

Simplified Machine Diagnosis Techniques by Impact Vibration using 4-th Moment of Absolute Deterioration Factor

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Abstract

Among many dimensional and dimensionless amplitude parameters, Kurtosis (4-th normalized moment of probability density function) is generally regarded as a sensitive good parameter for machine diagnosis. In this paper, a simplified calculation method is introduced when impact vibration arise in the observed data. The case in which the rolling elements number is nine is examined. Compared with the past papers' results, this new method shows a good result which fit well. This calculation method is simple enough to execute even on a pocketsize calculator and is very practical at the factory of maintenance field. This can be installed in microcomputer chips and utilized as a tool for early stage detection of the failure.

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1 Introduction

In mass production firms such as steel making that have big equipments, sudden stops of production processes by machine failure cause severe damages such as shortage of materials to the later processes, delays to the due date and the increasing idling time.

To prevent these troubles, machine diagnosis techniques play important roles. So far, Time Based Maintenance (TBM) technique has constituted the main stream of the machine maintenance, which makes checks for maintenance at previously fixed time. But it has a weak point that it makes checks at scheduled time without taking into account whether the parts are still keeping good conditions or not. On the other hand, Condition Based Maintenance (CBM) makes maintenance checks by watching the condition of machines. Therefore, if the parts are still keeping good condition beyond its expected life, the cost of maintenance may be saved because machines can be used longer than planned. Therefore the use of CBM has become dominant. The latter one needs less cost of parts, less cost of maintenance and leads to lower failure ratio.

However, it is mandatory to catch a symptom of the failure as soon as possible of a transition from TBM to CBM is to be made. Many methods are developed and examined focusing on this subject. In this paper, we propose a method for the early detection of the failure on rotating machines which is the most common theme in machine failure detection field.

So far, many signal processing methods for machine diagnosis have been proposed (Bolleter, 1998; Hoffner, 1991). As for sensitive parameters, Kurtosis,

Bicoherence, Impact Deterioration Factor (ID Factor) were examined (Yamazaki, 1977; Maekawa et al.1997; Shao et al.2001; Song et al.1998; Takeyasu, 1989). In this paper we focus our attention on the index parameters of vibration.

Kurtosis is one of the sophisticated inspection parameters which calculates normalized 4-th moment of Probability Density Function (PDF). In the industry, there are cases where quick reactions are required on watching the waveform at the machine site.

In this paper, we consider the case such that impact vibration occurs on the gear when the failure arises and introduce a simplified machine diagnosis technique to detect failure in early stages. The case in which the rolling elements number is nine is examined.

Kurtosis value is 3.0 under normal condition and when failure increases, the value grows big. Therefore, it is a relative index. On the other hand, Bicoherence is an absolute index which is close to 1.0 under normal condition and tends to be 0 when failure increases.

Here, we deal with the generalized 4-th moment. When theoretical value of 4-th moment is divided by calculated value of 4-th moment, it would behave as an absolute index. New index shows that it is 1.0 under normal condition and tends to be 0 when failure increases.

We introduce a simplified calculation method to this new index and name this as a simplified absolute index of 4-th moment.

This simplified method enables us to calculate the new index even on a pocketsize calculator and enable us to install it in microcomputer chips. We survey each index of deterioration in section 2. Simplified calculation method of Kurtosis is proposed and compared with the results of another papers in section 3. In section 4, we introduce a simplified absolute index of 4-th moment, which is followed by the numerical examples of section 5. Remarks is stated in section 6 and section 7 is a summary.

2 Factors for vibration calculation

In cyclic movements such as those of bearings and gears, the vibration grows larger whenever the deterioration becomes bigger. Also, it is well known that the vibration grows large when the setting equipment to the ground is unsuitable (Yamazaki, 1977). Let the vibration signal be presented by the function of time $x(t)$. And also assume that it is a stationary time series with mean \bar{x} . Denote the probability density function of these time series as $p(x)$. Indices for vibration amplitude are as follows. Here we especially suppose that $\bar{x} = 0$ for the simplicity of description.

$$X_{root} = \left[\int_{-\infty}^{+\infty} |x|^{1/2} p(x) dx \right]^2 \quad (1)$$

$$X_{rms} = \left[\int_{-\infty}^{+\infty} x^2 p(x) dx \right]^{1/2} \quad (2)$$

$$X_{abs} = \int_{-\infty}^{+\infty} |x| p(x) dx \quad (3)$$

$$X_{peak} = \lim_{n \rightarrow \infty} \left[\int_{-\infty}^{+\infty} x^n p(x) dx \right]^{1/n} \quad (4)$$

These are dimensional indices which are not normalized. They differ by machine sizes or rotation frequencies. Therefore, normalized dimensionless indices are required.

There are four main categories for this purpose.

- A. Normalized root mean square value
- B. Normalized peak value
- C. Normalized moment
- D. Normalized correlation among frequency domain

A. Normalized root mean square value

a. Shape Factor : SF

$$SF = \frac{X_{rms}}{\bar{X}_{abs}} \quad (5)$$

(\bar{X}_{abs} : mean of the absolute value of vibration)

B. Normalized peak value

b. Crest Factor : CrF

$$CrF = \frac{X_{peak}}{X_{rms}} \quad (6)$$

(X_{peak} : peak value of vibration)

c. Clearance Factor : CIF

$$CIF = \frac{X_{peak}}{X_{root}} \quad (7)$$

d. Impulse Factor : IF

$$IF = \frac{X_{peak}}{\bar{X}_{abs}} \quad (8)$$

e. Impact Deterioration Factor : ID Factor / ID

This is proposed in Maekawa *et al.* (1997).

$$ID = \frac{X_{peak}}{X_C} \quad (9)$$

(X_C : vibration amplitude where the curvature of PDF becomes maximum)

C. Normalized moment

f. Skewness : SK

$$SK = \frac{\int_{-\infty}^{+\infty} x^3 p(x) dx}{\left[\int_{-\infty}^{+\infty} x^2 p(x) dx \right]^{3/2}} \quad (10)$$

g. Kurtosis : KT

$$KT = \frac{\int_{-\infty}^{+\infty} x^4 p(x) dx}{\left[\int_{-\infty}^{+\infty} x^2 p(x) dx \right]^2} \quad (11)$$

D. Normalized correlation in the frequency domain

h. Bicoherence

Bicoherence shows the relationship between two frequencies and is expressed as

$$Bic_{,xxx}(f_1, f_2) = \frac{B_{xxx}(f_1, f_2)}{\sqrt{S_{xx}(f_1) \cdot S_{xx}(f_2) \cdot S_{xx}(f_1 + f_2)}} \quad (12)$$

Here

$$B_{xxx}(f_1, f_2) = \frac{X_T(f_1) \cdot X_T(f_2) \cdot X_T^*(f_1 + f_2)}{T^{3/2}} \quad (13)$$

means Bispectrum and

$$X_T(t) = \begin{cases} x(t), & 0 < t < T \\ 0, & else \end{cases}$$

T : Basic Frequency Interval

$$X_T(f) = \int_{-\infty}^{+\infty} X_T(t) e^{-i2\pi f t} dt \quad (14)$$

$$S_{xx}(f) = \frac{1}{T} X_T(f) X_T^*(f) \quad (15)$$

Range of Bicoherence satisfies

$$0 \leq Bic_{,xxx}(f_1, f_2) \leq 1 \quad (16)$$

when there exists a significant relationship between frequencies f_1 and f_2 , Bicoherence is near 1. Otherwise, the value of Bicoherence comes close to 0.

These indices are generally used in combination and machine condition is judged totally. Among them, Kurtosis is known to be one of the superior indices (Noda, 1987) and numerous researches have been conducted on Kurtosis (Maekawa *et al.*, 1997; Shao *et al.*, 2001; Song *et al.*, 1998).

Judging from the experiment we made in the past, we may conclude that Bicoherence is also a sensitive good index (Takeyasu, 1987, 1989).

In Maekawa *et al.* (1997), ID Factor is proposed as a good index. In this paper, focusing on the indices of vibration amplitude, we introduce a simplified calculation method for absolute index of 4-th moment and show that simplified absolute index of 4-th moment is a sensitive good index for machine diagnosis.

3 Simplified calculation method of kurtosis

Assume that we get N amount of data and then newly get l amount of data. Let mean, variance, $MT(3)$, $MT(4)$, SK , KT of $1 \sim N$ data state as \bar{x}_N , σ^2_N , $MT_N(3)$, $MT_N(4)$, SK_N , KT_N for each. And as for $N+1 \sim N+l$ data, we state above items as

$$\bar{x}_{N/l}, \sigma^2_{N/l}, MT_{N/l}(3), MT_{N/l}(4), SK_{N/l}, KT_{N/l}$$

For example, $MT_N(4)$ and $MT_{N/l}(4)$ are stated as follows.

$$MT_N(4) = \frac{1}{N} \sum_{i=1}^N (x_i - \bar{x})^4 \quad (25)$$

$$MT_{N/l}(4) = \frac{1}{l} \sum_{i=N+1}^{N+l} (x_i - \bar{x})^4 \quad (26)$$

When there arise failures on bearings or gears, peak values arise cyclically. In the early stage of the defect, this peak signal usually appears clearly. Generally, defects will injure another bearings or gears by contacting the covering surface as time passes. Assume that the peak signals of S times magnitude from normal signals arise during m times measurement of samplings. As for determining sampling interval, sampling theorem is well known (Tokumaru et al., 1982). But in this paper, we do not pay much attention on this point in order to focus on our proposal theme. Let $\sigma^2_{N/l}$ and $MT_{N/l}(4)$ of this case denote $\bar{\sigma}^2_{N/l}, \overline{MT}_{N/l}(4)$, then we get

$$\begin{aligned} \bar{\sigma}^2_{N/l} &= \frac{1}{l} \sum_{i=N+1}^{N+l} (x_i - \bar{x})^2 \\ &\cong \frac{l - \frac{l}{m}}{l} \sigma^2 + \frac{\frac{l}{m}}{l} S^2 \sigma^2_N \\ &= \sigma^2_N \left(1 + \frac{S^2 - 1}{m} \right) \end{aligned} \quad (27)$$

$$\begin{aligned} \overline{MT}_{N/l}(4) &= \frac{1}{l} \sum_{i=N+1}^{N+l} (x_i - \bar{x})^4 \\ &\cong \frac{l - \frac{l}{m}}{l} MT_{N/l}(4) + \frac{\frac{l}{m}}{l} S^4 MT_{N/l}(4) \\ &= \left(1 + \frac{S^4 - 1}{m} \right) MT_{N/l}(4) \end{aligned} \quad (28)$$

We assume that time series are stationary as is stated before. Therefore, even if sample pass may differ, mean and variance is naturally supposed to be same when the signal is obtained from the same data occurrence point of the same machine.

We consider such case that the impact vibration occurs. Except for the impact vibration, other signals are assumed to be stationary and have the same means and variances. Under this assumption, we can derive the simplified calculation method for machine diagnosis which is a very practical one.

For these equations, we obtain \overline{KT}_{N+l} as KT_{N+l} of the above case

$$\begin{aligned} \overline{KT}_{N+l} &\cong \frac{\frac{N}{N+l}MT_N(4) + \frac{l}{N+l}\left(1 + \frac{S^4 - 1}{m}\right)MT_N(4)}{\left\{\frac{N}{N+l}\sigma^2_N + \frac{l}{N+l}\sigma^2_N\left(1 + \frac{S^2 - 1}{m}\right)\right\}^2} \\ &= KT_N \frac{\left(1 + \frac{l}{N+l} \frac{S^4 - 1}{m}\right)}{\left(1 + \frac{l}{N+l} \frac{S^2 - 1}{m}\right)^2} \end{aligned} \quad (29)$$

While $KT_N = 3.0$ under normal condition.

n.b.

Under the following Normal distribution,

$$\phi(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\bar{x}}{\sigma}\right)^2} \quad (30)$$

Its moment is described as follows which is well known (Hino, 1977).

$$\begin{aligned} MT(2n-1) &= 0 \\ MT(2n) &= \prod_{k=1}^n (2k-1)\sigma^{2n} \end{aligned} \quad (31)$$

4 Simplified absolute index of 4-th moment

From Eq. (29), we can get the simplified absolute index of KURTOSIS($\bar{z}_{N+l}(4)$:4-th moment) as follows.

$$\bar{z}_{N+l}(4) \cong \frac{\left(1 + \frac{l}{N+l} \cdot \frac{s^2-1}{m}\right)^2}{1 + \frac{l}{N+l} \cdot \frac{s^4-1}{m}} \quad (32)$$

This is an absolute deterioration factor of which range is 1 to 0.

5 Numerical examples

If the system is under normal condition, we may suppose $p(x)$ becomes a normal distribution function. Under the assumption of 3, let $m=9$, considering the cases $S=2,4,6,8$ for 3, and setting $l=\frac{N}{10}$, we obtain Table 1 from the calculation of $\bar{z}_{N+l}(4)$. Here, m is the number of rolling elements.

Table 1: Transition of $\bar{z}_{N+l}(4)$, ($m=9, l=\frac{N}{10}$)

S=1	2	4	6	8
1	0.92185	0.37083	0.13011	0.06321

Next, setting $N \rightarrow 0$, $l \rightarrow N$, we obtain Table 2.

Table 2: Transition of $\bar{z}_{N+l}(4)$ ($m=9, N=\varepsilon(\varepsilon \rightarrow 0), l=N$)

S=1	2	4	6	8
1	0.66667	0.24242	0.16496	0.14035

As S grows large, the value decreases rapidly.

Subsequently, we examine Bicoherence. We made experiment in the past (Takeyasu, 1987, 1989).

Summary of the experiment is as follows. Pitching defects are pressed on the gears of small testing machine.

Small defect condition: Pitching defects pressed on 1/3 gears of the total gear.

Middle defect condition: Pitching defects pressed on 2/3 gears of the total gear.

Big defect condition: Pitching defects pressed on whole gears of the total gear.

We examined several cases for the f_1, f_2 in Eq.(12). We got best-fit result in the following case.

$$\left\{ \begin{array}{l} f_1 : \text{peak frequency of power spectrum} \\ f_2 : 2 f_1 \end{array} \right.$$

We obtained following Bicoherence values in this case (Table3).

Table3: Transition of Bicoherence value

Condition	Bicoherence
Normal	0.99
Small defect	0.38
Middle defect	0.09
Big defect	0.02

Thus, Bicoherence proved to be a very sensitive good index. Bicoherence is an absolute index of which range is 1 to 0.

Therefore it can be said that it is a universal index.

In those experiment, small defect condition is generally assumed to be $S=2$ and big defect condition is generally assumed to be $S=6$ (Maekawa,K. et al. 1997). Therefore, approximate comparison may be achieved, though the condition does not necessarily coincide.

Compared with these results, the proposed method shows rather similar results with those of Bicoherence. It could be said that this method would be

sensitive enough for the practical use. This calculation method is simple enough to be executed even on a pocket size calculator. Compared with Bicoherence which has to be calculated by Eq.(12)~(15), the proposed method is by far a simple one and easy to handle on the field deflection.

6 Remarks

Here, we introduced an absolute index of 4-th moment with a simplified calculation method of 4-th moment(Kurtosis). We compared proposed method with Bicoherence.

The result of this simplified calculation method is a reasonable one compared with the results obtained so far.

The steps for the failure detection by this method are as follows.

1. Prepare a standard \bar{Z} Table for each normal or abnormal level
2. Measure peak values by signal data and compare the peak ratio to the normal data
3. Calculate \bar{Z} by Eq(32)
4. Judge the failure level by the score of \bar{Z}

Generally, it is said that machine is under small defect condition when $S=2$. By precise diagnosis utilizing deterioration index such as Bicoherence or equation (32), much more detailed machine condition can be estimated.

How to set the abnormal level of the machine depends upon the level of control.

If the machine is very important and they must be maintained carefully, subtle change of deterioration index should not be neglected. They have to make maintenance at a rather small change of deterioration index. If not, maintenance may be executed at a rather big value of index (This is a case that big value of index means increasing damages).

Preparing standard Table of \bar{Z} for each normal and abnormal level, we can easily judge the failure level only by taking ratio of the peak value to the normal

level and calculating \bar{Z} by (32). This method is simple enough to be carried out even on a pocket-size calculator and is very practical at the factory of maintenance site. This can be installed in microcomputer chips and utilized as the tool for early stage detection of the failure.

7 Conclusions

We proposed a simplified calculation method for an absolute index of 4-th moment and named this as simplified absolute index of 4-th moment. The case in which the rolling elements number was nine was examined. As S increases, the value rapidly decreases. Compared with the results obtained so far, the results of numerical examples of this paper are reasonable. Judging from these results, our method is properly considered to be effective especially for early stage failure detection. This calculation method is simple enough to be executed even on a pocket-size calculator and is very practical at the factory of maintenance site. The effectiveness of this method should be examined in various cases.

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