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Out-of-sample performance of the Black-Litterman model

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Abstract

The aim of this paper is to test the out-of-sample performance of the Black Litterman (BL) model for a German stock portfolio compared to the traditional mean-variance optimized (MV) portfolio, the German stock index DAX, a reference portfolio, and an equally weighted portfolio. The BL model was developed as an alternative approach to portfolio optimization many years ago and has gained attention in practical portfolio management. However, in the literature, there are not many studies that analyze the out-of-sample performance of the model in comparison to other asset allocation strategies. The BL model combines implied returns and subjective return forecasts. In this study, for each stock, sample means of historical returns are employed as subjective return forecasts. The empirical analysis shows that the BL portfolio performs significantly better than the DAX, the reference portfolio and the equally weighted portfolio. However, overall, it is slightly outperformed by the MV portfolio. Nevertheless, the BL portfolio may be of greater interest to investors because -according to this study, where the subjective return forecasts are based on historical returns of a rather long past period of time- it could lead in most cases to lower absolute (normalized) values for the stock weights and for all stocks to smaller fluctuations in the (normalized) weights compared to the MV portfolio.

JEL classification numbers: C61, G11.
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1. Introduction

Today's professional asset allocation of stock portfolios is essentially based on the findings of modern portfolio theory and models derived from it. To determine the optimal portfolio according to the traditional mean-variance portfolio optimization (Markowitz, 1952), the necessary input parameters (expected return values, variances of the returns and covariances) must be known to the investor at the time of the portfolio decision as forecast values for the future. In practice, historical mean values are often used as estimates for the respective expected values. Future returns, however, are random variables whose actual characteristics may differ significantly from their expected value. As a result, the forecasts may be subject to considerable estimation errors. Thus, the theoretical composition of a portfolio may be ideal at the time of planning. At a later point in time, however, estimation errors may cause the portfolio to be far removed from the characteristics of an optimal portfolio, because even slight deviations in the forecasts can lead to completely different portfolio structures. However, investors want a portfolio structure defined at the beginning to remain as optimal as possible in a volatile market environment, so that no unnecessary shifts and associated transaction costs arise (Ernst/Schurer, 2015, pp. 424-425).

Various models have been developed to solve this estimation error problem. One of these models is the Black-Litterman (BL) model, which can be seen as a robust optimization method (Black/Litterman, 1991, Black/Litterman, 1992, He/Litterman, 1999). This model is also able to integrate different estimates and forecasts of expected returns into the portfolio optimization process. The BL model has been of increasing interest for quantitative portfolio managers. This raises the question to what extent this model is promising, especially in comparison to the traditional mean-variance portfolio optimization. In the academic literature, there have been only few empirical studies of the out-of-sample performance of the BL model (Bessler/Opfer/Wolff, 2014, p. 2, Allaj, 2020, pp. 465-468).

In fact, in the literature, there are some analyses of the rationale of the model, examples for applying the methodology or the suggestion of model extensions (Satchell and Scowcroft, 2000, Drobetz, 2003, Herold, 2003, Idzorek, 2007, Martellini/Ziemann, 2007, Da Silva/Lee/Pornrojnangkool, 2009, Zankl, 2009, Zhou, 2009, Cheung, 2010, Braga/Natale, 2012, He/Grant/Fabre, 2013, Walters, 2014, Chen/Da/Schaumburg, 2015, Geyer/Lucivjanská, 2016, Figelman, 2017, Harris/Stoja/Tan, 2017, Kolm/Ritter, 2017, Tee/Huang/Lim, 2017, van der Schans/Steehouwer, 2017, Martin/Sankaran, 2019, and Chen/Lim, 2020).

Bessler/Opfer/Wolff (2014) provide an out-of-sample performance analysis of the BL model. They test the performance relative to other asset allocation models. Their findings show significant higher Sharpe ratios of the BL portfolio than naïve strategies. Furthermore, the BL portfolios consistently outperform the traditional mean-variance portfolio optimization (Bessler/Opfer/Wolff, 2014, p. 3).

Harris/Stoja/Tan (2017) consider different BL portfolios and find that the dynamic BL portfolios and the risk-adjusted BL portfolios outperform the benchmark and

the equally weighted portfolio where they use different performance measures.
Allaj (2020) examins the out-of-sample performance of the BL model with respect to other asset allocation strategies and presents a new methodology for defining the investor's views. He finds that the strategies based on the BL model can lead to superior performance.
The aim of this paper is to test the out-of-sample performance of the Black Litterman (BL) model compared to the traditional mean-variance portfolio and an equally weighted portfolio for selected German stocks. The forecast values used in the BL model are based on sample means.
The BL model is presented in Chapter 2. After a short introduction, the determination of reference excess returns and the formation of a reference portfolio is shown. On this basis, subjective forecasts can then be considered for the subsequent determination of BL returns. With the help of these returns, the optimal portfolio according to BL can finally be determined.
Chapter 3 contains the empirical analysis, which should determine the success of the BL model when applied to a German stock portfolio by means of certain performance measures for a period of several years. The results are compared to the success of the traditional mean-variance optimized (MV) portfolio (Markowitz, 1952), the DAX, a reference portfolio, and an equally weighted portfolio.
Chapter 4 summarizes the main results of the paper.

2. The Black-Litterman (BL) model

2.1 Reference excess returns and reference portfolio

The BL model has been developed to solve the above mentioned estimation error problem by combining ‘neutral’ (‘implied’ or ‘reference’) returns and ‘subjective’ return estimates (‘views’). If only publicly available information can be used, the BL model uses the neutral returns as so-called reference excess returns, which are obtained, for example, with the help of the Capital Asset Pricing Model (CAPM). These equilibrium excess returns can be used as input variables in portfolio construction. A combination of these excess returns with subjective forecasts of returns can lead to the derivation of economically better-supported and more stable portfolio weightings. Accordingly, the BL model uses historical data, equilibrium considerations and individual assessments of the portfolio managers for the near future (Drobetz, 2003, p. 213, Ernst/Schurer, 2015, p. 497 and Bodie/Kane/Marcus, 2018, p. 918.).
Investors can provide return estimates for each asset in the portfolio. However, if they do not feel comfortable in making return forecasts they can also stay neutral for some assets. Besides, each forecast can be assigned a certain degree of uncertainty, which reflects the investor's imperfect conviction. Therefore, investors can distinguish between qualified forecasts and pure guesses. The idea behind this is that only in case of reliable return forecasts investors should deviate from the reference portfolio. This portfolio might be the market or another benchmark portfolio. Based on the asset weights of this reference portfolio, the above-

mentioned reference excess returns can be calculated. In the original BL model, the observable market or benchmark weights are assumed to be the result of a mean-variance optimization so that the reference excess returns can be derived using reverse optimization (Bessler/Opfer/Wolff, 2014, p. 7).
The reference excess returns determined as equilibrium excess returns can also be described as "baseline forecasts", which form the basis of a passive investment strategy. Accordingly, their definition as strategic potential returns is also possible. In this case, the reference excess returns reflect the long-term return expectations that implicitly result from a strategic asset allocation or benchmark determined in advance by the investor (Bodie/Kane/Marcus, 2018, p. 918 and Ernst/Schurer, 2015, p. 500). The background to the inclusion of a reference portfolio is practical experience. Thus, investors often use indices as benchmarks for asset allocation. The portfolio manager's task is to achieve outperformance by deviating from the benchmark, i.e. to exceed the return of the benchmark. The BL model also allows for the selection of a reference or initial portfolio weighted according to the market capitalization of the individual securities. The current actual portfolio would also be possible (Ernst/Schurer, 2015, pp. 503-504; Feilke/Gürtler, 2008, p. 5 and Walters, 2014, p. 6).
The reference excess returns can be calculated as implicit equilibrium excess returns by solving the quadratic utility function:

$$\text{Max } U_R = E(r_R) - \frac{A}{2} \cdot \sigma_R^2 \tag{1}$$

U_R stands for the utility of a risky portfolio, $E(r_R)$ is the expected return of the risky portfolio, A is the investor's average risk aversion coefficient, and σ^2_R is the variance of the returns of the risky portfolio.
The objective function is accordingly

$$U_R = w^T \Pi - \frac{A}{2} w^T \Sigma w \rightarrow \max! \tag{2}$$

Where: w is the n×1 vector of the asset weights in the portfolio, w^T is the 1×n vector of the asset weights in the portfolio (transposed vector w), Π is the n×1 vector of the (implied) reference excess returns and Σ is the n×n covariance matrix of the historical asset excess returns. Under the assumption that the weightings of the reference portfolio are determined on the basis of the respective market capitalization, the associated implicit reference excess returns can be determined from this reference portfolio. For this purpose, the above-mentioned objective function is derived according to the weightings and the derivation is set to zero (Ernst/Schurer, 2015, pp. 504-505, Walters, 2014, pp. 9-10 and Black/Litterman, 1991, p. 14):

$$\frac{\delta U_R}{\delta w} = \Pi - A \cdot \Sigma \cdot w_{opt} = 0 \qquad \Leftrightarrow \qquad \Pi = A \cdot \Sigma \cdot w_{opt} \tag{3}$$

Thus the vector Π results from the reverse optimization. The optimal weights w result – when a market portfolio of n investments is defined – by the market capitalization of the respective assets in relation to the total market capitalization (Feilke/Gürtler, 2008, p. 6.):

$$w_j = \frac{\eta_j \cdot P_j}{\sum_{i=1}^{N} \eta_i \cdot P_i} \tag{4}$$

Where: η_j is the number of the j-th asset of the market portfolio, P_j is the current price of asset j, and N is the number of assets in the market portfolio. The determination of the average risk aversion coefficient A across all investors can be done in different ways. For example, it can be determined by using the Sharpe ratio (SR) (Walters, 2014, p. 10 and He/Grant/Fabre, 2013, p. 452) or by assuming a certain average risk tolerance. For example, He/Litterman (1999, p. 4) use a value for A of 2.5 in their examples, while Drobetz (2003, p. 215) uses a risk aversion coefficient of 1. Tee/Huang/Lim (2017, p. 128) emphasize that a market-average risk aversion coefficient is not appropriate because, on the one hand, the portfolio under management may differ from the general market portfolio and, on the other hand, the style and characteristics and thus also the risk aversion of the respective portfolio managers may vary significantly.

Moreover, the covariance matrix of the reference excess returns can be determined. This provides information on the accuracy of these (estimated or implicitly expected) returns. This covariance matrix thus differs from the above matrix Σ, which is based on historical (realized) excess returns. For example, if the standard deviation of the expected portfolio's reference excess returns is assumed to be 10% of the standard deviation of the historical portfolio excess returns (or equivalently, a variance that is 1% of the variance of the historical returns), it would be correct to use a covariance matrix of the (estimated or implicitly expected) reference excess returns that is 1% of the covariance matrix of historical returns (Bodie/Kane/Marcus, 2018, pp. 918-919).

The 1% in this example can be referred to as the parameter τ. Basically, the smaller τ is, the less the (expected) reference excess returns spread around its expected value vector Π (= vector of the implied reference or equilibrium excess returns) and the more confidence the investor will have in the reference portfolio. Provided that the reference portfolio was selected plausibly, τ should be selected accordingly low. In addition, a low τ also seems to make sense from a statistical point of view, because the variance of (historical) returns should be higher than the variance of expected returns (Zankl, 2009, p. 45, Ernst/Schurer, 2015, p. 504 and Drobetz, 2003, p. 219).

Therefore, the value for τ should be less than 1 (Duqi/Franci/Torluccio, 2014, p. 1291). Black/Litterman (1992, p. 34) justify a value for τ close to 0 by stating that the uncertainty in the mean value (expected value) is much smaller than the uncertainty in the return itself. Allaj (2013, p. 229) proposes an econometric method for estimating τ. In the literature, τ typically ranges between 0.025 and 0.300 (Bessler/Opfer/Wolff, 2014, p. 8).

2.2 Investor's 'subjective' return forecasts ('views')

In the BL model, investors can integrate their own forecasts ('views') of future returns into the investment process, which can correct the above-mentioned implied reference excess returns. The estimates can be absolute or relative forecasts. While an absolute forecast refers to a concrete expected return on a particular stock in the coming period, a relative forecast makes statements about relationships between different stocks, such as the expectation that the return on stock A will be X% higher than the return on stock B in the coming period (Ernst/Schurer, 2015, p. 509; Drobetz, 2003, pp. 219-220 and Feilke/Gürtler, 2008, p. 7).
For example, an investor assumes that in the coming period stock A will generate a 0.7 percentage point higher excess return than stock B. This view can be expressed as follows:

$$1 \cdot R_A + (-1) \cdot R_B = 0.7\% \tag{5}$$

Where: R is the excess return of the stock.
In general, all forecasts that are a linear combination of the corresponding excess returns can be represented as matrix multiplication. In this example, the array of weights is P = (1, -1) while the array of the reference excess returns is E(R) = (R_A, R_B). The value of this linear combination is denoted Q and reflects the investor's forecast. Thus, in this case the value of 0.7% for Q would be included in the portfolio optimization process.
It should be noted that each forecast can be assigned a certain degree of uncertainty. Accordingly, a standard deviation should be included, which measures the precision of Q. Indeed, the investor's forecast is actually $Q + \varepsilon$, where ε represents the forecast error with an expected value of zero and a standard deviation that reflects the investor's imperfect confidence (Bodie/Kane/Marcus, 2018, p. 920).
More generally, it is assumed that the subjective forecasts can be expressed in the form of k different linear combinations of the n assets. The following equation serves as a basis (Ernst/Schurer, 2015, p. 509; Drobetz, 2003, pp. 219-220 and Feilke/Gürtler, 2008, p. 7):

$$P \cdot E(R) = Q + \varepsilon \tag{6}$$

Where: Q is the $k \times 1$ vector of the returns for each subjective forecast, ε is the $k \times 1$

vector of the subjective forecast errors, E(R) is the k×1 vector of the expected excess returns, and P is the k×n matrix of the asset weights within each subjective forecast. For a relative forecast, the sum of the weights will be zero while it will be one for an absolute view. In the literature the weights within the subjective forecasts are computed differently, e.g. a market capitalization weighted scheme, or an equal weighted scheme. Thus, P also contains the information for which asset a subjective excess return forecasts is available (Walters, 2014, p. 13, Bessler/Opfer/Wolff, 2014, p. 7).

It should be noted that the BL model does not require a subjective forecast for all assets in the portfolio. For example, if only 4 forecasts in a 10 asset portfolio are given, the subjective forecast vector Q would be a 4×1 column vector. The forecasts are subject to uncertainty. This uncertainty results in an error term vector ε, which is assumed to be random, unknown, and normally distributed with a mean of zero. Besides, the subjective forecasts are assumed to be independent of each other and also independent of the implied reference excess returns. Thus, a subjective forecast has the form Q + ε (Idzorek, 2007, p. 24):

$$Q+\varepsilon=\begin{bmatrix} Q_1 \\ \vdots \\ Q_k \end{bmatrix}+\begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_k \end{bmatrix} \tag{7}$$

Where: Q is the k×1 vector of the returns for each subjective forecast, ε is the k×1 vector of the subjective forecast errors. The value of the error term (ε) is different from zero unless there is a clairvoyant investor who is 100% confident in the expressed forecast, which is just a hypothetical case. The error term vector is not considered directly in the BL model, but the variance of each error term ($\sigma^2_{\varepsilon_i}$) enters the BL formula. These variances form the diagonal covariance matrix Ω in which all values except the diagonal positions are zero. As in the model it is assumed that subjective forecasts are independent of each other, the off-diagonal elements of Ω are zero. Thus, the uncertainty of the forecasts is represented by the variances of the error terms ($\sigma^2_{\varepsilon_i}$), i.e. the larger these variances, the greater the uncertainty of the forecasts. Ω, which is the k×k-matrix of the covariance of the subjective forecasts, can be represented as follows (Idzorek, 2007, p. 24):

$$\Omega=\begin{bmatrix} \sigma^2_{\varepsilon_1} & 0 & 0 & 0 \\ 0 & \sigma^2_{\varepsilon_2} & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \sigma^2_{\varepsilon_k} \end{bmatrix} \tag{8}$$

The variance of the subjective forecasts is thus inversely related to the confidence of the investors in their forecasts. However, the BL model does not provide an approach to quantify this variance. Accordingly, the investor himself has to calculate the variance of the forecasts. Four options are proposed to determine the diagonal matrix Ω (Walters, 2014, p. 14):

(1) The calculation is based on the assumption that the variance of the subjective forecasts is proportional to the variance of the asset returns. Thus, the following expression can be proposed for the diagonal matrix Ω (He/Litterman, 1999, p. 6):

$$\Omega = \mathrm{diag}\left(P \cdot \left(\tau \cdot \Sigma\right) \cdot P^{T}\right) \tag{9}$$

Where: P is the k×n matrix of the asset weights within each subjective forecast, Σ is the n×n covariance matrix of the historical asset excess returns, and τ is a measure of uncertainty of the equilibrium variance (Walters, 2014, p. 65).

(2) The investor can determine the variance of the subjective forecasts by using a confidence interval around the estimated mean return. If, for example, a mean return of 5% is forecast and it is expected with a probability of 80% that the mean return will be between 4% and 6%, a variance of the forecasts of 0.006089% can be calculated from this, assuming normally distributed returns:

$$\sigma_i^2 = \left(\frac{0.06 - 0.05}{1.28155157}\right)^2 = \left(0.007803\right)^2 = 0.006089\% = \left(\frac{0.04 - 0.05}{-1.28155157}\right)^2 \tag{10}$$

The denominator contains the Z-score of the normal distribution where Prob(Z ≤ 1.28155157) = 90% and Prob(Z ≤ -1.28155157) = 10%. Thus, in this example, the forecast uncertainty can be interpreted as a symmetrical 80% confidence interval around the expected mean return of 5%. Correspondingly, this confidence interval implies a variance of 0.006089% in the forecast return. It should be noted that the matrix Ω thus does not indicate the variances of the returns about the mean, but rather the uncertainty in the forecast of the mean (Walters, 2014, pp. 14-15 and Drobetz, 2003, p. 221).

(3) Another alternative for determining the matrix Ω is to use the variances of the residuals in case the investor uses a factor model to compute the subjective forecast (Walters, 2014, p. 15).

(4) Finally, it is proposed to determine the forecast uncertainty by a special method to determine an implied confidence level (Idzorek, 2007, pp. 32-36).

Since the first-mentioned alternative for the determination of the matrix Ω seems to represent the method most frequently used in the literature, in the following only this is considered in this paper.
Now the reference excess returns (as returns expected on a long-term basis) can be combined with the rather short-term, subjective forecasts of the investor. This is shown in the following section.

2.3 Solving for the new combined return vector

On the basis of a Bayesian approach the subjective return forecasts of the investor are combined with the vector of the reference excess returns to form a new vector of combined expected returns which will be denoted „BL returns“ in this paper. This n×1 vector ($E(R_{BL})$) can be derived after having specified the scalar (parameter τ) and the matrix Ω. It is shown in the following equation (Zankl, 2009, p. 40 and p. 52; Walters, 2014, p. 19; Idzorek, 2007, pp. 18, 21 and 27, Ernst/Schurer, 2015, p. 517):

$$E\left(R_{BL}\right)=\left[\left(\tau\cdot\Sigma\right)^{-1}+P^{T}\cdot\Omega^{-1}\cdot P\right]^{-1}\cdot\left[\left(\tau\cdot\Sigma\right)^{-1}\cdot\Pi+P^{T}\cdot\Omega^{-1}\cdot Q\right] \tag{11}$$

This formula can be transformed into the following (Zankl, 2009, pp. 52-53, Mankert, 2010, p. 48, Walters, 2014, p. 19, O’Toole, 2017, p. 582 and p. 586):

$$E\left(R_{BL}\right)=\Pi+\left(\tau\cdot\Sigma\right)\cdot P^{T}\cdot\left[\Omega+P\cdot\left(\tau\cdot\Sigma\right)\cdot P^{T}\right]^{-1}\cdot\left(Q-P\cdot\Pi\right) \tag{12}$$

Thus, the BL returns are derived from the implied reference excess returns (Π) and a correction term that takes into account the investor's subjective forecasts. On the basis of this formula it becomes also evident that $E(R_{BL})$ is independent of the parameter τ, because with the way of calculating the matrix Ω used in this paper (equation 9), each change of τ in the product of the square bracket and the term before the square bracket cancels itself out.

Finally, the posterior covariance matrix of returns (Σ_P) can be determined (Walters, 2014, pp. 19-20):

$$\Sigma_{P}=\Sigma+\left[\left(\tau\cdot\Sigma\right)^{-1}+P^{T}\cdot\Omega^{-1}\cdot P\right]^{-1} \tag{13}$$

In case that no subjective forecasts are available for the assets in the portfolio, this formula is reduced to

$$\Sigma_P = \Sigma + (\tau \cdot \Sigma) = (1+\tau) \cdot \Sigma \tag{14}$$

In the literature, this posterior covariance matrix is also expressed as follows, whereby the derivation from the above formula uses the Woodbury matrix identity (Bodie/Kane/Marcus, 2018, p. 930 und Walters, 2014, pp. 20-21):

$$\Sigma_P = \Sigma + \tau \cdot \Sigma - \tau \cdot \Sigma \cdot P^T \cdot \left(P \cdot \tau \cdot \Sigma \cdot P^T + \Omega\right)^{-1} \cdot P \cdot \tau \cdot \Sigma \tag{15}$$

The BL return vector will be used in the mean-variance optimizer to find the optimal BL portfolio, which is shown in the following section.

2.4 Optimal portfolio according to the BL model

The composition of the optimal portfolio according to the BL model is based on the traditional risk-return optimization that maximizes the investor's utility value as shown above. Provided, that neither short-selling nor budget constraints are imposed, the solution to this maximization problem is (Idzorek, 2007, pp. 20 and 27-28, Braga/Natale, 2012, p. 7):

$$w_{BL} = \frac{1}{A} \cdot \Sigma^{-1} \cdot E\left(R_{BL}\right) \tag{16}$$

Where: w_{BL} is the n×1 vector of asset weights in the BL portfolio, A is the investor's risk aversion coefficient, Σ^{-1} is the inverse of the n×n covariance matrix of historical excess returns, and $E(R_{BL})$ is the n×1 vector of the BL returns.
In addition, the optimal vector w_{BL} can also be determined without an explicit prior determination of the BL returns as follows (Zankl, 2009, pp. 55-56 and Mankert, 2010, p. 49):

$$w_{BL} = w_{Ref} + P^T \cdot \left(\frac{1}{\tau} \cdot \Omega + P \cdot \Sigma \cdot P^T\right)^{-1} \cdot \left(\frac{1}{A} \cdot Q - P \cdot \Sigma \cdot w_{Ref}\right) \tag{17}$$

Where: w_{Ref} is the n×1 vector of asset weights in the reference portfolio.
If the matrix Ω is calculated according to equation (9), the proportionality factor τ has no influence on the BL returns and thus on the optimal vector w_{BL}. The latter can be immediately recognized by this formula for w_{BL} because in the first parenthesis, Ω is divided by τ while in this case, the matrix Ω contains τ (Zankl, 2009, p. 74).

As formula (17) shows, the extent of the differences between w_{BL} and w_{Ref} is determined by, among others, the subjective forecasts and the level of confidence in these forecasts. This difference in weights could also be called active weights (Braga/Natale, 2012, p. 7).

Finally, it can be pointed out that in addition to the return adjustment by considering subjective forecasts, an additional risk adjustment would also be conceivable. In this case, the above-mentioned posterior covariance matrix would be included in the calculation of the optimal portfolio instead of the historical covariance matrix. If, however, the BL portfolio is to be compared to the traditional mean-variance optimized (MV) portfolio (Markowitz, 1952), only a return adjustment and the waiver of an additional risk adjustment would be useful (Zankl, 2009, p. 55).

3. Empirical Analysis

3.1 Methodology, performance measures, and data

In order to analyze the out-of-sample performance of the four portfolios included (BL, reference, MV, and equally weighted), optimized portfolios are computed every six months, i.e. the portfolios are rebalanced every six months. Thus, after each rebalancing, the computed number of stocks is retained in the portfolio until the next rebalancing. The semi-annual re-composition of the portfolios is intended to ensure a regular response to changing market situations. Transaction costs are not included in the analysis, because with a semi-annual rebalancing, they are less significant compared to a weekly or even daily rebalancing.

The calculation of the optimal weight of each stock in the portfolios is based on the (annualized) monthly logarithmic stock returns of the previous 5 years. These weights are used to compute the (ex post) monthly portfolio returns in the resulting (out-of-sample) period, which includes the time period from 30 December 2010 to 30 December 2019. Thus, monthly stock returns from December 2005 to December 2019 are used.

All models include the budget restriction that portfolio weights sum to one according to the following equation (Bessler/Opfer/Wolff, 2014, p. 3):

$$\sum_{i=1}^{n} w_i = 1 \tag{18}$$

Where: w_i is the weight of stock i in the portfolio and n is the number of stocks in the portfolio.

Furthermore, the BL portfolio and the MV portfolio are constructed both on the assumption that short sales are permitted. The resulting portfolio weights are normalized to a total of 100%. The stock weights of the reference portfolio are based on the market capitalization of the respective stock at the time of rebalancing.

In addition, for the BL portfolio and the MV portfolio, the risk aversion coefficient is set to 3. The risk-free rate is set to zero so that the excess returns equal the returns.

Furthermore, a value of $\tau = 0.2$ is assumed.
As shown above, the BL model combines implied returns and subjective return forecasts. The problem is how to derive subjective return forecasts. Simply assuming exogenously given forecasts seems not to be appropriate to evaluate the performance of the BL model compared to the MV portfolio and the equally weighted portfolio. In line with Bessler/Opfer/Wolff (2014, p. 9), sample means are employed as subjective return forecasts. In this paper, the annualized mean of the logarithmic monthly returns of the previous 5 years is used as subjective return forecast. It should be noted that, unlike the MV portfolio, the BL model also takes into account the reliability of these forecasts, which is expressed in the matrix Ω.
To evaluate the different portfolios, several performance measures are calculated based on the logarithmic portfolio returns. First, the Sharpe ratio of a portfolio (SR_{PF}) is computed as a performance measure frequently used in investment practice:

$$SR_{PF} = \frac{\bar{r}_{PF} - r_f}{\sigma_{PF}} \tag{19}$$

Where: $\bar{r}_{PF}$ is the mean portfolio return, r_f is the risk-free rate, and σ_{PF} is the standard deviation of the returns.
Besides the Sharpe ratio, the Treynor ratio (TR) is also included in the empirical analysis. This is a further key figure for comparing several portfolios. Unlike the Sharpe ratio, the Treynor ratio uses the beta factor of the portfolio ($ß_{PF}$) as a measure of systematic risk:

$$TR_{PF} = \frac{\bar{r}_{PF} - r_f}{ß_{PF}} \tag{20}$$

In contrast to the Sharpe ratio, the Treynor ratio is more appropriate if the portfolio to be evaluated is only part of a larger, well-diversified overall portfolio or is itself well-diversified (Bruns/Meyer-Bullerdiek, 2020, pp. 911-912).
In addition, the Jensen alpha (α_J) is used to measure performance, so that all three classical performance measures are taken into account. It can be described as a classical measure for determining the securities selection abilities of portfolio managers. Based on the Capital Asset Pricing Model (CAPM), it measures that part of the total return of a portfolio that is not correlated with the return of the benchmark. A linear regression can be used to measure the Jensen alpha (α_J):

$$\alpha_J = R_{PF} - R_{BM} \cdot ß_{PF} - \varepsilon_{PF} \tag{21}$$

Where: R_{PF} is the excess return of the portfolio, R_{BM} is the excess return of the benchmark, $ß_{PF}$ is the beta factor of the portfolio, and ε_{PF} is the residual return.
As a further measure, the Risk-Adjusted Performance (RAP) of a portfolio is used

in the empirical analysis. With this measure a ranking as well as a comparison of portfolio and benchmark return is possible. It can be calculated as follows (Modigliani/Modigliani, 1997, p. 47 and Fischer, 2010, pp. 461-463):

$$\mathrm{RAP}_{\mathrm{PF}} = r_f + \frac{\overline{r}_{\mathrm{PF}} - r_f}{\sigma_{\mathrm{PF}}} \cdot \sigma_{\mathrm{BM}} = r_f + \mathrm{SR}_{\mathrm{PF}} \cdot \sigma_{\mathrm{BM}} \tag{22}$$

Where: σ_{BM} is the standard deviation of the benchmark.
Using this performance measure leads to the same ranking as the Sharpe ratio when comparing different portfolios. However, a reference to a common risk level is established at the same time. The Risk-Adjusted Performance represents the portfolio return if the portfolio risk corresponds to the benchmark risk (Bruns/Meyer-Bullerdiek, 2020, p. 921).
The performance of the different portfolios is determined both for the entire period from 30 December 2010 to 30 December 2019 and for the following periods:

30 December 2010 – 30 December 2013
30 December 2013 – 30 December 2016
30 December 2016 – 30 December 2019

The portfolio construction is based on 10 stocks that are included in the German DAX index: Allianz, Bayer, Daimler, Deutsche Bank, Deutsche Telekom, Deutsche Post, E.ON, Henkel, SAP, and Siemens. To calculate the monthly logarithmic stock returns, monthly closing stock prices are used which are adjusted for stock splits and interim income (such as dividend income or income from subscription rights).

3.2 Empirical results

The stock weights in the different portfolios at each rebalancing date are presented in table 1, whereby the above-mentioned normalization of the sum of the portfolio weights to 100% was used for all portfolios.
In the BL portfolio (permitting short sales), the sum of all stock weights was below 100% in 6 periods (thereof below 50% in 3 periods). For the MV portfolio (permitting short sales), this was the case in 5 periods (thereof in 1 period under 50%). The corresponding normalization to 100% resulted in the individual absolute stock weights being correspondingly higher.
The invested capital is initially allocated to the individual stocks according to the weightings determined for each portfolio. The number of stocks determined in each case is then retained until the next portfolio reallocation, which takes place after six months ("rebalancing"). The weights for each stock in the equally weighted portfolio are set at 10% at each rebalancing date.

Table 1: (Normalized) Stock weights in the MV portfolio

	Allianz	Bayer	Daimler	Dt. Bank	Dt. Post	Dt.Telek.	E.ON	Henkel	SAP	Siemens	Total
30 Dec. 10	-4.91%	162.25%	12.79%	-98.17%	-77.98%	-81.30%	-57.39%	140.40%	-27.43%	131.75%	**100%**
30 June 11	59.78%	105.49%	37.90%	-78.11%	-67.92%	19.51%	-124.85%	73.64%	-18.45%	93.00%	**100%**
30 Dec. 11	-18.03%	221.06%	-5.71%	-201.89%	-86.30%	-92.57%	-233.64%	153.05%	93.44%	270.60%	**100%**
29 June 12	6.16%	32.16%	-36.65%	-73.92%	10.02%	26.52%	-87.07%	114.72%	108.92%	-0.85%	**100%**
28 Dec. 12	67.86%	19.94%	-59.48%	-57.55%	-19.81%	20.11%	-148.62%	137.54%	141.62%	-1.61%	**100%**
28 June 13	52.34%	1.11%	-11.29%	-21.18%	-12.02%	45.11%	-92.75%	136.62%	32.17%	-30.13%	**100%**
30 Dec. 13	21.85%	12.37%	-15.46%	-9.24%	20.63%	24.78%	-62.92%	71.52%	35.28%	1.20%	**100%**
30 June 14	67.07%	7.79%	-29.54%	-56.07%	66.80%	39.50%	-65.58%	73.21%	-11.92%	8.74%	**100%**
30 Dec. 14	85.91%	13.75%	-43.30%	-69.89%	89.64%	31.91%	-71.12%	46.47%	9.27%	7.36%	**100%**
30 June 15	85.85%	48.75%	-47.69%	-62.68%	102.64%	35.65%	-65.45%	27.47%	26.66%	-51.20%	**100%**
30 Dec. 15	89.97%	29.51%	-44.54%	-53.21%	119.20%	42.87%	-42.08%	1.23%	58.91%	-101.87%	**100%**
30 June 16	101.99%	14.54%	-64.87%	-77.07%	129.88%	30.52%	-37.74%	38.49%	50.68%	-86.43%	**100%**
30 Dec. 16	72.57%	21.74%	-39.32%	-45.99%	88.46%	25.98%	-31.09%	-2.66%	28.81%	-18.50%	**100%**
30 June 17	91.95%	30.24%	-45.43%	-48.12%	78.75%	19.88%	-26.82%	-18.35%	21.19%	-3.29%	**100%**
29 Dec. 17	91.86%	17.24%	-34.13%	-57.67%	112.95%	25.81%	-29.73%	-27.05%	25.20%	-24.48%	**100%**
29 June 18	122.50%	2.57%	-49.80%	-78.66%	70.33%	23.95%	-21.14%	-14.24%	24.06%	20.43%	**100%**
28 Dec. 18	227.45%	-36.65%	-48.09%	-109.99%	34.95%	54.68%	-12.98%	-38.17%	-1.29%	30.09%	**100%**
28 June 19	197.82%	7.71%	-62.70%	-81.97%	44.37%	20.58%	-18.43%	-44.49%	42.45%	-5.35%	**100%**

Table 2: (Normalized) Stock weights in the BL portfolio

	Allianz	Bayer	Daimler	Dt. Bank	Dt. Post	Dt.Telek.	E.ON	Henkel	SAP	Siemens	Total
30 Dec. 10	-9.32%	79.27%	17.80%	-27.15%	-26.77%	-15.37%	-11.45%	52.74%	0.32%	39.94%	**100%**
30 June 11	2.93%	58.14%	19.15%	-20.40%	-21.42%	21.16%	-32.42%	36.72%	3.57%	32.56%	**100%**
30 Dec. 11	-27.65%	125.71%	5.52%	-69.62%	-43.24%	-22.26%	-90.17%	73.14%	61.83%	86.73%	**100%**
29 June 12	-15.33%	69.12%	-20.89%	-55.63%	-10.81%	5.22%	-83.23%	99.98%	106.22%	5.34%	**100%**
28 Dec. 12	14.59%	63.95%	-20.29%	-34.82%	-13.06%	-21.22%	-128.12%	97.99%	130.36%	10.63%	**100%**
28 June 13	5.62%	24.75%	0.79%	-9.15%	0.09%	22.26%	-54.28%	82.09%	26.25%	1.58%	**100%**
30 Dec. 13	4.30%	21.29%	1.04%	-2.62%	8.58%	13.53%	-28.63%	46.20%	27.66%	8.65%	**100%**
30 June 14	8.60%	18.83%	2.15%	-16.04%	25.74%	19.25%	-22.22%	43.76%	9.00%	10.93%	**100%**
30 Dec. 14	12.83%	21.27%	1.17%	-19.27%	32.11%	17.18%	-29.55%	36.84%	19.57%	7.85%	**100%**
30 June 15	13.97%	34.77%	1.70%	-17.90%	35.86%	20.61%	-28.20%	27.88%	19.27%	-7.96%	**100%**
30 Dec. 15	27.75%	26.85%	-0.86%	-24.25%	39.15%	30.85%	-33.26%	24.39%	31.50%	-22.12%	**100%**
30 June 16	21.16%	21.64%	-9.36%	-38.90%	43.16%	27.87%	-30.84%	47.84%	30.07%	-12.65%	**100%**
30 Dec. 16	25.29%	14.46%	-0.18%	-17.45%	32.92%	22.59%	-20.35%	18.69%	16.71%	7.31%	**100%**
30 June 17	33.87%	15.28%	-3.03%	-16.96%	27.57%	16.17%	-16.40%	14.09%	16.47%	12.93%	**100%**
29 Dec. 17	36.21%	5.61%	2.51%	-22.38%	44.11%	22.20%	-13.98%	8.38%	13.65%	3.68%	**100%**
29 June 18	45.76%	1.35%	-4.29%	-36.35%	22.10%	22.83%	-11.94%	10.68%	30.04%	19.83%	**100%**
28 Dec. 18	106.10%	-37.57%	-16.26%	-62.94%	5.11%	52.96%	-15.48%	4.61%	45.44%	18.04%	**100%**
28 June 19	96.58%	-17.66%	-19.46%	-41.26%	11.52%	28.07%	-15.00%	-15.91%	64.94%	8.18%	**100%**

Table 3: Stock weights in the reference portfolio

	Allianz	Bayer	Daimler	Dt. Bank	Dt. Post	Dt.Tele k.	E.ON	Henkel	SAP	Siemens	Total
30 Dec. 10	11.58%	13.33%	13.67%	7.71%	3.02%	8.23%	12.19%	2.28%	9.58%	18.39%	**100%**
30 June 11	12.14%	13.40%	12.08%	10.36%	3.03%	9.05%	10.78%	2.29%	10.58%	16.29%	**100%**
30 Dec. 11	11.54%	13.07%	10.00%	8.74%	3.28%	9.32%	10.81%	2.65%	13.33%	17.25%	**100%**
29 June 12	11.42%	14.58%	10.26%	8.46%	3.75%	8.29%	9.85%	2.91%	13.70%	16.77%	**100%**
28 Dec. 12	12.59%	15.93%	10.82%	8.00%	3.95%	6.71%	7.05%	2.86%	14.39%	17.69%	**100%**
28 June 13	12.76%	17.33%	11.54%	8.10%	4.52%	6.98%	6.59%	3.28%	13.13%	15.77%	**100%**
30 Dec. 13	12.34%	17.39%	12.99%	7.33%	5.32%	7.92%	5.33%	3.06%	11.88%	16.44%	**100%**
30 June 14	11.92%	17.45%	14.76%	6.05%	5.29%	8.20%	6.09%	3.09%	10.90%	16.26%	**100%**
30 Dec. 14	13.02%	17.02%	14.27%	6.73%	5.31%	8.55%	5.75%	3.23%	10.99%	15.14%	**100%**
30 June 15	12.69%	16.92%	15.93%	6.79%	4.92%	9.33%	4.73%	3.47%	11.39%	13.83%	**100%**
30 Dec. 15	14.44%	16.03%	15.02%	5.89%	4.84%	10.25%	3.22%	3.49%	13.27%	13.56%	**100%**
30 June 16	13.62%	16.33%	12.84%	4.12%	5.46%	10.19%	3.83%	3.86%	14.38%	15.37%	**100%**
30 Dec. 16	13.42%	15.20%	13.26%	4.72%	5.54%	9.53%	2.40%	3.67%	14.70%	17.55%	**100%**
30 June 17	13.90%	17.44%	11.41%	3.93%	5.35%	9.16%	2.98%	3.80%	15.53%	16.49%	**100%**
29 Dec. 17	14.83%	14.88%	11.96%	5.20%	6.30%	8.26%	3.44%	3.30%	15.91%	15.92%	**100%**
29 June 18	14.32%	15.17%	10.31%	3.17%	5.17%	8.19%	3.73%	3.44%	19.37%	17.13%	**100%**
28 Dec. 18	16.32%	12.58%	9.19%	2.95%	5.00%	10.63%	4.22%	3.73%	17.87%	17.51%	**100%**
28 June 19	17.85%	10.16%	8.98%	2.34%	5.49%	9.87%	4.27%	2.87%	22.04%	16.15%	**100%**

The values in tables 1-2 show very high fluctuations in the respective stock weights. This becomes clear when looking at the mean values (μ_{weight}) and standard deviations (σ_{weight}) of the periodic weights (table 4).

Table 4: Mean and standard deviations of (normalized) stock weights

MV portfolio			BL portfolio		
	$\mu_{weight.}$	$\sigma_{weight.}$		$\mu_{weight.}$	$\sigma_{weight.}$
Allianz	78.89%	62.09%	Allianz	22.40%	34.09%
Bayer	39.53%	62.58%	Bayer	30.39%	37.71%
Daimler	-32.63%	27.04%	Daimler	-2.38%	11.47%
Deutsche Bank	-71.19%	40.64%	Deutsche Bank	-29.62%	18.37%
Deutsche Post	39.14%	68.88%	Deutsche Post	11.82%	26.24%
Deutsche Telekom	17.42%	39.26%	Deutsche Telekom	15.77%	18.87%
E.ON	-68.30%	55.36%	E.ON	-36.97%	32.10%
Henkel	48.30%	67.26%	Henkel	39.45%	32.43%
SAP	35.53%	43.77%	SAP	36.27%	34.76%
Siemens	13.30%	83.97%	Siemens	12.86%	23.43%
Mean	**10.00%**	**55.08%**	**Mean**	**10.00%**	**26.95%**

Table 5: Mean and standard deviations of stock weights (reference portfolio)

Reference portfolio		
	$\mu_{weight.}$	$\sigma_{weight.}$
Allianz	13.37%	1.71%
Bayer	15.23%	2.06%
Daimler	12.18%	2.05%
Deutsche Bank	6.14%	2.27%
Deutsche Post	4.75%	0.95%
Deutsche Telekom	8.82%	1.08%
E.ON	5.96%	3.02%
Henkel	3.18%	0.48%
SAP	14.05%	3.24%
Siemens	16.31%	1.27%
Mean	**10.00%**	**1.81%**

The stock weights for the BL portfolio shown above are based on subjective return forecasts that are derived from the respective (annualized) mean of the monthly logarithmic returns from the 5 years prior to the respective rebalancing of the portfolio. The analysis was also performed for subjective return forecasts based on the monthly logarithmic returns of 1 year and 0.5 years before the portfolio rebalancing. However, in these two cases, the sum of all stock weights in the BL portfolio was negative in some periods, which is equivalent to a short position in this portfolio. For this reason, no further analysis of these two cases was performed in this paper.

Using the stock weights shown above at each time of rebalancing, and the prices of all stocks at the end of the month, the monthly values and the respective monthly logarithmic returns for each portfolio can be calculated. This is done for all periods mentioned above. It should again be pointed out that no actual stock prices are used in the analysis, but stock prices that are adjusted for dividend payments, payments from the proceeds of subscription rights, stock splits, etc. The monthly logarithmic portfolio returns determined from the portfolio values form the basis of the performance analysis. In order to calculate beta, Treynor ratio, and Jensen alpha, the DAX serves as the benchmark. The performance values are shown in the following tables, with each period considered separately.

Table 6: Performance results for the period 30 Dec. 2010 – 30 Dec. 2019

	DAX	MV	BL	Reference	EW
Mean return	0.6022%	2.2254%	1.6562%	0.6037%	0.5979%
Standard deviation	4.7508%	10.1097%	7.1462%	4.9292%	4.7622%
Sharpe ratio	12.6752%	22.0125%	23.1753%	12.2482%	12.5555%
Correl. with DAX	1.0000	0.2186	0.4896	0.9835	0.9744
Beta	1.0000	0.4651	0.7365	1.0204	0.9767
Treynor ratio	0.6022%	4.7847%	2.2488%	0.5917%	0.6122%
Jensen alpha	0.0000%	1.9453%	1.2127%	-0.0107%	0.0098%
RAP	0.6022%	1.0458%	1.1010%	0.5819%	0.5965%

Table 7: Performance results for the period 30 Dec. 2010 – 30 Dec. 2013

	DAX	MV	BL	Reference	EW
Mean return	0.8978%	2.1284%	2.6137%	1.0366%	1.1691%
Standard deviation	5.5440%	13.9671%	9.2517%	5.6190%	5.3516%
Sharpe ratio	16.1932%	15.2383%	28.2516%	18.4473%	21.8457%
Correl. with DAX	1.0000	0.2070	0.4378	0.9840	0.9834
Beta	1.0000	0.5215	0.7306	0.9973	0.9493
Treynor ratio	0.8978%	4.0815%	3.5774%	1.0393%	1.2315%
Jensen alpha	0.0000%	1.6602%	1.9578%	0.1412%	0.3169%
RAP	0.8978%	0.8448%	1.5663%	1.0227%	1.2111%

Table 8: Performance results for the period 30 Dec. 2013 – 30 Dec. 2016

	DAX	MV	BL	Reference	EW
Mean return	0.5109%	1.7792%	1.1434%	0.4909%	0.4234%
Standard deviation	5.0077%	7.6790%	5.5608%	5.1701%	5.1034%
Sharpe ratio	10.2026%	23.1704%	20.5613%	9.4944%	8.2955%
Correl. with DAX	1.0000	0.2720	0.7331	0.9860	0.9736
Beta	1.0000	0.4171	0.8141	1.0180	0.9922
Treynor ratio	0.5109%	4.2662%	1.4045%	0.4822%	0.4267%
Jensen alpha	0.0000%	1.5662%	0.7274%	-0.0292%	-0.0836%
RAP	0.5109%	1.1603%	1.0297%	0.4755%	0.4154%

Table 9: Performance results for the period 30 Dec. 2016 – 30 Dec. 2019

	DAX	MV	BL	Reference	EW
Mean return	0.3978%	2.7686%	1.2114%	0.2838%	0.2013%
Standard deviation	3.6117%	7.6089%	6.1799%	3.9578%	3.7576%
Sharpe ratio	11.0154%	36.3866%	19.6015%	7.1702%	5.3576%
Correl. with DAX	1.0000	0.2077	0.3365	0.9805	0.9623
Beta	1.0000	0.4375	0.5758	1.0745	1.0012
Treynor ratio	0.3978%	6.3278%	2.1038%	0.2641%	0.2011%
Jensen alpha	0.0000%	2.5945%	0.9823%	-0.1437%	-0.1970%
RAP	0.3978%	1.3142%	0.7079%	0.2590%	0.1935%

The tables show that the mean returns and standard deviations vary, in part significantly, among the respective portfolios. For the overall period, the BL portfolio achieves the highest Sharpe ratio but the MV portfolio is only just behind. The DAX, the equally weighted portfolio and the reference portfolio follow at a greater distance, reaching quite similar values. These portfolios are very similar in terms of composition, so that the values for beta, correlation and Treynor ratio are very close together. For the MV portfolio, the Treynor ratio is higher compared to the BL portfolio, whereby all Treynor Ratios of these portfolios are significantly higher than for the DAX, reference portfolio and equally weighted portfolio. These results are also reflected in the Jensen alpha in a similar way. For the RAP measure -as theoretically shown above- the same order is obtained as for the Sharpe ratio.
The following tables show a ranking of the individual portfolios in terms of performance measures for each period. The performance measure RAP is not included in the ranking because it leads to the same order as the Sharpe ratio.

Table 10: Ranking of the portfolios for the period 30 Dec. 2010 – 30 Dec. 2019

	DAX	MV	BL	Reference	EW
Sharpe ratio	3	2	1	5	4
Treynor ratio	4	1	2	5	3
Jensen alpha	4	1	2	5	3

Table 11: Ranking of the portfolios for the period 30 Dec. 2010 – 30 Dec. 2013

	DAX	MV	BL	Reference	EW
Sharpe ratio	4	5	1	3	2
Treynor ratio	5	1	2	4	3
Jensen alpha	5	2	1	4	3

Table 12: Ranking of the portfolios for the period 30 Dec. 2013 – 30 Dec. 2016

	DAX	MV	BL	Reference	EW
Sharpe ratio	3	1	2	4	5
Treynor ratio	3	1	2	4	5
Jensen alpha	3	1	2	4	5

Table 13: Ranking of the portfolios for the period 30 Dec. 2016 – 30 Dec. 2019

	DAX	MV	BL	Reference	EW
Sharpe ratio	3	1	2	4	5
Treynor ratio	3	1	2	4	5
Jensen alpha	3	1	2	4	5

As these tables show, overall, the best performing portfolios are the MV portfolio and the BL portfolio, with the former mostly coming first. One exception is the period 30 Dec. 2010 – 30 Dec. 2013, in which the BL portfolio is superior. During this period, the large ranking difference of the MV portfolio is noticeable. While this portfolio has the lowest Sharpe ratio, its Treynor ratio is the highest. This is due to the relatively high standard deviation and the relatively low beta. The latter in turn is related to the low correlation with the DAX.

The comparative portfolios (DAX, reference portfolio, equally weighted portfolio) share the last three places in most cases. The reference portfolio leads to the lowest performance values over the entire period, although it is usually in 4th place in the respective single periods. In the last two subperiods, the DAX achieves better results than the reference portfolio and the equally weighted portfolio, although these differences are only very small overall, which is particularly true for the overall period.

4. Conclusion

The BL model uses historical data, equilibrium considerations and individual assessments of the portfolio managers for the near future. It can therefore be assumed that a combination of the ("neutral") reference excess returns with individual return forecasts leads to the derivation of economically better and more stable stock weights in the portfolio. In this study, the success of the BL portfolio is examined by means of an empirical analysis of a German stock portfolio.

In conclusion, the BL portfolio performs significantly better than the DAX, the reference portfolio and the equally weighted portfolio. However, overall, it is slightly outperformed by the traditional mean-variance optimized (MV) portfolio. Nevertheless, it may be of greater interest to investors because – as shown above – the BL portfolio could lead in most cases to lower absolute (normalized) values for the stock weights and for all stocks to smaller fluctuations in the (normalized) weights (measured as standard deviations of the weights) compared to the MV portfolio. Although high short positions also occur, the absolute (normalized) values for the weights, on average, are lower overall than in the MV portfolio. However, in this study, this is true if 5-year periods of historical monthly returns are used for the subjective return forecast determination. In the case of 1-year or 0.5-year periods as the basis for determining these forecasts, the sum of all stock weights in the BL portfolio was negative in some periods, which is equivalent to a short position in this portfolio. Therefore, no further analysis of these two cases was performed in this paper.

According to this paper, the BL portfolio leads to better results than the benchmark index (DAX), the reference portfolio, or the equally weighted portfolio. The complex approach to the asset allocation should not be a problem in practice.

However, it should be noted that the results of this study cannot be generalized. Further investigations with alternative forecasting methods have to be conducted. Furthermore, this study assumes a risk aversion parameter of 3, a factor τ of 0.2, and a risk-free rate of 0%. To determine the matrix Ω as a diagonal matrix, it is assumed in this study that the variance of the subjective forecasts is proportional to the variance of the asset returns. Furthermore, this study refers only to the selected stock portfolios and only to certain selected periods. Further investigations are therefore necessary for alternative input parameters, stock markets and periods, and should include an exclusion of short sales as many investors are restricted to long only positions.

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