely been accepted by both academics and practitioners. Optimal portfolio choice in a continuous-time setting has been studied by Samuelson (1969) and Merton (1969, 1971). With constant coefficient diffusion process characterizing risky asset prices, the optimal portfolio choice can be solved analytically by solving the relevant Hamilton-Jacobi-Bellman (HJB) equation. Merton (1973) analyses optimal portfolio decision in a setting in which the mean returns depends on a set of time-varying state variables. Recent literature has considered the state variables dynamics from which either the explicit solution can be derived (Liu 2006, Watcher 2002) or the numerical computation is implemental (Campbell et Viceira 1999).

This literature assumes that the investment opportunity set is observable and the parameters of the stochastic process governing its dynamics are known to the investor. However, in reality the investor does not in fact know the parameters of the probability distribution from which the returns are drawn. Rather, the investor would estimate the parameters by observing the past market data. Thus estimates may be treated as time varying state variables. The estimates may deviate from their true values and give rise to the estimation error, which subsequently causes the ”estimation risk” that investors must take into account in making portfolio decisions. The fact that mean variance “optimal” portfolios are sensitive to small changes in input data is well documented in the literature. (Chopra and Ziemba 1993) shows that even slight changes to the estimates of expected returns or risk can produce vastly different mean-variance optimized portfolios. Instead of focusing on the weights of the assets in optimal portfolio, others have focused on the financial impact of mean variance efficient portfolios computed from estimates. Jobson and Korkie (1980) show that even an equal-weighted portfolio can have a greater Sharpe ratio than an optimal mean-variance portfolio computed using estimated inputs. Broadies (1999) shows how the estimated efficient frontier overestimates the expected returns of portfolios for varying levels of estimations errors. Since mean-variance efficient portfolio weights are very sensitive to the level of the expected returns, it is widely believed that most of the estimation risk in optimal portfolios is due to errors in estimates of expected returns, and not in the estimates of risk (Chopra and Ziemba 1993). In order to cope with the effect of estimation errors in the estimates of expected returns, attempts have been made to create better and more stable mean-variance optimal portfolio by utilizing expected return estimators that have a better behaviour when used in the context of the mean-variance framework.

The Bayesian approach is potentially attractive. First, it can employ useful prior information about quantities of interest. Second, it accounts for estimation risk and model uncertainty. Third, it facilitates the use of fast intuitive and easily implementable numerical algorithms for the simulation of otherwise complex economic quantities. In addition, three building blocks underlie Bayesian portfolio analysis: Firstly, the formation of prior beliefs which are typically represented by a probability density function on the stochastic parameters underlying the stock-return evolution. Secondly, the prior density can reflect information about events, macro economy news, asset pricing factors, and forecasting variables. Thirdly the recovery of the predictive distribution of future asset returns, analytically or numerically, incorporating prior information, law of motion, as well as estimation risk and model uncertainty. The predictive distribution, which integrates out the parameter space, characterizes the entire uncertainty about future asset returns. The bayesian optimal portfolio rule is obtained by maximizing the expected utility with respect to the predictive distribution. Zellner and Chetty (1965) pioneer the use of predictive distribution in decision making in general. Appearing during the 1970, the first applications in finance are entirely based on uninformative or data-based priors. Jorion (1986) introduces the hyper parameter prior approach in the spirit of the Bayes-stein shrinkage prior, whereas Black and Litterman (1992) advocate an informal Bayesian analysis with economic views and equilibrium relations. Recent studies by Pastor (2000) and Pastor and Stambaugh (2000) centre prior beliefs on values implied by asset pricing theories. Tu and Zhou (2010) argue that the investment objective provides a useful prior for portfolio selection.

2 Portfolio optimisation strategies: Markowitz MV / Michaud resample efficiency

2.1 Markowitz MV optimisation

The standard mean variance method of portfolio selection, pioneered by Markowitz (1952) has long attracted the attention of financial economics and researchers. In the framework of Markowitz, under the hypothesis of multivariate normal distributed returns, the investor maximizes the following preference function:

$$w'\mu - \frac{\lambda}{2} w'\Sigma w$$

where $w = (w_1, ..., w_m)'$ represents the vector of portfolio shares of m risky assets, μ is the vector of expected excess returns, Σ is the variance covariance matrix and λ is the risk aversion coefficient. The difference $1 - w_1 - ...... - w_m$ is invested in the riskless asset. Since the investor does not know the true parameters μ and Σ of the return distribution, he has to estimate them. Classical portfolio selection uses least squares estimates of (μ, Σ).

2.2 Michaud's resampling efficiency

The Michaud resampling efficiency comprises three points:
(i) Generation of a sequence of returns, which are statistically equivalent to the actual time series of returns, through a Monte Carlo simulation;
(ii) Determination of portfolio weights for every sample;
(iii)Averaging over the obtained portfolio weights to obtain the optimal portfolio weights according to Michaud.
The procedure of resample efficiency aims at minimizing the impact of estimation risk on the portfolio composition. This approach can be summarized in the following steps:

1. Estimate the input parameters $\hat{\mu}$ and $\hat{\Sigma}$.
2. Resample from the inputs of (1) by taking T draws from a multivariate normal distribution $N(\hat{\mu}, \hat{\Sigma})$ and estimate new input parameters $\hat{\mu}_n$ and $\hat{\Sigma}_n$.
3. Identify the optimal portfolio composition $\hat{w}_n$ with the estimators $\hat{\mu}_n$ and $\hat{\Sigma}_n$.
4. Repeat steps (2) and (3) 500 times.
5. Calculate the average portfolio weight vector $\hat{w}$ from the 500 different optimal weight vectors and chose $\hat{w}$ as the optimum.

In order to analyse the performance of the approach of Michaud, some studies have dealt with the comparison between traditional mean variance optimization by Markowitz and the resample efficiency by Michaud. The results are ambiguous. In a simulation study of Michaud and Michaud (2008b), the resample efficiency leads to the best outcomes. Markowitz and Usmen (2003) also find strong evidence for a better performance of the resample efficiency compared to a bayesian estimator using a diffused prior. However, Harvey et al. (2008) and Scherer (2006) found a completely different result when they use different prior distribution. In this work, we compare the two strategies of optimization in a bayesian framework without specifying any prior density for the parameters by applying an implicit estimator of the mean vector and covariance matrix of the returns. Our optimization will account for estimation risk through the estimations of the parameters and the optimization of the portfolio.

3 Competing estimation approach: Bayesian / Implicit

In order to account for estimation risk in parameters of asset returns, a Bayesian framework can be considered. In this theory, the unknown parameter is assumed to be a random variable with a known prior distribution. The prior distribution shrinks value of parameter estimates in order to have an equilibrium value or significant mean. Prior information allows enriching the data distribution. Considered together, they allow generating a posterior distribution for the parameters of the model. The use of Bayesian theory to estimate mean vector and covariance matrix of the returns have been advocated by several researchers (Brown 1976, Bawa et al. 1979, Frost and Savarino 1986). Greyseman et al. 2006 consider hierarchical priors for the parameters. Ando 2009 compares the performance of a portfolio when using Bayesian theory and the standard mean variance method. This research shows that M-V-P using Bayesian estimates dominate Mean Variance portfolio using classical least squares estimates. Formally, we have a parameter θ, with a prior density π, such that $\theta \sim \pi$, the likelihood density is such that $X / \theta \sim P(x,\theta)$. So the posterior distribution of θ is

$$\theta / X = x = P(\theta / X = x).$$

The estimator is :

$$\hat{\theta} = E(\theta / X = x)$$

However, the choice of prior information in Bayesian approaches has already been problematic. Alternative to the posterior distribution, the concept of the implicit distribution has been recently introduced. The latter is considered like a posterior density in the Bayesian method without a need to specify any prior distribution. (Hassairi et al., 2005).

Let

$$P_\theta(dx) = p(x,\theta)\nu(dx), \quad \theta \in \Theta$$

a statistical model parameterized by $\theta \in \Theta$.

The density $p(x,\theta)$ is the conditional distribution of x given θ.

The implicit method consists in determining the implicit distribution:

$$Q_x(d\theta) = (c(x))^{-1} p(x,\theta),$$

where $c(x) = \int_\theta p(x,\theta)\sigma(d\theta)$.

In the implicit approach, $Q_x(d\theta)$ plays the role of a posterior distribution of θ given x in the Bayesian method. This fact implies the uniqueness of the implicit distribution. The implicit estimator $\hat{\theta}$ of θ is the mean of the implicit distribution associated with the quadratic risk.

Mukhopadhyay (2006) claimed that the implicit inference is nothing new and that it is non informative Bayesian method. Ben Hassen et al. (2008) showed that the implicit inference is a new paradigm in statistical inferences. In many cases, implicit distribution does not coincide with the Bayesian distribution. The concept of implicit distribution differs from usual Bayesian analysis. In fact, an implicit distribution resembles a posterior distribution of θ but without the presence of a specific prior distribution of θ. In the implicit method, Hassairi et al. (2005) didn't consider a prior law of θ like in a Bayesian method, but they simply considered a measure which is neither a probability nor a prior law, and this scenario is new in statistical reasoning.

The inference problem of the implicit approach is solved by determining a norming constant function $c(x)$ in x such that

$$\frac{p(x/\theta)\sigma(d\theta)}{c(x)}$$

becomes a probability distribution of θ, that is $\int \frac{p(x/\theta)}{c(x)} \sigma(d\theta) = 1$.

It comes that the implicit distribution of θ given x is:

$$Q_x(d\theta) = \left(c(x)\right)^{-1} p\left(x,\theta\right)\sigma(d\theta)$$

4 Simulation study

After explaining the two optimization approaches, we will compare the two competing estimation techniques we will explore. In this section we will develop our simulation study and present our results.

4.1 Settings

In order to test the performance of the portfolio optimization techniques of Markowitz and Michaud with Bayesian and implicit estimation techniques, we perform a two step simulation study. In the first step, “True” parameters μ and Σ are generated. In the second step, we draw realizations of excesses returns from a multivariate normal distribution with parameters μ and Σ. On the basic of these drawn excess returns, we estimate the parameters μ and Σ using the Bayesian and the implicit estimation and apply the optimization technique under consideration (MV of Markowitz, Michaud’s Resampling Efficiency). So, we can see that the second step corresponds to an application, where the input parameters μ and Σ have to be estimated.

The true parameters corresponding to a simulate step 1) allow the evaluation of the performance of different approaches with respect to the true resulting performance values.

i. The choice of the true parameters μ and Σ is based on 266 monthly returns of 8 stocks (6 risky assets and 2 bonds assets).
ii. In order to achieve results that do not rely on one specific parameter setting, for each of these true parameter, we generate 100 ”observable” time series each consisting of 266 monthly returns. These returns are drowning from a multivariate normal distribution:

$$r_{i,t} \sim N_8\left(\mu_i, \Sigma_i\right),\ t = 1,\dots 266 \quad \text{and} \quad i = 1, \dots, 100$$

The mean vector μ_s and the covariance matrix $\sum_i$ of the simulated database are considered true parameters.

iii. As a next step, we estimate the expected return vector $\hat{\mu}_i$ and the covariance matrix $\hat{\sigma}_i$. For this purpose, we implement the Bayesian and the implicit estimation method. The estimation is based on the Monte Carlo technique. In order to measure the estimation error caused by these two approaches, we compute the mean squared error of the estimated return vector and the estimated covariance matrix. More accurate estimator will have the minimum mean squared error.

iv. The Markowitz portfolio optimization and the Michaud's resampling efficiency are applied at each estimated parameters. As a result, we get optimal portfolio weights $\hat{W}_i$ for each approach and estimated parameters. We also compute the true optimal portfolio weights $\hat{W}_s^*$ which would be resulted if the true parameters were applied. These weights are considered like a benchmark portfolio.

v. In order to assess and compare the performance of the Michaud's resampling efficiency relatively with mean variance Markowitz, we calculate the portfolio weights that will maximize the expected utility under three different utility functions. These functions are given by:

$$U\lambda\left(w, r_{n+1}\right) = w' r_{n+1} - \lambda\left(\omega'\left(r_{n+1} - E\left[r_{n+1/H}\right]\right)\right)^2$$

where

$E\left[r_{n+1/H}\right]$ is the predictive mean given one stimulated data base (H).

ω is the portfolio weight.

r_{n+1} are the predictive returns.

λ reflect risk aversion $\lambda = \{2,5;\ 3;\ 5\}$.

The Michaud weights are equal to the average of the best Σ weights for each resampling history. These weights form a discrete estimate of the efficient frontier for each resampling history for a discrete grid of 101 equally spaced portfolios. For each value of λ, the Michaud optimal weight is as follows:

$$w_{\lambda it} = \arg\max\left\{w' \mu_{i,t} - \lambda w' \Sigma_{it}\, w\right\}$$

where $\omega > 0$ and $\Sigma\omega_p = 1$.

The Bayesian weights will maximize the expected utility with respect to the predictive moments for each history. The approximation of the expected utility is calculated as follows:

$$E\left[\mu_\lambda\left(w, r_{n+1}\right)/H\right] \cong \frac{1}{M}\sum w' r_{n+1}^m - \lambda\left(w'(r_{n+1}^m - \hat{\mu})\right)^2$$

where

$\hat{\mu} = \frac{1}{M}\sum r_{n+1}^{m}$.

$r_{n+1}^{m} \sim f(r/\mu^{m}, \Sigma^{m})$ is the predictive return simulated from the predictive density.

$\mu^{m}, \Sigma^{m} \sim f(\mu, \Sigma/H, \bar{\mu}, r^{2}, n, SS)$ represents the posterior distribution.

4.2 Simulation results

The mean squared error calculated for the Bayesian estimator, the implicit one and the estimator from the resampling efficiency approach are summarized in Table 1.

Table 1

MSE				
		Bayesian	Implicit	RE
$\lambda = 2.5$	μ	0.0038	10^{-7}	0.011
	Σ	1.68	2.49	1.98
$\lambda = 3$	μ	0.0102	10^{-7}	0.665
	Σ	1.13	2.90	1.53
$\lambda = 5$	μ	0.0102	10^{-7}	0.0358
	Σ	1.13	2.74	2.03

We present in the following table a comparison results between the utility obtained when we apply the true parameters (reference utility) and the utility obtained with the estimated parameters with the three estimation approach.

The results using Bayesian, implicit and efficient resampling for inference and using original performance criteria (i.e. evaluating each weight using the proposed true parameter values as the predictive mean and covariance) differ from the results reported by existing experiment.

Markowitz and Usmen (2003) found that an investor who uses the Monte Carlo based resampling approach advocated in Michaud, always gain a utility larger than an investor who uses Bayesian methods for determining portfolio weights. Harvey et al. (2008) found that this result depends on the risk aversion parameter, and that sometimes resampling approach gives a maximum utility, sometimes Bayesian methods provide the maximum utility for the investor.

Table 2

	Resample Efficiency		
	$\lambda = 2$	$\lambda = 3$	$\lambda = 5$
Utility (Ref)	71.041	62.83	52.34
Utility (Max)	76.162	67.13	56.041

	Bayesian		
	$\lambda = 2$	$\lambda = 3$	$\lambda = 5$
Utility (Ref)	46.52	44.213	35.15
Utility (Max)	58.80	57.70	52.39

Table 3

	Implicit		
	$\lambda = 2$	$\lambda = 3$	$\lambda = 5$
Utility (Ref)	46.43	60.82	57.78
Utility (Max)	93.38	92.7	90.82

	Utility (Max)		
	ER	Bayesian	Implicit
$\lambda = 2$	76.17	58.80	93.38
$\lambda = 3$	67.13	57.70	92.7
$\lambda = 5$	56.041	52.39	90.82

In addition to the Michaud's approach and the Bayesian one, we implement the implicit inference method. Our results prove the accuracy of the implicit method when estimating the mean vector of the returns.

The comparison of the optimal weight found with the efficient resampling approach with that found with the Bayesian approach prove that when we applied the implicit estimators on the Bayesian optimization, we found a better utility compared with the Bayesian estimators. The efficient resampling approach gives a better utility than the Bayesian approach for the three risk aversion parameter.

The Bayesian allocation applied with the implicit estimator give better utility than the efficient resampling approach.

Our results confirm the view that risk estimation of the mean return affects the portfolio optimization more than the risk associated with the estimation of the covariance matrix.

References

[1] Tomohiro Ando, Bayesian portfolio selection using a multifactor model, *International Journal of forecasting*, **25**, (2009), 550-566.

[2] Vijay Bawa, Stephen Brown and Roger Klein, *Estimation risk and optimal portfolio choice*, New York, North Holland, 1979.

[3] Fischer Black and Robert Litterman, Global portfolio optimization, *Financial Analysts Journal*, **48**(5), (1992), 28-43.

[4] Michael Broadie, Computing efficient frontiers using estimated parameters, *Annals of operations research: financial engineering*, **45**, (1999), 21-58.

[5] Stephen Brown, *Optimal portfolio choice under uncertainty: A Bayesian Approach*, *Ph. Dissertation*, University of Chicago, 1976.

[6] John Campbell and Luis Viceira, Consumption and Portfolio Decisions When Expected Returns Are Time Varying, *The Quarterly Journal of Economics,* **114**, (1999), 433-495.

[7] Vijay Kumar Chopra and William Ziemba, The effect of errors in means, variances and covariances on optimal portfolio choice, *Journal of Portfolio Management*, (Winter, 1993), 6-11.

[8] Peter Frost and James Savarino, An empirical Bayes approach to efficient portfolio selection, *Journal of Financial and Quantitative Analysis*, **21**, (1986), 293-305.

[9] Alex Greyserman, Douglas Jones and William Strawderman, Portfolio selection using hierarchical Bayesian analysis and MCMC methods, *Journal of Banking and Finance*, **30**, (2006), 669-678.

[10] Campbell Harvey, John Liechty and Merrill Liechty, Bayes vs. Resampling: A Rematch, *Journal of Investment Management*, **6**, (2008), 29-45.

[11] Abdelhamid Hassairi, Afif Masmoudi and Clestin Kokonendji, Implicit distributions and estimation Communications, *Statistics-Theory and Methods*, **34**, (2005), 245-252.

[12] Hanen Ben Hassen, Afif Masmoudi and Ahmed Rebai, Causal inference in biomolecular pathways using a Bayesian network approach and an Implicit method, *Journal of Theoretical Biology*, **253**, (2008), 717-724.

[13] Jose Jobson and Bob Korkie, Estimation for Markowitz efficient portfolios, *Journal of the American Statistical Association*, **75**, (1980), 544-554.

[14] Philippe Jorion, Bayes-Stein estimation for portfolio analysis, *Journal of Financial and Quantitative Analysis*, **21**(3), (1986), 279-292.

[15] Jun Liu, Portfolio Selection in stochastic Environments, *The Review of Financial Studies*, **20**, (2006), 1-39.

[16] Harry Markowitz, Portfolio selection, *Journal of finance*, **7**, (1952), 177-181.

[17] Harry Markowitz and Nilufer Usmen, Resampled frontiers versus diffuse bayes: an experiment, *Journal of investment Management*, **1**, (2003), 9-25.

[18] Robert Merton, Lifetime Portfolio Selection under Uncertainty: The Continuous-Time Case, *The Review of Economics and Statistics*, **51**, (1969), 247-257.

[19] Richard Michaud and Robert Michaud, Estimation Error and portfolio optimization: A resampling solution, *Journal of Investment Management*, **6**, (2008), 8-28.

[20] Nitis Mukhopadhyay, Bayesian Inference Some Comments on Hassairi et al.s. Implicit Distributions and Estimation, Communications, *Statistics-Theory and Methods*, **35**, (2006), 293-297.

[21] Lubos Pastor, Portfolio selection and asset pricing models, *Journal of Finance*, **55**, (2000), 179-223.

[22] Lubos Pastor and Robert Stambaugh, Comparing asset pricing models: an investment perspective, *Journal of Financial Economics*, **56**, (2000), 335-381.

[23] Paul Samuelson, Lifetime Portfolio Selection by Dynamic Stochastic Programming, *The Review of Economics and Statistics*, **51**, (1969), 239-246.

[24] Bernd Scherer, Note on the out-of-sample performance of resampled efficiency, *Journal of Asset Management*, **7**, (2006), 170-178.

[25] Jun Tu and Guofu Zhou, Incorporating economic objectives into Bayesian priors: portfolio choice under parameter uncertainty, *Journal of Financial and Quantitative Analysis*, **45**, (2010), 959-986.

[26] Jessica A. Wachter, Portfolio and consumption decisions under mean-reverting returns: An exact solution for complete markets, *Journal of Financial and Quantitative Analysis*, **37**, (2002), 63-91.

[27] Arnold Zellner and Karuppan Chetty, Prediction and decision problems in regression models from the Bayesian point of view, *Journal of the American Statistical Association*, **60**, (1965), 608-616.