Journal of Computations & Modelling, vol.7, no.3, 2017, 1-19
ISSN: 1792-7625 (print), 1792-8850 (online)
Scienpress Ltd, 2017

A quantitative model for the distribution operations in defense logistics

Vasileios Vrysagotis[1], Michail Vidalis[2] and Anastasia Paris[3]

Abstract

The paper analyzes the operation of a defense logistics system. In more detail, the examined system consists of a one warehouse, the first supply echelon and four battlefields, the second supply echelon, which face four non-identical demand rates. Characterized as a divergent dynamic supply network, the system is analyzed by modeling it as markovian process with discrete space. Major outputs of the modeling process are the evaluation of the performance measures of Work In Process (WIP) , Fill Rate (FR) and the presentation of the behavior of the performance measures in relation with a number of variables such as safety stock (s) , replenishment rates (μ) and demand rates (λ).

Keywords: Two-echelon defence supply network; divergent system; Markov analysis; performance measures

---

[1] Scientific Associate Technologícal Education Instífute of Sterea Ellada, Logistics Department, Thiva, Greece.
E-mails: vvrisagotis@mail.teiste.gr , v.vrysagotis@gmail.com
[2] Business Administration Department, University of Aegean, Chios, Greece.
E-mail: michael.vidalis@gmail.com
[3] Hellenic Army Academy,Vari, Attica, Greece. E-mail: kinshasa@otenet.gr

Article Info: *Received:* March 12, 2017. *Revised:* April 19, 2017.
Published online : July 10, 2017.

1 Introduction

This paper deals with the analytical modelling of a dynamic supply system with two stages (warehouse, customers). The operation of every system is depended by the system's strucure, system's behavior and system's interconnectivity. The system's strucure is a divergent one (the final stages are connected with a warehouse). System's behavior involves inputs, processing and outputs of military material, information or data. The replenishment times between the members are random and follow a Coxian-2 phase type distribution. Further, the customers face a demand distributed according to Poisson distribution and the warehouse is never starved.

Based on the above assumptions, the performance of this supply chain system is explored. The system is modelled as continuous time Markov process with discrete space. The structure of the transition matrices of these specific systems is examined and a computational algorithm is developed to generate it for different values of system characteristics.The proposed algorithm allows the calculation of performance measures from the derivation of the steady state probabilities.

The system performance is evaluated by the metrics of special interest from the view point of achieving customer service targets, viz., fill rates, cycle times. Performance measures such as the average inventory (WIP),fill rate (FR) of the military supply chain are examined as a function of system characteristics i.e. number of battlefields (R) and replenishment time characteristics μ_{i1}, μ_{i2}, d_{i1},d_{i2},d_1,d_2 i=1,2 and demand characteristics λ_i.

The authors aim to provide a modelling process for the analysis of defence supply systems which are characterized of high variability. Further, a computational process for stochastic models is presented. Last but not least, a number of managerial insights concerning defence supply operation are stressed.

The outline of our paper is as follows: in the second part the relating literature is presented. In the third part and the fourth part, the system and the model are presented correspondingly whereas in the fifth part the behaviour of the performance measures is offered.

2 Literature Review

The literature on the split out inventory models is categorized in two categories a) Pure split out inventory models and b) Applied in defense operations split out inventory models.

2.1. Pure split out inventory models

Anderson and Melchiors (2001) present an inventory system with one distribution center and multiple retailers with independent Poisson demands, (S, S-1) base stock policy, lost sales and deterministic lead times. Moutqatipkul and Yerandee (2008) deal with a supply network with one distribution center multiple retailers and (S,s) periodic review inventory policies. Axsater (2003) deals with a two echelon inventory system with a central warehouse and a number of retailers following a continuous review installation stock (R,Q) policies. Axsater et al (2004) consider a one-warehouse and N-retailers inventory system. A heuristic method to optimize policy parameters is developed. Ahire and Schimdt (1996) analyze a Mixed Continuous Periodic One –Warehouse N- retailers inventory system. An analytical approximate model is processed to predict system performance under different operating conditions. Marklund (2002) introduces a new replenishment policy of a inventory system consisting of one warehouse and an arbitrary number of non-identical retailers. Further, a technique for the exact evaluation of the expected inventory holding and backorder costs of the system is presented. Rifai and Rossetti (2007) analyze an inventory system of one warehouse and N identical retailers and implement the reorder point, the order quantity (R.Q) inventory policy. They develop an iterative heuristic optimization algorithm in order to minimize the total annual inventory investment subjected to average annual ordering frequency and expected number of backorder constraints. Last but not least, Thangam and Uthayakumar (2008) propose a split-out supply network with independent retailers and a single supplier following a continuous review policy (R.Q). Based on the assumptions of 1) Poisson demand at retailers 2) constant transportation times 3) partial backlogging an approximate cost function to find potential reorder points is presented.

2.2. Applied in defense operations split out inventory models

A system of a split out supply network of repairable spare parts for fighting airplanes is popular concerning the applied models in defense logistics. Besaler and Veinott (1996) analyze a network of a central warehouse and a number of bases facing random demand. An ordering policy which minimizes expected costs is sought. Lau et al (2006) propose a Monte Carlo simulation model to study a multi –echelon repairable item inventory system. Muckstadt (1979) develops a mathematical model for a system consisting of a group Air Forces bases and a central depot. The model determines the stock levels at each air base. Rappold and Roo (2009) deal with a model corresponding to joint problem of facility location, inventory allocation and capacity investment in a two echelon single item service parts supply chain with stochastic demand. Gupta and Albright (1992) model a split out inventory system for the repairs of military spare parts. Owning to Markovian approach, they evaluate steady state operating characteristics of the inventory system.

3 The System

Supplying military units is a paramount task for any military administration: the quantity of supplies and their replenishment rate directly affects how effective a team is operating during a military operation. Replenishing the supplies efficiently is thus a very important factor of success in the field

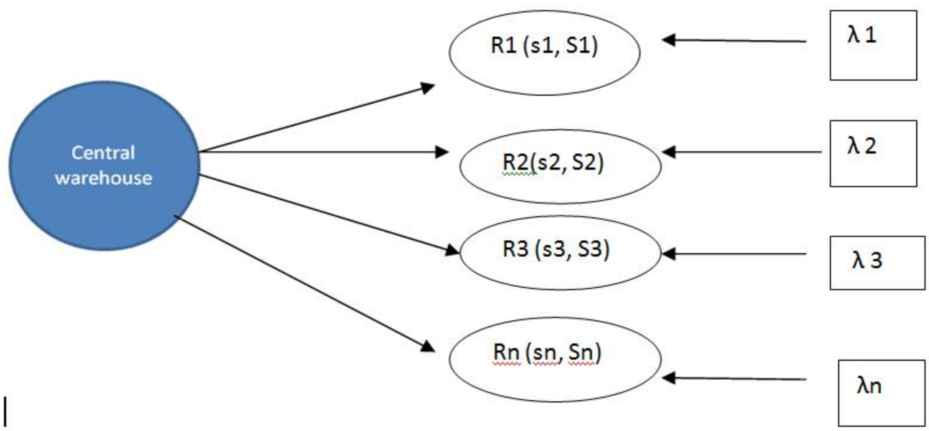

Figure 1: the distribution system

.

In this study, major aim is to evaluate how time variability in replenishing the supplies can affect military units during an operation problem under consideration. A central replenishment source and K scattered military units -each facing different pressures is assumed. The central source has sufficient supplies to cover any demand of the supported units, which communicate with it when the fire power is critically limited.

The normal time of replenishment varies, following an exponential distribution with a mean value μ_1. Unpredictable events result in a delay of replenishment, which then follows an exponential distribution with a mean value $\mu_{2.}$ Thus, the variability of replenishment time can be expressed as a Coxian distribution with two phases (1^{st} phase: normal replenishment, 2^{st} phase: replenishment facing difficulties).

4 The Model

The two-echelon supply network consists of R+1 members: R military units who receive orders from the external enviroment and one manufacturer that serves the military units' orders. Independent Poisson demands with mean rates $\lambda 1$, $\lambda 2$, $\lambda 3,\ldots,\lambda n$ .The military unit's orders are replenished by the manufacturer, who is never starved. The replenishment time intervals are random variables with high variability which is enforced by unexpected events (such as enemy attacks, transportation breakdowns etc.). A fraction of the military units' orders dR_1, $(0\leq d_1\leq 1)$ needs a random time to be served, exponentially distributed with rate μR_1. The rest of the fraction, dR_2, $(0 \leq d_2=1-d_1\leq 1)$ faces an additional time of delay, also exponentially distributed with rate μR_2. Thus, the total replenishment time follows the Coxian distribution with two phases (Coxian-2). Due to the fact that the military unit's ordering is triggered by the reorder point s, it is assumed that *there is never more than one order outstanding*. After an order is placed, the military unit serves the external demand until its remaining inventory reaches zero or the outstanding order is arrived. When an order by manufacturer arrives, the military unit re-examines the inventory at hand to decide whether a new order needs to be triggered The military units follow (s, S) inventory policy meaning that whenever the inventory drops below s the military units order S-s quantity.

4.1. The Modeling Process

It is important to describe the 'physics' of the problem before we attempt to solve it. The modeler's art is to incorporate in the solution as much of the 'physics' of the problem as possible. After the problem definition, the next step is to formulate a model that is an accurate representation of reality. Then we can use the model to see which decisions produce the best outputs. The last step is to implement the model and update it over time.

Modeling involves

- abstraction,
- simplification, and
- formalization, in light of particular methods and assumptions, in order to better understand a particular part or feature of the world, and to potentially intervene.

We based our modeling on Markov stochastic processes, which allowed us to evaluate different scenarios, each with certain parameters in place, such as:

1. spot of replenishment of each team si, where i = 1, 2, 3,…, K
2. quantity of supplies qi each unit demands
3. mean time value of normal replenishment μi1
4. mean time value of delayed replenishment μi2
5. probability of unpredictable events occurring di
6. rate of consumption λi for the team

4.2. Assumptions of the model

The two-echelon supply chain consists of i+1 members, i nodes who react with the external enviroment and one Distribution center (DC) that serves the military units' orders

Independent Poisson demands with mean rates λ_1, λ_2, λ_3,…, λ_n at 1st echelon The node's orders are replenished by the DC, who is never starved. The replenishment time intervals are random variables with high variability which is enforced by unexpected events (such as enemy attacks, transportation breakdowns etc.). A fraction of the nodes' orders d_{i1}, ($0 \leq d_{i1} \leq 1$) needs a random time to be served, exponentially distributed with rate μ_{i1}. The rest of the fraction, d_{i2}, ($0 \leq d_{i2} = 1 - d_{i1} \leq 1$) faces an additional time of delay, also exponentially distributed with rate μ_{i2}. Thus, the total replenishment time follows the Coxian distribution with two phases (Coxian-2). Due to the fact that the node's ordering is triggered by the reorder point s, it is assumed that *there is never more than one order outstanding*. After an order is placed, the military unit serves the external demand until its remaining inventory reaches zero or the outstanding order is arrived.

When an order by DC arrives, the node re-examines the inventory at hand to decide whether a new order needs to be triggered. The nodes follow (s, S) inventory policy meaning that whenever the inventory drops below s the node order a S-s quantity.

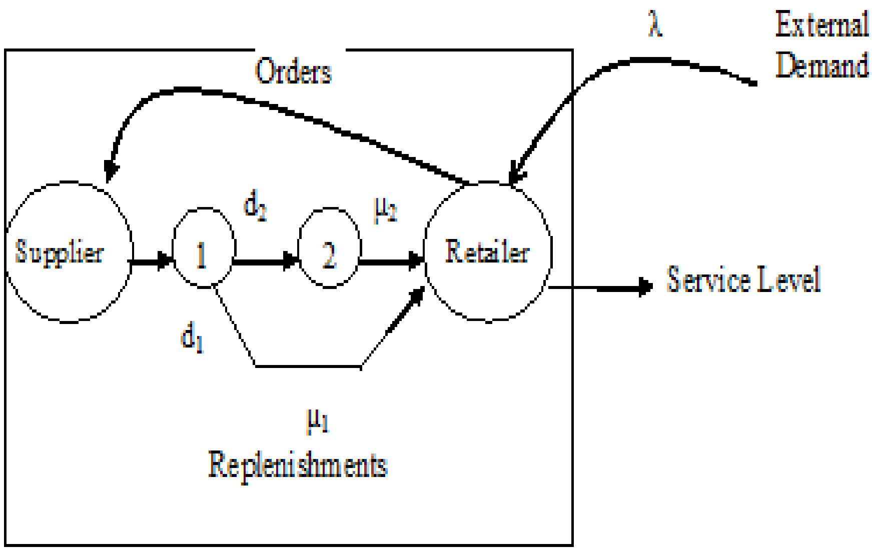

Figure 2 : A graphical representation of Coxian distribution

4.3. Notation

λ_R : external demand rate at the military unit R

μ_{R1} : mean replenishment rate from the manufacturer to the military unit during the first phase

μ_{R2} : mean replenishment rate from the manufacturer to the military unit during the second phase

d_{R1}: probability that the order will arrive having only one phase of delay, which is the usual case

d_{R2}: probability that the order will arrive having two phases of delay, which happens more rarely

S_R*:* order-up-to level at the military unit R

s_R*:* re-order point at the military unit R

q_r=S-s: size of the order placed by the military unit to the manufacturer.

I_{Rt}: inventory at hand at military unit at time t.

C_p: product price per unit

: holding cost at the military unit per unit

: shortage cost at the military unit per unit

FRR: fill rate at the military unit R

WIPR: average inventory at the military unit R, on transport or in the system

THRR: average output rate of the military unit R

4.4. Markovian model

We model the inventory system as a continuous time Markov process with finite number of states. The state of the system is defined by the R vector (pR_t, $\mathbf{IR_t}$) with $\boldsymbol{0 \leq I_t \leq s}$ or by the number $\boldsymbol{I_t}$ for $\boldsymbol{s} < I_t \leq \boldsymbol{S}$ where:

$I R_t$: military unit's inventory level (on hand) at time t, $0 \leq I_t \leq S$ and

$p R_t$: the number of phase of replenishment process, at time t, p=1,2 for $0 \leq I_t \leq s$

p=1,2,…,k if Coxian distribution with k phases is adopted.

4.4.1. State Transitions

Occurrence of an external demand at time *t*: the state of the military unit R I_t jumps from *n* to *n*-1. The probability of exactly one customer arriving in a small interval Δt is $\lambda \cdot \Delta t$, while the probability of more than one customers arriving is considered as $0(\Delta t)$ and hence it is disregarded.

Shipment arrival at the military unit Rat time t that has faced only one phase of delay: I_t increases by an amount q. The probability of a shipment arriving at the military unit with one phase of delay in the interval Δt is $d_1 \cdot \mu_1 \cdot \Delta t$.

Shipment arrival at the military unit Rat time t faces an additional (2^{nd}) phase of delay: I_t remains stable. The probability that a shipment will pass to the 2^{nd} phase in the interval Δt is $d_2 \cdot \mu_1 \cdot \Delta t$.

Shipment arrival at the military unit Rat time t that has faced two phases of delay: I_t increases by an amount q. The probability of a shipment arriving at the military unit in the interval Δt is $\mu_2 \cdot \Delta t$.

The total number of states, i.e. the dimension of the transition matrix, is given

$$2(s_R+1)a_{R-1} \mp (S_R - s_R)a_{R-1} \quad (1)$$

where a_{R-1} the states for R-1 military units

4.4.2. Structure of Transition Matrix

The structure of the transition matrix is affected by the military units replenishment policy: up-to-order level SR and reorder point sR. This matrix is a tri-diagonal matrix and consists of three sets of sub-matrices:

1. the set of sub-matrices in the main diagonal, denoted by D_κ,
2. the set of sub-matrices under the main diagonal, denoted by L_κ and
3. the set of sub-matrices above the main diagonal, denoted by U_κ

4.4.3. Dimensions and number of submatrices

Submatrices D

s_{R+1}submatrices with dimensions: $(k_R a_{R-1}) \times (k_R a_{R-1})$ (2)

$S_R - s_R$ submatrices with dimensions:$a_{R-1} \times a_{R-1}$ (3)

Submatrices A

s_R submatrices with dimensions: $k_R \times a_{R-1}$ (4)

one (1) submatrix with dimensions: $(k_R a_{R-1}) \times a_{R-1}$ (5)

Submatrices K

s_R submatrices with dimensions:$(k_R a_{R-1}) \times (k_R a_{R-1})$ (6)

one (1) submatrix with dimensions $(k_R a_{R-1}) \times a_{R-1}$ (7)

$S_R - s_R$ with dimensions $a_{R-1} \times a_{R-1}$ (8)

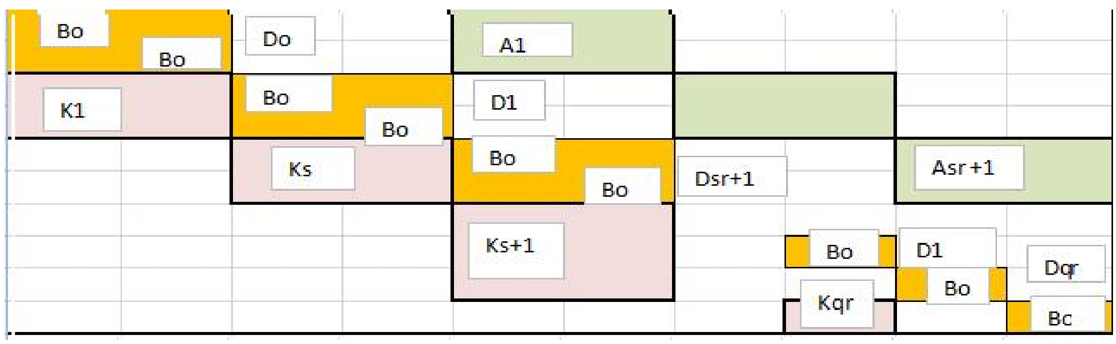

Figure 3: Structure of Transition Matrix

5 Numerical Results

5.1. Fill rate of each military unit and time rates (military unit 1)

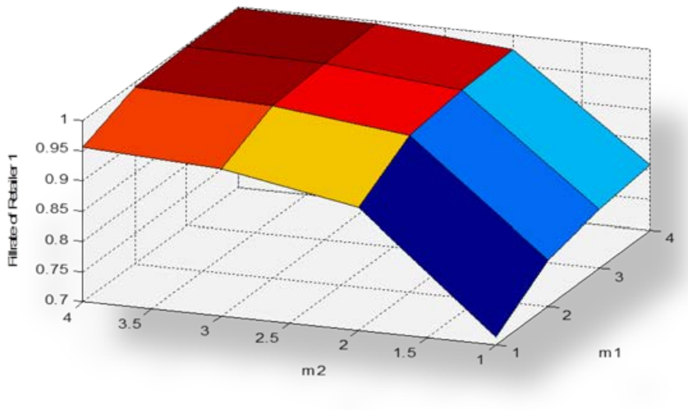

Figure 4 : Fill rate of each military unit and time rates (military unit 1)

For each battle point, we assume R=4 and we point out the impact on fillrate of each military unit (fll rate local) replenishment time rates ($\mu 1$,$\mu 2$) have. It is profound that as time rates increase fill rate also increase. Fill Rate reaches the maximum of about 1 (less than 100).

5.2. Fill rate of each military unit and time rates (military unit 2)

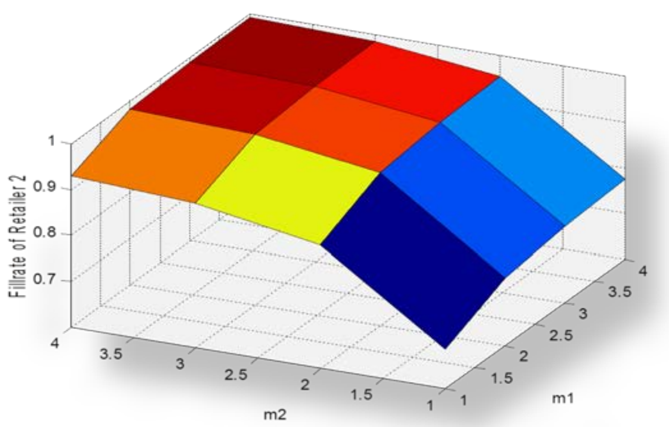

Figure 5 : Fill rate of each military unit and time rates (military unit 2)

5.3. Fill rate of each military unit and time rates (military unit 3)

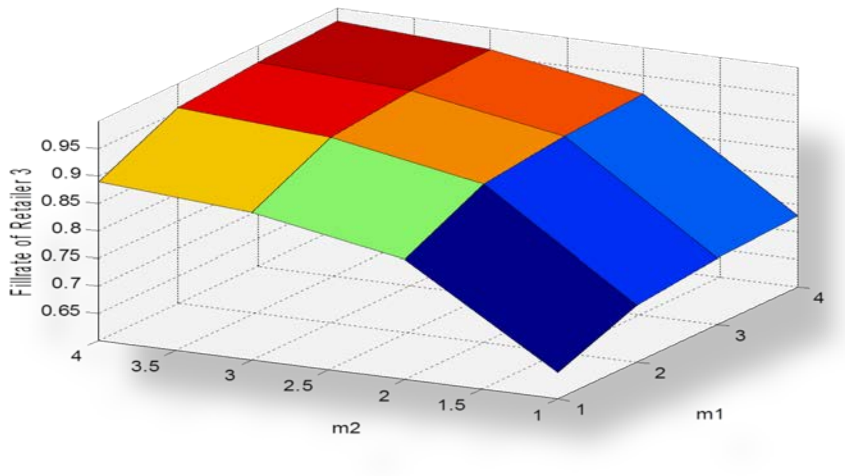

Figure 6 : Fill rate of each military unit and time rates (military unit 3)

5.4. Fill rate of each military unit and time rates (military unit 4)

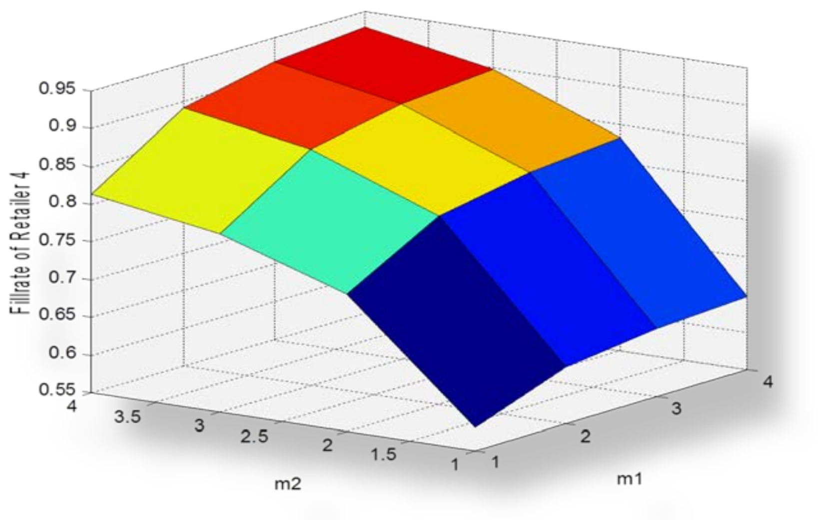

Figure 7: Fill rate of each military unit and time rates (military unit 4)

5.5. Fill rate of each military unit and external demand rate(λ) military unit 1

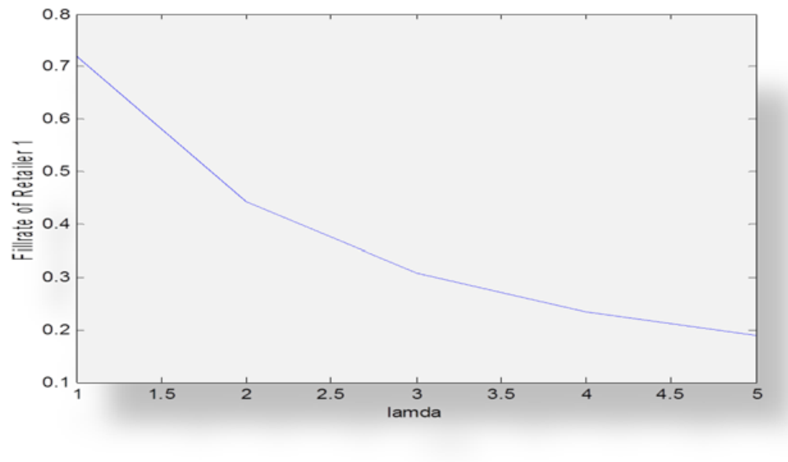

Figure 8: Fill rate of each military unit and external demand rate (λ) military unit 1

5.6. Fill rate of each military unit and external demand rate(λ) (military unit 2)

We have same behavior as in the case of military unit 1 with the minimum value of about 20%.

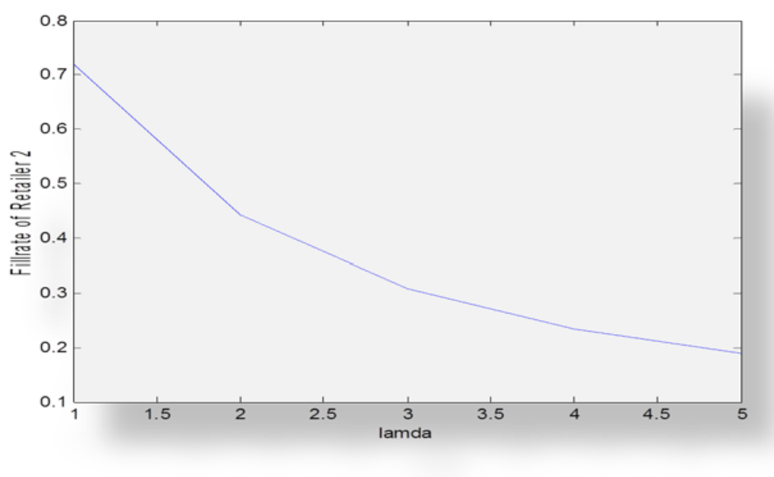

Figure 9 : Fill rate of each military unit and external demand rate(λ) (military unit 2)

5.7. Fill rate of each military unit and external demand rate(λ) (military unit 3)

We have same behavior as in the case of military unit 1 with the minimum value of about 20 %.

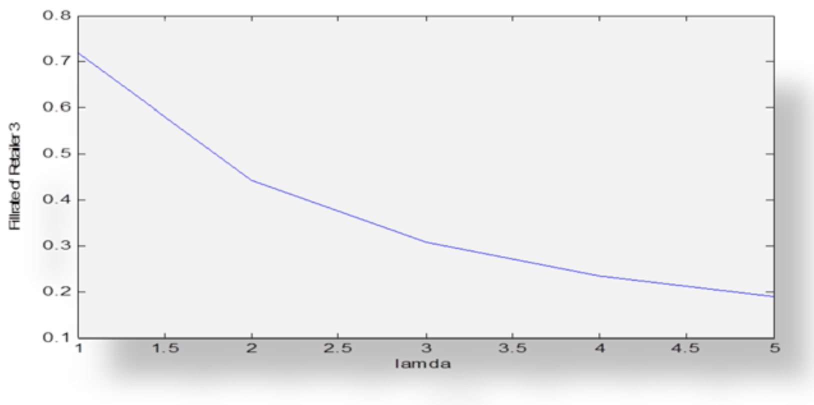

Figure 10: Fill rate of each military unit and external demand rate(λ) (military unit 3)

5.8. Fill rate of each military unit and external demand rate(λ) (military unit 4)

We have same behavior as in the case of military unit 1 with the minimum value of about 20%.

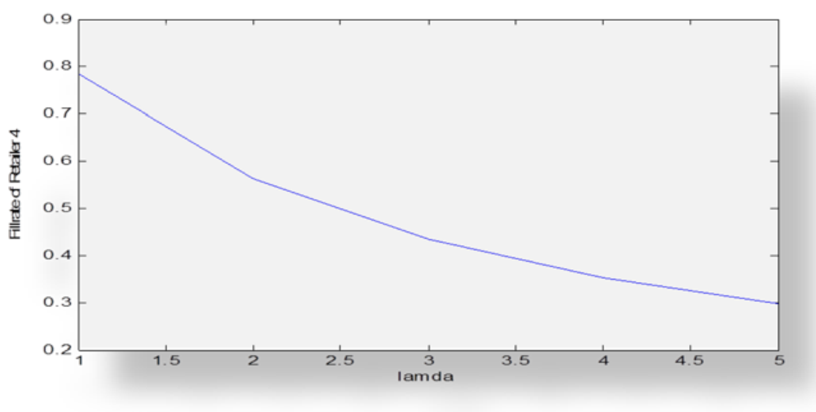

Figure 11: Fill rate of each military unit and external demand rate(λ)(military unit 4)

5.9. Fill rate of each military unit and safety stock (s) (military unit 1)

We assume R=4. As safety stock increases from 1 to 5 fill rate increases from more than 35% to about 65%.

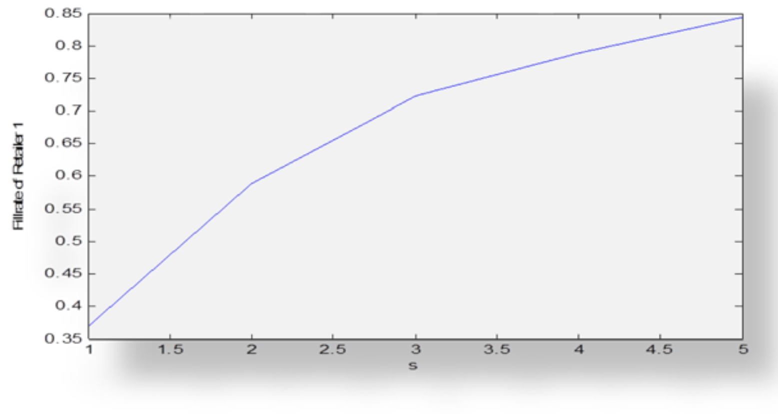

Figure 12: Fill rate of each military unit and safety stock (s) (military unit 1)

5.10. Fill rate of each military unit and safety stock (s) (military unit 2)

We have same behavior as in the case of military unit 1. Fill rate increases from 37% to 41% as safety stock increases from 1 to 5.

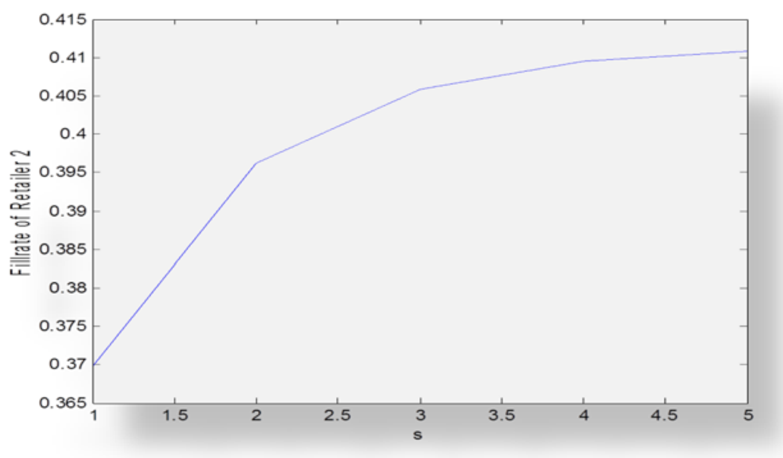

Figure 13: Fill rate of each military unit and safety stock (s)- military unit 2

5.11. Fill rate of each military unit and safety stock (s) (military unit 3)

We have same behavior as in the case of military unit 1.Fill rate increases from 37% to 41% as safety stock increases from 1 to 5.

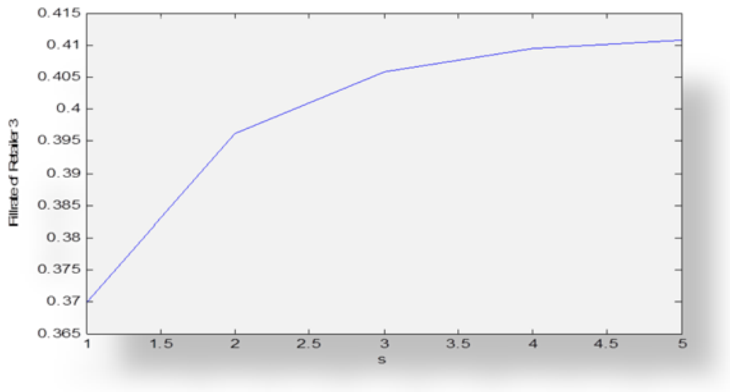

Figure 14: Fill rate of each military unit and safety stock (s) (military unit 3)

5.12. Fill rate of each military unit and safety stock (s) (military unit 4)

We have same behavior as in the case of military unit 1. Fill rate increases from 37% to 41% as safety stock increases from 1 to 5.

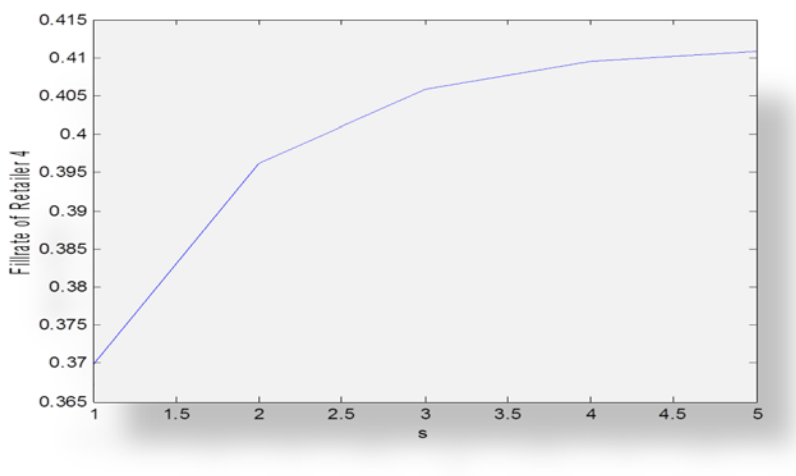

Figure 15 : Fill rate of each military unit and safety stock (s) (military unit 4)

6 Conclusions

Concluding, a number of relations between the operating variables is stressed. First, the fill rate of each military unit increases as mean replenishment time rates ($\mu 1$,$\mu 2$) increases. In contrast the fill rate of each military unit decreases as mean demand rate increase. Last but not least, the fill rate of each military unit increases as safety stock (s) increases. For further research, it is proposed the development of a model with more than one distribution centers and a number of suppliers procured the distribution centers.

References

[1] J. Andersson., P. Melchiors, A two-echelon inventory model with lost sales. *International Journal of Production Economics*, **69**, (2001), 307–315.

[2] C.Monthatipkul, P.Yenradee Inventory/distribution control system in a one-warehouse multi-military unit supply chain, *International Journal of Production Economics*, **114**, (2008), 119–133.

[3] S, Axsater, Approximate optimization of a two-level distribution inventory system, *International Journal of Production Economics*, **81-82**, (2003), 545–553.

[4] S.Axsater, M. Kleun T.G.De Kok. Stock Rationing in a Continuous Review Two-Echelon Inventory Model, *Annals of Operations Research*, **126**, (2004), 177–194.

[5] S.L.Ahire, C.P.Schmidt,.,A model for a Mixed Continuous-Periodic review One-Warehouse, N-Military unit inventory system.*European Journal of Operational Research*, **92**, (1996), 69-82.

[6] J. Marklund, Centralized Inventory Control in a Two-Level Distribution System with Poisson Demand. *Naval Research Logistics*, **49**, (2002), 798–822.

[7] M.H. Al-Rifai. M Rossetti., An efficient heuristic optimization algorithm for a two-echelon (R, Q) inventory system, *International Journal Production Economics*, **109**, (2007), 195–213.

[8] A. Thangam, R. Uthayakumar. A two-level supply chain with partial backordering and approximated Poisson demand, *European Journal of Operational Research*, **187**, (2008), 228–242.

[9] S.A. Bessler., A. Veinott., Optimal policy for a dynamic multi-echelon inventory model, *Naval Research Logistics*, **13**, (1966), 355 -387.

[10] H.C. Lau, H. Song, C.T. See, S.Y. Cheng, Evaluation of time-varying availability in multi-echelon spare parts systems with passivation, *European Journal of Operational Research,* **170**, (2006), 91–105.

[11] J. A. Muckstadt,,Three echelon multi-item model for recoverable items. *Naval Research Logistics,* **26**, (1979), 199-221.

[12] J.A. Rappold, B.D.Van Roo, Designing multi-echelon service parts networks with finite repair capacity, **199**, (2009), 781–792.

[13] A. Gupta. S.C. Albright,, Steady-state approximations for a multi echelon multi-indentured repairable-item inventory system, *European Journal of Operational Research*, **62**, (1992), 340-353.