Journal of Computations & Modelling, vol.7, no.1, 2017, 171-190
ISSN: 1792-7625 (print), 1792-8850 (online)
Scienpress Ltd, 2017

The quantum theory in targeting decision

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Abstract

Humans don't always make the most rational decisions. As studies haveshown, even when logic and reasoning point in one direction, sometimes humans "walk" to the opposite route, motivated by personal bias or simply "wishful thinking." This paradoxicalhuman behavior has resisted explanation by classical decision theory for over a decade. Scientists have shown that a quantum probability model can provide a simpleexplanation for human decision-making. In military,and especially in artillery the decision-makingconcering targeting process is considered to be the most neuralgic one. With the recent interest in quantum computing and quantum information theory, there has been an effort to recast classical game theory using quantum probability amplitudes, and hence study the effect of quantum superposition, interference and entanglement on the agents' optimal strategies. Apart from unsolved problems in quantum information theory, quantum game theory and decision –making, may be useful in studying quantum communication since that can be considered as a game where the objective is to maximize effective artillery targeting.

Article Info: *Received* : October 30, 2015. *Revised* : January 20, 2016.
Published online : January 20, 2017.

This paper discusses the idea of using quantum theory to decision making for targeting.

1 Introduction

No decision by a military commander engaged in hostilities has more profound consequences than the decision to launch an attack. The decision to attack almost always sets in motion the use of deadly combat power and routinely produces loss of life or grievous bodily injury, often to individuals and property not the intended object of attack. This targeting processis made in a myriad of contexts, sometimes involving split-second decisions, sometimes involving complex and deliberate processes at high levels of command, and sometimes involving summarized processes at those same levels of command to address time-sensitive targeting requirements. Artillery's targeting is one of the most significant contexts.

The classical theory of decision-making is based on the relevant theory formalized by Von Neumann and Morgenstern [1] [2]. In spite of its normative appeal, since then, many researchers have discovered many systematic violations of expected utility theory especially in experiments involving real human beings. Some well-known works are the one of Allais [2], Edwards [3] and Ellsberg [4]. The most significant conclusion of all these works is the verification that human beings' decision does not meet the principles of the classical decision making principles rather than paradoxes.

This paper presents, the co-said "Quantum decision theory" as a game where the objective is to maximize the effectiveness of artillery targeting. The roots of the aforementioned theory are found in quantum theory; in this frame this work is trying to show how the strict mathematical structure of quantum mechanics can provide a general description of quantum measurements and of quantum information processing [1,7,8,9,10].

In order to achieve this purpose, the following methodology is developing into ontological parts. In the first part the fundamentals of the game theory are presented; in the second one the basics of the quantum theory are also presented; in the third part the literature review of the works associated with the decision making by using quantum theory are noted. Finally some useful examples are outlined in order reader can understand the precise nature of the engagement. Conclusions and future research are closing this paper.

2 The decision making theory (or gaming theory)

Decision theory is theory concerning decisions. The subject is not considered to be a very unified one. To the contrary, there are many different ways to theorize about decisions, and therefore also many different research traditions. Instead, the starting-point of the modern discussion is generally taken to be John Dewey's [1][2][3] exposition of the stages of problem solving. According to Dewey, problem solving consists of five consecutive stages:

(1) A felt difficulty,
(2) The definition of the character of that difficulty,
(3) Suggestion of possible solutions,
(4) Evaluation of the suggestion, and
(5) Further observation and experiment leading to acceptance or rejection of the suggestion.

Game theory attempts to mathematically model a situation where agents interact. The agents in the game are called players, their possible actions moves, and a prescription that specifies the particular move to be made in all possible game situations a strategy [9]. That is, a strategy represents a plan of action that contains all the contingencies that can possibly arise within the rules of the game. In response to some particular game situation, a pure strategy consists of always playing a given move, while a strategy that utilizes a randomizing device to select between different moves is known as a mixed strategy [9][8]. The utility to a player of a game outcome is a numerical measure of the desirability of that outcome for the player. A payoff matrix gives numerical values to the players' utility for all the game outcomes. It is assumed that the players will seek to maximize their utility within the given rules of the game. Games in which the choices of the players are known as soon as they are made are called games of perfect information.

A dominant strategy is one that does at least as well as any competing strategy against any possible moves by the other player(s).

The Nash equilibrium (NE) is the most important of the possible equilibria in game theory. It is the combination of strategies from which no player can improve his/her payoff by a unilateral change of strategy. A Pareto optimal outcome is one from which no player can obtain a higher utility without reducing the utility of another. Strategy A is evolutionary stable against B if, for all sufficiently small, positive ϱ, A performs better than B against the mixed strategy $(1 - ϱ)A + ϱB$. An evolutionary stable strategy (ESS) [36] is one that is evolutionary stable against all other strategies. The set of all strategies that are ESS is a subset of the NE of the game. A two-player zero-sum game is one where the interests of the players are diametrically opposed. That is, the sum of the payoffs for any game result is zero. In such a game a saddle point is an entry in the payoff matrix for (say) the row player that is both the minimum of its row and the maximum of its column [9].

2.1 The known prisoner dilemma in the artillery

A two player game where each player has two possible moves is known as a 2×2 game, with obvious generalizations to larger strategic spaces or number of players. As an example, consider one such game that has deservedly received much attention: the prisoners' dilemma.

In the artillery procedures the same example can be transported as follows; the players' moves are known as firing (R) or not firing - waiting (W). The payoff matrix is such that there is a conflict between the NE and the Pareto optimal outcome. The payoff matrix can be written as

Friend (F)	Bob (W)	
Enemy : F	(3, 3)	(0, 5)
Enemy : W	(5, 0)	(1, 1)

3 The quantum theory

Quantum mechanics departs from classical mechanics primarily at the quantum realm of atomic and subatomic length scales. Quantum mechanics provides a mathematical description of much of the dual particle-like and wave-like behavior and interactions of energy and matter. In advanced topics of quantum mechanics, some of these behaviors are macroscopic and only emerge at extreme (i.e., very low or very high) energies or temperatures.The name quantum mechanics derives from the observation that some physical quantities can change only in discrete amounts (Latin quanta), and not in a continuous (cf. analog) way. For example, the angular momentum of an electron bound to an atom or molecule is quantized. In the context of quantum mechanics, the wave–particle duality of energy and matter and the uncertainty principle provide a unified view of the behavior of photons, electrons, and other atomic-scale objects.

The simplest equation expression about the energy's state for a quantum particle prisoned in a box is presented in the following one.

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\psi$$

The probabilistic nature of quantum mechanics thus stems from the act of measurement. This is one of the most difficult aspects of quantum systems to understand. It was the central topic in the famous Bohr-Einstein debates, in which the two scientists attempted to clarify these fundamental principles by way of thought experiments. Newer interpretations of quantum mechanics have been formulated that do away with the idea of "wavefunction collapse" (see, for example, the relative state interpretation). The basic idea is that when a quantum system interacts with a measuring apparatus, their respective wavefunctions become entangled, so that the original quantum system ceases to exist as an independent entity. For details, see the article on measurement in quantum mechanics [9].

Since its inception, the many counter-intuitive aspects and results of quantum mechanics have provoked strong philosophical debates and many interpretations. Even fundamental issues, such as Max Born's basic rules concerning probability amplitudes and probability distributions took decades to be appreciated by society and many leading scientists [10].

In the Copenhagen interpretation, the probabilistic nature of quantum mechanics is not a temporary feature, which will eventually be replaced by a deterministic theory, but instead must be considered a final renunciation of the classical idea of "causality". It is also believed therein that any well-defined application of the quantum mechanical formalism must always make reference to the experimental arrangement, due to the complementarity nature of evidence obtained under different experimental situations.

4 Decision theory anomaly's indications

4.1 The Physics.org example

A very well known example having indications about a non rational way of deciding is the one given by researchers (Physics.org). In this example it is said that if you were asked to gamble in a game in which you had a 50/50 chance to win $200 or lose $100, would you play? Moreover in the referred study, participants were told that they had just played this game, and then were asked to choose whether to try the same gamble again.

One-third of the participants were told that they had won the first game, one-third were told they had lost the first game, and the remaining one-third did not know the outcome of their first game.

Most of the participants in the first two scenarios chose to play again (69% and 59%, respectively), while most of the participants in the third scenario chose not to (only 36% played again).

These results violate the “sure thing principle,” which says that if you prefer choice A in two complementary known states (e.g., known winning and known losing), then you should also prefer choice A when the state is unknown.

4.2 Another example (MIT review)

Suppose you receive the following questionnaire in an email:

Imagine an urn containing 90 balls of three different colors: red balls, black balls and yellow balls. We know that the number of red balls is 30 and that the sum of the black balls and the yellow balls is 60. Our questions are about the situation where somebody randomly takes one ball from the urn (MIT Review , 2012).

(1) The first question is about a choice between two bets: Bet I and Bet II. Bet I involves winning ’10 euros when the ball is red’ and ‘zero euros when it is black or yellow’. Bet II involves winning ’10 euros when the ball is black’

and ‘zero euros when it is red or yellow’. The first question is: Which of the two bets, Bet I or Bet II, would you prefer?

(2) The second question is again about a choice between two different bets, Bet III and Bet IV. Bet III involves winning ‘10 euros when the ball is red or yellow’ and ‘zero euros when the ball is black’. Bet IV involves winning ‘10 euros when the ball is black or yellow’ and ‘zero euros when the ball is red’.The second question is: which of the two bets, Bet III or Bet IV, would you prefer?

This are exactly the questions sent out by Diederik Aerts and pals at the Brussels Free University in Belgium. They received replies from 59 people which broke down like this: 34 respondents preferred Bets I and IV, 12 preferred Bets II and III, 7 preferred Bets II and IV and 6 preferred Bets I and III. That most respondents preferred Bets I and IV.

Decision theory assumes that any individual tackling this problem would do it by assigning a fixed probability to the chance of picking a yellow or black ball and then stick with that probability as they chose their bets. This approach leads to the conclusion that if you prefer Bet I, then you must also prefer Bet III. But if you prefer Bet II, then you must also prefer Bet IV.

Of course, humans don’t generally think like that, which is why most people prefer Bets I and IV (and why modern economic theory has served us so badly in recent years). At the heart of the Ellsberg paradox are two different kinds of uncertainties. The first is a probability: the chance of picking a red ball versus picking a non-red ball, which we are told is 1/3. The second is an ambiguity: the chance of the non-red ball being black or yellow that is entirely uncertain. Conventional decision theory cannot easily handle both types of uncertainty. But various researchers in recent years have pointed out that quantum theory can cope with with both types and what’s more, can accurately model the patterns of answers that humans come up with.

4.3 Another example

Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in antinuclear demonstrations.

Which is more probable?

- Linda is a bank teller.
- Linda is a bank teller and is active in the feminist movement.

It turns out that 85 per cent of people choose the second option. But the probability of two events occurring together (in conjunction) is always less than or equal to the probability of one of them alone.

4.4 The anomality conclusions

This is the conjunction fallacy (humans show a similar problem over the probability of one event OR another being true, called the disjunction fallacy). The question is how to explain the problem humans have with this kind of reasoning. Until now, psychologists have turned to classical probability theory to study the concept of probability judgement error. This allows them to build a mathematical model of human reasoning that allows for errors in judgement. Quantum probability theory leads to more realistic predictions about the type of errors humans make. “Quantum probability theory is a general and coherent theory based on a set of (von Neumann) axioms which relax some of the constraints underlying classic (Kolmogorov) probability theory,” [6].

According to classical theory, before the vote is cast, the voter is in a mixed state. Thinking about the voter in a superposition of states is a better model. The same think could be used in artillery targeting decision. In this case there are 2 main states opposed: fire and wait. Before the targetting plan is examined, the executives are in a mixed or superposition state, just like it happens in the atomic world.

4.5 Applying decision theory to quantum events

A two state system, such as a coin, is one of the simplest gaming devices. If we have a player that can utilize quantum moves we can demonstrate how the expanded space of possible strategies can be turned to advantage. Lets think wider with this coin and take it as a target to be fired.

Lets examine the problem by the classical game theory point of view. Friends (Bob) prepare their plans to fire. Bob wins if the coin is in head state. The enemies (Alice) can fire or wait as having a coin in the heads state (fire), while in the same time Friends (Bob), without knowing the state of the coin that Alice has in her hands, can choose to either fire or leave their state unaltered, and Alice, without knowing Bob's action, can do likewise; throw the coin. Finally, Boband Alice have a second turn at the coin. The coin is now examined and Alice wins if it shows heads. A classical coin clearly gives both players an equal probability of success unless they utilize knowledge of the other's psychological bias, and such knowledge is beyond analysis by standard game theory [10,11,12,26].

In a quantum artillery enviroment, we replace the choise to fire or not – or a coin by a two state quantum system such as a spin one-half particle. Now Friends (Bob) are given the power to make quantum moves while Enemies (Alice) is restricted to classical ones. Can Friends profit from their increased strategic space, that uses quantum concept to decision about relesasing firing or not (fire is the heads state and wait is the tails one).

In mathematics terms, Bob's quantum game is giving him an advantage. Let $|0\rangle$ represent the "heads" state and $|1\rangle$ the "tails" state. Alice initially prepares the system in the $|0\rangle$ state (fire). She has not won yet, because Bob has to play firstly. Bob proceeds by first applying the Hadamard operator,

$$\widehat{H} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

Putting the system into the equal superposition of the two states:
$1/\sqrt{2}(|0\rangle + |1\rangle)$.

Now Alice can leave the "coin" alone or interchange the states |0⟩ and |1⟩, but if we suppose this is done without causing the system to decohere either action will leave the system unaltered, a fact that can be exploited by Bob. In his second move he applies the Hadamard operator again resulting in the pure state |0⟩ thus winning the game. Bob utilized a superposition of states and the increased latitude allowed him by the possibility of quantum operators to make Alice's strategy irrelevant, giving him a certainty of winning [10,11].

Where a player has a choice of two moves they can be encoded by a single bit. To translate this into the quantum realm we replace the bit by a quantum bit or qubit that can be in a linear superposition of the two states. The basis states |0⟩ and |1⟩ correspond to the classical moves. The players' qubits are initially prepared in some state to be specified later. We suppose that the players have a set of instruments that can manipulate their qubit to apply their strategy without causing decoherence of the quantum state. That is, a pure quantum strategy is a unitary operator acting on the player's qubit. Unitary operations on the pair of qubits can be carried out either before the players' moves, for example to entangle the qubits, or afterwards, for example, to disentangle them or to chose an appropriate basis for measurement. Finally, a measurement in the computational basis {|0⟩, |1⟩} is made on the resulting state and the payoffs are determined in accordance with the payoff matrix. Knowing the final state prior to the measurement, the expectation values of the payoffs can be calculated. The identity operator Iˆ corresponds to retaining the initial choice while

$$\widehat{H} = i\widehat{\sigma_x}\begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$$

corresponds to a bit flip. The resulting quantum game should contain the classical one as a subset. We can extend the list of possible quantum actions to include any physically realizable action on a player's qubit that is permitted by quantum mechanics. Some of the actions that have been considered include projective measurement and entanglement with ancillary bits or qubits. A quantum game of the above form is easily realized as a quantum algorithm. Physical simulation of such an algorithm has already been performed for a quantum prisoners' dilemma in a two qubit nuclear magnetic resonance computer [13].

4.6 The base of Artillery firing quantum decision theory

In the modern warfare environment, quantum decision theory can contribute enough. Considering the simple example that two opponents exist and every one has just a single move, to fire or wait.

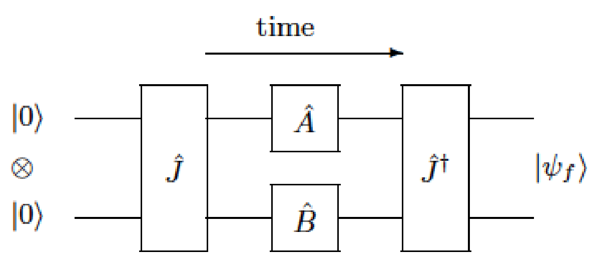

Figure 1

In traditional 2 × 2 games where each player has just a single move, creating a superposition by utilizing a quantum strategy will give the same results as a mixed classical strategy. In order to see non-classical results Eisert et al [14] produced entanglement between the players' moves. Keeping in mind that the classical game is to be a subset of the quantum one, Eisert created the protocol in Figure 1 for a quantum game between two players, Alice and Bob. The final state can be computed by

$$|\psi f >= \widehat{J}^{\dagger}(\hat{A} \bigotimes \widehat{B})\hat{J}|\psi i >$$

where |ψi⟩ = |00⟩ represents the initial state of the qubits and |ψf ⟩ the final states, Jˆ is an operator that entangles the players' qubits, and Aˆ and Bˆ represent Alice's and Bob's move, respectively.

A disentangling gate Jˆ† is applied prior to taking a measurement on the final state and the payoff is subsequently computed from the classical payoff matrix. Since we require the classical game to be a subset of the quantum one, of necessity Jˆ commutes with the direct product of any pair of classical moves. In the quantum game it is only the expectation value of the players' payoffs that is important. For Alice (Bob) we can write

$$< \$ > = P_{oo}|< \psi f|00 > |^2 \ + \ P_{o1}|< \psi f|01 > |^2 \ + P_{11}|< \psi f|11 > |^2$$

where Pij is the payoff for Alice (Bob) associated with the game outcome ij, i,j {0,1}. If both players apply classical strategies the quantum game provides nothing new. However, if the players adopt quantum strategies the entanglement provides the opportunity for the players' moves to interact in ways with no classical analogue.

A maximally entangling operator Jˆ, for an N × 2 game, may be written, without loss of generality [15], as

J = √ 2 (I + i σˆ x) .

An equivalent form of the entangling operator that permits the degree of entanglement to be controlled by a parameter γ [0, π/2] is

$$\hat{J} = \frac{1}{\sqrt{2}} \left(\widehat{I^{\otimes N}} + \ i\widehat{\sigma_{\chi}{}^{\otimes N}}\right)$$

with maximal entanglement corresponding to γ = π/2. The full range of pure quantum strategies are any Uˆ SU(2). We may writen

$$\widehat{U}(\theta,\alpha,\beta) = \begin{pmatrix} e^{ia}\cos\left(\frac{\theta}{2}\right) & ie^{ia}\sin\left(\frac{\theta}{2}\right) \\ e^{-i\beta}\sin\left(\frac{\theta}{2}\right) & e^{-ia}\cos\left(\frac{\theta}{2}\right) \end{pmatrix}$$

where $\theta \in [0, \pi]$ and $\alpha, \beta \in [-\pi, \pi]$. The strategies $\tilde{U}(\theta) \equiv \hat{U}(\theta, 0, 0)$ are equivalent to classical mixtures between the identity and bit flip operations. When Alice plays $\tilde{U}(\theta A)$ and Bob plays $\tilde{U}(\theta B)$ the payoffs are separable functions of θA and θB and we have nothing more than could be obtained from the classical game by employing mixed strategies.

In quantum prisoners' dilemma a player with access to quantum strategies can always do at least as well as a classical player. If cooperation is associated with the $|0\rangle$ state and defection with the $|1\rangle$ state, then the strategy "always cooperate" is $\hat{C} \equiv \tilde{U}(0) = \hat{I}$ and the strategy "always defect" is $\hat{D} \equiv \tilde{U}(\pi) = \hat{F}$. Against a classical Alice playing $\tilde{U}(\theta)$, a quantum Bob can play Eisert's "miracle" move [22]

$$\widehat{M} = \widehat{U}\left(\frac{\pi}{2},\frac{\pi}{2},0\right) = \frac{\iota}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

that yields a payoff of $\langle \$B \rangle = 3+2\sin\theta$ for Bob while leaving Alice with only $\langle \$A \rangle = 1/2(1-\sin\theta)$. In this case the dilemma is removed in favor of the quantum player. In the partially entangled case, there is a critical value of the entanglement parameter γ = arcsin(1/ sqrt(5)), below which the quantum player should revert to the classical dominant strategy $\hat{D}$ to ensure a maximal payoff [9]. At the critical level of entanglement there is effectively a phase change between the quantum and classical domains of the game [17, 18].

In a space of restricted quantum strategies, corresponding to setting β = 0 in Eq. (8), Eisert demonstrated that there was a new NE that yielded a payoff of three to both players, the same as mutual cooperation. This NE has the property of being Pareto optimal. Unfortunately there is no a priori justification to restricting the space of quantum operators to those of with β = 0.

With the full set of three parameter quantum strategies every strategy has a counter strategy that yields the opponent the maximum payoff of five, while the player is left with the minimum of zero [13]. This result arises since for any Aˆ=Uˆ(θ,α,β) there exists Bˆ=Uˆ(θ,α,- π/2 - β) such that,

$$(A \otimes I)\sqrt{2}(|00\rangle + i|11\rangle) = (I \otimes B)\sqrt{2}(|00\rangle + i|11\rangle) .$$

That is, on the maximally entangled state any unitary operation that Alice carries out on her qubit is equivalent to a unitary operation that Bob carries out on his. So for any strategy Uˆ(θ,α,β) chosen by Alice, Bob has the counter Dˆ Uˆ (θ, −α, π/2-β), essentially "undoing" Alice's move and then defecting. Hence there is no equilibrium amongst pure 2 quantum strategies. We still have a (non-unique) NE amongst mixed quantum strategies [14]. A mixed quantum strategy is the combination of two or more pure quantum strategies using classical probabilities. This is in contrast to a superposition of pure quantum strategies which simply results in a different pure quantum strategy. The idea is that Alice's strategy consists of choosing the pair of moves

$$\widehat{A_1} = \hat{C} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \ \widehat{A_2} = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$$

with equal probability, while Bob counters with the corresponding pair of optimal answers

$$\widehat{B_1} = \hat{D} = \begin{pmatrix} i & 0 \\ 0 & i \end{pmatrix}, \ \widehat{A_2} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

with equal probability. The combinations of strategies {Ai,Bj} provide Bob with the maximum payoff of five and Alice with the minimum of zero when i = j, while the payoffs are reversed when i ≠ j. The expectation value of the payoffs for each player is then the average of PCD and PDC, or 2.5. There is a continuous set of NE of this type, where Alice and Bob each play a pair of moves with equal probability, namely

Aˆ1=Uˆ(θ,α,β), Aˆ2=Uˆ(θ,π/2+α,π/2+β),

$$\hat{B}_1=\hat{U}(\pi-\theta,\pi/2+\beta,\alpha),\quad \hat{B}_2=\hat{U}(\pi-\theta,\pi+\beta,\pi/2+\alpha)$$

If other values of the payoffs were chosen in Eq. (1), while still retaining the conditions for a classical prisoners' dilemma [12], the average quantum NE payoff may be below (as is the case here) or above that of mutual cooperation [16]. In the latter case the conflict between the NE and the Pareto optimal outcome has disappeared, while in the former we have at least an improvement over the classical NE result of mutual defection.

The prescription provided by Eisert et al is a general one that can be applied to any 2 × 2 game, with the generalization to 2 × n games being to use SU(n) operators to represent the players' actions.

This method of quantization is not unique. Another way of achieving similar results is simply to dispense with the entanglement operators and simply hypothesize various initial states, an approach first used by Marinatto and Weber [17] and since used by other authors [25, 22]. The essential difference to Eisert's scheme is the absence of a disentangling operator. Different games are obtained by assuming different initial states. The classical game (with quantum operators representing mixed classical strategies) is obtained by selecting $|\psi\rangle = |00\rangle$, while an initial state that is maximally entangled gives rise to the maximum quantum effects. In references [21, 25] the authors restrict the available strategies to probabilistic mixtures of the identity and bit flip operators forcing the players to play a mixed classical strategy. The absence of the $\hat{J}^\dagger$ gate still leads to different results than playing the game entirely classically.

5 Discussion

Rationality is the only way in which the concept of 'probability' makes contact with the physical world. But probability is just relative frequency in the limit. If by 'limit' it is meant after an actual infinity of experiments, no-one can actually carry out an actual infinity of experiments. In practice, probabilities are predicted from the relative frequencies of outcomes in finitely many experiments. This isn't automatically correct — probability theory itself predicts a finite probability that it will fail — so it is needed some account of why it's rational to do it. Also, once these "probabilities" exist, what it actually do with them is use them as a guide to expectations about the future. Arguably, if we could do both we'd have a completely satisfactory analysis of probability.

Decision theory is a tool for decision-making under uncertainty. It doesn't introduce a primitive concept of (quantitative) probability at all, incidentally — it just shows that rational decision-making requires us to assign probabilities to events, values to consequences, and then use them together to maximise expected utility. Decision theory provides a framework in which we can understand what is involved in deducing quantitative probabilities for quantum branching, and then shows us that this can be done satisfactorily even when questionable assumptions like additivity are abandoned. Furthermore, the relevant links between quantum probability and non-probabilistic facts can then be satisfactorily established. Just as interesting are the implications of quantum theory for decision theory and the general philosophy of probability. On the technical side,it is noteworthy that the structure axioms required throughout classical decision theory can be very substantially weakened. To be sure, this is only because the mathematical structure of the physical theory (i. e. , QM) in which the decision problem is posed is so rich, but it seems far more satisfactory to have a richly structured physical theory (whose structure is clearly required on directly empirical grounds and in any case is ontologically on a par with any other postulate of physical theorising) rather than introduce axioms governing rationality which are not self-evident and which fairly clearly are introduced purely to guarantee a representation theorem.

On a more conceptual level, quantum mechanics seems to provide a novel route by which the concept of objective chance can be introduced. An account of how these chances connect with credence is available that is at least as secure as the frequency-based account — indeed, though we do not have a full derivation of the Everett Principal Principle, we have come close. What would be a simple way of seeing how all this is possible: how quantum mechanics can have these consequences for decision theory, and how the derivation of the quantum probability rule was possible in the first place? It's long been recognised that the most fruitful guides to allocation of probability have been frequencies and symmetries, but the latter has always been somewhat suspect, and it is easy to see why: how are we to choose which symmetries are respected by the chances? Appeal to the symmetries of the physical laws seems the obvious method, but obviously this just begs the question if those laws are probabilistic. Even for deterministic laws, though, the situation is problematic: for if the situation is completely symmetric between two outcomes, how is it that one outcome happens rather than the other? In classical mechanics, for instance, knowledge of the exact microstate of a flipped coin breaks the symmetry of that coin and tells us with certainty which side it will land. The symmetry only enters because we assume the coin's microstate to be distributed randomly with 50% probability of leading to each result, but this introduces probability in advance rather than deriving it from symmetry. In a sense, then, this interpretation reverses the primacy of frequency over symmetry: the frequency of outcomes is an excellent guide to the symmetry of the state being measured, but ultimately it is the symmetries which dictate which events are equiprobable. But inevitably the question will be asked. If the principles of quantum information processing better describe the way humans make decisions, what does that imply about the way the brain works? In general, quantum extension of a standard (classical) game is not unique. Most of the published analyses explore completely positive trace-preserving maps as admissible quantum operations (tactics or strategies).

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